

My Thoughts on Teaching Math (the problems and the solutions)

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1 The problem with teaching

The biggest disservice a teacher can do a student is to simply help them “get by” with math, without teaching them to think in the process. Failure to teach certain concepts to students can be corrected - students can pick-up most of the concepts later. However, failure to teach the students to think leads to a downward spiral of understanding. Students grow older without developing these crucial skills, and it becomes increasingly harder to correct their attitude and approach to math. They stiffen up, lose their creativity, and through a rule-book approach to math, are trapped in an incomprehensible world. They freeze-up when confronted with a novel problem (if it’s not already in their rule-book), and therefore, quite naturally, throw their hands up in the air and proclaim “I’m just bad at math”. My philosophy (perhaps naive, but nevertheless optimistic) is that no reasonably logical human being can be “bad at math” - they can fail to understand the way math concepts are taught (and I don’t blame them!), but not the concepts themselves (if taught to look at them correctly).

2 Moving away from the rule-based approach

When teaching my students, my goal is to make mathematical thinking a very intuitive process, removing any notions of ‘math as a rule-book’ that may have been sown into the students at their schools (an unfortunate result of many teachers’ own discomfort with mathematics being propagated on to further generations). A rule-based approach to math is inherently limited in that students are thrown into a panic as soon as a problem becomes unrecognizable (in terms of the problems they have already seen and memorized sequences of rules for). At some point, they will run out of rules, and math will seem a mysterious subject matter. Instead, I advocate an approach to math whereby students learn to explore mathematical entities from a variety of angles - learning to interact with the entities in a variety of ways. What this does for the student is ground them in mathematical concepts, placing in their hands a whole range of tools and giving them a language with which to describe what they are doing. They become fearless, knowing that they will have a plan of attack for any problem thrown their way: at least one way of looking at things will help

them tackle a problem; if they find multiple ways of looking at the problem, then they can be confident that they have understood the problem well.

3 Kicking it up a notch

I do not hide terminology or notation from my students, and I do not attempt to oversimplify mathematical concepts - I merely make them more intelligible. If introduced to proper terminology and notation early on, things should become clearer rather than more complicated, as the logic and clarity of the mathematical language comes to the students' aid. Introducing concepts as they come up (without hiding too many of the details) will prevent sudden surprises later on, which often make students feel misguided or deceived. In fact, I have found that introducing more advanced concepts to students (if done properly), will leave them feeling good about themselves - after all, they have been entrusted with higher-level knowledge! Moreover, when exposed to 'real math' earlier on, they form no misconceptions. Many things are not beyond the students' understanding capabilities - they are more likely to be beyond the teachers' explaining capabilities, in which case the latter is the issue to be addressed. After all, a concept taught poorly is better not taught at all (for correcting student's misconceptions and adjusting their attitude towards a subject matter is more difficult than building understanding from scratch).

4 Thinking about math...

Mathematical reasoning is a beautiful thing. If properly taught, one can see that there is nothing mysterious about it. We start with some 'entity' and mold it, using all the mathematical tools at our disposal, until we have a form we can recognize and discuss. For instance, our entity may be an equation: it may start out looking complicated, but, through a sequence of transformations, we end up with a form we recognize and know how to deal with. It is as if we are given a new object, which we can rotate, poke, smell, and observe. After a while, our brain recognizes the identity of the object - and having recognized it as such, we are able to discuss its properties and features. Note that through our manipulations, we have not changed anything about our entity - we have merely extracted the information that was there all along. Similarly, when we work with mathematical entities, we manipulate them in such a way that we never really change what was there initially. Thus, a student who understands this will not be puzzled as to why we can do one thing to a mathematical entity and not another. The student will understand which manipulations are legal and maintain the state of our entity, and which are not applicable (and why).

5 Taking matters into one's own hands

I think that we are very lucky with the state of technology these days. Free online teaching materials, full courses, and video and audio lectures are readily available. With enough searching,

a student can find material capable of explaining concepts at just about any level. I strongly believe that students should take advantage of this, and not allow their school math classes to limit their learning or cause incorrect notions to form. I further believe that the more sources used by a student to learn about a concept, the more complete the understanding. Thus, a few mediocre (but correct) explanations can have the effect of a thorough and complete one.