Are Constants Constant?

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Following an overview of the history and experimental status of time-varying fundamental constants, I summarize methods of constraining time-variation in the fine structure constant α and the gravitational constant G using CMB and BBN-based observations. The limitations and merits of these constraints are evaluated in comparison to each other and to results from non-cosmological studies.

I. HISTORY: DIRAC'S LARGE NUMBER HYPOTHESIS

In 1937, Dirac noticed that the estimated age of the universe t_0 in atomic units was of approximately the same order of magnitude as the ratio of the electromagnetic force to the gravitational force between an electron and a proton. At the time, t_0 was believed to be about 2×10^9 years, which gave $t_0/(e^2/m_e c^2) \approx 10^{39} \approx e^2/Gm_e m_p$. The coincidence was so striking that Dirac wrote, "Such a coincidence we may presume is due to some deep connexion in Nature between cosmology and atomic theory." Dirac's certainty in this connection lead him to hypothesize that the aforementioned relation must hold at all times and therefore G must vary as t^{-1}. (For some reason, Dirac firmly believed that the atomic parameters must be constant whereas the cosmological parameter G could vary in time.) Furthermore, he noticed that all large dimensionless constants apppearing in cosmology were of order 10^{39} or 10^{78} and thus should vary as t or t^2, respectively [1]. Dirac's hypothesis, which clearly violates general relativity and has been widely criticized as numerology, nonetheless has motivated investigation into the constancy of fundamental constants in the decades since [2].

II. OVERVIEW OF EXPERIMENTS

In general, the strategy of experiments testing the variation of fundamental constants is pick a physical system that depends dramatically on the values of some set of constants, constrain the variation of an observable as precisely as possible and then relate the observable to the set of constants. In reality, most observables are coupled to several fundamental constants, making it difficult to independently constrain variations.

The physical system also determines the time scale over which variation can be probed. Some values of this time scale are 1-12 months for atomic clock measurements in the laboratory, 1 billion years for isotopic ratios from the Oklo reactor, 1-10 billion years for quasar spectra, and 10 billion years for cosmological (CMB and BBN) measurements [3].

For the purposes of this study, we will consider only the constraints on α and G from CMB and BBN observations.

III. CMB CONSTRAINTS

III.0.1. *Fine-structure constant*

The redshift of last scattering is defined as the redshift where g, the visibility function, is maximized:

$$g = \frac{d\tau}{dx} exp(-\tau)\,, \tag{1}$$

where τ is the photon optical depth, or the fraction of photons rescattered during reionization, with the differential optical depth defined by

$$\frac{d\tau}{dx} = x_e n_e c \sigma_T\,. \tag{2}$$

Here, n_e is the electron density, $\sigma_T \propto \alpha^2/m_e$ is the Thompson cross section, and x_e is the ionization fraction, which has a complicated dependence on α. It turns out that increasing α pushes z_{rec} to earlier times, which has two effects on the observed CMB anisotropy power spectrum [3]:

1. The universe is expanding faster at recombination, so the sound horizon size r_s is smaller. The anisotropy peaks are shifted toward higher multipole moments.
2. Photons have a shorter diffusion length (distance traveled before scattering), so Silk damping is reduced and the amplitude of the power spectrum increases.

Analysis of data from WMAP alone is able to constrain $\Delta\alpha/\alpha_0$ since recombination to within 10^{-2}, which is as good as all other previous CMB datasets combined [4].

III.1. Gravitational constant

G comes into the Friedmann Equations and thus affects calculations of the age of the universe. A higher value

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of G at earlier times, as postulated by Dirac, results in earlier t_{rec}. However, a complete study of the variation of G on the CMB power spectrum is difficult since G also plays a role in the Integrated Sachs-Wolfe effect. As such, one must know the value of G at all times, not only at recombination, in order to predict the resulting power spectrum [3].

IV. BBN-BASED CONSTRAINTS

Light elemental abundances produced during BBN provide another way to constrain α and G. We consider the abundance of ^{4}He varying G and $Q = m_n - m_p$, and assume that Q has some dependence on α.

IV.1. Gravitational constant

Recall that the abundance of ^{4}He is a simple function of the neutron to proton ration $(n/p)_N$ at t_N, the age of the universe when $n + p$ reactions proceed faster than their inverse dissociation:

$$\left(\frac{n}{p}\right)_N = exp(-Q/T_f)exp(-t_N/\tau_n)\,, \tag{3}$$

where T_f is the weak interaction freezeout temperature (0.8 eV) and τ_n is the neutron decay time. The first exponential accounts for the neutron-proton fraction at statistical equilibrium (before freezeout) and the second accounts for the decrease in n/p due to neutron beta decay after freezeout. The G dependence enters via $t_N \propto G^{-1/2}$ and $T_f \propto G^{1/2}$ [3].

Simulations by Kolb et al. show that, in fact, increasing G leads to higher production of ^{4}He [5]. This effect can be understood by noting that increasing G leads to earlier freezeout at a higher temperature, so more ^{4}He exists at equilibrium before freezeout [3].

BBN constrains the variation of G to $|\dot{G}/G| \leq 9 \times 10^{-13} yr^{-1}$.

IV.1.1. Fine-structure constant

In order to constrain α using BBN, one may assume a functional dependence of Q on α. In a seminal study, Kolb et al. apply the relation $\alpha/\alpha_0 \approx Q/Q_0$ to deduce that $(n/p)_N \approx (n/p)_{N,0}(1 - \Delta\alpha Q_0/T_f\alpha_0)$ and thus the ^{4}He abundance is a strongly decreasing function of α/α_0. Kolb concludes that $|\dot{\alpha}/\alpha| \leq 1.5 \times 10^{-14} h\ yr^{-1}$.

V. EVALUATION

V.1. Limitations

In the previous sections, we made the unjustified assumption that variations in α and G are independent of all other relevant fundamental constants, such as m_e/m_p and the weak interaction coupling constant. In reality, since observable quantities usually entangle a set of fundamental constants, the most reliable constraints on α and G come from physical systems with the least degenerate dependences on fundamental constants.

CMB observations at first seem to provide a fair method of measuring $\Delta\alpha$, since α is decoupled from the weak and strong interaction coupling parameters; on the other hand, it has been shown that α is entangled with m_e/m_p and the baryonic density. Furthermore, the CMB anisotropy power spectrum alone is not a viable means of constraining ΔG due to the Integrated Sachs-Wolfe effect.

Unfortunately, nucleosynthesis observables are degenerate in all four fundamental couplings, and so most studies are necessarily dependent on the assumed model for the relationship between these couplings. Even in the simplest case where we considered only independent variation in α, we had to assume a dependence of Q on α in order to constrain $\Delta\alpha$ from helium abundance observations [3].

V.2. Comparison to other methods of constraining G and α

We now compare the cosmological constraints on G and α to constraints from other competitive methods (detailed tables given at the end of this report). CMB and BBN observations are about equally powerful in constraining α, yielding $|\dot{\alpha}/\alpha| \leq 10^{-15} yr^{-1}$. This is two orders of magnitude worse than observations of isotopic yield abundances at the Oklo natural nuclear reactor, dating back 2 billion years, and an order of magnitude better than atomic spectra from quasars, dating back 1-10 billion years. On the other hand, BBN observations provide some of the tightest constraints on G, rivaled only by constraints from 14 months of Viking spacecraft orbit data [3].

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$\lvert\dot\alpha/\alpha\rvert$	Method	$\Delta\tau$[a]
5×10^{-15} yr^{-1}	^{187}Re/^{187}Os	5×10^{9} yr
1×10^{-17} yr^{-1}	Oklo reactor	1.8×10^{9} yr
$13\times10^{-13}h$ yr^{-1}	Radio galaxies	$2\times10^{9}h^{-1}$ yr
$2\times10^{-14}h$ yr^{-1}	QSO[b]	$5\times10^{9}h^{-1}$ yr
$15\times10^{-15}h$ yr^{-1}	Primordial nucleosynthesis	$6.6\times10^{9}h^{-1}$ yr

FIG. 1. Comparison of constraints on $|\dot\alpha/\alpha|$, where $\Delta\tau$ is the look-back time. From [5].

Reference	Constraint (yr^{-1})	C.L.	Method
(Teller, 1948)	$(0\pm2.5)\times10^{-11}$	x	Earth temperature
(Shapiro *et al.*, 1971)	$(0\pm4)\times10^{-10}$	x	Planetary ranging
(Morrison, 1973)	$(0\pm2)\times10^{-11}$	x	Lunar occultations
(Dearborn and Schramm, 1974)*	$<4\times10^{-11}$	x	Clusters of galaxies
(Van Flandern, 1975)	$(-8\pm5)\times10^{-11}$	x	Lunar occultations
(Heintzmann and Hillebrandt, 1975)	$(0\pm1)\times10^{-10}$	x	Pulsar spin-down
(Reasenberg and Shapiro, 1976)	$(0\pm1.5)\times10^{-10}$	x	Planetary ranging
(Mansfield, 1976)*	$<(3.4\pm3.4)\times10^{-11}$	2σ	Pulsar spin-down
(Williams *et al.*, 1976)	$(0\pm3)\times10^{-11}$	1.1σ	Planetary ranging
(Blake, 1977b)	$(-0.5\pm2)\times10^{-11}$	x	Earth radius
(Muller, 1978)	$(2.6\pm1.5)\times10^{-11}$	x	Solar eclipses
(McElhinny *et al.*, 1978)*	$<8\times10^{-12}$	x	Planetary radii
(Barrow, 1978)	$(2\pm9.3)h\times10^{-12}$	x	BBN
(Reasenberg *et al.*, 1979)*	$<10^{-12}$	x	Viking ranging
(Van Flandern, 1981)	$(3.2\pm1.1)\times10^{-11}$	x	Lunar occultation
(Rothman and Matzner, 1982)	$(0\pm1.7)\times10^{-13}$	x	BBN
(Hellings *et al.*, 1983)	$(2\pm4)\times10^{-12}$	x	Viking ranging
(Reasenberg, 1983)	$(0\pm3)\times10^{-11}$	x	Viking ranging
(Damour *et al.*, 1988)	$(1.0\pm2.3)\times10^{-11}$	2σ	PSR 1913+16
(Shapiro, 1990)	$(-2\pm10)\times10^{-12}$	x	Planetary ranging
(Goldman, 1990)*	$(2.25\pm2.25)\times10^{-11}$	x	PSR 0655+64
(Accetta *et al.*, 1990)	$(0\pm9)\times10^{-13}$	2σ	BBN
(Müller *et al.*, 1991)	$(0\pm1.04)\times10^{-11}$	x	Lunar laser ranging
(Anderson *et al.*, 1992)	$(0.0\pm2.0)\times10^{-12}$	x	Planetary ranging
(Damour and Taylor, 1991)	$(1.10\pm1.07)\times10^{-11}$	x	PSR 1913+16
(Chandler *et al.*, 1993)	$(0\pm1)\times10^{-11}$	x	Viking ranging
(Dickey *et al.*, 1994)	$(0\pm6)\times10^{-12}$	x	Lunar laser ranging
(Kaspi *et al.*, 1994)	$(4\pm5)\times10^{-12}$	2σ	PSR B1913+16
(Kaspi *et al.*, 1994)	$(-9\pm18)\times10^{-12}$	2σ	PSR B1855+09
(Demarque *et al.*, 1994)	$(0\pm2)\times10^{-11}$	x	Heliosismology
(Guenther *et al.*, 1995)	$(0\pm4.5)\times10^{-12}$	x	Heliosismology
(Garcia-Berro *et al.*, 1995)*	$(2\pm2)\times10^{-11}$	1σ	White dwarf
(Williams *et al.*, 1996)	$(0\pm8)\times10^{-12}$	x	Lunar laser ranging
(Thorsett, 1996)	$(-0.6\pm4.2)\times10^{-12}$	2σ	Pulsar statistics
(Del'Innocenti *et al.*, 1996)	$(-1.4\pm2.1)\times10^{-11}$	1σ	Globular clusters
(Guenther *et al.*, 1998)	$(0\pm1.6)\times10^{-12}$	x	Heliosismology

FIG. 2. Comparison of constraints on $|\dot G/G|$. From [3].