Subject 24.118. Paradox and Infinity. Homework due Thursday, October 4.

1. Imagine the following situation: An electric bulb is connected to a switch. At time 0, the lamp is off. At time $t = ½$, the lamp is turned on. At time $t = {}^{3}/_{4}$, the lamp is turned off. At time $t = {}^{7}/_{8}$, the lamp is again turned on. At time $t = {}^{15}/_{16}$, the lamp is turned back off. Pretend, for the sake of argument, that the lamp turns on or off instantly when the switch is pulled, and that pulling the switch requires no energy. What can you say about the state of the bulb at time $t = 1$?

2. For the following questions, let B = {Al, Bruce} and G = {Xochitl, Yolanda, Zaida}.
   a) List the functions from B to B, indicating which are one-one and which onto.
   b) List the functions from B to G, indicating which are one-one and which onto.
   c) List the functions from G to B, indicating which are one-one and which onto.
   d) List the functions from G to G, indicating which are one-one and which onto.

3. For the following, let B = {Al, Bruce} and let C = the set of current justices of the U.S. Supreme Court, of whom there are nine.
   a) How many functions are there from B to C? Of these, how many are one-one, and how many onto?
   b) How many functions are there from C to B? Of these, how many are one-one and how many onto?

4. Let α be the number of functions from the natural numbers to the real numbers, and let β be the number of functions from the real numbers to the natural numbers. Which is greater, α or β? Or are they the same? You may use any of the results cited in the attached "Cardinal Arithmetic: Definitions and Key Results." Briefly explain your answer.