Single Color-Octet Scalar Production at the LHC

: Factorization and Resummation


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Motivation

- **Scalar Sector in Standard Model (SM)**
  - All the matter fields discovered so far are fermions.
  - Hidden local symmetry in Nature requires Spontaneous Symmetry Breaking Mechanism
    - Usually the potential is expressed in terms of scalar fields.
  - The Scalar Sector has not been tested experimentally.
  - **Why not colored scalar particles?**

\[
V(\phi) = \frac{\lambda}{4} \left( \frac{\phi^2}{\lambda} - \frac{m^2}{\lambda} \right)^2
\]
Various NP Models predicting Color-Octet Scalar

- Color-octet scalar fields arose in GUTs.

<table>
<thead>
<tr>
<th>Model</th>
<th>SU(5) Adjoint Perez et al., 2008</th>
<th>SU(5)-Type II Seesaw Dorsner and Mocioiu, 2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper mass bound</td>
<td>440 TeV</td>
<td>250 TeV</td>
</tr>
</tbody>
</table>

- Compared to GUT scale, surprisingly light Color-Octet Scalars

- SUSY
  - Scalar gluon (MRSSM) Plehn and Tait, 2008

- Hidden color symmetry breaking

  \[ SU(3)_1 \times SU(3)_2 \rightarrow SU(3)_C \]

  - Chiral Color Model Frampton and Glashow, 1987
  - Top Color Hill, 1991

- Leptoquark Model Popov et al., 2005, 2007
  - based on Pati-Salam Unification

  \[ SU(4)_V \times SU(2)_L \times U(1) \rightarrow SU(3)_c \times SU(2)_L \times U(1) \]
**Manohar-Wise Model**  
*Manohar and Wise, 2006*

- Generic NP model satisfying the constraints of MFV

  ❖ **Minimal Flavor Violation (MFV) Hypothesis**

  If we expect new physics at a few TeV scale,
  - Generic flavor-violating interactions (Ex: FCNC) at a few TeV scale are not supported by experiments.
  - SM Yukawa couplings should be only sources for quark flavor symmetry breaking.

\[
\lambda_{ij}^U, D \propto g_{ij}^U, D
\]

- Allowed 
  \[\text{SU}(3)_c \times \text{SU}(2)_w \times \text{U}(1)_Y\]

  representations for scalar fields with Yukawa couplings to SM fermions consistent with MFV are

  1) \((1, 2)_{1/2}\) : SM Higgs doublet
  2) \((8, 2)_{1/2}\) : Another possible representation in quark Yukawa sector.

- Yukawa Sector for Color-Octet Scalar

\[
\mathcal{L}_{\text{Yukawa}} = -\eta_U g_{ij}^U \bar{U}_{Ri} S^i Q_{Lj} - \eta_D g_{ij}^D \bar{D}_{Ri} \tilde{S}^i Q_{Lj} + \text{H.C.}, \quad S = T^A S^A
\]

The same coupling as SM
- **Gluon fusion process gives a dominant contribution** \((\sqrt{s} = 14 \text{ TeV})\)
  - Single Production: loop induced processes, depends on new physics parameters
  - Pair Production: Tree level process, Model-independent

- **QCD Lagrangian for color-octet scalar fields**

\[
\mathcal{L}_{\text{QCD},S} = -S^a (D^2)^{ab} S^b - m_S^2 S^a S^a
\]
• **Constraints from Tevatron experiments** \( (\sqrt{s} = 1.96 \, \text{TeV}) \) are weak.
  
  - Quark-antiquark initial state contributions are dominant.
  
  - From the comparison of measurement of 4b-jets, \( m_S \geq 200 \, \text{GeV} \) suggested.
    (Under the assumption \( S^0 \rightarrow b\bar{b} \) is a dominant decay channel)

• **As mass increases, the single production becomes dominant.**

![Graph showing the cross-section (\( \sigma \)) for gg \( \rightarrow SX \) vs. mass (\( m_S \))](image)

- \( m_S < 1 \, \text{TeV} \)
  - Pair production > Single production

- \( m_S > 1 \, \text{TeV} \)
  - Pair production < Single production

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Gresham and Wise, 2007
Factorization for the single production

- **Kinematics**
  - Approximation to the partonic threshold region \( z = \frac{m_s^2}{\hat{s}} \to 1 \)
  - Gluon PDF is dominant in small x region \( \tau = \frac{m_s^2}{s} \ll 1 \).
  - Color-octet scalar field can be described as a heavy field to do only soft interactions \( \sim m_s(1 - z) \).

- **Three Separate Scales**
  
  \[
  \begin{align*}
  \mu_H & \sim m_S \\
  \mu_S & \sim m_S (1 - z) \\
  \Lambda_{QCD} & \sim m_P
  \end{align*}
  \]

  Virtual \( \to \) Real & Virtual \( \to \) PDFs
- Description of the factorization theorem

\[ \sigma(pp \to SX) \sim \tau H(m_S, \mu_F) \ S(m_S(1-z), \mu_F) \otimes F\left(\frac{\tau}{z}, \mu_F\right) \]

- Resummation of large logarithms such as \( \ln \left[ \frac{\mu_H}{\mu_S} \right] \) is necessary.
• Factorization formula can be applicable to any other new physics model

\[ = C(m_S, \mu) \otimes \]

- Long distance strong interactions like collinear or soft interactions.

All the information on short distance interactions including EW/new physics interactions.

• Precise Results enable us to extract correct information on unknown parameters.
  - New physics parameters
  - Better mass bounds if we do not see
Large K-factor for the single production

- Production Mechanism is similar to Higgs
  - Factorization formalism is similar
  - A difference is a strong interaction of color-octet scalar particle.

- Strong Corrections to Higgs production are Huge

Catani et. al, 2003
• Where do large color corrections come from?
  
  - Large log resummed effects at the threshold region
  \[ \text{Ex}) \alpha_s \frac{\ln(1-z)}{1-z} ..., \]
  
  ※ As mass increases, the corrections become larger.
  
  - \( \pi^2 \)-enhancement in timelike process
  \[ \text{Ex}) C^{(i)}(Q, \mu) \sim -\alpha_s \ln^2 \frac{\mu^2}{-Q^2 - i\varepsilon} \sim \alpha_s \left(-\ln^2 \frac{\mu^2}{Q^2} + \pi^2\right) \]
  
  - Higgs production: \( \pi^2 \)-enhancement contribution is dominant.  
  Ahrens, Becher, Neubert, and Yang, 2008
  
  - Color-Octet Production: more complicated.
    ※ Timelike and spacelike processes are mixed.
    ※ Resummation effects are larger than Higgs \((m_S \gg m_H)\)
**SCET Operator for Color-Octet Production**

- **Collinear gauge-invariant building block**
  - Describes two incoming gluons from protons

\[
\mathcal{B}^a_{n\perp} = i\bar{n}^\rho g_{\perp}^{\mu\nu} G^{b}_{n,\rho\nu} W^b_n, \mathcal{B}^a_{\vec{n}\perp} = i\bar{n}^\rho g_{\perp}^{\mu\nu} G^{b}_{\vec{n},\rho\nu} W^b_{\vec{n}}. 
\]

※ Lightcone vectors
\[n^2 = \vec{n}^2 = 0, n \cdot \vec{n} = 2.\]

※ Gluon PDF in SCET →

\[
f_{g/p}(x) = \frac{1}{x(n \cdot P)^2} \langle p_n | \mathcal{B}^{a,\mu}_{n\perp} \delta \left( x - \frac{\vec{n} \cdot P}{n \cdot P} \right) B^a_{n\perp\mu} | p_n \rangle
\]

**Bauer, Fleming, Pirjol, Rothstein, and Stewart, 2002**

- **Heavy Scalar Effective Theory**: analogous to HQET
  - After integrating out heavy mass, describes soft interactions.

\[
\mathcal{L}_{\text{HSET}} = S^{a*}_v \left( v \cdot iD^a_s \right) S^c_v - \frac{1}{2m_s} S^{a*}_v \left( D^2_s \right)^{ac}_v S^c_v,
\]

\[
S^a_v(x) = \frac{1}{\sqrt{2m_s}} \left( e^{-im_s v \cdot x} S^a_v + e^{+im_s v \cdot x} S^{a*}_v \right)
\]
• Decoupling soft-interactions

\[ \mathcal{B}_{n\perp}^{a,\mu} \rightarrow \mathcal{Y}_{n}^{ab} \mathcal{B}_{n\perp}^{b,\mu}, \quad \mathcal{B}_{n\perp}^{a,\mu} \rightarrow \mathcal{Y}_{n}^{ab} \mathcal{B}_{n\perp}^{b,\mu}, \quad S_{v}^{a} \rightarrow \mathcal{Y}_{v}^{ab} S_{v}^{b} \]

- Soft Wilson lines

\[ \mathcal{Y}_{k}(x) = \text{P} \exp\left(ig \int_{-\infty}^{x} dsk \cdot A_{S}^{a}(sk^{\mu})t^{a}\right), \quad k = v, n, \bar{n}. \]

Bauer, Pirjol, and Stewart, 2001

• Structure of SCET Operators

Soft gluon Wilson lines

\[ \mathcal{B}_{n\perp} \quad \text{\quad} \mathcal{S}_{v}^{*} \quad \times \quad \left(\mathcal{Y}_{n}, \mathcal{Y}_{\bar{n}}, \mathcal{Y}_{v}\right) \]
• Effective Theory Lagrangian for single color-octet production

\[ \mathcal{L}_{\text{EFT}} = \frac{1}{m_s} \left( C_f (\mu) O_f (\mu) + C_d (\mu) O_d (\mu) \right) + O \left( \frac{1}{m_s} \right) \]

- **Generic SCET Operators**

\[ O_{(f,d)} = \frac{\left( i f^{abc}, d^{abc} \right)}{\sqrt{2 m_s}} \left( Y \gamma_5 S_v \right)^a \left( Y \gamma_\mu B_{\mu \perp} \right)^b \left( Y n B_{n \mu} \right)^c. \]

- In case of Pseudoscalar color-octet

\[ O^p_{(f,d)} = \frac{\left( i f^{abc}, d^{abc} \right)}{\sqrt{2 m_s}} \varepsilon^{\mu \nu \rho \sigma} \left( Y \gamma_5 S_v \right)^a \left( Y \gamma_\mu B_{\mu \perp} \right)^b \left( Y n B_{n \mu} \right)^c, \]

\[ \varepsilon_{\mu \nu} = \frac{1}{2} \varepsilon_{\mu \nu \rho \sigma} n^\rho \bar{n}^\sigma. \]

✿ Coloron (Axigluon)

\[ O^\chi_{(f,d)} = \frac{\left( i f^{abc}, d^{abc} \right)}{\sqrt{2 m_s}} \varepsilon^{\mu \nu \rho \sigma} \left( Y \gamma_5 A_{X_v} \right)^a \left( Y \gamma_\mu B_{\mu \perp} \right)^b \left( Y n B_{n \mu} \right)^c \]

- Factorization and Resummation formulae are independent of the Lorentz Structure.
- There is no mixing between \( O_f \) and \( O_d \).
**Factorization Theorem**

\[
\sigma(pp \to SX) = \tau \sum_{i=f,d} H_i(m_S, \mu_F) \int_{1}^{Z} \frac{dz}{z} S_i(m_S(1-z), \mu_F) F\left(\frac{\tau}{Z}, \mu_F\right)
\]

\[
H_{(f,d)}(m_S, \mu) = \frac{\pi}{m_S^2} \left(\frac{5}{16} - \frac{3}{48}\right) |C_{(f,d)}(m_S, \mu)|^2, F(x, \mu_F) = \int_{x}^{1} \frac{dy}{y} f_{g/p}(y, \mu_F) f_{g/p}(\frac{x}{y}, \mu_F),
\]

\[
S_{(f,d)}(m_S(1-z)) = \left\{ \begin{array}{l}
\frac{f^{abc}_{def}}{24}, \frac{3d^{abc}_{def}}{40} \\
0
\end{array} \right\} \begin{vmatrix}
\gamma_v^{ak} & \gamma_n^{bl} & Y^c_{\pi} \delta \left(1 - z + 2 \frac{i\partial_0}{m_S}\right) Y_v^{*+kd} & Y_n^{*+le} & Y_{\pi}^{*+mf}
\end{vmatrix}.
\]

**Scaling evolution**

\[
\frac{dC_{f,d}(m_S, \mu)}{d \ln \mu} = \gamma_H(\mu) C_{f,d}(m_S, \mu),
\]

\[
\frac{dS_{f,d}(m_S(1-z), \mu)}{d \ln \mu} = 2 \int_{x}^{1} \frac{dz}{z} [\gamma_g(z, \mu) - \text{Re}[\gamma_H(\mu)] \delta(1-z)] S_{f,d}(m_S(1-z), \mu),
\]
• **Anomalous Dimensions for NLL Resummation**

\[
\gamma_H(\mu) = -\left(\frac{1}{2} \Gamma_{cusp} \left(\ln \frac{\mu^2}{m_S^2} + \ln \frac{\mu^2}{-m_S^2 - i\epsilon}\right) + B^S\right), \quad \gamma_g(z, \mu) = \frac{2\Gamma_{cusp}}{(1-z)_+} + B^g \delta(1-z).
\]

\[
\Gamma_{cusp} = \sum_{k=1} A_k \left(\frac{\alpha_s}{4\pi}\right)^k, \quad B^{(S,g)} = \sum_{k=1} B_k^{(S,g)} \left(\frac{\alpha_s}{4\pi}\right)^k.
\]

\[
\ln \frac{\mu^2}{-m_S^2 - i\epsilon} \rightarrow \nabla \quad \text{Timelike}
\]

\[
\ln \frac{\mu^2}{m_S^2} \rightarrow \nabla \quad \text{Spacelike}
\]

• **Soft function at NLO**

\[
S_f(m_S^2(1-z), \mu) = S_d(m_S^2(1-z), \mu)
\]

\[
= \delta(1-z) \left(1 + \frac{\alpha_s}{\pi} C_A \left(1 - \frac{\pi^2}{4} + \ln \frac{\mu}{m_S} + 2 \ln^2 \frac{\mu}{m_S}\right)\right) + \frac{\alpha_s}{\pi} C_A \left(-4 \ln \frac{\mu}{m_S} \frac{1}{(1-z)_+} + 4 \left(\frac{\ln(1-z)}{1-z}\right)_+\right).
\]
**Resummation**

**Resummation in Moments Space**: $N \to \infty (\tau & z \to 1)$

$$\sigma_N = \int_0^1 \! d\tau \tau^{N-1} \sigma(pp \to SX) \sim H(m_S, \mu_H, \mu_F)S_N(\mu_S, \mu_F)[f_{g/p}^N(\mu_F)]^2$$

$$= \sigma_0 \exp[G_S(m_S, \mu_F)][f_{g/p}(\mu_F)]^2$$

- The scales are chosen as $\mu_H = m_S, \mu_S = \frac{m_S}{Ne^{y_E}} = \frac{m_S}{N}$

$$G_S(m_S, \mu_F) = g_S^{(0)} \ln \bar{N} + g_S^{(1)}(m_S, \mu_F) + g_S^{(2)}(m_S, \mu_F) \alpha_S(m_S) + ...$$

**LL** $\lambda = \frac{\alpha_S}{4\pi} \beta_0 \ln \bar{N}$, $\alpha_S \ln \bar{N} \sim O(1)$

$$g_S^{(0)} = \frac{A_1}{\lambda \beta_0} \left[ 2\lambda + (1 - 2\lambda) \ln(1 - 2\lambda) \right],$$

$$g_S^{(1)} = \frac{1}{\beta_0} \left[ (B_1^g - B_1^S) \ln(1 - 2\lambda) + 2A_1 \lambda \ln \frac{\mu_F^2}{M_S^2} - \frac{A_2}{\beta_0^2} \left[ 2\lambda + \ln(1 - 2\lambda) \right] ight]$$

$$+ \frac{\beta_1 A_1}{2\beta_0^2} \left[ 4\lambda + (2 + \ln(1 - 2\lambda)) \ln(1 - 2\lambda) \right],$$
Resummation in Momentum Space

Becher and Neubert, 2006
Becher, Neubert, and Xu, 2008

\[ \sigma(pp \rightarrow SX) = \sigma_0 \tau \int_{z}^{1} \frac{dz}{z} V(z, m_s, \mu_F) F \left( \frac{\tau}{z}, \mu_F \right), \]

\[ V(z, m_s, \mu_F) = \hat{H}(m_s, \mu_H) U_S(\mu_H, \mu_s, \mu_F) \frac{z^{-\eta}}{(1-z)^{1-2\eta}} \tilde{S}(\partial_\eta, \mu_s) \frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}. \]

\[ \tilde{S}(L, \mu) = \int_0^\infty d\omega \ e^{-s\omega} \bar{S}(\omega, \mu), \quad \bar{S}(\omega) = S(\omega) / m_s, \quad \omega = m_s (1-z), \quad s = 1 / (e^{\gamma_E + L / 2} \mu_s) \]

\[ \hat{H} = H / \sigma_0 \]

Up to NLL \( (\hat{H} = \tilde{S} = 1) \),

\[ \ln U_S(\mu_H, \mu_s, \mu_F) = 4SU_{NLL}(\mu_H, \mu_s) + \frac{B^S}{\beta_0} \ln \frac{\alpha_S(\mu_s)}{\alpha_S(\mu_H)} + \frac{B^g}{\beta_0} \ln \frac{\alpha_S(\mu_F)}{\alpha_S(\mu_s)}, \quad r = \frac{\alpha_S(\mu_s)}{\alpha_S(\mu_H)} \]

\[ SU_{NLL}(\mu_H, \mu_s) = \frac{A_1}{4\beta_0^2} \left[ \frac{4\pi}{\alpha_S(\mu_H)} \left( 1 - \frac{1}{r} - \ln r \right) + \left( \frac{A_2}{A_1} - \frac{\beta_1}{\beta_0} \right) (1 - r + \ln r) + \frac{\beta_1}{2\beta_0} \ln^2 r \right]. \]

\[ -\frac{1}{2} < \eta(\mu_s, \mu_F) = \frac{A_1}{\beta_0} \ln \frac{\alpha_S(\mu_F)}{\alpha_S(\mu_s)} < 0 \]

: In the integral, we subtract the singular values at \( z=1 \), and then add the same quantity keeping the analytic continuation in \( \eta \). Ex) \( x^{-2\eta} \bigg|_{x \to 0} = 0 \).
Exponentiation of the large $\pi^2$ corrections

- In case of Higgs, Ahrens, Becher, Neubert, and Yang, 2009

\[ |C_H(\mu)|^2 \approx |C_H^{(0)}(\mu)|^2 \text{Re} \left[ 1 + 2\frac{\alpha_S C_A}{4\pi} \left( -\ln^2 \frac{\mu^2}{-m_H^2 - i\varepsilon} + \ldots \right) \right] \]

\[ \rightarrow |C_H^{(0)}(\mu)|^2 \left( 1 + \frac{\pi\alpha_S C_A}{2} \right) \]

\[ \rightarrow |C_H^{(0)}(\mu)|^2 \bigg|_{\mu^2 = m_H^2}^{} \bigg|_{\mu^2 = -m_H^2} : \text{Choice of the hard scale to minimize the perturbative corrections} \]

- Exponentiation of $\pi^2$-enhancement

\[ U_H(\mu_H = -im_H, \mu_S, \mu_F) = U_\pi(-im_H, m_H) \times U_H(m_H, \mu_S, \mu_F) \]

\[ U_\pi(-im_H, m_H) \sim e^{\frac{\pi\alpha_S C_A}{2}} \sim \left( 1 + \frac{\pi\alpha_S C_A}{2} \right)^{\mu^2 = m_H^2} \]

Evolution from $\mu = m_H e^{-im/2}$ to $m_H$
• Complicated Situation of the Color-Octet Scalar Production

\[ |C_{f,d}^{(0)}(\mu)|^2 = |C_{f,d}^{(0)}(\mu)|^2 \Re \left[ 1 + 2 \frac{\alpha_s C_A}{4\pi} \left( -\frac{1}{2} \ln^2 \frac{\mu^2}{-m_S^2 - i\epsilon} - \frac{1}{2} \ln^2 \frac{\mu^2}{m_S^2} + \ldots \right) \right] \]

\[ \rightarrow |C_{f,d}^{(0)}(\mu)|^2 \left( 1 + \frac{\pi \alpha_s C_A}{4} \right)_{\mu^2 = m_S^2} \quad \text{or} \quad |C_{f,d}^{(0)}(\mu)|^2 \left( 1 + \frac{\pi \alpha_s C_A}{4} \right)_{\mu^2 = -m_S^2}. \]

- We cannot eliminate the large \( \pi^2 \) term with the choice of \( \mu = m_S e^{i\alpha} \ (-\pi / 2 \leq \alpha \leq 0) \)

- Minimizing the \( \pi^2 \) corrections: \( \mu = m_S e^{-i\pi/4} \)

\[ \rightarrow U_{\pi} = \left( 1 + \frac{\pi \alpha_s C_A}{8} \right)_{\mu^4 = -m_S^4} \sim \left( 1 + \frac{\pi \alpha_s C_A}{4} \right)_{\mu^2 = m_S^2} \]

Evolution from \( \mu = m_H e^{-i\pi/4} \) to \( m_H \)

- Reflecting \( \pi^2 \)-corrections fully,

\[ U_S(\mu_H, \mu_S, \mu_F) = U_{\pi} \times U_S(m_S, \mu_S, \mu_F) \]

\( \sim 20\% \) enhancement
- **K-factors**

  - UP to NLL frequency, K-factor is universal.

  \[
  K(m_S, \tau) = \int_{z}^{1} \frac{dz}{z} V(z, m_S, \mu_F) F \left( \frac{\tau}{z}, \mu_F \right) / F_{\text{LO}} \left( \frac{\tau}{z}, \mu_F \right)
  \]

  - At \( m_S = 500 \) GeV, \( K = 2.4 \).
  - At \( m_S = 3 \) TeV, \( K = 3.6 \).
• **Scale Dependence**
  - Choice of the soft scale
    \[ \mu_S = \frac{\mu_S^I + \mu_S^II}{2} \]
    \[ \mu_S^I : S^{(1)}(m_S (1 - z), \mu_S^I) \sim 0.15 \times S^{(0)}(m_S (1 - z), \mu_S^I) \]
    \[ \mu_S^II : S^{(1)}(m_S (1 - z), \mu_S^II) \sim \text{Min} \left[ S^{(1)}(m_S (1 - z), \mu_S) \right] \]
  - Default choice of the factorization scale: \( \mu_F = m_S \)

- Uncertainty (500 GeV \( \leq m_S \leq 3\) TeV)
  - 15% variance for the choice of \( \mu_S \)
  - 25% variance for the choice of \( \mu_F \)
Application to Manohar-Wise Model

- At $m_S = 1$ TeV,

<table>
<thead>
<tr>
<th>Process</th>
<th>NLL</th>
<th>LO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$pp \rightarrow S_R^0 X$</td>
<td>$\sigma_{\text{NLL}} = 57$ fb</td>
<td>$\sigma_{\text{LO}} = 21$ fb</td>
</tr>
<tr>
<td>$pp \rightarrow S_I^0 X$</td>
<td>$\sigma_{\text{NLL}} = 73$ fb</td>
<td>$\sigma_{\text{LO}} = 26$ fb</td>
</tr>
</tbody>
</table>

Gresham and Wise, 2007
Conclusion

• The single color-octet production at the LHC can be factorized and described successfully when SCET applied.

• Systematic resummation enables us to calculate universal $K$ factor with NLL accuracy, which increases the cross section 2-4 times.

• The large corrections arising from the timelike process gives an additional 20% enhancement.

• Scaling dependences up to NLL accuracy are not small, but they can be reduced significantly with the higher order corrections.