Intensity preception. VI. Summary of recent data on deviations from Weber’s law for 1000-Hz tone pulses

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Data from 15 modern studies are used to characterize the deviations from Weber’s law for intensity discrimination of 1000-Hz tone pulses. The results show that Weber’s law is satisfied in the region 10–40 dB SL, that resolution decreases with decreasing intensity below 10 dB SL and increases with increasing intensity above 40 dB SL, and that resolution at 90 dB SL is roughly 2.5 times better than at 40 dB SL.

Subject Classification: [43]65.50, [43]65.75.

INTRODUCTION

It is now generally accepted that Weber’s law does not hold for pulsed tones: Resolution, rather than being constant as a function of intensity, tends to improve at higher levels. As part of our work on intensity per-ception (e.g., Durlach and Braida, 1969; Lippmann, Braida, and Durlach, 1975), we have found it necessary to specify the form of the deviations from Weber’s law for pulsed 1000-Hz tones. The purpose of this letter is to present our estimate of this form and describe the method by which it was derived. In general, a knowledge of the dependence of resolution on intensity is important for characterizing auditory sensitivity, for constructing loudness scales based on resolution measures, and for relating auditory performance to auditory physiology.

I. DATA REDUCTION

In considering the various resolution studies that have recently appeared in the literature, two problems are immediately evident. First, the data are not reported in a uniform manner. Therefore, prior to averaging across studies, it is necessary to convert the data to a common resolution metric. Second, because the studies differ with respect to both paradigm and parameter values, considerable variation in resolution exists among the studies. Thus, the averaging process to be used across studies must be chosen judiciously.

A. Resolution metric

The resolution metric that we have chosen is sensitivity per Bel:

\[ \delta'(I) = \delta'(I, I + \Delta I) / \log(1 + \Delta I/I). \]

In this equation, \( I \) denotes intensity, \( \Delta I \) denotes intensity increment, and \( \delta'(I, I + \Delta I) \) denotes the sensitivity between \( I \) and \( I + \Delta I \).

The quantity \( \delta'(I) \) is an appropriate measure of resolution provided that the psychometric function describing the dependence of \( \delta'(I, I + \Delta I) \) on \( \log(1 + \Delta I/I) \) is linear and passes through the origin. If this condition is satisfied, then \( \delta'(I) \), which measures the slope of this function, constitutes a complete specification of the listener’s ability to resolve \( I \) and \( I + \Delta I \).

There are now available a great deal of data which show that this condition is well satisfied for reasonable values of \( \Delta I \). An illustration of this fact is shown in Fig. 1 where we have plotted some results obtained by Rabinowitz (1970). In this experiment, sensitivity measurements were made for four subjects, five intensities, and three increments per intensity, yielding 20 three-point psychometric functions. Two such functions are plotted in Fig. 1(a). Clearly, these functions satisfy the above-mentioned condition to a high degree of precision. In order to illustrate the extent to which all 20 functions satisfy the condition, we normalized each function such that the normalized increments lie in the range 0 to 1 and such that the unweighted average of the slopes of the three lines passing through the origin and the three points of the psychometric function is unity. These normalized functions were then overlayed on the same graph. With this normalization, the above-mentioned condition is satisfied to the extent that all 60 points lie on the line of unit slope passing through the origin. Of the 20 functions measured, 17 had the property that all three points fell within the \( \pm 10\% \) slope sector illustrated in Fig. 1(b). The results for the remaining three functions (the worst cases) are also plotted in Fig. 1(b). In general, we believe that the results of this study are typical and that the condition is suf-

![Fig. 1. Psychometric functions (from Rabinowitz, 1970): (a) shows sample functions for \( I = 36 \) and 72 dB SL [\( \Delta I \) dB means \( 10 \log(1 + \Delta I/I) \)]; (b) summarizes the normalized psychometric functions. See text for details.](image-url)
ficiently well satisfied to ensure that $\delta'(I)$ constitutes a reasonable measure of resolution.

Often the resolution results reported by other investigators have been expressed in terms of the value of the increment $\Delta/I$ required to achieve a certain threshold of performance. These increments have been reported in terms of JND functions [$10 \log(1 + \Delta/I)$ vs $10 \log/I$] or in terms of "masking functions" [$10 \log \Delta$ vs $10 \log/I$]. Also, performance has often been measured in terms of percent correct rather than in terms of $d'$. We have chosen to represent the results in terms of $\delta'$ because (a) $\delta'$ is independent of response bias; (b) $\delta'$ does not require the specification of an arbitrary performance threshold; (c) $\delta'$ can be evaluated and easily compared for different experimental paradigms; (d) significant variations in resolution appear as significant variations in $\delta'$; and (e) we need the results in this form for reference in future papers of this series. For those who find it more convenient to work with JND functions, it should be noted that $\delta'(I)$ is simply related to JND(I): $\delta'(I) = 10/\text{JND}(I)$, where the JND is defined as the value of $10 \log(1 + \Delta/I)$ required to produce a $d'$ of unity. It should also be noted that all of the results on $\delta'(I)$ presented in this letter correspond to one-interval paradigms (when two-interval paradigms were used, the resolution measures were divided by $\sqrt{2}$). Furthermore, in those cases for which the results were reported in terms of percent correct, we have assumed in computing $d'$ that the listeners had no significant response bias.

### B. Averaging across studies

The studies considered, together with some of the more important features of the studies, are given in Table I. The final column of this table gives $\delta'$ for $I = 40$ dB SL. As noted earlier, there are variations in paradigm and parameter values that lead to substantial variations in the value of $\delta'$. In particular, note that the three lowest values of resolution occur for the studies in which the pulse duration $T_p$ is very short ($T_p \leq 0.1$ sec). For these studies, resolution is uniformly low at all intensities relative to the resolution obtained in studies for larger values of $T_p$. In all cases, however, the results indicate a strong dependence of resolution on intensity.

In averaging across these studies, we have assumed that the shape of the function describing the dependence of $\delta'(I)$ on $I$ (in dB) is an invariant (i.e., that the differences in the functions for different paradigms and different parameter values can be described merely by a multiplicative constant). Using this assumption, we have constructed an average resolution function by first normalizing each function with respect to its value at $40$ dB SL, i.e., transforming the function $\delta'(I)$ into the function $10 \delta'(I)/\delta'(40 \text{ dB})$ and then averaging linearly (with equal weights) across studies.

### Table I. Experimental studies.

<table>
<thead>
<tr>
<th>Study</th>
<th>Paradigm*</th>
<th>$T_p$ (sec)</th>
<th>$T$ (sec)**</th>
<th>Feedback</th>
<th>No. of Subjs.</th>
<th>$I_{\text{min}} - I_{\text{max}}$</th>
<th>$\delta'(40)$d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berliner (1973)*</td>
<td>2I-2AFC</td>
<td>0.65</td>
<td>0.2</td>
<td>N</td>
<td>3</td>
<td>1800</td>
<td>36-90</td>
</tr>
<tr>
<td>Berliner and Durlach (1973)*</td>
<td>2I-2AFC</td>
<td>0.8</td>
<td>1.4</td>
<td>Y</td>
<td>2</td>
<td>~300</td>
<td>41-94</td>
</tr>
<tr>
<td>Braila and Durlach (1972)*</td>
<td>1I-10AFC</td>
<td>0.5</td>
<td>7</td>
<td>Y</td>
<td>1</td>
<td>1000</td>
<td>33-85</td>
</tr>
<tr>
<td>Callahan et al. (1973)*</td>
<td>2I-2AFC</td>
<td>0.5</td>
<td>0.25</td>
<td>Y</td>
<td>5</td>
<td>1200</td>
<td>34-90</td>
</tr>
<tr>
<td>Campbell (1969)</td>
<td>2IFC</td>
<td>1.0</td>
<td>~0.3</td>
<td>N</td>
<td>4</td>
<td>120</td>
<td>0-85</td>
</tr>
<tr>
<td>Campbell and Lasky (1967)</td>
<td>2IFC</td>
<td>0.02</td>
<td>0.95</td>
<td>N</td>
<td>10</td>
<td>120</td>
<td>10-90</td>
</tr>
<tr>
<td>Haughey (1970f)</td>
<td>2I-2AFC</td>
<td>0.5</td>
<td>0.3</td>
<td>Y</td>
<td>4</td>
<td>1125</td>
<td>6-68</td>
</tr>
<tr>
<td>Johnston (1972)</td>
<td>2I-2AFC</td>
<td>0.5</td>
<td>0.25</td>
<td>Y</td>
<td>4</td>
<td>1800</td>
<td>0-20</td>
</tr>
<tr>
<td>Luce and Green (1974)</td>
<td>2IFC</td>
<td>0.5</td>
<td>1.25</td>
<td>Y</td>
<td>2</td>
<td>~300</td>
<td>4-95</td>
</tr>
<tr>
<td>McGill and Goldberg (1968a)</td>
<td>1I-2AFC</td>
<td>0.02</td>
<td>8.4</td>
<td>Y</td>
<td>3</td>
<td>~800</td>
<td>0-80</td>
</tr>
<tr>
<td>McGill and Goldberg (1968b)</td>
<td>1I-2AFC</td>
<td>0.15</td>
<td>8.4</td>
<td>Y</td>
<td>3</td>
<td>~600</td>
<td>4-70</td>
</tr>
<tr>
<td>Penner et al. (1974)</td>
<td>2IFC</td>
<td>0.1</td>
<td>0.5</td>
<td>Y</td>
<td>3</td>
<td>140</td>
<td>30-75</td>
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<tr>
<td>Rahbnowitz (1970f)</td>
<td>2I-2AFC</td>
<td>0.5</td>
<td>0.25</td>
<td>Y</td>
<td>4</td>
<td>1350</td>
<td>12-74</td>
</tr>
<tr>
<td>Schacknow and Raab (1973)</td>
<td>2IFC</td>
<td>0.25</td>
<td>0.55</td>
<td>Y</td>
<td>2</td>
<td>~</td>
<td>30-70</td>
</tr>
<tr>
<td>Vienmeier (1972)</td>
<td>2I-2AFC</td>
<td>0.16</td>
<td>0.5</td>
<td>Y</td>
<td>2</td>
<td>800</td>
<td>30-85</td>
</tr>
</tbody>
</table>

*Paradigm definitions as follows:
2I-2AFC: Two interval, two-alternative (fixed increment), forced choice.
1I-2AFC: One interval, two-alternative (fixed increment), forced choice.
2IFC: Two interval, forced choice, staircase type method (adaptive increment).
1I-10AFC: One interval, ten-alternative, forced choice identification experiment with $\frac{1}{10}$ dB between adjacent stimuli, and a total range of $2\frac{1}{2}$ dB.

**Time between intervals of 2I tests, and between trials for 1I tests.

Sensation level (dB SL) and sound pressure level (dB SPL re 0.0002 dynes/cm$^2$) used interchangeably when no thresholds are given for data reported in dB SPL. $I_{\text{min}} - I_{\text{max}}$ are minimum and maximum intensities tested.

Interpolated when 40 dB SL not employed, and extended from 20 dB SL for Johnston (1972).

From our laboratory.

Average of 0.2, 0.5, and 1.25 sec.

Average of 0.2 and 2.5 sec.

Test frequency of 950 Hz.

tion constrains the final normalized value of $\delta'(40\, \text{dB})$ to be 10 [corresponding to the reasonable value $\text{JND (40 dB)} = 1\, \text{dB}$]. If the three studies in which $T_p \leq 0.1\, \text{sec}$ are excluded, the unnormalized average of the remaining studies considered is $\langle \delta'(40\, \text{dB}) \rangle = 9.4$ with a $\sigma$ of 1.7. [If all the studies are included, then $\langle \delta'(40\, \text{dB}) \rangle = 8.2$ with a $\sigma$ of 2.9.] In general, this average normalized function is useful only for specifying the shape of the resolution function (i.e., the relative dependence on level). In order to obtain the function that is relevant to a particular paradigm and set of parameter values, the given values of $\delta'(I)$ must be multiplied by an appropriate constant.

II. RESULTS AND COMMENTS

The average normalized resolution function, together with $\pm 1\sigma$ bounds, is shown in Fig. 2. The function exhibits three distinct regions. From 10 to 40 dB SL, resolution is nearly constant (i.e., Weber's law is valid). Below 10 dB, resolution decreases. Note, however, that it is still roughly 5 at threshold. Above 40 dB, resolution steadily improves, reaching a value at 90 dB that is 2.5 times greater than the value at 40 dB. Assuming that the dashed line represents a reasonable fit to the data, the increase in resolution with intensity above 40 dB is characterized by a linear increment of $0.3\, \text{dB}^2$. Note also that if the dashed line is extrapolated to higher intensities, it is predicted that $\delta'(120\, \text{dB}) = 34$ [corresponding to $\text{JND (120 dB)} = 0.29\, \text{dB}$]. Unfortunately, there are very little data available at these higher intensities.

In older data, not explicitly considered here, a wide spectrum of behavior can be found. Riesz's (1928) classic amplitude-modulation detection results are similar to those shown in Fig. 2 with respect to the general tendency for resolution to increase with intensity; however, the details are different. Above 40 dB SL the slope of Riesz's resolution function continually decreases, being greater than 0.3 in the region 40–70 dB SL and less than 0.3 in the region 80–110 dB SL. Furthermore, Riesz's results do not indicate a plateau in the region 10–40 dB SL; resolution continues to decrease as the intensity decreases below 40 dB SL. The results of Dimmick and Olson (1941) also show a decrease in resolution below 40 dB SL, but almost constant resolution in the region 40–70 dB SL. However, responses of "equal loudness" were permitted in their paradigm, thereby increasing the contaminating influences of response criteria. Not surprisingly, the JND's obtained in their study were unusually large (e.g., 2.9 dB at 54 dB SL). Harris (1963) employed an "optimized" procedure in which the subject controlled the initiation of trials and was permitted not to respond (if unsure). In this study, resolution was found to be nearly constant from 10–80 dB SL and to be exceptionally good [e.g., $\text{JND (40 dB)} = 0.6\, \text{dB}$ or, equivalently, $\delta'(40\, \text{dB}) = 17$].

Despite the fact that the results of some studies are inconsistent with the results shown in Fig. 2, and despite the fact that the data shown in Fig. 2 evidence considerable variability (and include only a small number of entries at the most extreme intensities), we believe that the form of the resolution function shown in Fig. 2 represents a reasonable approximation. With regard to the studies that contradict the results shown in Fig. 2, it should be noted that most of these studies are rather old and that many of them used psycho-physical methods that would now be regarded as questionable. With regard to the variability of the data used in constructing Fig. 2, the following points should be noted. First, despite the magnitude of $\sigma$ at the higher and lower intensities ($\sigma$ is constrained to be zero at 40 dB SL by the normalization procedure), the mean data points follow an orderly progression. Second, for intensities in the region 40–90 dB SL, the orderly progression of the mean data points, combined with the values of $\sigma$, leave little doubt that $\delta'(I)$ steadily increases with $I$ in this region and that the rate of this increase is of the order of $0.3\, \text{dB}^2$ (it seems extremely unlikely that the rate is less than 0.25 or greater than 0.36). Third, the relatively large values of $\sigma$ at very low intensities are to be expected since estimates of $\delta'(I)$ near threshold [unlike the estimates of $\delta'(I)$ at higher intensities] are extremely sensitive to the estimate of threshold. This fact, combined again with the orderly progression of the mean data points [and with the fact that $\delta'(I)$ must necessarily decrease rapidly to zero below threshold], leaves little doubt that the sharp fall in $\delta'(I)$ at very low intensities pictured in Fig. 2 is roughly correct. Finally, the fact that $\delta'(I)$ is relatively flat in the region 10–40 dB SL is indicated both by the orderly progression of mean data points in this region and by the fact that the mean value of $\delta'(I)$ at 10 dB is approximately equal to the mean value of $\delta'(40\, \text{dB})$ (of the eight entries at 10 dB, five of them are in the interval $9 \leq \delta' \leq 11$). Although a number of investigators have observed a dip in $\delta'(I)$ between 10 and 40 dB (e.g., Campbell and Lasly, 1967; McGill
and Goldberg, 1968b; Rabinowitz, 1970) and roughly half of the individual curves that we have examined show such a dip, the mean data points shown in Fig. 2 suggest that, on the average, the dip is very slight.

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We implicitly assume here that the underlying probability densities associated with the measure of sensitivity are Gaussian and of equal variance (so that the receiver operating characteristics are straight lines of unit slope on normal-normal coordinates). Also, we assume that the magnitude of $\Delta f / f$ is restricted to reasonable values (i.e., values for which $d'^2(1,1+\Delta f) -< 3.0$). For very large values of $\Delta f / f$, it is obvious that experimental artifacts will cause the psychometric function to be concave downward (i.e., no matter how large $\Delta f / f$ is chosen, the value of $d'^2(1,1+\Delta f)$ will always be limited by occasional response errors due to lapses of attention, inadvertently pushing the wrong response button, etc.). In addition, it should be noted that the statement that $d'^2(1,1+\Delta f)$ vs $\log f + \Delta f / f$ is linear for small values of $\Delta f / f$ is equivalent to the statement that Weber's law is valid locally (i.e., that for any given value $f_0$ of $f$, $b'(f)$ is approximately constant in a small region around $f_0$).

Some experimental data on the ratio $b'(f)/b'(f_0)$ (1 denotes one-interval and 2 the two-interval) suggest that this ratio exceeds $\sqrt{2}$ (e.g., see Jesteat and Bilger, 1974). We have chosen $\sqrt{2}$ because of theoretical considerations and because it lies within the range of values measured experimentally. Also, since we are primarily interested in the shape of the function $b'(f)$ rather than its overall level (and the overall level depends strongly on the details of the experimental procedure), the precise value that we assume for this ratio is unimportant.

If this type of normalization was omitted, the shape of the average resolution function would be distorted by the fact that different studies measured resolution over different ranges of intensity. Furthermore, estimates of the variability of the shape of the average resolution function would be confounded by variability due to the overall level of the function. The choice of 40 dB SI for normalization was motivated by the fact that all of the studies (except Johnston's) included measurements at this level and that most of the observed functions showed a change in slope at this level.

The value of resolution $b'$ at threshold can be best understood by considering the relation between $b'(f)$ and the absolute detection function $d'(0,f)$ describing sensitivity between the intensities $f$ and 0 (silence). From the additivity property of $d'(0,f)$ it follows that $d'(0,f+\Delta f) = d'(0,f) + d'(0,f+\Delta f)$ and therefore that $b'(f)$ is equal to the derivative of $d'(0,f)$ with respect to $f$. Thus, the result that $b'(f) = 5$ at threshold is equivalent to the statement that the slope of $d'(0,f)$ vs $f$ is 5 at threshold. This result is reasonable provided that the performance criterion defining threshold is $d'(0,f) = 5$ for all studies (and $d'(0,f)$ is roughly 5 in the region 0.4 $\leq d'(0,f) \leq 2.0$, see, for example, Watson et al., 1972.) Note also that since the slope of $d'(0,f)$ vs $f$ decreases rapidly as $f$ decreases below threshold, the value of $b'(f)$ drops rapidly to zero as $f$ decreases below threshold.

In terms of masking functions (i.e., $10 \log f + 10 \log f'$, the average resolution data for 40 $\leq f \leq 90$ dB are well fit by a straight line of slope 0.91. This result is consistent with that typically cited for the "near-miss" to Weber's law (McGill and Goldberg, 1968a, 1968b; Penner et al., 1974; Schacknow and Raab, 1973; Viemeister, 1972).


