Systemic Risk and the Refinancing Ratchet Effect

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Abstract

The confluence of three trends in the U.S. residential housing market—rising home prices, declining interest rates, and near-frictionless refinancing opportunities—led to vastly increased systemic risk in the financial system. Individually, each of these trends is benign, but when they occur simultaneously, as they did over the past decade, they impose an unintentional synchronization of homeowner leverage. This synchronization, coupled with the indivisibility of residential real estate that prevents homeowners from deleveraging when property values decline and homeowner equity deteriorates, conspire to create a “ratchet” effect in which homeowner leverage is maintained or increased during good times without the ability to decrease leverage during bad times. If refinancing-facilitated homeowner-equity extraction is sufficiently widespread—as it was during the years leading up to the peak of the U.S. residential real-estate market—the inadvertent coordination of leverage during a market rise implies higher correlation of defaults during a market drop. To measure the systemic impact of this ratchet effect, we simulate the U.S. housing market with and without equity extractions, and estimate the losses absorbed by mortgage lenders by valuing the embedded put-option in non-recourse mortgages. Our simulations generate loss estimates of $1.5 trillion from June 2006 to December 2008 under historical market conditions, compared to simulated losses of $280 billion in the absence of equity extractions.

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1 Introduction

Home mortgage loans—one of the most widely used financial products by consumers in the United States—are collateralized mainly by the value of the underlying real estate.\(^1\) This feature makes the market value of that collateral very important in measuring the risk of a mortgage.\(^2\) To reduce the risk of default, mortgage lenders usually ask for a down payment of 10% to 20% of the value of the home from the borrower, creating an “equity buffer” that absorbs the first losses from home-price declines. Any event that reduces the value of this buffer, e.g., an equity extraction or a drop in home values, increases the risk to the lending institution.

Over the past two decades, institutional changes in the U.S. mortgage market, including the increased efficiency of the refinancing process and the growth of the refinancing business, have made it much easier for homeowners to refinance their mortgages to take advantage of declining interest rates, increasing housing prices, or both. Some of the consequences of this institutional change have been documented by Greenspan and Kennedy (2008, p. 120), who observe that “… since the mid-1980s, mortgage debt has grown more rapidly than home values, resulting in a decline in housing wealth as a share of the value of homes”. They attribute most of this effect to discretionary equity extractions via home sales, “cash-out” refinancing (where the homeowner receives cash after the refinancing), and home-equity loans.

In this paper, we focus on a previously unstudied dimension of risk in the mortgage market: the interplay among the growth of the refinancing business, the decline in interest rates, and the appreciation of property values. Each of these three trends is systemically neutral or positive when considered in isolation, but when they occur simultaneously as they did over the past decade, the results can be explosive. In particular, we show that

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\(^1\) Although most models of household finance assume that residential mortgages are non-recourse loans, the legal procedure for foreclosure and obtaining a deficiency judgment is complex, varying greatly from state to state. In fact, Ghent and Kudlyak (2009, Table 1) observe that home mortgages are explicitly non-recourse in only 11 states. Not surprisingly, some of those states are experiencing severe foreclosure problems in the current crisis, such as Arizona and California. However, in certain populous states with recourse, generous homestead-exemption laws can make it virtually impossible for lenders to collect on deficiency judgments because borrowers can easily shield their assets, e.g., Florida and Texas. Ghent and Kudlyak (2009) study the effect of lender recourse on mortgage defaults across the U.S. and conclude that recourse does decrease the probability of default for homeowners who have negative equity.

\(^2\) See, for example, Danis and Pennington-Cross (2005), Downing, Stanton and Wallace (2005), Gerardi, Shapiro and Willen (2007), Doms, Furlong, and Krainer (2007), Bajari, Chu and Park (2008), Bhardwaj and Sengupta (2008a), and Gerardi et al. (2008).
refinancing-facilitated home-equity extractions alone can account for the dramatic increase in systemic risk posed by the U.S. residential housing market, which was the epicenter of the Financial Crisis of 2007–2008.

It is obvious that the value of collateral is an important feature of any risky loan, and it has been suggested that the loan-to-value ratio should be incorporated into risk-based capital requirements for home mortgages (Calem and LaCour-Little, 2004). However, two particular features of refinancing activity have far-reaching implications for systemic risk in the U.S. residential housing market. The first is the unintentional synchronization that refinancing activity imposes on homeowner leverage and mortgage duration, and the second is the fact that refinancing-related increases in leverage cannot be symmetrically reduced when property values decline because homes are indivisible.

Although refinancing activity is not new, it naturally becomes more widespread and competitive during periods of falling interest rates and rising home prices, inadvertently increasing the leverage and duration of borrowers in a coordinated fashion and at the same point in time. Once property values decline, a wave of defaults becomes unavoidable because mortgage lenders have no mechanism such as a margin call to compel homeowners to add more equity to maintain their leverage ratio, nor can homeowners reduce their leverage in incremental steps by selling a portion of their homes and using the proceeds to reduce their debt. This self-synchronizing “ratchet effect” of the refinancing market can create significant systemic risk in an otherwise geographically and temporally diverse pool of mortgages, steadily increasing the aggregate leverage of the housing market until it reaches a systemically critical threshold.

The impact of indivisibility can be seen more clearly by contrasting an investment in residential real estate with a leveraged investment in a typical exchange-traded instrument such as common stock. The latter is subject to an initial margin requirement, a maintenance margin requirement, and margin calls by lenders that, if unanswered, can trigger forced liquidations of some or all of the investor’s position. Therefore, indivisibility in residential housing is also related to type of debt used to finance such purchases. Nevertheless, the fact

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3It is hard to imagine homeowners willing to finance large capital purchases using short-term debt like margin accounts. In fact, long-term debt has become the standard method for financing home purchases precisely because of the indivisible nature of the collateral. The indivisibility problem is also related to the fact that, in contrast to commercial real-estate, residences are typically owned by a single equityholder, i.e., the homeowner. Also, due to the U.S. Constitution’s 13th amendment prohibiting involuntary servitude, and unlike corporations, homeowners cannot raise additional capital by issuing equity if they become over-
that it is impossible to liquidate a portion of one’s home and use the proceeds to reduce the mortgage creates an important asymmetry in the housing market that does not exist in most financial markets. While over-leveraged homeowners can decide to sell their homes, recognize their capital losses, and move into less expensive properties that satisfy lenders’ minimum loan-to-value ratio requirements, the enormous costs—both financial and psychological—of such a transaction make it a highly impractical (and generally unenforceable) response to incremental and frequent increases in homeowner leverage during housing-market downturns.

The refinancing ratchet effect is most clearly illustrated by the hypothetical scenario in which all homeowners decide to maximize their leverage by reducing their home equity to the lowest possible levels via refinancing, and suppose the refinancing market is so competitive, i.e., refinancing costs are so low and capital is so plentiful, that homeowners are able to extract any equity above the minimum each month. In such an extreme case, during periods of rising home prices and falling interest rates, cash-out refinancing has the same effect as if all mortgages were re-originated at the peak of the housing market, with homeowners extracting all their capital gains, ratcheting up their leverage at successively lower interest rates, and resetting the duration of their loans to the maximum levels allowable during the housing-price run-up. Then, as home prices fall and interest rates rise, the ratchet “locks” because homeowners cannot easily unwind their real-estate positions and de-leverage due to indivisibility and illiquidity. The unintentional synchronization of leverage and duration during the market’s rise naturally leads to an apparent shift in regime during the market’s decline, in which historically uncorrelated defaults now become almost perfectly correlated.4 This refinancing ratchet effect can lead to a destructive feedback loop of correlated foreclosures, forced sales, and ultimately, a market crash. And the most insidious aspect of this phenomenon is its origin in three benign market conditions, each of which is usually considered a harbinger of economic growth. In fact, lower interest rates, higher home prices, and easier access to mortgage loans have appeared separately in various political platforms and government policy objectives over the years, and their role in fostering economic growth makes it virtually impossible to address the refinancing ratchet effect within the current regulatory framework.

4Of course, if mortgages were recourse loans and borrowers had uncorrelated sources of income, the aggregate risk of the mortgage market would be lower. However, as discussed in footnote 1, recourse does not exist in all states, hence this diversification channel is not always available.
In this paper, we propose to gauge the magnitude of the refinancing ratchet effect by creating a numerical simulation of the U.S. mortgage market. By calibrating our simulation to the existing stock of real estate, and by specifying reasonable behavioral rules for the typical homeowner’s equity extraction decision, our simulation can match some of the major trends in this market over the past decade such as the rapid rise in the amount of mortgages outstanding and the massive equity extractions from U.S. residential mortgages during this period.

We then use this simulation to document the effect of equity extractions on the aggregate amount of equity in the U.S. residential housing system after the decline in prices started in mid-2006, and on the cross-sectional distribution of loan-to-value ratios. In our simulations, approximately 18% of all mortgage loans exhibit negative equity as of December 2008, which is nearly identical to the actual figure reported by industry sources. The comparable simulated percentage in the absence of equity extractions during the housing boom would have been about 3%, implying that the 15-percentage-point difference may be attributed to the refinancing ratchet effect.

Using a simple derivative pricing model, we construct an estimate of losses absorbed by mortgage lenders—banks, asset management firms, and GSEs—due to the decline in real-estate prices over the last two years, and compare these estimates with the scenario of no equity extractions over the same period. Our simulation yields an approximate loss of $1.5 trillion from the housing-market decline since June 2006 under historical equity-extraction rules, compared to a loss of $280 billion if no equity had been extracted from U.S. residential real estate during the boom.

While we have attempted to construct as realistic a simulation as possible, we acknowledge at the outset that our approach is intended to capture “reduced-form” relations, and is not based on a general equilibrium model of households and mortgage lenders. An empirically accurate stochastic dynamic general equilibrium model of the housing and mortgage markets is currently computationally intractable, hence our choice to simulate simple heuristics calibrated to the data instead. Also, we do not model the supply of refinancing and the behavior of lenders, but assume that households can refinance as much as they wish at prevailing historical interest rates. While this may have been close to reality during the decade leading up to the peak of the housing market in June 2006, our motivation for this assumption is to study the impact of the refinancing ratchet effect by itself. Although lending behavior
no doubt contributed significantly to the magnitude of the Financial Crisis of 2007–2008, the focus of this paper is different. We show that systemic risk in the housing and mortgage markets can arise quite naturally from the confluence of three apparently salutary economic conditions, a more subtle form of systemic risk that is not simply the result of dysfunctional individual and institutional behavior such as excessive borrowing or lending.

We begin in Section 2 with a brief review of the literature. In Section 3, we outline the design of our simulation and describe the various time series that we use to calibrate the simulation’s parameters. Section 4 contains the results of our simulation, including a comparison of the time series produced by the simulation with their historical counterparts. We will show that even a relatively simple simulation can capture the observed trends in the U.S. mortgage market surprisingly well. We use the results of this simulation in Section 5 to estimate the impact of mortgage refinancing on the aggregate risk of the U.S. mortgage market as home prices declined from 2006 to 2008. We provide some qualifications for and discussion of our results in Section 6, and conclude in Section 7.

2 Literature Review

We start by reviewing the literature on the modeling of aggregate and microeconomic risks of residential mortgages, and then turn to several studies that support our hypothesis of technological and institutional forces that changed the behavior of mortgage borrowers in the last two decades.

Given the magnitude of the subprime mortgage crisis in 2007–2008, a number of recent papers have attempted to trace the root causes of the crisis. For example, the impact of the decline in housing prices on delinquency and default rates after 2007 has been studied extensively (see footnote 2 for references). Similarly, the contribution of lax lending standards and greater availability of credit to the current crisis has been considered by Dell’Ariccia, Igan, and Laeven (2008), Demyanyk and Van Hemert (2008), Bhardwaj and Sengupta (2008b), and Keys et al. (2008).

The impact of institutional changes and market structure have also been considered. For example, Mian and Sufi (2008) find that “the expansion in mortgage credit to subprime zip codes and its dissociation from income growth is closely correlated with the increase in securitization of subprime mortgages”. Dell’Ariccia, Igan and Laeven (2008) show that “lending standards declined more in areas with higher mortgage securitization rates” and
“underlying market structure mattered, with entry of new, large lenders triggering declines in lending standards by incumbent banks.” And using household data on debt and defaults from 1997 to 2008, Mian and Sufi (2009) show that borrowing against home equity is responsible for a significant fraction of the sharp rise in household leverage from 2002 to 2006 and the increase in defaults from 2006 to 2008. They also find that “[m]oney extracted from increased home equity is not used to purchase new real estate or pay down high credit card debt, which suggests that real outlays (i.e., consumption or home improvement) are likely uses of borrowed funds”. However, none of these studies have considered the implications of refinancing and depreciating collateral values on aggregate or systemic risk in the U.S. residential mortgage market.

The uncertain durations of mortgages—due to prepayment or default by the borrower—make their risks different from other fixed-income products. The decision by the borrower to prepay or default on a non-recourse mortgage can be modeled as an option written by the lender and held by the borrower. Prepayment may be viewed as a call option that allows the borrower to buy back the remaining mortgage payments from the lender at the prevailing mortgage rate, while default may be viewed as a put option that gives the borrower the right to terminate his or her mortgage by transferring the collateral property to the lender. LaCour-Little (2008) provides a recent review of literature dealing with sources of mortgage termination risk.5

The approach to modeling these embedded options can be divided into two categories: structural and reduced-form models. Structural models focus on the underlying dynamics of the collateral value and the interest rates paid for new mortgages should the borrower decide to refinance. Default or prepayment events are triggered by random movements in the home price or interest rates. By modeling the dynamics of asset prices and interest rates, and the optimizing behavior of agents, the structural approach links option-exercise events to the underlying fundamentals faced by the borrower. Kau, Keenan, Muller and Epperson (1992, 1995) are examples of this approach, and Kau and Keenan (1995) provide a review of option-theoretic pricing models of mortgages.

In the reduced-form approach, an atheoretical relation between the decision to prepay or default and various input variables summarizing general economic conditions is hypothesized and estimated, e.g., Schwartz and Torous (1989), Deng, Quigley and Van Order (2000), and

5See, also, Quercia and Stegman (1992) and Vandell (1993) for a review of earlier research on this topic.
Deng and Quigley (2002). While this flexible approach may be successful in fitting the historical data, the lack of structure may reduce its out-of-sample predictive power. For this reason, we adopt a structural approach in this paper.

The earliest structural models adopted simplifying assumptions that yielded elegant closed-form solutions, but at the expense of certain stylized facts of the U.S. mortgage market that could not be captured by those assumptions (Schwartz and Torous, 1989). For example, consider the decision to default on a mortgage. The value of the underlying real estate is obviously the most important factor in driving this decision, however, while negative equity may be a necessary condition to trigger default, it is apparently not sufficient (Foote, Gerardi, and Willen, 2008). The notion that a homeowner would continue making monthly mortgage payments after the market value of his house has fallen below the remaining balance seems odd at first, and has been attributed to the owner’s sentimental attachment to the home, moving costs, a desire to preserve reputational capital, or default penalties. The combined effect of these factors has been termed “transactions costs”. On the other hand, homeowners seeking to refinance into a lower interest-rate mortgage when rates decline may be constrained by their financial circumstances or insufficient amounts of equity in their homes.

While such frictions and financial constraints certainly influence the homeowner’s decision to default or prepay a mortgage, these factors do not provide a complete picture of all the economic forces behind such decisions. As argued by Kau, Keenan and Kim (1994), the decision to terminate a mortgage results in the loss of the option to default or prepay in the future. Therefore, the value of a house must fall below zero equity before a rational homeowner would decide to default. Gerardi, Shapiro, and Willen (2007) propose a model that retains the basic structure of rational decision-making, but yields the intuitive prediction that financially strapped borrowers are more likely to default. Foote, Gerardi, and Willen (2008) incorporate the impact of actual or imputed income from not defaulting on a homeowner’s decision to default, and argue that most homeowners with negative equity will probably not end up losing their homes. Moreover, according to Gent and Kudlyak (2009), the legal structure of the foreclosure process and the ability of the lender to obtain deficiency

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6 See, for example, Downing, Stanton and Wallace (2005) and the references in footnote 2.
7 Stanton (1995) and Downing, Stanton and Wallace (2005) incorporate these costs into models of mortgage termination.
8 Archer, Ling, and McGill (1997) and Peristiani et al. (1997) consider the impact of household financial conditions such as income, credit history, and the amount of homeowner’s equity on the ability to refinance.
judgments seem to have a detectable impact on the behavior of borrowers.

The invention of new mortgage products and corresponding institutional, social, and political changes over the last decade also contributed to the increase in the systemic risk in the mortgage system. For example, the so-called “subprime” and “Alt-A” mortgage products were designed to allow households with lower credit scores, smaller down-payments, and little documentation of income to purchase homes. However, as Mayer and Pence (2008, p. 1) observe, “… these new products not only allowed new buyers to access credit, but also made it easier for homeowners to refinance loans and withdraw cash from houses that had appreciated in value”. Moreover, some of the more exotic products such as non- or negative-amortization mortgages are contractual equivalents to dynamic strategies involving frequent cash-out refinancings to maintain a desired leverage ratio. These product innovations may have facilitated large-scale equity extractions by making refinancing significantly easier, cheaper, and virtually automatic.

By comparing the refinancing decision of homeowners in the 1980’s relative to the 1990’s, Bennett, Peach, and Peristiani (2001) find evidence that over time, a combination of technological, regulatory, and structural changes has reduced the net benefit needed to trigger a refinancing decision. They conjecture that homeowners’ familiarity with the refinancing

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9 They go on to point out that “subprime mortgages are used a bit more for refinancing than home purchase” and “almost all subprime refinance are cash-out refinances” (Mayer and Pence, 2008, p. 10). Similarly, Gerardi, Shapiro, and Willen (2007, p. 4) highlight the use of subprime mortgages for (cash-out) refinancing of mortgages that were originally classified as prime at the time of purchase, and observe that “[a]pproximately 30 percent of the 2006 and 2007 foreclosures in Massachusetts were traced to homeowners who used a subprime mortgage to purchase their house. However, almost 44 percent of the foreclosures were of homeowners whose last mortgage was originated by a subprime lender. Of this 44 percent, approximately 60 percent initially financed their purchase with a mortgage from a prime lender.”

10 See, for example, Haugh and Lo (2001) for more general applications of derivatives to implement dynamic portfolio strategies.

11 Many of these innovations may also have important tax or transaction-cost benefits to the borrower, hence they may have been demand-driven rather than the result of overly aggressive mortgage lenders. In fact, these products may be essential to achieving optimal risk-sharing. For example, in their theoretical study of optimal mortgage design, Piskorski and Tchistyi (2006) find that the “optimal allocation can be implemented using either a combination of an interest-only mortgage with a home equity line of credit or an option adjustable-rate mortgage”. See Chomsisengphet, Murphy and Pennington-Cross (2008) for an empirical analysis of the factors that determine the type of loan used to finance home purchases.

12 Specifically, they compare the refinancing behavior during two major refinancing cycles: 1986–1987 and 1992–1993. They find that measurable transactions costs such as points and fees are quite important in the refinancing decision, and these costs have declined over time due to competition and growth in the refinancing market. However, even after controlling for these costs and other factors that are known to impact the refinancing decision, the estimated refinancing probability is still considerably higher in the later part of their sample (9% vs. 14%; see Bennett, Peach, and Peristiani, 2001, pp. 970–971). Motivated by this analysis, we will propose refinancing rules with a structural break in the year 1988 (see Section 4.3).
process and their increased financial sophistication are possible drivers behind this phenomenon.

The behavioral and social aspects of the decision to default on a residential mortgage is considered by Guiso, Sapienza and Zingales (2009) using surveys of American households in late 2008 and early 2009. They find that those who consider it immoral to default are 77% less likely to declare their intention to do so. They also find that households who have been exposed to defaults are more willing to default strategically, i.e., to default even though they can afford their mortgage payments. For example, holding social stigma constant, individuals who know someone who defaulted strategically are 82% more likely to declare their intention to do so. And as defaults become more common within a given social network, the social stigma of default is likely to decline, lowering the threshold for new defaults to occur.

Perhaps a similar set of forces were at play during the most recent cash-out refinancing boom. Institutional changes, heightened competition, and technological advances made it materially easier and cheaper for consumers to engage in mortgage refinancing, and increased awareness of and familiarity with the refinancing process made it more popular. Even though many homeowners were undoubtedly aware of the potential dangers of equity extractions, the fact that many of their neighbors or co-workers were extracting equity from their homes made it socially more acceptable to do so in the midst of the height of the housing boom.

3 Simulating the U.S. Mortgage Market

This section outlines our approach to simulating the behavior of the entire U.S. residential mortgage market for the period from 1919 to 2008. We begin in Section 3.1 with some basic empirical facts about the U.S. mortgage market that are most relevant for our simulations. The overall design of our simulation is described in Section 3.2. The simulation of the dynamics of a single home is outlined in Section 3.3, and in Section 3.4 we describe the process by which the single-home simulations are aggregated.

3.1 Basic Facts About the U.S. Mortgage Market

We begin with some basic facts about the overall size and trends of the U.S. mortgage market that are most relevant for our simulation. Figure 1 shows the time series of conventional 30-year fixed-rate mortgage rates, and purchase and refinancing mortgage-origination
volumes in the U.S. from the first quarter of 1991 (1991Q1) to the fourth quarter of 2008 (2008Q4). The data depicted in this figure were obtained from a number of public sources. The interest-rate data is the Federal Home Loan Mortgage Corporation (“Freddie Mac”) 30-Year fixed-rate mortgages series. The Mortgage Origination Volume data is obtained from Mortgage Bankers Association (MBA) publications. The data collected by the Mortgage Bankers Association breaks down origination volume into two components: origination of loans intended for new purchase, and those intended for refinancing purposes. Refinancing volume can be further broken down by loan type based on the data collected by the U.S. Federal Housing Finance Agency (FHFA), formerly the Office of Federal Housing Enterprise Oversight (OFHEO). In particular, FHFA data classifies loans into the following three categories: Purchase, Cash-Out Refinancing, and Rate/Term Refinancing (where the homeowner receives no cash from the refinancing, but merely changes the terms and/or reduces the interest rate to be paid on the remaining balance of the mortgage). We have used this data to break down the refinancing volume reported by the MBA into Cash-Out Refinancing and Rate/Term Refinancing volume.

Several prominent themes emerge from Figure 1. While purchase volume is highly seasonal, the increase and subsequent decline closely matches the trend in overall real-estate prices. There is also a clear relationship between decline in mortgage rates and rate refinancing volume. For example, the decline in interest rates in the early 1990’s is followed by a period of high rate-refinancing activity from 1992 to 1993. The next period of increased rate refinancing occurs in 1998, again coinciding with a drop in mortgage rates. However,

13See http://www.freddiemac.com/pmms/.
14See the page “MBA Mortgage Origination Estimates” at the website: http://www.mbaa.org/ResearchandForecasts/EconomicOutlookandForecasts.

This data is available quarterly from 1990Q1 to 2008Q4 at the time of this study.
15This data is available at http://www.fhfa.gov/Default.aspx?Page=87, in the section titled “Loan Purposes by Quarter”, and is available from 1991Q1 through 2008Q4 at the time of this study. Similar data is available from a number of public sources and the values reported can differ substantially on occasion (see Chang and Nothaft, 2007, for further discussion).
16The three categories reported in this FHFA data—Purchase, Cash-Out Refinancing, and Rate/Term Refinancing—account for about 98% of all originations. To calculate the Cash-Out and Rate/Term Refinancing volume, we use the relative ratio reported in the FHFA to breakdown the refinancing volume reported in the MBA data. For example, for 1991Q1 the MBA reports $32B in refinancing volume and the FHFA data indicates that 3.8% of the total volume was due to Cash-Out Refinancing and 27.0% was due to Rate/Term Refinancing, hence we conclude that $32 billion $3.8/(3.8 + 27.0) = $3.95 billion is the appropriate figure for Cash-Out Refinance volume and $32 billion $27.0/(3.8 + 27) = $28.05 billion is the corresponding figure for Rate/Term Refinancing volume.
the most active period of rate refinancing takes place in 2001Q4 through 2003Q3, where the average volume is $342 billion per quarter, far exceeding the peak of each of the previous two refinancing booms. There is also indirect evidence that mortgage-lending competition increased during this period—according to Freddie Mac’s surveys (see www.freddiemac.com), the average number of points associated with conventional 30-year fixed-rate mortgages declined from 1.8 in December 1997 to 1.0 in December 1998 to 0.6 in December 2002, and is 0.7 in the most current weekly survey of June 25, 2009.

Home-equity extraction is a process in which a homeowner converts a portion of the equity in the home into cash by retiring the existing loan and taking out a new and larger loan. Such loans are categorized as “cash-out” refinancing in the FHFA data set, and it is not surprising that equity extraction is more common in a rising real-estate market because during such periods, homeowners’ equity increases dollar-for-dollar with home prices, giving homeowners more equity to extract. Figure 1 documents a seemingly permanent increase in cash-out refinance volume in the second half of the sample. The first peak in cash-out refinancings occurs in 1998Q4, when volume surpasses $100 billion for the first time. Although the volume in the following 9 quarters (1999Q1 to 2001Q1) was less than $100 billion per quarter, the average value of cash-out refinancings per quarter was $204 billion in the subsequent 30 quarters (2001Q2 to 2008Q3), far exceeding the average value in the preceding 41 quarters from 1991Q1 to 2001Q1.\(^\text{17}\) Not surprisingly, as home prices fell from 2006 to 2008, cash-out refinancing volume rapidly subsided, declining to only $84 billion in the last quarter of 2008.

Figure 2 shows the relation between gross equity extraction and aggregate U.S. home prices during the period from 1991Q1 to 2008Q4.\(^\text{18}\) The increase and subsequent decline in the gross equity extraction closely mirrors the pattern of aggregate U.S. residential real-estate prices. According to this estimate, U.S. homeowners extracted an average of $160 billion in each of the 32 quarters between 1999Q3 to 2007Q2, far outstripping the $87 billion extracted during the previous peak in 1998Q4.

Other things equal, equity extraction leads to a larger mortgage on a given home, implying a link between the amount of equity extracted and the volume of mortgages outstanding that

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\(^{17}\)Of course, aggregate refinancing activity is expected to grow as the economy grows. However, the time period of interest is sufficiently short that even as a percentage of GDP, the total housing stock, or other macroeconomic variables, the qualitative patterns of cash-out refinancing are similar.

\(^{18}\)The estimates of gross equity extractions are from Greenspan and Kennedy (2005). We are grateful to Jim Kennedy for providing us with updated estimates.
Figure 1: 30-year fixed-rate mortgage rates, and purchase, cash-out refinancing, and rate-refinancing origination volume from 1991Q1 to 2008Q4.

Figure 2: The appreciation in and subsequent decline of U.S. residential real-estate values, as measured by the Case-Shiller Composite 10 Index, and the corresponding growth of equity extractions from 1991Q1 to 2008Q4. The equity extraction data is based on Greenspan and Kennedy (2005).
is confirmed in Figure 3. This figure shows that outstanding mortgages grew from $2,648 billion in 1991Q1 to its peak of $11,142 billion in 2008Q1. During this period, homeowners extracted $6,720 billion in equity. These figures suggest that equity extractions represent a non-trivial portion of outstanding mortgages, and the risk transferred from homeowners to the financial sector due to these extractions may have had a significant impact on the overall risk exposure of this sector to real-estate prices. The objective of the simulation in this paper is to quantifiy this effect.

Figure 3: Cumulative equity extractions and the growth in the volume of U.S. residential mortgages outstanding from 1991Q1 to 2008Q4. The equity extraction data is based on Greenspan and Kennedy (2005), and outstanding residential-mortgage volume is reported by the Federal Reserve as “Personal Sector Home Mortgages Liability”.

An important issue affecting the link between equity extractions and risk is what homeowners do with the equity they extract. If, for example, the extracted equity is invested in liquid assets that are not highly correlated with property values, then indivisibility may be less of an issue and the ratchet effect may not be nearly as pronounced. A subsequent decline in home prices will still increase homeowner leverage, but the homeowner can easily liquidate a portion of the invested equity that was previously extracted to pay down the mortgage or to make mortgage payments. However, using the credit files of a sample of almost 70,000 individuals from a national consumer credit rating agency, Mian and Sufi (2009) conclude that equity extractions are used primarily for consumption or home improvement, neither of
which reduces the risks inherent in cash-out refinancing.

3.2 Simulation Design

To design a simulation of the behavior of homeowners in the face of changing interest rates and home prices, we propose a two-step procedure. First, we calibrate our simulation to be representative of the actual stock of homes in the U.S. by using data on new home construction and sales, and the average prices of new homes sold. Each house enters our simulation when it is first sold, and stays in our simulation until its mortgage is fully paid. In the process, the house may be refinanced one or more times. We parameterize the refinancing decision—for example, as a function of the loan-to-value ratio—and seek to calibrate this decision rule to reproduce related observable macroeconomic data such as the total value of residential mortgages outstanding or the total value of equity extracted from homes during this period.

As in Figure 1, our simulation also distinguishes between two types of refinancing: rate and cash-out refinancing. In the former case, the homeowner refines to take advantage of lower interest rates by reducing the monthly payment without altering the maturity of the mortgage or the mortgage amount, and in the latter case, the owner refines to take advantage of capital appreciation, in which case the newly originated mortgage will reflect a larger principal amount than that of the existing mortgage, with the difference paid out to the homeowner.\(^\text{19}\) Even this relatively simple set of refinancing decisions, and the corresponding impact on aggregate risk exposures, poses significant computational challenges. To see why, consider the computational complexity involved in determining the amount of loans outstanding \(N\) years after a single cohort of homeowners purchase their homes in a given year (we shall refer to this cohort as a single “vintage”). In the simplest case, we assume that all homes in this vintage are purchased at the same initial loan-to-value ratio—this will be 85% in our simulations—using conventional 30-year fixed-rate mortgages. In addition, assume that homeowners do not move, refinancing is possible only once a year, homeowners extract the maximum amount of equity if they do decide to refinance, and if they refinance, their new mortgages are also conventional 30-year fixed-rate mortgages. In this highly simplified

\(^\text{19}\)Another possible refinancing arrangement is for the homeowner to extract an amount of equity and increase his mortgage so as to maintain the same level of monthly payments. However, the amount of equity that can be extracted in this manner is typically very small, and not enough to explain the massive rise in mortgages outstanding and equity extractions shown in Figure 3. For this reason, and to keep our simulations as simple as possible, we limit our analysis to the two refinancing arrangements described above.
world, the refinancing decision is a binary choice, hence \( N \) years after the initial purchase, there are \( 2^N \) possible decision paths that each homeowner might have followed. All \( 2^N \) paths for all homeowners must be evaluated to compute the total value of mortgages outstanding from this single vintage\(^{20}\).

Also, note that there is no possibility of using re-combining binary trees to simplify the computations since the outcomes—in terms of magnitude and timing of equity extractions—are quite distinct across each “slice” of the tree, hence the computations grow exponentially with the horizon of the simulation. Therefore, in designing our simulation, we must balance the desire for realism against the tractability of the computations required. To that end, we make the following assumptions:

(A1) Each house is purchased at a loan-to-value ratio of \( \text{LTV}_o \) which is set to 85% in our simulations.

(A2) All homes are purchased with conventional 30-year fixed-rate mortgages that are non-recourse loans.

(A3) The market value of all houses that are in “circulation”, i.e., that have outstanding mortgages and may be refinanced at some point, grows at the rate given by the Home Price Index (HPI), which will be discussed in Section 4.

(A4) We use only national data on home sales, price appreciation, and mortgages outstanding to calibrate and test our simulation, hence potentially important geographical differences are not reflected.

\(^{20}\) For concreteness, consider the case where the annual interest rate is 5%, the initial home price is $100,000, purchased with a 30-year mortgage with initial LTV of 85% and, for further simplicity, assume that payments are made once a year at year end. Based on these assumption, the initial outstanding loan is $85,000 which, assuming no refinancing and equity extractions, will decline to $83,720 after 1 year and to $82,377 after two years due to the payments made by the owner. Let home prices appreciate by 3% per year in each of the next two years, so the home is worth $103,000 by the second year and $106,090 by the third year. Consider a homeowner who decides to refinance at the end of first year. In this case, the new mortgage will be $103,000 \times 85% = $87,550 and the difference between this amount and $83,720—$3,829—can be extracted via a cash-out refinancing. By the end of the second year, there are four possibilities to consider in calculating the total loan value and total equity extractions: homeowners who never refinanced, those who refinanced after one year but did not refinance in the second year, those who refinanced only in the second year, and lastly those who refinanced in both year to take advantage of home-price appreciation. Each of these cases is associated with a different value and timing of equity extractions, and results in a different value for the outstanding loan amount at the end of the second year. It should be clear that each additional year multiplies the number of paths that need to considered by a factor of 2, resulting in \( 2^N \) paths after \( N \) years.
(A5) We assume that a homeowner’s decision to refinance is made each month, and is only a function of the current equity in the home and the potential savings from switching to a lower interest-rate mortgage. In particular, we assume that the refinancing decision does not depend on factors such as the price and age of the home, or the time elapsed since the last refinancing.

(A6) For rate refinancing, we assume that the owner will refinance when rates have fallen by more than the rate-refinancing threshold (RRT) from the rate on an existing mortgage. For all of our simulations, we set RRT to be 200 basis points. The new mortgage is assumed to have the same maturity as the existing mortgage, and the principal of the new mortgage is equal to the remaining value of the existing mortgage. Therefore, the homeowner will save in monthly payments due to the lower mortgage rate, but there is no equity extracted from the house.

(A7) For cash-out refinancings, we assume the homeowner will refinance to take out the maximum amount of equity possible. Therefore, the loan-to-value ratio will be brought back to LTV₀ after each refinancing, and a new loan with a maturity of 30 years will be originated.

(A8) We assume that the refinancing decisions by homeowners are random and independent of each other, apart from the dependence explicitly parameterized in the refinancing rule. For example, given a pool of 10MM homes, if the refinancing rule yields a probability of 1% for refinancing based on the prevailing inputs (e.g., the current loan-to-value ratio, the current interest rate, etc.), we assume that 100,000 homeowners will refinance their homes during that period.

(A9) We assume that once fully owned, a home will not re-enter the market, i.e., the probability of a cash-out refinancing is zero after a mortgage is paid off.

(A10) We do not incorporate taxes or transactions costs explicitly into our simulations.

Given the central role that these assumptions play in our simulations and their interpretation, a few words about their motivation are in order.

Assumptions (A1) and (A2) determine the initial leverage and type of mortgage we assume for new homeowners, and we chose a loan-to-value ratio of 85% and 30-year conventional fixed-rate mortgages partially based on the data reported in Tables 3–14 and 3–15 of
the American Housing Survey regarding the behavioral patterns of homeowners.\textsuperscript{21} Of course, in the years leading up to the peak of the housing market in June 2006, considerably more aggressive and exotic loans were made, including the now-infamous NINJA (“no income, no job or assets”) mortgages and many others with embedded options. Assumptions (A1) and (A2) are motivated by our desire for simplicity, but also because we wish to err on the conservative side of default-related loss implications wherever possible. Assuming that all mortgages are conventional 30-year fixed-rate loans with 15% downpayments instead of a more realistic mix of conventional and exotic mortgage products is likely to yield under-estimates of loan-to-value ratios, default correlations, and potential losses from falling home prices.

Assuming that mortgages are non-recourse loans greatly simplifies our simulations because we do not need to model the dynamics of other sources of collateral. However, by assuming that lenders have no recourse to any other sources of collateral, our simulation may yield over-estimates of potential losses, and may also oversimplify the behavior of borrowers (see Ghent and Kudlyak, 2009). To take on the more complex challenge of matching the mix of recourse and non-recourse loans in the mortgage system in our simulations, we require information about the types of recourse that are permitted and the practicalities of enforcing deficiency judgments in each of the 50 states, as well as cross-sectional and time-series properties of homeowner income levels, assets, and liabilities.\textsuperscript{22} While this is beyond the scope of our current study, it is not an insurmountable task given sufficient time, resources, and access to financial data at the household level.

Assumptions (A3) and (A4) allow us to calibrate the price dynamics of our simulated housing stock, and our decision to ignore potentially significant individual and regional and differences in housing prices, mortgage refinancing behavior, and default rates is also likely to yield conservative loss estimates. To see why, consider the case where the national average home price shows little growth, masking the fact that certain regions have experienced large price gains that are offset by comparable declines in other regions. Simulations based on

\textsuperscript{21}See http://www.census.gov/hhes/www/housing/ahs/ahs07/ahs07.html.

\textsuperscript{22}Mian and Sufi (2009) provide some interesting empirical insights into these cross-sectional and time-series properties using individual-level data on homeowner debt and defaults from 1997 to 2008: “Home equity-based borrowing is stronger for younger households, households with low credit scores, and households with high initial credit card utilization rates. Homeowners in high house price appreciation areas experience a relative decline in default rates from 2002 to 2006 as they borrow heavily against their home equity, but experience very high default rates from 2006 to 2008. Our estimates suggest that home equity-based borrowing is equal to 2.8% of GDP every year from 2002 to 2006, and accounts for 34% of new defaults.”
the national average would produce little equity extraction during such periods. In a more realistic model disaggregated by region, those regions enjoying positive growth will generate significant equity extractions incrementally as housing prices rise. However, this regional increase in leverage will not be offset by other regions experiencing comparable price declines because the indivisibility of housing makes it impossible for over-leveraged homeowners to reduce their leverage incrementally. Thus, aggregate equity extractions would likely be more significant in a regionally disaggregated simulation.

Assumptions (A5)–(A7) are simple behavioral rules meant to encapsulate the economic deliberations in which homeowners engage to decide whether or not to refinance. Accordingly, implicit in these rules are many factors that we do not model explicitly such as transactions costs, opportunity costs, homeowner characteristics such as income and risk preferences, macroeconomic conditions, and social norms. While it may be possible to derive similar rules from first principles, the computational challenges may outweigh the benefits, especially from the perspective of producing estimates of potential losses from the aggregate housing sector. In particular, Assumption (A5) addresses the usual “curse of dimensionality” that plagues many computational problems by structuring the simulation so that all homes purchased or refinanced at a given time can be “combined” into a single simulation path. Adding other state-dependent factors such as house age or price can increase the dimensionality of the problem geometrically, quickly rendering the simulation computationally infeasible.

Assumptions (A6) and (A7) outline the two polar opposites of our simulated refinancing activities—(A6) describes refinancing behavior that does not extract any equity or increase the size of the mortgage, and (A7) describes refinancing behavior that extracts the maximum amount of equity possible and resets the leverage to the initial 85% loan-to-value ratio. Clearly a number of intermediate cases can be considered, but we focus only on these two extremes to delineate the boundaries that span them. The choice of 200 basis points for RRT is arbitrary, and does not affect the loss estimates for cash-out refinancing, but this behavioral rule does yield a reasonable approximation to the historical rate/term refinancing origination volumes shown in Figure 1.

Also, implicit in (A5)–(A7) is the assumption that the supply of credit to households is infinitely elastic at prevailing market rates, which is motivated by our interest in measuring the impact of household refinancing behavior by itself. The complexities of consumer credit markets warrant a separate simulation study focusing on just those issues.
Assumption (A8) requires some clarification because clearly the refinancing rules in (A6) and (A7) imply that refinancing decisions are not independent across households. Assumption (A8) simply states that there are no other sources of dependence, such as social pressures arising from neighbors refinancing. Therefore, the only channel by which we allow refinancing decisions to be correlated across households is through interest rates and home prices via the behavioral rules in (A6) and (A7). Remarkably, this single source of commonality is sufficient to generate an enormous amount of synchronized losses when home prices decline.

Assumption (A9) is motivated primarily by expositional convenience, and can easily be amended to allow fully paid houses to re-enter the real-estate market, but as shown in Figure 7, our simulations imply that 35% of homes are fully owned as of December 2007, which is identical to the reported value in American Housing Survey of 2007. Also, ignoring issues such as relocation or renting vs. owning are not likely to affect our estimates of aggregate risk and losses. For example, consider the case of an individual who decides to rent after selling his home for $200,000 which was recently purchased for $100,000 with a downpayment of $15,000. Assuming a 0% interest rate for simplicity, this fortunate individual has taken $115,000 of equity out of the housing market. However, the new buyer of this home will likely borrow all but 10% to 20% of the purchase price, e.g., under our Assumption (A1), the buyer will borrow $170,000. It is not hard to see that the aggregate effect of this transaction is virtually identical to a cash-out refinancing by the original homeowner.

Assumption (A10) is a standard simplification, but is not equivalent to the usual “perfect markets” assumption where taxes and transactions costs are assumed to be zero. In fact, assuming away market frictions may seem particularly incongruous in the context of a simulation of refinancing activity, which some consider to be driven largely by transactions costs. In fact, Assumption (A10) does not assert that these frictions do not exist, but merely that we do not model their impact on behavior explicitly. Instead, our behavioral rules for the homeowner’s refinancing decision implicitly incorporates these costs into our simulation in a “reduced-form” manner.

If, instead of positing simple heuristics, we simulated optimal refinancing behavior of the kind proposed by Pliska (2006), Fortin et al. (2007), and Agarwal, Driscoll, and Laibson (2008), then the specific type and degree of transactions costs and taxes could be incorporated explicitly. The complexities of such stochastic dynamic optimal control policies would make our simulations considerably less tractable and transparent, but we conjecture that
the qualitative properties may not change a great deal. Our intuition is derived from the vast literature on optimal portfolio policies with transactions costs in which the presence of fixed and proportional trading costs reduces the frequency of trades and makes trading volume more “lumpy” (see, for example, Constantinides, 1976, 1986; Davis and Norman, 1990; Grossman and Laroque, 1990; and Lo, Mamaysky, and Wang, 2004). Our behavioral refinancing rules capture the spirit of these optimal policies, hence our calibration to historical refinancing levels should implicitly reflect the impact of transactions costs, taxes, and other factors that enter into a homeowner’s decision to refinance.

With these assumptions in place, we can now turn to the specific components of the simulation. In Section 3.3 we describe the simulation of the evolution of a single home over time, and in Section 3.4, we consider the corresponding simulation for the entire pool of mortgages in the U.S. housing market.

3.3 Single-Home Simulation

In this component of the simulation, we calculate the monthly payment, mortgage rate, outstanding loan, market values, and LTV ratios for each vintage of homes using monthly data from January 1919 to December 2008 (there are 1,080 vintages all together). The initial purchase price is set to the average price of new homes sold in that month. According to (A1) and (A2), the starting loan size is set so that the loan-to-value ratio is equal to $\text{LTV}_0$ and all homes are purchased with conventional 30-year fixed-rate mortgages.

At this stage, we assume that homeowners refinance their mortgages if rates have dropped by more than the RRT parameter. In this part of the simulation, we assume that no cash-out refinancing takes place. The effect of cash-out refinancing will be captured through our aggregation scheme described in the next section. Given the assumptions outlined above, the new mortgage has the exact same maturity and principal amount as the existing mortgage and no equity is extracted. This step of the simulation produces the following quantities:

- $\text{VALUE}_{i,t}$ is the value of a house from vintage $i$ by time $t$, $i \leq t$. $\text{VALUE}_{i,i} = \text{NHP}_i$ where $\text{NHP}_i$ is the average price for all new homes sold in vintage $i$. The value for all subsequent time periods grows at the rate given by HPI (see Section 4 for the specific time series used).

- $\text{LOAN}_{i,t}$ is the loan value for a home initially purchased in vintage $i$ by time $t$, $i \leq t$. 
Note that $LOAN_{i,t} = LTV_o \times VALUE_{i,t}$. Also, since there is no change in the amount of the loan during the life of the mortgage (however, the rate may change via rate refinancings), the magnitude of $LOAN_{i,t}$ simply follows the standard amortization schedule of a conventional 30-year fixed-rate mortgage with a monthly payment frequency.

- $LTV_{i,t}$ is the loan-to-value ratio at time $t$ for a home initially purchased in vintage $i$, $i \leq t$. It is simply given by $LTV_{i,t} = \frac{LOAN_{i,t}}{VALUE_{i,t}}$.

- $GRT_{i,t}$ is the value of the embedded guarantee a non-recourse mortgage provides to the homeowner, essentially giving the owner the right (but not the obligation) to sell the property back to the lender if the home price falls below the mortgage’s outstanding balance (see Section 5 for further discussion).

### 3.4 Aggregate Simulation

The aggregation component combines the effect of cash-out refinancing and the time series obtained from the single-home simulation component to yield aggregate series for the amounts of loans outstanding and equity extractions, as well as several systemic risk measures.

By Assumption (A5), refinancing behavior depends only on the current loan-to-value ratio and the potential savings due to lower rates, which makes the computation of aggregate quantities much quicker. This efficiency gain is due to the fact that, starting from the same initial loan-to-value ratio, $LTV_o$, and with the same price appreciation determined by HPI, all homes refinanced in period $t$ will have the same refinancing behavior going forward as homes purchased in period $t$. Therefore, the time series produced by the single-home simulation can serve as the starting point for our aggregation simulation, and as long as we keep track of the number of houses that were refinanced in each period, we can construct our aggregate measures accordingly.

As the starting point of our aggregate simulations, we define the variable $TOTALV_t$, which is the total value of all homes that were either directly purchased at time $t$ or were purchased in an earlier period, survived until time $t-1$, and cash-out refinanced at time $t$. This variable can be computed recursively through the relation:

$$TOTALV_t = NH_t \times VALUE_{t,t} +$$
\[ \sum_{i=1}^{t-1} \text{TOTALV}_i \times \text{SURVIV}_{i,t-1} \times \text{REFI}_{i,t} \times \frac{\text{VALUE}_{i,t}}{\text{VALUE}_{i,i}} \]  

(1)

where \( \text{NH}_t \) is the number of new homes entering the system in vintage \( t \), \( \text{SURVIV}_{i,t-1} \) is the probability that a new home from vintage \( i \) has not undergone a cash-out refinancing by time \( t - 1 \), and \( \text{REFI}_{i,t} \) is the probability that a home entering the system in vintage \( i \) undergoes a cash-out refinancing at date \( t \), conditioned on the event that it has not yet been refinanced by date \( t \). The multiplier \( \frac{\text{VALUE}_{i,t}}{\text{VALUE}_{i,i}} \) is an adjustment factor that reflects time variation in housing prices. From \( \text{TOTALV}_t \), we can compute the total value of loans outstanding, \( \text{TOTALM}_t \), and the total equity extractions, \( \text{TOTALX}_t \), using the following two sums:

\[
\text{TOTALM}_t = \sum_{i=1}^{t} \text{TOTALV}_i \times \text{SURVIV}_{i,t} \times \frac{\text{LOAN}_{i,t}}{\text{VALUE}_{i,i}}
\]  

(2a)

\[
\text{TOTALX}_t = \sum_{i=1}^{t} \text{TOTALV}_i \times \text{SURVIV}_{i,t-1} \times \text{REFI}_{i,t} \times \frac{\text{VALUE}_{i,t}}{\text{VALUE}_{i,i}} \times (\text{LTV}_o - \text{LTV}_{i,t})
\]  

(2b)

The logic behind (2) is as follows. The total amount of mortgage debt outstanding is simply the sum—over all previous vintages of homes—of the product of three terms: (1) the total value of all homes requiring mortgage financing, i.e., the value of all homes that were either directly purchased or refinanced in each previous period; (2) the probability that homes in (1) have survived, i.e., have not undergone cash-out refinancings, until time \( t \) (note that the value of the mortgages associated with homes that have gone through cash-out refinancing will be reflected through the recursive relation given in (1)); and (3) the ratio \( \frac{\text{LOAN}_{i,t}}{\text{VALUE}_{i,i}} \), which accounts for normal mortgage amortization. The total amount of equity extraction given in (2b) is the sum—also over all previous vintages—of the product of four terms: (1) the total value of all new and refinanced homes in each previous vintage; (2) the probability that the homes in (1) did not undergo cash-out refinancings until time \( t - 1 \) and were refinanced at date \( t \); (3) the multiplier \( \frac{\text{VALUE}_{i,t}}{\text{VALUE}_{i,i}} \), to take into account changes in the market value of homes; and (4) the difference \( \text{LTV}_o - \text{LTV}_{i,t} \), which is the amount of equity that can be extracted from refinancing.

The process of calibrating the aggregate simulation consists of specifying a particular
behavioral rule for the refinancing decision and selecting parameters for that rule that can generate realistic histories of the aggregate time series defined in (2). We turn to this task in Section 4.

4 Calibrating the Simulation

To properly calibrate our simulation, we first describe the data we use as inputs to the calibration in Section 4.1, and then focus on the two primary time series we intend to match in our simulation in Section 4.2. We present the results of the calibration in Section 4.3.

4.1 Input Data

The following five time series are used as the main inputs to our simulation:

1. Home Price Appreciation, HPI_t. We use three sources to assemble this series. For the most recent history (since January 1987), we use the S&P/Case-Shiller Home Price Composite-10 Index.\(^{23}\) From 1975Q1 to 1986Q4, we use the national house price index from the FHFA.\(^{24}\) Prior to 1975Q1, we use the nominal home price index collected by Robert Shiller.\(^{25}\) These last two series are only available at a quarterly and annual frequency, respectively, and to be consistent with the rest of our simulation, we convert them into monthly series assuming geometric growth.\(^{26}\) Given the importance of this variable for our simulation, we considered two other home-price series using different data in the more recent period, but because the results did not differ significantly from those based on HPI_t, we have omitted them to conserve space.\(^{27}\)

\(^{23}\)See Standard and Poor’s website.
\(^{24}\)See \url{http://www.fhfa.gov/Default.aspx?page=87}. The relevant data may be found in the “All-Transactions Indexes” section.
\(^{25}\)See \url{http://www.econ.yale.edu/~shiller/data.htm}.
\(^{26}\)Specifically, for months other than March, June, September, and December, HPI_t is computed as:

\[
HPI_t = \exp \left[ \log(HPI_{Q^-}) + (t - t_{Q^-}) \log \left( \frac{HPI_{Q^+}}{HPI_{Q^-}} \right) \right]
\]

where HPI_{Q^-} denotes the quarterly index value from the previous quarter, and HPI_{Q^+} denotes the quarterly index value from the current quarter. The approach for interpolating monthly observations from annual data is similar.

\(^{27}\)Specifically, we define CSNAT-HPI_t and NAR-HPI_t using the same data as HPI_t for the earlier part of the simulation period, but CSNAT-HPI_t uses the Case-Shiller National Home Index price since 1987Q1, and NAR-HPI_t uses the appreciation in the median price of existing homes sold as reported by the National Association of Realtors, which is available since January 1999. Because the Case-Shiller National Price...
2. New Homes Entering the Mortgage System, $NH_t$. We construct this time series from a variety of sources. The time series of “New One-Family Houses Sold” available from the U.S. Census Bureau is the starting point.\(^{28}\) This series is available monthly since January 1963. However, it only includes homes built for sale and, for example, excludes homes built by homeowners and contractors. To take such cases into account, we use data collected by the U.S. Census Bureau on the intent of completed home constructions.\(^{29}\) This construction data separates the completed units by their intent—units in the “Built for Sale” category correspond to homes that will be reported in the “New One-Family Houses Sold” upon the completion of a sale transaction. We take the sum of construction numbers reported under the “Contractor-Built”, “Owner-Built”, and “Multi-Units Built for Sale” categories, and use the ratio of this sum to the number of “One-Family Units Built for Sale” to adjust the “New One-Family Houses Sold” series.\(^{30}\) For example, in 1974 this ratio is 1.06, therefore we multiply the monthly “New One-Family Houses Sold” by a factor of $1 + 1.06 = 2.06$ in each month during 1974 to estimate the total number of units entering the mortgage system that year. For the period from 1963 to 1973, this ratio is not available (see footnote 29), so we will use the average of the adjustment factor from 1974 to 1983 to make the adjustments prior to 1974. This yields values of $NH_t$ back to January 1963.

However, the useful life of a typical home is often greater than 46 years (1963 to 2008), hence we may be omitting a significant fraction of homes with current mortgages if $NH_t$ only starts in January 1963. Based on data from the American Housing Survey 2007, approximately 93% of homes surveyed were built after 1919.\(^{31}\) Therefore, we choose to

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\(^{28}\)See [http://www.census.gov/const/www/newressalesindex.html](http://www.census.gov/const/www/newressalesindex.html).

\(^{29}\)See [http://www.census.gov/const/www/newresconstindex_excel.html](http://www.census.gov/const/www/newresconstindex_excel.html). The relevant data may be found in “Quarterly Housing Completions by Purpose of Construction and Design Type”. We use the annual series since it is available since 1974 (quarterly data only goes back to 1999).

\(^{30}\)We have excluded the “Multi-Units Built for Rent” category because our focus is the mortgage liability of the Personal sector. Mortgages for multi-units built for rent, such as large apartment buildings, are typically held outside of the Personal sector. However, some of these units may eventually be converted into condominiums and sold to individual buyers, which will not be captured in our simulations. This exclusion is yet another reason we consider our simulated mortgage-default losses to be conservative estimates of actual losses.

\(^{31}\)See [http://www.census.gov/hhes/www/housing/ahs/nationaldata.html](http://www.census.gov/hhes/www/housing/ahs/nationaldata.html). The relevant data is given in Table 1A-1.
extend $N_{Ht}$ back to January 1919 to yield a more realistic time series for the stock of U.S. residential real estate in more recent years. Appendix A.2 outlines our method for extending this series back to January 1919 by using the statistical relationship between $N_{Ht}$ from 1963 to 2008 and factors such as population and real home prices.

3. **New House Prices, $N_{HPt}$.** We will use the average home price available from the U.S. Census Bureau for “New One-Family Houses Sold” in the period from January 1975 to December 2008. For the period from January 1963 to January 1975, the average price is not available. However, the Census reports the median sale price for this period, which we will use as our starting point. From January 1975 to December 2008, when both mean and median home prices are available, we observe that the mean price is typically higher than the median by approximately 5%, and the ratio has increased in the more recent history. To make our simulations more accurate, we multiply the reported median prices in the January-1963-to-December-1974 sample by 1.05, i.e., we inflate the median by 5%, and use the resulting values to complete the $N_{HPt}$ series. From January 1919 to December 1962, we will use the growth rate of $HPI_t$ to extrapolate sales prices, starting with the sales price in January 1963.

4. **Long-Term Risk-Free Rates, $RF_t$.**

We will use the yield on the 30-year constant maturity U.S. Treasury securities, which is available from February 1977 to December 2008, but with a gap from March 2002 to January 2006. We fill this gap using yields on 20-year constant maturity Treasury securities.\(^{32}\) For the period prior to February 1977, we will use the “Long Rate” collected by Robert Shiller,\(^{33}\) which is only available annually, so we use linear interpolation to obtain monthly observations.

5. **Mortgage Rates, $MR_t$.** We will use the series constructed by Freddie Mac for the 30-year fixed-rate mortgage rate, which starts in April 1971.\(^{34}\) For the earlier period, we will simply add 150 basis points to the long-term risk-free rates $RF_t$ (see above), which is approximately the average spread between 30-year mortgage rates and $RF_t$ for the period from April 1971 to December 2008.


\(^{34}\)See [http://www.freddiemac.com/dlink/html/PMMS/display/PMMSOutputYr.jsp](http://www.freddiemac.com/dlink/html/PMMS/display/PMMSOutputYr.jsp). The section “30-Year Fixed-Rate Historic Tables” contains the relevant data.
4.2 Calibration Reference Series

As described in Section 3.4, for a given model of refinancing behavior, our simulation can generate the hypothetical time series of total mortgages outstanding and total equity extractions. Our goal is to calibrate the parameters of that refinancing model so that these two hypothetical series come as close as possible to the following two historical time series, which will serve as our calibration reference series:

1. Outstanding Mortgage Volume. We will use the value of residential mortgage liabilities as reported in the Federal Reserve *Flow of Funds Accounts*. This data is available at a quarterly frequency from 1951Q4 to 2008Q4, and annually from 1945 to 1951.

2. Equity Extractions. We will use the series produced by Greenspan and Kennedy (2005), which is available at a quarterly frequency from 1968Q1 to 2008Q4. While their approach decomposes the “Total Gross Equity Extractions” into three components (home sales, home equity loans net of unscheduled payments, and cash-out refinancings), we will use their “Total Gross Equity Extractions” series in our calibration process. This is motivated by the fact that home sales and cash-out refinancings have a similar impact on the aggregate risk of the housing market (see Appendix A.1 and the discussion of Assumption (A9) in Section 3.2 for further details).

4.3 Calibration Results

The calibration exercise consists of finding a decision rule for the cash-out refinancing decision that most closely matches the two reference time series of Section 4.2. The simulation described in Section 3.4 is suitable for simulating situations where the cash-out refinancing decision depends on any state variable that is not specific to a particular vintage of houses. For example, the refinancing decision can incorporate changes in interest rates or expectations about future residential real-estate prices. Therefore, in the interest of simplicity and feasibility, we propose the most parsimonious rule that can match the gross properties of the two reference series.

To establish a baseline for comparison, we first present the results of our simulation if homes are never cash-out refinanced after their initial purchase. In this scenario, the $\text{REFI}_{i,t}$ probability is 0 and the $\text{SURVIV}_{i,t}$ probability is 1 for all $i$ and $t$, which greatly simplifies (1).
and (2). Since no equity is extracted in this case, the main metric for assessing the results of the simulations is based on the total value of outstanding mortgages. The result of this exercise is shown in Figure 4 for \( \text{LTV}_o \) set to 85%. Such an approach clearly falls short of reproducing the total outstanding mortgage volume by a wide margin.

![Figure 4: Volume of outstanding mortgages using our simulation approach versus the actual values as reported by the Federal Reserve. The simulation is based on an initial loan-to-value ratio of 85% with no cash-out refinancing. The Federal Reserve mortgage data is available at a quarterly frequency from 1951Q4 to 2008Q4 and at an annual frequency from 1945 to 1951. The simulation data is at a monthly frequency since January 1919.](image)

We now turn to the more interesting case where each home has some positive probability of being cash-out refinanced. In the interest of simplicity, we assume that this probability depends only on the loan-to-value ratio and is constant across all loan-to-value ratios. Within this class of refinancing rules, the following rule is able to match the calibration reference series reasonably well:

\[
\text{REFI}_{i,t} = \begin{cases} 
    0 & \text{if } \text{LTV}_{i,t} = 0 \\
    0.6\% & \text{if } \text{LTV}_{i,t} \in (0\%, 85\%]
\end{cases}
\]

We shall refer to this refinancing rule as the “Uniform Rule”. Figure 5 shows the simulated time series of outstanding mortgage volume and cumulative equity extractions, respectively, based on (3) along with the historical series described in Section 4.2. Clearly this rule is relatively successful in matching the historical data.

However, for both time series, it seems that the simulations run ahead of the historical
Figure 5: Simulated and actual time series of mortgages outstanding and cumulative equity extractions where homes are purchased at an initial loan-to-value ratio of 85% and cash-out refinanced based on the Uniform Rule (3). The Federal Reserve mortgage data is available at a quarterly frequency from 1951Q4 to 2008Q4 and annually from 1945 to 1951. Greenspan and Kennedy’s (2005) equity extraction data is available at a quarterly frequency from 1968Q1 to 2008Q4. The simulated time series are monthly since January 1919.
data. To address this issue, we propose an alternative heuristic motivated by Bennett, Peach, and Peristiani (2001) in which we assume that the probability of refinancing exhibits a structural shift (see footnote 12):

$$\text{REFI}_{i,t} = \begin{cases} 
0 & \text{if } \text{LTV}_{i,t} = 0 \\
0.3\% & \text{if } t \leq 1988 \text{ and } \text{LTV}_{i,t} \in (0\%, 85\%]
0.9\% & \text{if } t \geq 1989 \text{ and } \text{LTV}_{i,t} \in (0\%, 85\%]
\end{cases}$$

The parameters 0.3% and 0.9%—the probability of refinancing before and after the structural break—were selected to be symmetrically spaced around 0.6%, which is the refinancing probability in the Uniform Rule (3). We will refer to this new refinancing rule as the “Uniform Rule with Structural Break”. Figure 6 contains the calibration results for this rule. By adding this simple structural break, the simulated time series matches the historical data much more closely.\(^{35}\)

The depth of data collected by various organizations on the housing market allows us to test some other features of our simulation system. In particular, some data on the distribution of loan-to-value ratios is collected as a part of the American Housing Survey.\(^{36}\) Figure 7 displays this data based on the 2007 survey, and also includes the corresponding simulations based on no cash-out refinancings as well as the refinancing rules (3) and (4). It is apparent that the Uniform Rule with Structural Break is reasonably successful in generating similar loan-to-value ratios to those in this survey. For example, the refinancing rule implies that 35% of the homes in the simulation are owned without any mortgage, which is identical to the 35% reported in the survey. Also, in the simulation, 58% of the homes had loan-to-value ratios of 40% or less, which is again identical to 58% based on the 2007 survey results.

Since the time series used in these calibration are non-stationary, traditional measures such as correlation and \(R^2\) may be misleading indicators of goodness-of-fit. A simpler alternative is to compute the mean of the quarterly absolute deviations between the simulated

\(^{35}\)However, note the discrepancy between the actual and simulated outstanding mortgages in 2008 (see Figure 6a). This difference is likely the result of two factors: (1) the wave of foreclosures in 2008 may have forced lenders to write down significant amounts of principal after foreclosed homes were sold to new buyers at lower market prices, and this is not captured in our simulations; and (2) the supply of credit declined dramatically in 2007 and 2008, but this effect is not reflected in the simulations because of our exclusive focus on the demand side of the mortgage market.

\(^{36}\)See [http://www.census.gov/hhes/www/housing/ahs/nationaldata.html](http://www.census.gov/hhes/www/housing/ahs/nationaldata.html). The relevant data is given in Table 3-15.
Figure 6: Simulated and actual time series of outstanding mortgages and cumulative equity extractions for the simulation where homes are purchased at an initial loan-to-value ratio of 85% and cash-out refinanced based on the Uniform Rule with Structural Break (4). The Federal Reserve mortgage data is available at a quarterly frequency from 1951Q4 to 2008Q4, and annually from 1945 to 1951. Greenspan and Kennedy’s (2005) equity extraction data is available at a quarterly frequency from 1968Q1 to 2008Q4. The simulated time series are monthly since January 1919.
Figure 7: Distribution of simulated loan-to-value ratios (assuming an initial loan-to-value ratio of 85%) in December 2007 vs. the results from the 2007 American Housing Survey, based on the Uniform Rule (3) and the Uniform Rule with Structural Break (4).

and actual series as a percentage of the actual quarterly values:

\[
\text{Mean Absolute Deviation} \equiv \frac{1}{T} \sum_{k=1}^{T} \frac{|\text{Simulated}_t - \text{Actual}_t|}{\text{Actual}_t}.
\]

These mean absolute errors are reported for various sample periods in Table 1, and for the refinancing rule (4), we see that the mean absolute error ranges from 12% for the 1980–2000 sample period to just 5% during the more recent 2000–2008 period.

The values in Table 1 confirm the patterns in Figures 6 and 7, namely that our simulation under the refinancing rule (4) is now properly calibrated to assess the impact of refinancing on the systemic risk of the U.S. residential mortgage market. We shall adopt this specification in our analysis of such risks in Section 5. While this rule implies a uniform probability of cash-out refinancing, other refinancing rules where the probability of a cash-out decision depends on the loan-to-value ratio can also match the calibration reference series relatively well. We have summarized the results for two such rules—where the refinancing probability is linearly decreasing in the loan-to-value ratio—in Appendix A.3. The fact that the empirical implications of these other refinancing rules are similar suggests that our findings are robust to the particular behavioral hypothesis regarding refinancing.\(^{37}\)

\(^{37}\)When calibrated against the other two home-price indexes CSNA-T-HPI\(_t\) and NAR-HPI\(_t\) (see footnote
<table>
<thead>
<tr>
<th>Time Period</th>
<th>No Cash-Out</th>
<th>Uniform Rule</th>
<th>Uniform Rule with Break</th>
<th>Linear Rule</th>
<th>Linear Rule with Break</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean Absolute Deviation of Total Mortgages Outstanding (% of Actual)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980 to 2008</td>
<td>45%</td>
<td>33%</td>
<td>11%</td>
<td>29%</td>
<td>10%</td>
</tr>
<tr>
<td>1990 to 2008</td>
<td>51%</td>
<td>21%</td>
<td>10%</td>
<td>17%</td>
<td>8%</td>
</tr>
<tr>
<td>2000 to 2008</td>
<td>57%</td>
<td>7%</td>
<td>4%</td>
<td>4%</td>
<td>3%</td>
</tr>
<tr>
<td></td>
<td>Mean Absolute Deviation of Total Cumulative Equity Extractions (% of Actual)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980 to 2008</td>
<td>N/A</td>
<td>59%</td>
<td>12%</td>
<td>54%</td>
<td>13%</td>
</tr>
<tr>
<td>1990 to 2008</td>
<td>N/A</td>
<td>26%</td>
<td>7%</td>
<td>22%</td>
<td>9%</td>
</tr>
<tr>
<td>2000 to 2008</td>
<td>N/A</td>
<td>6%</td>
<td>5%</td>
<td>5%</td>
<td>9%</td>
</tr>
</tbody>
</table>

Table 1: Mean absolute deviations of simulated vs. actual time series for total mortgages outstanding and total cumulative equity extractions under five different refinancing behavioral rules: rate/term refinancing (where no equity is extracted), the Uniform Rule (3), the Uniform Rule with Structural Break (4), the Linear Rule (A.5), and the Linear Rule with Structural Break (A.6).

5 Measuring the Refinancing Ratchet Effect

Armed with a properly calibrated simulation of the U.S. residential mortgage market, we can now turn to the main focus of this paper: an assessment of the systemic risk posed by the refinancing ratchet effect. We first consider the impact of equity extractions on loan-to-value ratios in Section 5.1, and then construct a more dynamic measure of risk in Sections 5.2 and 5.3 using a simple derivatives pricing model.

5.1 Aggregate Loan-to-Value Ratios

Figure 8 shows the simulated aggregate value of mortgages outstanding and the market value of the underlying residential real estate for the no-refinancing case and for the cash-out refinancing case based on the Uniform Rule with Structural Break (4). For the purposes of risk assessment, the market value of homes that serve as collateral for outstanding mortgages is most relevant, hence this is the measure we have displayed in these figures. Note that

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27) we do obtain slightly different parameters for the refinancing heuristic but the qualitative properties are the same.
this series is lower in Figure 8(a) because a larger number of mortgages are eventually fully paid under the no-cash-out-refinancing simulation, and all those homes are excluded from this calculation since they are no longer actively used as collateral for any mortgage.

It can be seen that a much smaller aggregate equity remains in the mortgage system in the second case. In the case of no refinancing, the total value of mortgages outstanding in December 2008 is $4,105 billion, which is supported by $10,154 billion of real estate, representing an aggregate loan-to-value ratio of approximately 40%. In the case where cash-out refinancings are possible, the amount of loans outstanding is $12,018 billion and this larger amount of mortgages is supported by $16,570 billion of real estate, for a loan-to-value ratio of 72%. While a 72% aggregate loan-to-value ratio may suggest that even with refinancing, a relatively large level of homeowners’ equity is left in the mortgage system at the end of 2008, the cross-sectional distribution of loan-to-value ratios tells a very different story.

Figure 9 contains the cross-sectional distribution of the loan-to-value ratios at the end of our sample (December 2008) and at the peak of the housing market (June 2006), respectively. The simulation results indicate that with cash-out refinancing, approximately 18% of homes are in negative-equity territory by December 2008, which is remarkably consistent with the actual figure of 20% contained in First American CoreLogic’s Negative Equity Report. The same figure without cash-out refinancing is 3%. Figure 9(b) shows that cash-out equity extractions shift the loan-to-value-ratio distribution to the right during the housing boom, causing a much larger proportion of homeowners to have negative equity as housing prices declined.

### 5.2 Option-Implied Losses

Given our assumption that all mortgages in our simulations are non-recourse loans—collateralized only by the value of the underlying real estate—the homeowner has a guarantee or put option that allows him to put or “sell” the home to the lender at the remaining value of the loan if the value of the home declines below the outstanding mortgage. Naturally, this guarantee is worth less for higher amounts of equity that the homeowner has in the property.

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38The collateral value is lower in the first case because in that scenario, a larger fraction of mortgage loans are fully paid off, in which case the values of the corresponding homes are excluded from the total since they are no longer used as collateral in the mortgage system.

Figure 8: Simulated value of outstanding mortgages vs. the value of the underlying real-estate collateral for no-cash-out refinancing vs. cash-out refinancing using the Uniform Rule with Structural Break (4).
Figure 9: Histograms of simulated loan-to-value ratios for the no-cash-out refinancing and cash-out refinancing (under the Uniform Rule with Structural Break (4)) cases for the end of our sample period, December 2008, and for the peak of the housing market, June 2006, where the initial loan-to-value ratio is set to 85%.
However, by engaging in a cash-out refinancing, the homeowner effectively increases his loan-to-value ratio, thereby increasing the value of the guarantee. Such guarantees can be evaluated using derivatives pricing theory as described in Merton (1977) and Merton and Bodie (1992), and can be applied to quantifying macro-level risks as described in Gray, Merton and Bodie (2006, 2007a, 2007b, 2008) and Gray and Malone (2008).

As mortgages are placed in various structured products like collateralized mortgage obligations and sold and re-sold to banks, asset management firms, or government sponsored enterprises (see Figure 10), the ultimate entities exposed to these guarantees may be masked. In fact, some have blamed the Financial Crisis of 2007–2008 on the complexity of these multi-layered risk-sharing arrangements (see, for example, Gorton, 2008, 2009). However, it is clear that all mortgage lenders must, in aggregate, be holding the guarantees provided to all homeowners.\(^40\) Therefore, we can circumvent the complexities of these intermediate transactions—those in the dotted box of Figure 10—by calculating the aggregate value of the guarantees on the underlying real estate that mortgage lenders have extended to homeowners.

Figure 10: Interlinked balance sheet of entities backed by the underlying real estate, based on Gray, Merton, and Bodie (2008). Intermediate risk redistributions (through, for example, CDOs) will be ignored in our simulations.

\(^{40}\)Of course, to the extent that some owners may be liable for the deficiency in their collateral value through recourse, those owners share some of the burden of the loss caused by a decline in home prices. See footnote 1 and Ghent and Kudlyak (2009) for further discussion.
We measure the value of the guarantee of each mortgage as the value of the put option written on the underlying real estate. Since Merton’s (1977) analysis of deposit insurance, the use of derivatives pricing models to value guarantees has become standard. Such an approach is forward-looking by construction, providing a consistent framework for estimating potential losses based on current market conditions—in particular, the price and volatility of the guaranteed asset—rather than on historical experience. Of course, derivatives pricing models do require additional assumptions, e.g., complete markets and a specific stochastic process (one that is consistent with completeness, such as geometric Brownian motion). We adopt a discrete-time version of these assumptions in (A11):

\[ (A11) \text{Housing-price dynamics can be approximated by a discrete-time geometric random walk represented by a recombining binomial tree, and markets are dynamically complete so options on property values can be priced by no-arbitrage arguments alone.} \]

Whether aggregate home prices follow random walks is debatable, and a number of studies have documented departures from geometric Brownian motion in several financial assets (see Lo and MacKinlay, 1999, and the many references they provide to this burgeoning literature). However, for constructing an initial benchmark for valuing the embedded option in non-recourse mortgages, Assumption (A11) is a natural starting point from which more sophisticated models can be built.

Under (A11), we model the guarantee in non-recourse mortgages as a “Bermuda” put option—an option that can be exercised at certain dates in the future, but only on those fixed dates—and we set these exercise dates to be once a month, just prior to each mortgage payment date. The exercise price is the amount of the outstanding loan, which declines

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41See Cox and Rubinstein (1985) for further discussion of the binomial option-pricing method.

42One immediate extension is to consider price dynamics that reflect the U.S. real-estate “bubble”. However, developing a precise definition of a bubble is not a simple task, and while some studies concluded that real estate prices were too high in 2004–2006 (Shiller, 2006), other studies came to the opposite conclusion (McCarthy and Peach, 2004, and Himmelberg, Mayer and Sinai, 2005). Even ex post, estimating the appropriate “price correction” is not obvious, as Wheaton and Nechayev (2007) illustrate. This lack of consensus underscores the empirical challenges in identifying stable relations between prices and the most obvious fundamentals (in particular, see Gallin, 2004, 2006). But to the extent that a “bubble” refers, instead, to an impending “black swan” or tail event, this case can easily be accommodated by assuming a jump component in the stochastic process of the aggregate home price index, and then using Merton’s (1976) jump-diffusion option-pricing model to price the guarantee. Although this is a simple extension, it is likely to lead to larger loss estimates than Assumption (A11) because of the additional tail risk component, hence we adopt the simpler assumption in the spirit of conservatism.

43We use the Cox-Ross-Rubinstein binomial tree algorithm to price these options, and implement it in Matlab (version 7.2) using the Financial Derivatives Toolbox (Version 4.0) and the functions: \texttt{crrtimespec}, \texttt{crrpriceopt}, \texttt{crrpriceoptjump}, \texttt{crrpriceoptjumpsh}, \texttt{crrpriceoptjumpshsh}, \texttt{crrpriceoptjumpshshsh}, and \texttt{crrpriceoptjumpshshshsh}.
over time due to the monthly mortgage payments.

Before we can implement this option-pricing model, we must determine the volatility of the underlying asset on which the option is written, as well as any “dividend yield” that may affect the value of that asset. In this case, the underlying assets are individual homes, hence we require the volatility of individual home prices, and the yield from housing service flows which can be viewed as a “rental yield”. The observed volatility of national home-price indexes such as the Case-Shiller Index masks much of the volatility of individual home prices since, by construction, an index averages out the idiosyncratic noise in its constituents. However, Calhoun (1996) provides volatility estimates for individual home prices as part of the process of calculating the repeated sales index. Based on these results and the volatility estimates provided by the FHFA,\textsuperscript{44} the annualized volatility of individual homes falls between 6.5% and 10% per year. We will use a volatility of 8% per year in our option-pricing model.\textsuperscript{45}

The estimation of the rental yield for individual homes is challenging. Simply ignoring it is clearly inappropriate because homes do provide considerable service flows to their owners, but these flows must be balanced against maintenance costs and property taxes. Also, the economic value of such service flows is presumably affected by prevailing rental rates, the tax deductibility of mortgage-interest payments, and property taxes. Himmelberg, Mayer, and Sinai (2005) provides an excellent discussion of these considerations, and based on their analysis, we propose a rental yield of 4% for our option-pricing analysis.

Let \( GRT_{i,t} \) be the value of the guarantee calculated using the method and parameters just described for a single home from vintage \( i \) at time \( t \). We can combine these values to find the aggregate value of the guarantees through the following sum:

\[
\text{TOTALGR}_t = \sum_{i=1}^{t} \text{TOTALV}_i \times \text{SURVIV}_{i,t} \times \frac{\text{GRT}_{i,t}}{\text{VALUE}_{i,i}}
\]

where \( \text{TOTALV}_i \) is given by (1). The logic behind (5) is similar to the calculations of Section \texttt{crrsens}, \texttt{crrtree}, \texttt{instoptstock}, \texttt{intenvset}, and \texttt{stockspec}. See \url{http://www.mathworks.com} for documentation and additional details.

\textsuperscript{44}See \url{http://www.fhfa.gov/Default.aspx?Page=87}. The relevant data is given in the sections “Purchase Only Indexes Volatility” and “All-Transactions Indexes Volatility”.

\textsuperscript{45}Note that negative correlation between volatility and prices has been documented in several asset classes (see, for example, Bekaert and Wu, 2000, for the equities case). To the extent that this holds in real-estate markets as well, our volatility parameter—which is an approximate long-term average—is likely to under-estimate the realized volatility during a market downturn, which, in turn, will under-estimate aggregate losses.
3.4. \( \text{TOTALV}_i \) is the combined value of all homes that were either directly purchased or refinanced in period \( i \). To contribute to the value of the aggregate guarantees, a home must have survived until date \( t \), hence we must multiply \( \text{TOTALV}_i \) by \( \text{SURVIV}_{i,t} \). \( \text{GR}_t \) is the value of the guarantee per house, i.e., per \( \text{VALUE}_{i,i} \) dollars of initial investment, hence the multiplicative factor \( \frac{\text{GR}_t}{\text{VALUE}_{i,i}} \) in (5) adjusts for this fact.

Figure 11 shows the simulated time series of \( \text{TOTALGR}_t \) for the cases of no-cash-out vs. cash-out refinancing using Uniform Rule with Structural Break, and Table 2 reports these values for each quarter between 2006Q1 and 2008Q4.

During normal times, homeowners’ equity absorbs the first losses due to a decline in the residential real-estate prices (see Figure 10). However, the process of equity extraction causes the width of this buffer to decrease, resulting in a larger portion of the losses transferred to the equityholders and debtholders of various lending entities (through the complex risk redistribution methods shown in the dotted section of Figure 10). Our simulations show that with the downturn in the value of residential real estate in 2007 and 2008, the value of the guarantees extended to homeowners by mortgage lenders increased substantially. Clearly a loss of $1,543 billion is too large to be absorbed by the equity of these entities alone (for example, based on the data collected by the FDIC, as of December 2007 all U.S. banks had approximately $1,144 billion in equity), creating the need for government intervention to address these losses.\(^{46}\)

Moreover, our loss estimates do not include the impact of residential mortgage defaults on the many related mortgage-backed securities such as collateralized debt obligations (CDOs), credit default swaps (CDSs), and other derivatives contracts based on associated indexes such as the ABX.HE. Although these contracts may be viewed as pure side-bets in the sense that they are in zero net-supply (hence one party’s losses are another party’s profits), the sheer size of the portion of the derivatives industry affected by the U.S. residential real-estate market implies that large losses are likely to have major economic repercussions. Such effects include the distressed unwinding of illiquid assets, defaults and collapses of major financial institutions, and a general loss of confidence in the financial system.\(^{47}\) Accordingly, our estimated loss of $1.5 trillion may be a significant under-estimate of actual losses from the

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\(^{46}\)Simulations calibrated to the other two home-price indexes, CSNAT-HPI, and NAR-HPI, (see footnote 27), generated similar levels of losses: $1.481 billion in the former case, and $1.279 billion in the latter.

\(^{47}\)To develop a sense of the scale of the potential repercussions and how quickly these markets can grow, note that in June 2004 the notional amount of credit derivatives estimated by the Bank for International Settlements was $4.5 trillion, and by June 2007, this figure increased to $51.1 trillion (BIS, 2007).
Figure 11: The simulated value of the guarantee in non-recourse residential mortgages, with and without cash-out refinancing (under the Uniform Rule with Structural Break (4)), and the home price index level from January 1919 to December 2008.

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Total Guarantee Value without Refinancing ($B)</th>
<th>Total Guarantee Value with Cash-Out Refinancing ($B)</th>
<th>Case-Shiller Composite-10 Index Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006Q1</td>
<td>42.21</td>
<td>228.85</td>
<td>223.75</td>
</tr>
<tr>
<td>2006Q2</td>
<td>47.73</td>
<td>258.22</td>
<td>226.29</td>
</tr>
<tr>
<td>2006Q3</td>
<td>59.98</td>
<td>328.09</td>
<td>225.09</td>
</tr>
<tr>
<td>2006Q4</td>
<td>71.04</td>
<td>397.81</td>
<td>222.39</td>
</tr>
<tr>
<td>2007Q1</td>
<td>79.33</td>
<td>451.79</td>
<td>219.67</td>
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<td>2007Q2</td>
<td>89.07</td>
<td>501.58</td>
<td>217.37</td>
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<td>2007Q3</td>
<td>105.73</td>
<td>595.31</td>
<td>212.72</td>
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<td>2007Q4</td>
<td>135.79</td>
<td>766.90</td>
<td>200.67</td>
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<tr>
<td>2008Q1</td>
<td>176.71</td>
<td>991.88</td>
<td>186.12</td>
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<tr>
<td>2008Q2</td>
<td>194.03</td>
<td>1,084.30</td>
<td>180.52</td>
</tr>
<tr>
<td>2008Q3</td>
<td>223.53</td>
<td>1,242.20</td>
<td>173.36</td>
</tr>
<tr>
<td>2008Q4</td>
<td>279.93</td>
<td>1,543.10</td>
<td>162.17</td>
</tr>
</tbody>
</table>

Table 2: The simulated value of the guarantee in non-recourse residential mortgages, with and without cash-out refinancing (under the Uniform Rule with Structural Break (4)), and the Case-Shiller Index, from 2006Q1 to 2008Q4.
refinancing ratchet.

Table 3: The simulated value of the guarantee in non-recourse residential mortgages for various values of rental yields and home-price volatilities, with and without cash-out refinancing (under the Uniform Rule with Structural Break (4)), from 2006Q1 to 2008Q4.

To develop some sense of the robustness of the loss estimates in Table 2, we perform additional simulations for rental yields of 3% and 5%, and home-price volatilities of 6% and 10%. The various combinations are reported in Table 3. These results show that in the fourth quarter of 2008, the loss estimates range from a low of $1,039 billion (3% rental yield, 6% volatility) to a high of $2,073 billion (5% rental yield, 10% volatility). As volatility increases, or as the rental yield increases, ceteris paribus, the embedded guarantee is more valuable, hence the potential losses are greater. While rental yields may be relatively stable over time, it can be argued that home-price volatility is more variable. In particular, as the national home-price index reached its peak in June 2006 and began its decline, home-price volatility is likely to have increased significantly beyond its previous historical levels, another reason that the estimated losses in Table 2 may under-estimate actual losses.
5.3 Option-Implied Sensitivities

If \( \text{TOTALGRT}_t \) is used to measure the potential losses realized after the housing market downturn, then its sensitivity to changes in home prices can serve as a measure of systemic risk. This sensitivity can be easily derived from the embedded option’s “delta” or partial derivative with respect to the price of the underlying asset (see Gray, Merton, and Bodie, 2006). Let \( \text{DLT}_{i,t} \) be the sensitivity of the value of the guarantee, \( \text{GRT}_{i,t} \), to changes in the price of real estate for a single home from vintage \( i \) at time \( t \). This measure can be easily calculated as a part of the Single-Home Simulation step of our analysis discussed in Section 3.3. We can combine these single-home sensitivities using an approach similar to (5) to obtain the aggregate sensitivity, \( \text{TOTALDLT}_t \), as follows:

\[
\text{TOTALDLT}_t = \sum_{i=1}^{t} \text{TOTALV}_i \times \text{SURVIV}_{i,t} \times \frac{\text{DLT}_{i,t}}{\text{VALUE}_{i,i}}
\]  

(6)

\( \text{TOTALDLT}_t \) measures the dollar change in the aggregate value of the guarantees of non-recourse mortgages given an incremental change in the value of the underlying real estate. Higher sensitivities imply greater losses for mortgage lenders for the same incremental decline in real-estate values.

To measure the impact of cash-out refinancing on the aggregate sensitivity \( \text{TOTALDLT}_t \), in Figure 12 we plot the ratio of this sensitivity in the cash-out refinancing case (under the Uniform Rule with Structural Break (4)) to the sensitivity in the case of no-cash-out refinancing. This ratio may be used as a forward-looking measure of systemic risk in the housing market. Figure 12 shows that this ratio increased steadily over the course of the housing boom from the late 1980’s until the peak in mid-2006.\(^{48}\) According to this measure, just prior to that peak in June 2006, cash-out equity extractions increased the magnitude of losses in the event of a decline in real-estate prices by a factor of 5.5. This is consistent with the ratio of the simulated $1,543 billion loss in the case of cash-out refinancings to the simulated $280 billion for no-cash-out refinancing (see Table 2), and well within the range that should have been expected if the magnitude of systemic risk was measured correctly.

Moreover, while the magnitude of this delta-based measure of systemic risk exposure to the housing market reached its highest level in early 2006, its lowest level since 1990 was only slightly below 4. In other words, cash-out equity extractions will greatly multiply

\(^{48}\)Note that the jump from 2 to 4 occurs in 1989 due to the structural break modeled in (4).
the potential losses from any real-estate market downturn, so that even a small decline in housing prices is magnified into much larger losses for mortgage lenders. And after an extended period of rising home prices, falling interest rates, and refinancings, the ratchet effect can create so much irreversible leverage in the housing market that the question is no longer whether large-scale defaults will occur, but rather when they will occur (which amounts to the question of when housing prices will begin to fall). The Financial Crisis of 2007–2008 was an accident waiting to happen.

Figure 12: The ratio of cash-out-refinancing to no-cashout-refinancing aggregate sensitivities of the value of guarantees to changes in the price of real estate, based on quarterly data from 1990Q1 to 2008Q4. The cash-out refinancing simulation is based on the Uniform Rule with Structural Break (4) versus the case of no-cash-out refinancing.

The embedded options in non-recourse mortgages also imply that the magnitude of potential losses to lenders may not be linearly related to housing prices. Because a put option's value is a nonlinear, convex, and decreasing function of its underlying assets price, its risk sensitivity (delta) increases as the underlying assets price decreases. The options “gamma” measures this change in sensitivity, also known as “convexity”. The put option’s convexity
implies that the losses on mortgages experienced by lenders from an initial housing price decline will be substantially smaller than subsequent losses if prices were to decline again by the same dollar amount.

Now add to this feature the additional complexities introduced by securitization, banking regulation, credit ratings, and default insurance, and the implications of nonlinearity for systemic risk become apparent. Convexity can surprise even sophisticated and experienced managers, investors, and regulators who may have developed their market intuition over the past several decades, because they would not have experienced a significant national-level housing market decline during that period, and because the popularity and efficiency of refinancing is a relatively new phenomenon. Surprise—particularly negative surprise—can quickly lead to panic, massive coordinated unwindings, illiquidity and market dislocation, and, ultimately, financial crisis.

6 Discussion

In this section, we present a number of qualifications, extensions, and implications of our simulation of the U.S. residential housing market. In Section 6.1, we contrast the heuristic nature of our simulations to traditional general equilibrium analysis, and acknowledge in Section 6.2 that we have not modeled the behavior of lenders in our simulations. We distinguish between market risk and systemic risk in Section 6.3, and in Section 6.4 we observe that the welfare implications of the recent financial crisis, and the events leading up to it, are not yet fully understood.

6.1 Heuristic vs. General Equilibrium Analysis

Our simulations are based on a number of simplifying assumptions. While we have attempted to err on the side of lower implied losses whenever possible, some assumptions may have the opposite effect, e.g., assuming that all mortgages are non-recourse loans. By incorporating more realistic features of the housing market such as adjustable-rate and negative amortization mortgages with teaser rates, NINJA loans, and regional differences in the U.S. residential real-estate market, bankruptcy laws, homeowner asset and income dynamics, the accuracy of the simulation may be increased.

However, our analysis is not designed to capture feedback effects among all endogenous
variables such as home prices, interest rates, household income, and borrowing and lending behavior. Therefore, standard comparative-statics questions such as “how much would home prices have risen if the Fed did not cut interest rates from 2000 to 2003” are not addressed in our simulations. Instead, our narrower “reduced-form” focus has been to gauge the magnitude of the refinancing ratchet effect on mortgage lenders. A more formal general-equilibrium analysis of these markets would begin with optimizing households from which the demand for housing and mortgages are derived, aggregated, and equilibrated with optimizing builders and lenders that supply the homes and mortgages, respectively, to households. While computable general-equilibrium models have become considerably more sophisticated in recent years (see, for example, Dixon and Rimmer, 2002), the dynamic and stochastic nature of the demand and supply decisions are sufficiently complex even for a single agent that constructing a true stochastic dynamic general equilibrium model of the entire U.S. housing market seems computationally impractical. Nevertheless, some useful insights may be gleaned from considering special cases of such optimizing behavior and equilibrium, e.g., Pliska (2006), Fortin et al. (2007), and Agarwal, Driscoll, and Laibson (2008), and may be worth pursuing.

6.2 Lending Behavior

Any analysis of the Financial Crisis of 2007–2008 would be incomplete without some discussion of the behavior of mortgage lenders and associated businesses. Our simulations assume that all household demand for mortgages and refinancing is satisfied at prevailing historical rates, i.e., the supply of funds to borrowers is infinitely elastic at all times. While this may have been a reasonable approximation to reality during the decade prior to the peak of the housing market in 2006, we were motivated by the objective to isolate and measure the impact of the refinancing ratchet effect by itself. However, supply shocks certainly must have had an impact on systemic risk in recent years as well. Therefore, an important open question is how lenders behaved during the course of our simulations, and what economic or behavioral forces led them to engage in such behavior?

A tractable and empirically plausible model of lending behavior is beyond the scope of our current simulation, and deserves a separation set of simulation studies in its own right (one possible starting point is Thurner, Farmer, and Geanakoplos, 2009). However, it is not difficult to speculate about the factors those simulations might include. In addition to modeling the behavior of banks, which are the traditional sources of home loans, such
a simulation must also account for a host of financial innovations that have emerged only recently, including securitized debt (e.g., CDOs and CDO-squareds), credit default swaps and related insurance products, Internet-based marketing of consumer-finance products, the growth of the “shadow banking industry” and illiquidity, and the globalization of financial markets. Chan et al. (2006), Rajan (2006), Khandani and Lo (2007, 2008), Gorton (2008, 2009), Brunnermeier (2009), and Gorton and Metrick (2009) provide overviews of some of these developments. In addition, these simulations must incorporate the impact of rating agencies, government sponsored enterprises, and broader government policies in promoting cheap financing for would-be homeowners, as well as the increasing competition for yield among asset-managers and asset-owners. Collectively, these developments contributed to the enormous supply of funds available to homeowners during the past decade, but further analysis is needed before we can determine the relative importance of each.

However, the challenge in constructing a simulation with all of these features is the fact that there is precious little history on which to calibrate many of the parameters. In contrast to typical simulations that assume a statistically stationary environment, simulating the supply of funds for residential real-estate purchases involves the historically unique financial innovations described above. This may provide a clue as to the magnitude of the current crisis, as well as its apparent uniqueness in recent history.

More importantly, the main thrust of our analysis is that the refinancing ratchet effect is a wholly separate mechanism that operates irrespective of the supply of credit, and must be considered a potential source of systemic risk in its own right.

6.3 Market Risk vs. Systemic Risk

While the $1.5 trillion figure seems imposing, large financial losses do not necessarily imply significant systemic risk. For example, on April 14, 2000, the CRSP value-weighted stock market index (excluding dividends) declined by 6.63%, implying an aggregate one-day loss of approximately $1.04 trillion to corporate America. While certainly unfortunate, this event was not particularly memorable, nor was it a cause for national alarm or emergency government intervention. Market risk is distinct from systemic risk; the latter arises when large financial losses affect important economic entities that are unprepared for and unable to withstand such losses, causing a cascade of failures and widespread loss of confidence. This element of surprise lies at the heart of the current financial crisis. The three conditions for
the refinancing ratchet effect—rising house prices, falling interest rates, and easy access to refinancing opportunities—are individually innocuous, and often viewed positively as signs of economic growth and prosperity. But when all three occur at the same time, aggregate homeowner leverage can increase rapidly, and may even contribute to upward pressure on home prices. Without any “safety valve” to release this pressure gradually, the result when home prices inevitably decline can be explosive, as we witnessed in 2007–2008.

6.4 Welfare Implications

Although much has already been written about the Financial Crisis of 2007–2008, its welfare implications for homeowners, lenders, and intermediaries are not yet fully understood. While many homeowners have been adversely affected by higher interest rates, foreclosures, and falling property values, there are other satisfied and solvent homeowners who are homeowners only because of the business practices, government policies, and economic circumstances that contributed to the refinancing ratchet. Eliminating or otherwise restricting these business practices and policies may benefit some groups, but will no doubt disadvantage others. Moreover, as discussed above, we have not attempted to model the supply side of the refinancing industry, which no doubt contributed to the growth of home prices, leverage ratios, and systemic risk. Many have criticized the role of securitization, insurance, and financial innovation in creating the crisis, but during the decade leading up to the peak of the housing market in 2006, these developments were responsible for the low-interest-rate and easy-credit environment that was so conducive to global economic growth and the “ownership society”. Any policy recommendations with respect to the crisis must balance these myriad trade-offs between individual and institutional stakeholders.

7 Conclusions

During periods of rising house prices, falling interest rates, and increasingly competitive and efficient refinancing markets, cash-out refinancing is like a ratchet, incrementally increasing homeowner leverage as real-estate values appreciate without the ability to symmetrically decrease leverage by increments as real-estate values decline. Using a numerical simulation calibrated to the basic time-series properties of U.S. residential housing market, we show that this ratchet effect is capable of generating the magnitude of losses suffered by mortgage
lenders during the Financial Crisis of 2007–2008. During normal times, and in the absence of cash-out refi-\nnancings, the cross-sectional distribution of leverage among homeowners is relatively heterogeneous, with newer homeowners more highly leveraged than those who have had the opportunity to build up more equity. Heterogeneity of leverage in the cross section implies less correlated defaults among bor\nrowers, and lower systemic risk.

However, during periods of falling interest rates and rising house prices, most homeowners will have an incentive to refinance. If the refinancing market is so competitive and efficient that homeowners refinance frequently, this pattern of behavior has a similar effect on systemic risk as if these homeowners all purchased their homes at the same time, at peak prices, with newly issued mortgages at the highest allowable loan-to-value ratios. A coordinated increase in leverage among homeowners during good times will lead to sharply higher correlations in defaults among those same homeowners in bad times. Our simulations show that this effect alone is enough to generate $1.5 trillion in losses for mortgage-lending institutions since June 2006.

These observations have important implications for risk management practices and reg-\nulatory reform. The fact that the refinancing ratchet effect arises only when three market conditions are simultaneously satisfied demonstrates that the current financial crisis is subtle, and may not be attributable to a single cause. Moreover, a number of the activities that gave rise to these three conditions are likely to be ones that we would not want to sharply curtail or outright ban because they are individually beneficial. While excessive risk-taking, overly aggressive lending practices, pro-cyclical regulations, and political pressures surely contributed to the recent problems in the U.S. housing market, our simulations show that even if all homeowners, lenders, investors, insurers, rating agencies, regulators, and policymakers behaved rationally, ethically, and with the purest of motives, financial crises can still occur. Therefore, we must acknowledge the possibility that no easy legislative or regulatory solutions exist such as prohibiting the Fed from cutting interest rates below a certain threshold, or placing a ceiling on housing prices, or putting “sand in the gears” of the refinancing system and limiting consumer credit. Successfully managing systemic will require flexible, creative, and well-trained professionals that understand the fundamental drivers of such risk, not static rules meant to prevent history from repeating.

The complexities illustrated by our simulation underscores the need for an independent organization devoted solely to the study, measurement, and public notification of systemic
risk, not unlike the role that the National Transportation Safety Board plays with respect to airplane crashes, train wrecks, and highway accidents (see Getmansky, Lo, and Mei, 2004). Currently, no single regulatory body is responsible for monitoring the three refinancing-ratchet conditions; in fact, on occasion, each of these conditions has been associated with the policy objectives of one part of government or another. Therefore, it is difficult to imagine any existing regulatory agency raising red flags over any of these conditions. For example, in response to the bursting of the Tech Bubble in 2000 and the threat of recession, the U.S. Federal Reserve systematically lowered its Federal Funds target rate from 6.50% in May 2000 to 1.00% in June 2003. During this period, the Fed could hardly have been expected to sound the alarm regarding the refinancing ratchet, despite the fact that quarterly cash-out refinancing activity more than tripled over this period (see Figure 1). Similarly, lower interest rates and new mortgage products allowed more households to purchase homes that were previously unaffordable; rising home prices generated handsome wealth gains for those households; and greater refinancing opportunities allowed those households to liquify their gains, fueling consumer demand and general economic growth. Which politician or regulator would seek to interrupt such a “virtuous” cycle, and how could such a maverick accomplish this task without broad-based support from colleagues and constituents?

The subtle and multi-faceted nature of the refinancing ratchet effect is just one example of the much broader challenge of defining, measuring, and managing systemic risk in the financial system. As Reinhart and Rogoff (2008a,b) have documented, financial crises occur on a regular basis throughout the world, and is often tied to economic growth, capital inflows, and financial liberalization and innovation. In the current crisis, systemic risk has taken the form of three factors affecting the housing market; in the next crisis, it may be other combinations of factors that create vulnerabilities to the global financial system. The very nature of crisis is that the underlying drivers are difficult to spot in advance. Establishing an independent institution dedicated to systemic risk analysis—with the capabilities and mandate to construct a variety of simulations and risk measures across instruments, markets, institutions, industries, and economic activities to produce early warning signals of over-leverage, illiquidity, and potential market dislocations—can provide invaluable support to all regulatory bodies that are collectively responsible for managing systemic risk. We believe that systemic risk can and should be managed (see, for example, Lo, 2008), but it can only be managed if it is first properly measured and anticipated. Institutionalizing the ongoing
effort to construct simulations such as those in this paper can contribute to that effort.
A Appendix

In this Appendix, we describe the components of Greenspan and Kennedy’s (2005) gross equity extraction time series used in our analysis (Section A.1), our statistical approach to estimating home sales from 1919 to 1962 (Section A.2), and provide calibration results for additional refinancing rules (Section A.3).

A.1 Components of Gross Equity Extractions Series

Greenspan and Kennedy (2005) propose a method for disaggregating the net change in outstanding home mortgage debt into its constituent gross flows. For our purposes, the most important series produced by their approach is the Gross Equity Extractions series, and for completeness, we provide a brief overview of this series in this section. Please see Greenspan and Kennedy (2005) for further details.

Greenspan and Kennedy (2005) define Gross Equity Extractions as “extraction of equity on existing homes as the discretionary initiatives of home owners to convert equity in their homes into cash by borrowing in the home mortgage market”. To calculate Gross Equity Extractions, they hypothesize that the change in home mortgage debt outstanding in the absence of discretionary initiative would have been equal to the mortgage origination to purchase new homes minus the scheduled amortization. Accordingly, they define Gross Equity Extractions as the difference between the actual change in total home mortgage outstanding and this quantity. More precisely, they use the following relationship:

\[
\text{Gross Equity Extractions} \equiv \text{Change in home mortgage outstanding excluding construction loans} - \text{Origination for new homes} + \text{Scheduled amortization}.
\]

Using a variety of sources, they are able to estimate this quantity at a quarterly frequency since 1968Q1. This series is one of the two primary reference time series that we use to calibrate our simulations.

With a more detailed set of data sources that are only available since 1991, Greenspan and Kennedy (2005) further decompose Gross Equity Extractions into the following three components from 1991Q1 to 2008Q4:

\[
\begin{align*}
\text{Turnover Extractions} & \equiv \text{Origination to purchase existing homes} - \text{Cancellation of home sellers mortgage} \\
\text{Gross Cash-Out} & \equiv \text{Origination for refinancing} - \text{Cancellation of refinanced loans} \\
\text{Net Change in Home Equity Loans} & \equiv \text{Change in home equity loans outstanding} - \text{Unscheduled repayments}.
\end{align*}
\]

51
Figure A.1 plots these three components at a quarterly frequency from 1991Q1 to 2008Q4.

Figure A.1: Three component of the Gross Equity Extractions series of Greenspan and Kennedy (2005) from 1991Q1 to 2008Q4. The three components are: “Turnover Extractions” (origination to purchase existing homes minus cancellation of home sellers mortgage), “Gross Cash-Out” (defined as origination for refinancing minus cancellation of refinanced loans), and “Net Change in Home Equity Loans” (defined as change in home equity loans outstanding minus unscheduled repayments). See Greenspan and Kennedy (2005) for further details.

Our choice to calibrate our simulations to the Gross Equity Extractions series rather than to one of the three subcomponents is motivated by our focus on systemic risk. In particular, our refinancing behavioral rules (3) and (4) are meant to capture the aggregate effects of all three components of the Gross Equity Extractions series. As we discussed in Section 3.2 in reference to Assumption (A9), this broader focus is more relevant for aggregate risk measurement because all three components contribute to the total leverage of the residential housing market. In particular, in the example of Section 3.2 in which a homeowner decides to sell his home and rent thereafter, he would fall into the “Turnover Extraction” case but the buyer of his home presumably finances the purchase with a similarly leveraged loan, yielding virtually the same impact on aggregate leverage as if the original homeowner continued owning after engaging in a cash-out refinancing (place him in the “Gross Cash-Out” category). Therefore, for our purposes, combining the three disaggregated series of Greenspan and Kennedy (2005) seems more appropriate.
A.2 Estimating Home Sales from 1919 to 1962

New home sales data is only available from January 1963 to December 2008. Since outstanding loans and equity extractions depend on the value of the underlying stock of real estate, we would like to extend our analysis farther back to better approximate the current stock of real estate. We select 1919 as a starting point based on data from the American Housing Survey 2007, which shows that approximately 93% of all homes surveyed were built after 1919. We now describe the statistical model used to “back-fill” the new home sales time series from January 1919 to December 1962.

We begin by hypothesizing that the growth rate in NH$_t$ is related to the growth in population (see Section 4.1 for a definition of this variable). Higher values of NH$_t$ also seem to be correlated with periods of high real home-price appreciation, such as the earlier part of this decade. Accordingly, we first collect data for annual new home sales, population, and real home prices. The population data are obtained from two sources: data from 1900 to 1999 are obtained from

http://www.census.gov/popest/archives/1990s/popclockest.txt

and data from 2000 to 2008 are obtained from


The Real Home Price Index is obtained from Robert Shiller at


We then transform each of these series into growth rates by taking the first difference of the natural logarithms of the original time series, i.e.,

\[
\Delta \text{NH}_t = \log(\text{NH}_{t+1}) - \log(\text{NH}_t) \quad \text{for } t \in \{1963, \ldots, 2008\} \\
\Delta \text{POP}_t = \log(\text{POP}_{t+1}) - \log(\text{POP}_t) \quad \text{for } t \in \{1919, \ldots, 2008\} \\
\Delta \text{HPI}_t = \log(\text{HPI}_{t+1}) - \log(\text{HPI}_t) \quad \text{for } t \in \{1919, \ldots, 2008\}
\]

We then estimate the following linear model:

\[
\Delta \text{NH}_t = \alpha + \beta \Delta \text{POP}_t + \gamma \Delta \text{HPI}_t. \quad (A.1)
\]

The estimated parameters, $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ and the data for $\Delta \text{POP}_t$ and $\Delta \text{HPI}_t$ from 1919 to 1962 are used to construct left-hand-side variable $\Delta \text{NH}_t$ for this period. Finally, we use
and \( \hat{\Delta}N_H_t \) to backfill \( N_H_t \) from 1919 to 1962 through the following calculation:

\[
\log(\hat{N}_H_t) = \log(N_H_{1963}) - \sum_{i=t}^{1962} \hat{\Delta}N_H_i \quad \text{for} \quad t \in \{1919, \ldots, 1962\}. \quad (A.2)
\]

In the final step, we use the empirical distribution of monthly new home sales estimated using monthly data from 1963 to 2008 to construct monthly estimates based on the annual estimates of \( \hat{N}_H_t \) constructed using the method above. Given \( N_{H_{i,t}} \), i.e., the sales in month \( i \) of year \( t \) for years 1963 to 2008, we estimate the proportion of houses constructed in month \( i \) using the following estimator:

\[
P_i = \frac{1}{2008 - 1963 + 1} \sum_{t=1963}^{2008} \frac{N_{H_{i,t}}}{N_H_t}
\]

(A.3)

Combining (A.2) and (A.3), we set the number of new houses entering the mortgage system in month \( i \) of year \( t \) over the period from 1919 to 1962 to be:

\[
\hat{N}_{H_{i,t}} = \hat{N}_H_t \times P_i \quad \text{for} \quad t \in \{1919, \ldots, 1962\}. \quad (A.4)
\]

Figure A.2 shows the actual and estimated time series of the number of new units entering the mortgage system from January 1919 to December 2008. This approach implies that 101.5MM units enter our simulation. Of this total, 52.6MM is based on actual data from January 1963 to December 2008, and 48.9MM is based on the estimation approach outlined above for the period from January 1919 to December 1962. The total of 101.5MM seems reasonable given our objective of capturing more than 90% of the homes in the U.S.

**A.3 Calibrating Other Refinancing Rules**

In this section, we provide the calibration results for two alternative refinancing rules. Recall that in Section 4.3, the refinancing probability was assumed to be uniform for all loan-to-value ratios. Here we change this assumption so that the refinancing probability is linear in the loan-to-value ratio. We also consider the case of a structural shift after 1989. Specifically, we propose:

**Linear Rule:**

\[
\text{REFI}_{i,t} = \begin{cases} 
0 & \text{if} \; \text{LTV}_{i,t} = 0 \\
0.9\% \times \frac{85\% - \text{LTV}_{i,t}}{85\%} & \text{if} \; \text{LTV}_{i,t} \in (0\%, 85\%]
\end{cases}
\]

(A.5)
Figure A.2: Time series of the number of units of new homes entering the mortgage system since January 1919. This series is used as an input to our simulations. The data since January 1963 is available from the U.S. Census Bureau, and we have extrapolated the data back to 1919 based on a linear-regression model using population growth and a real-estate price index as regressors.

**Linear Rule with Structural Break:**

\[
\text{REFI}_{i,t} = \begin{cases} 
0 & \text{if } \text{LTV}_{i,t} = 0 \\
0.45\% \times \frac{85\% - \text{LTV}_{i,t}}{85\%} & \text{if } t \leq 1988 \text{ and } \text{LTV}_{i,t} \in (0\%, 85\%] \\
1.35\% \times \frac{85\% - \text{LTV}_{i,t}}{85\%} & \text{if } t \geq 1989 \text{ and } \text{LTV}_{i,t} \in (0\%, 85\%] 
\end{cases}
\] (A.6)

Figures A.3 and A.4 show the calibration results for these two rules. In both cases, the simulations can reproduce the outstanding mortgage volume and the cumulative equity extractions with reasonable accuracy. The Linear Rule with Structural Break given in (A.6) seem to match the earlier parts of the calibration reference series more closely.

Figure A.5 compares the simulation results based on the refinancing rules (A.5) and (A.6) with the results of the 2007 American Housing Survey. The refinancing levels generated by (A.6) comes relatively close to the survey data. This is similar to the data in Figure 7 based on the Uniform Rule (3) and the Uniform Rule with Structural Break (4).

Table A.1 reports the estimated values of losses according to the approach described in Section 5.2 and equation (5) at a quarterly frequency each quarter between 2006Q1 and 2008Q4 for the case of no cash-out refinancing as well as the Linear Rule with Structural Break (A.6). This is the counterpart to the results in Table 2 for the Uniform Rule with Structural Break (4). While these estimates are somewhat smaller than the values given in Table 2, the qualitative patterns, and the increase in the magnitude of losses due to equity
Table A.1: The simulated value of the guarantee in non-recourse residential mortgages, with and without cash-out refinancing (under the Linear Rule with Structural Break (A.6)), and the Case-Shiller Index, from 2006Q1 to 2008Q4.

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Total Guarantee Value without Refinancing ($B)</th>
<th>Total Guarantee Value with Cash-Out Refinancing ($B)</th>
<th>Case-Shiller Composite-10 Index Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006Q1</td>
<td>42.21</td>
<td>203.96</td>
<td>223.75</td>
</tr>
<tr>
<td>2006Q2</td>
<td>47.73</td>
<td>230.53</td>
<td>226.29</td>
</tr>
<tr>
<td>2006Q3</td>
<td>59.98</td>
<td>293.36</td>
<td>225.09</td>
</tr>
<tr>
<td>2006Q4</td>
<td>71.04</td>
<td>356.34</td>
<td>222.39</td>
</tr>
<tr>
<td>2007Q1</td>
<td>79.33</td>
<td>404.47</td>
<td>219.67</td>
</tr>
<tr>
<td>2007Q2</td>
<td>89.07</td>
<td>449.70</td>
<td>217.37</td>
</tr>
<tr>
<td>2007Q3</td>
<td>105.73</td>
<td>535.20</td>
<td>212.72</td>
</tr>
<tr>
<td>2007Q4</td>
<td>135.79</td>
<td>691.85</td>
<td>200.67</td>
</tr>
<tr>
<td>2008Q1</td>
<td>176.71</td>
<td>898.16</td>
<td>186.12</td>
</tr>
<tr>
<td>2008Q2</td>
<td>194.03</td>
<td>982.19</td>
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<tr>
<td>2008Q3</td>
<td>223.53</td>
<td>1,126.90</td>
<td>173.36</td>
</tr>
<tr>
<td>2008Q4</td>
<td>279.93</td>
<td>1,404.50</td>
<td>162.17</td>
</tr>
</tbody>
</table>

extractions, are comparable.
Figure A.3: Simulated vs. actual outstanding mortgages and cumulative equity extractions, where homes are purchased at an initial loan-to-value ratio of 85% and cash-out refinanced based on the Linear Rule (A.5). The Federal Reserve mortgage data is available at a quarterly frequency from 1951Q4 to 2008Q4, and annually from 1945 to 1951. Greenspan and Kennedy’s (2005) cumulative equity extraction data is available at a quarterly frequency from 1968Q1 to 2008Q4. The simulated time series is monthly, from January 1919.
Figure A.4: Simulated vs. actual outstanding mortgages and cumulative equity extractions, where homes are purchased at an initial loan-to-value ratio of 85% and cash-out refinanced based on the Linear Rule with Structural Break (A.6). The Federal Reserve mortgage data is available at a quarterly frequency from 1951Q4 to 2008Q4, and annually from 1945 to 1951. Greenspan and Kennedy’s (2005) cumulative equity extraction data is available at a quarterly frequency from 1968Q1 to 2008Q4. The simulated time series is monthly, from January 1919.
Figure A.5: Distribution of simulated loan-to-value ratios (assuming an initial loan-to-value ratio of 85%) in December 2007 vs. the results from the 2007 American Housing Survey, based on the Linear Rule for refinancing behavior (A.6) and the Linear Rule with Structural Break (A.6).
References


