24.118: Paradox and Infinity
PS 2: Suggested solutions

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1.

In (a)-(c), solve for a,b. In (d), express d and c as functions of a and b.

(a) (2 pts) $a = \frac{1}{2}$, $b = 0$

(b) (2 pts) $a = \pm 1$, $b = 0$. Note that if $a = 0$, then $b = \sqrt{-1}$, which is not a real.

(c) (3 pts) $a = b = \frac{1}{\sqrt{2}}$ or $a = b = -\frac{1}{\sqrt{2}}$

(d) (3 pts) $c = \frac{a}{a^2+b^2}$, $d = -\frac{b}{a^2+b^2}$

2.

(a) (3 pts) $P(7, 3) = 58$

(b) (3 pts) $P(2, 8) = 63$

(c) (4 pts) The desired polynomial is: $\frac{1}{2}x^2 + xy + \frac{1}{2}y^2 + \frac{1}{2}x + \frac{3}{2}y$

3.

(a) (2 pts) $Code(\{1, 3, 6\}) = 74$

(b) (2 pts) $Code(\{8, 10\}) = 1280$

(c) (3 pts) $Code(\{0, 5, 4\}) = 2^0 + 2^5 + 2^4 = 49$

(d) (3 pts) $Code(\{0, 2, 6\}) = 2^0 + 2^2 + 2^6 = 69$

4.

(3 pts)

In both this and the next problem you need to apply the identity-condition for sets ($x = y \Leftrightarrow \forall z(z \in x \leftrightarrow z \in y)$) until you get down to urelements (a, b, c, d). It is not enough to just appeal to the 'look' of the two sets. The longwinded and painfully explicit argument follows.

Suppose

$$\{\{\{a\}, \emptyset\}, \{\{b\}\}\} = \{\{\{c\}, \emptyset\}, \{\{d\}\}\}$$

By the identity-conditions for sets, this is true if either

$$\{\{a\}, \emptyset\} = \{\{d\}\} \text{ and } \{\{b\}\} = \{\{c\}, \emptyset\}$$

or

$$\{\{a\}, \emptyset\} = \{\{c\}, \emptyset\} \text{ and } \{\{b\}\} = \{\{d\}\}$$

The former cannot hold, because the sets flanking the identity sign have a different number of elements. So

$$(\star) \ \ \{\{a\}, \emptyset\} = \{\{c\}, \emptyset\} \text{ and } \{\{b\}\} = \{\{d\}\}$$

From this it follows, again by the identity-conditions for sets that

$$\{b\} = \{d\}$$

from which it follows, once again by the identity-conditions for sets that

$$b = d$$

Further, it follows from $(\star)$ that either

$$\{a\} = \emptyset \text{ and } \emptyset = \{c\}$$

or

$$\{a\} = \{c\} \text{ and } \emptyset = \emptyset$$

The former fails (because $\{a\}$ has an element that $\emptyset$ lacks), so $\{a\} = \{a\}$, so $a = c$.

5.

(3 pts) The argument here is similar to the previous one.