Source Coding with Fixed Lag Side Information

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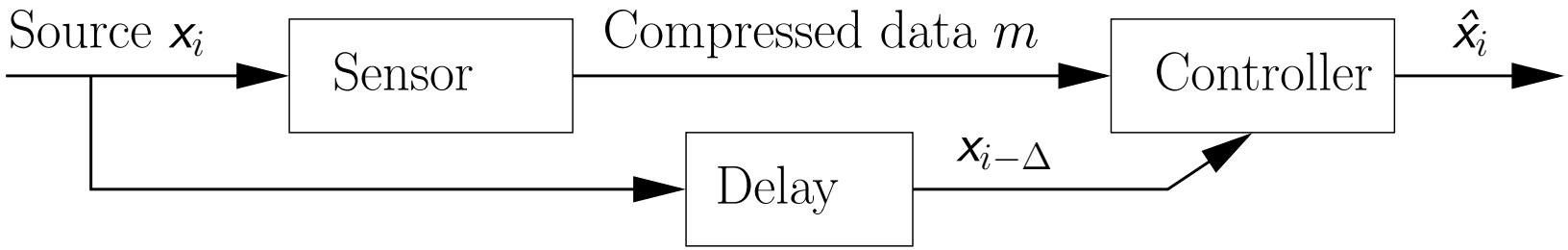
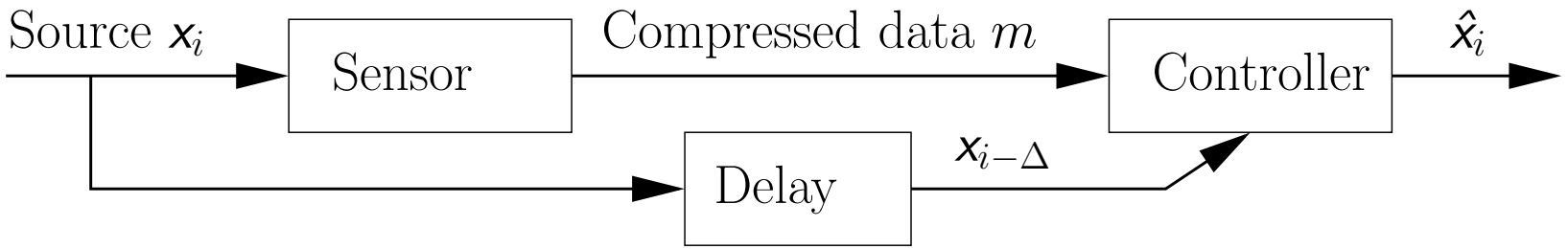
Source Coding for a Remote Control Scenario

- Sensor (*e.g.*, satellite/aircraft) sends observations to controller
- Data encoded via lossy compression to save power/bandwidth



- (Tatikonda, Sahai, Mitter 1998), (Borkar *et al.* 2001), (Elia *et al.* 2001), (Murray, Astrom, Boyd, Brockett, Stein 2003)

Source Coding for a Remote Control Scenario



- Control uses lossy, compressed data
- Controller takes action to anticipate upcoming conditions
- True, uncompressed data received later with delay Δ

Can getting true data *after* control action aid compression?

Rate distortion model by Pradhan, ISIT 2004

- Memoryless source sequence $\mathbf{x} = x_1, x_2, \ldots, x_n$
- Rate R encoder maps $\mathbf{x}$ to $m \in \{1, 2, \ldots, 2^{nR}\}$
- Decoder maps m and past samples to ith reconstruction $\hat{x}_i$
- Distortion: $D = \frac{1}{n} \sum_{i=1}^{n} d(x_i, \hat{x}_i)$

Concrete Example:

- Alice to be asked K Yes/No questions & gets prize if correct.
- There are $K/2$ valuable prizes and $K/2$ worthless prizes.
- *After* each question, Alice learns correct answer & prize value.

Can we make $K/2$-bit cheat-sheet so Alice gets all valuable prizes?

Making the Optimal Cheat-Sheet

Bad prizes don't matter so map answer & prize to $\{0, 1, *\}$:

Answers:	1	1	0	0	1	0	0	0	1	0	...
Prizes:	G	B	G	B	B	G	B	B	G	G	...
Data:	1	$*$	0	$*$	$*$	0	$*$	$*$	1	0	...

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Result: Binary Erasure Quantization (dual to Erasure Channel)

Goal: compress $\{0, 1, *\} \to \{0, 1\}$ by only changing $*$'s

True Data: 1 * 0 * * 0 * * 1 0
Compressed: m = 1 0 0 1 0

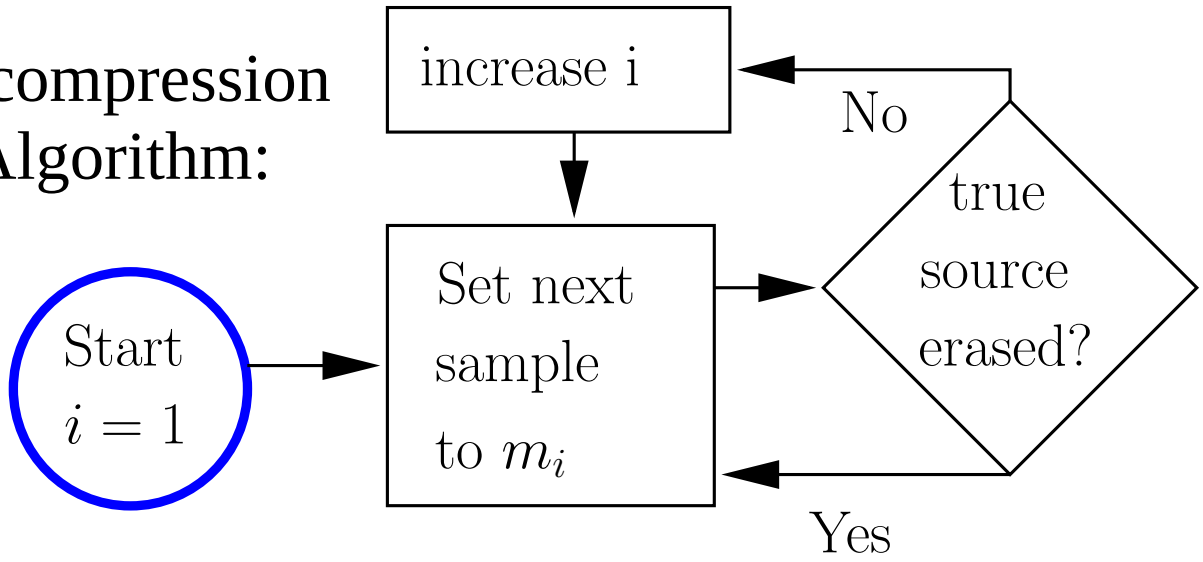
Compression: delete *'s to get m

Decompression: Use an answer
until it wins an important prize

True Data: 1 * 0 * * 0 * * 1 0

Compressed: 1 0 0 1 0

Decompression Algorithm:



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True Data:

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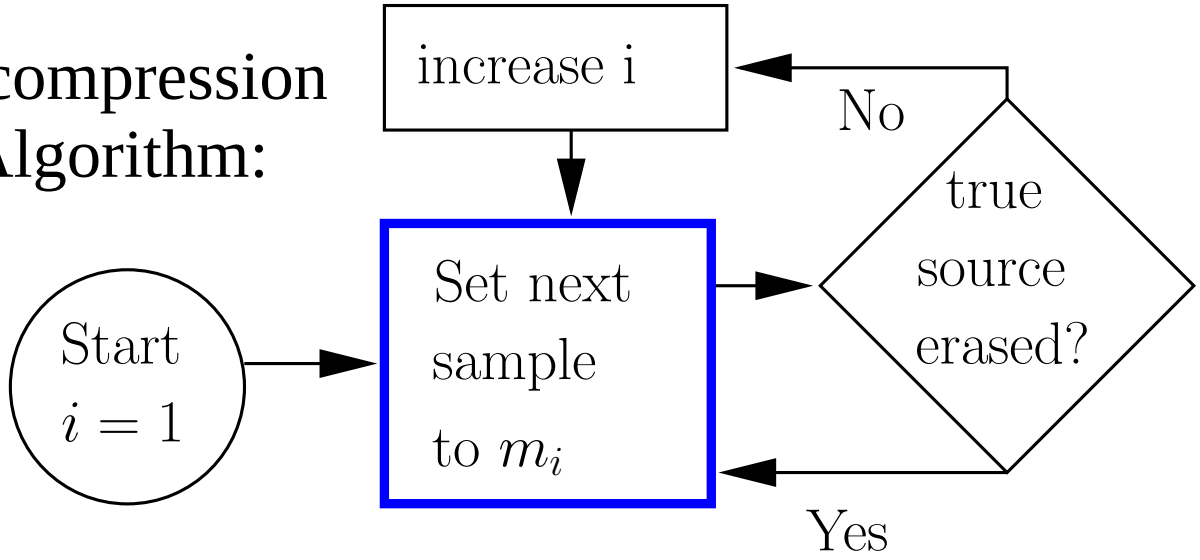

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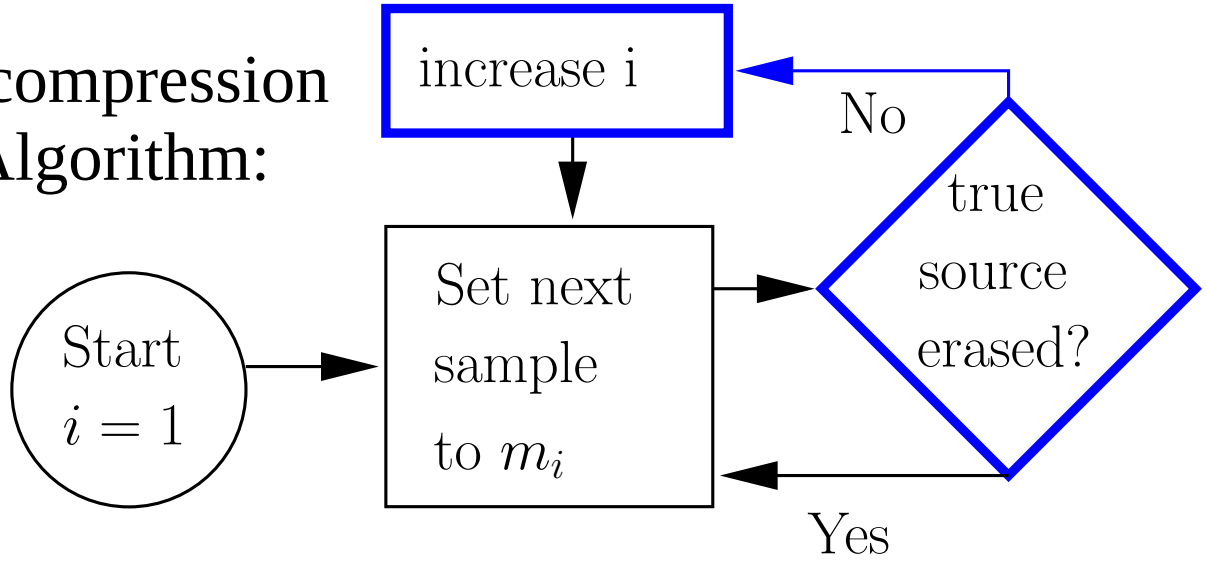

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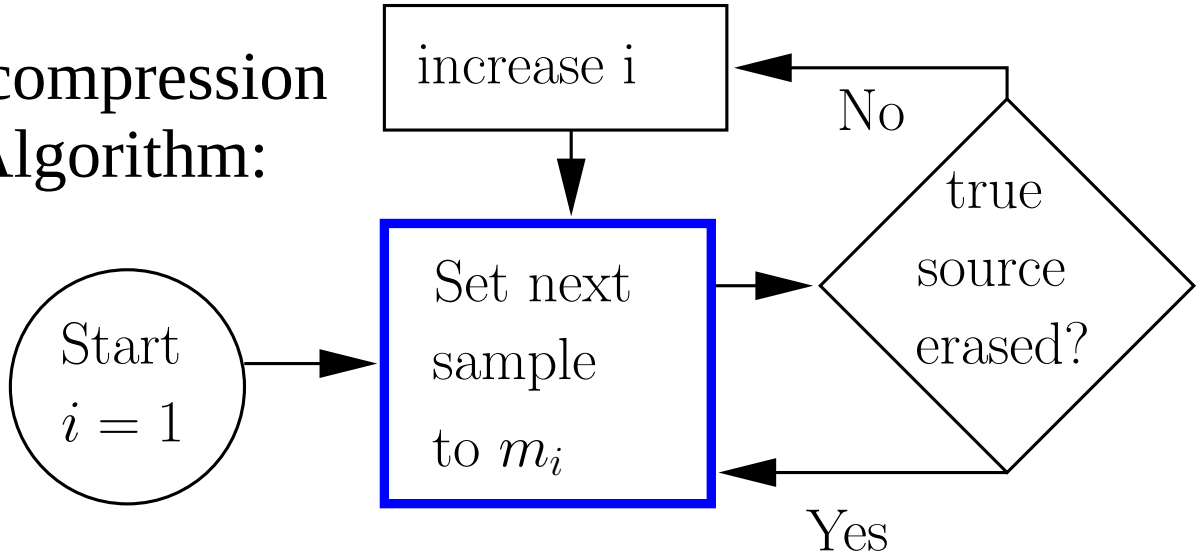

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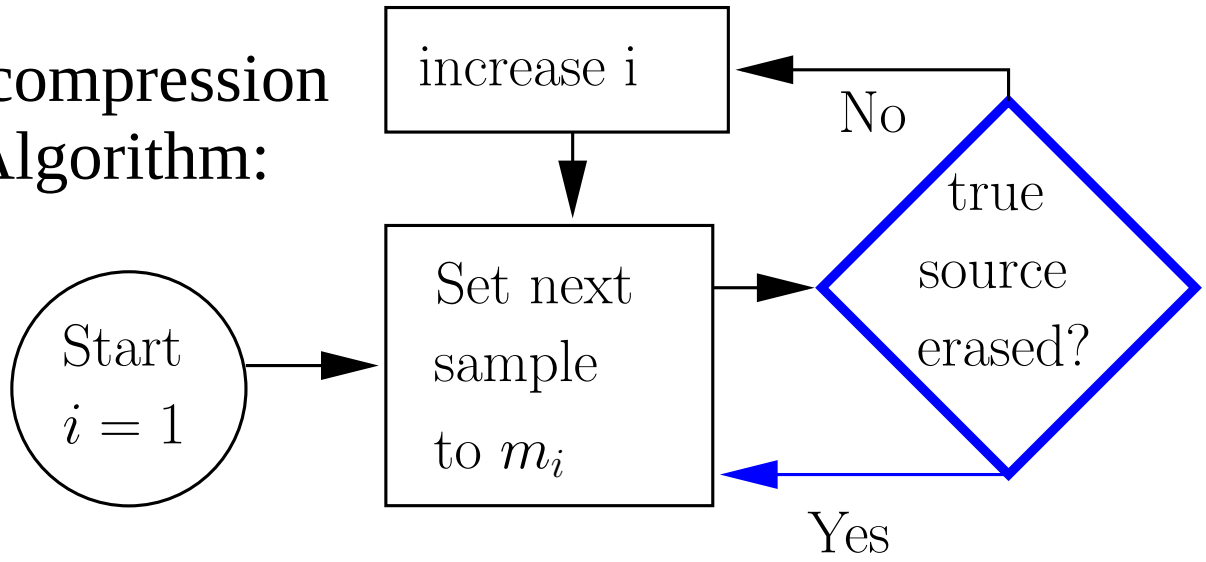

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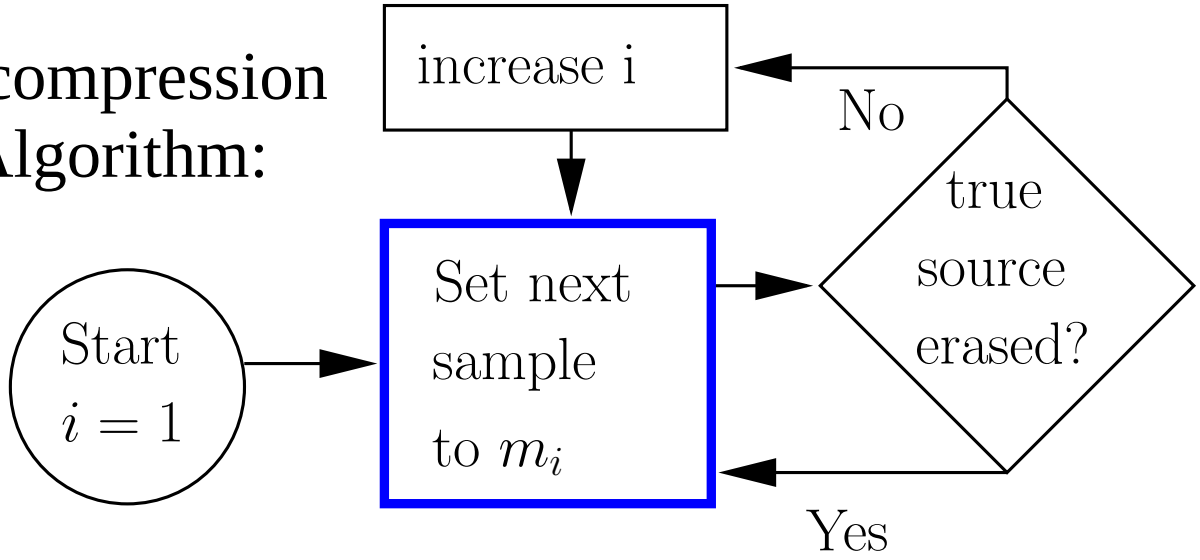

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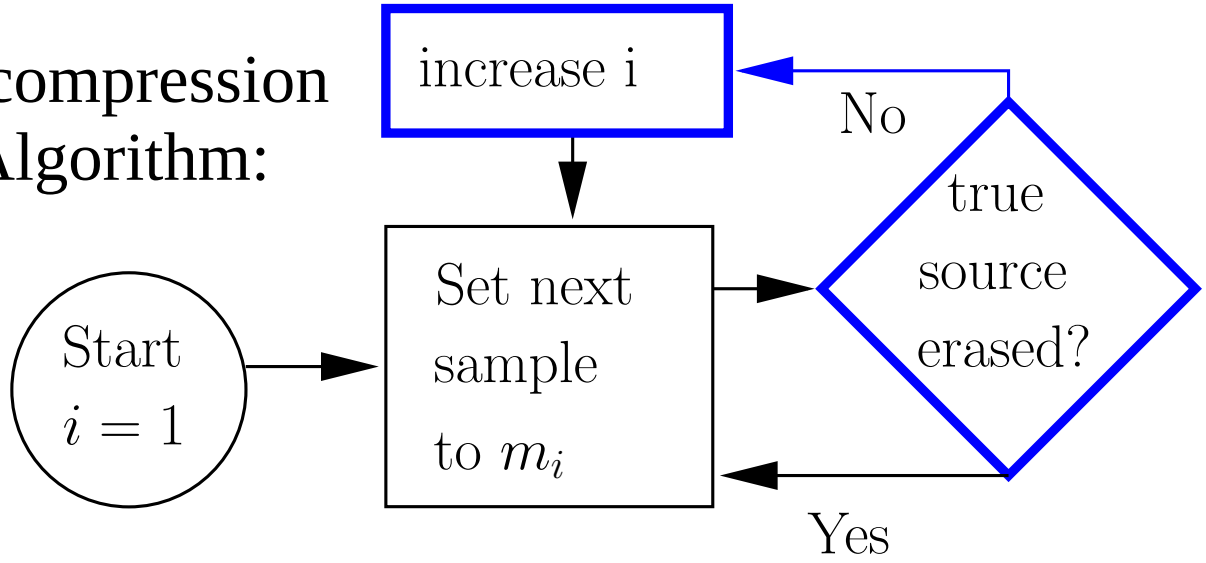

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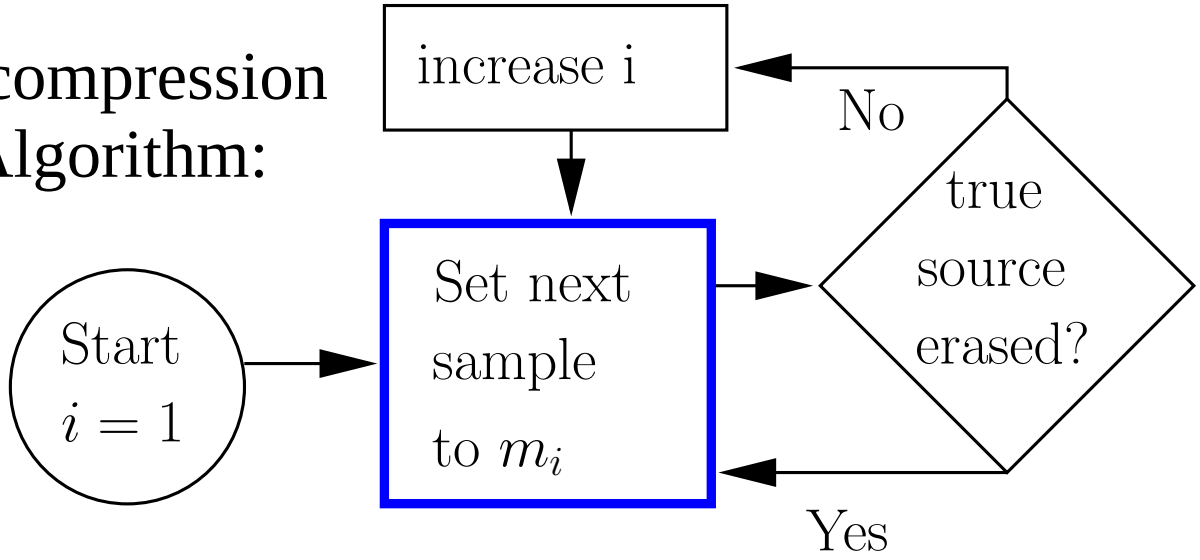

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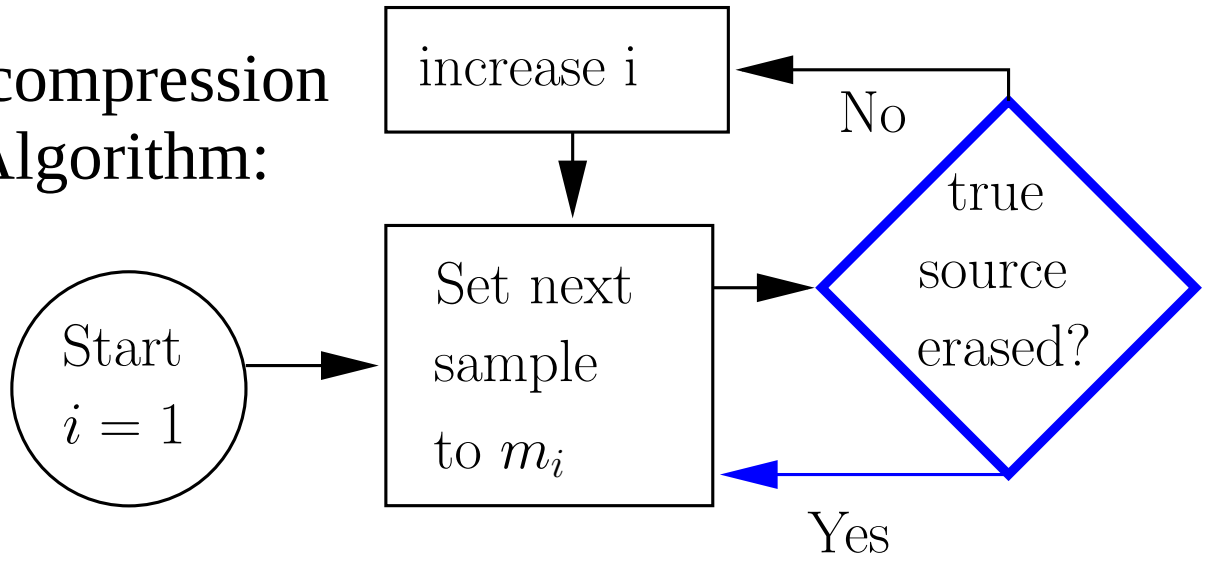


Decoded: 1 0 0 0

True Data:1 * 0 *

Channel Feedback/Source Feedforward Are Duals

- Memoryless $\Rightarrow$ side information doesn't affect Capacity/$R(D)$
- Feedback $\Rightarrow$ achieve C w/ linear complexity
  - Kailath-Schalkwijk Gaussian, Ooi-Wornell finite alphabet
- Feedforward $\Rightarrow$ achieve $R(D)$ w/ linear complexity
  - Pradhan Gaussian, this paper finite alphabet sources

Preliminaries

Lossless compression: map non-uniform symbols to uniform:

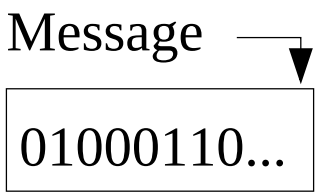
$$00010000010001 \longrightarrow 0110$$

Shaping code: map uniform symbols to non-uniform symbols

$$baabba \longrightarrow aaaaabaaaababaaaaaa$$

(shaping and lossless compression are duals)

Ooi-Wornell Channel Coding w/ Feedback



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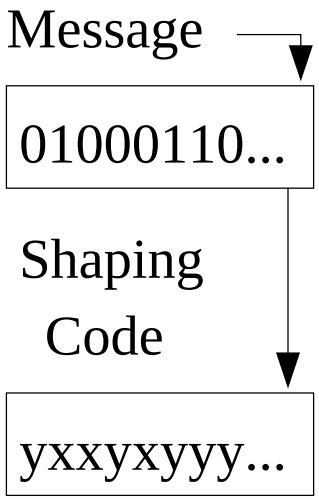

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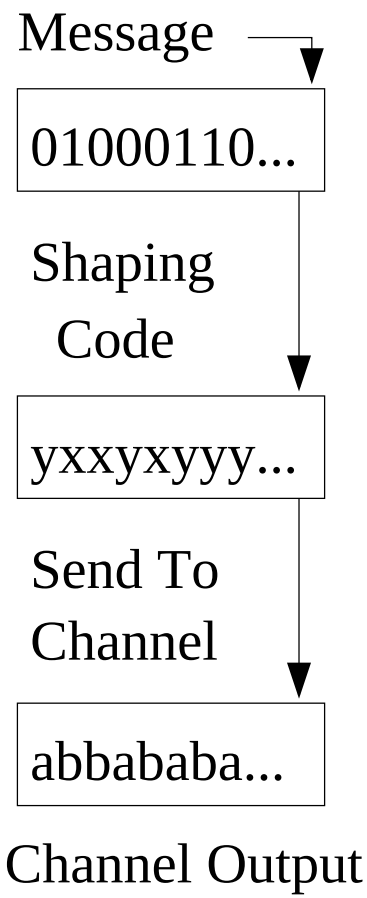

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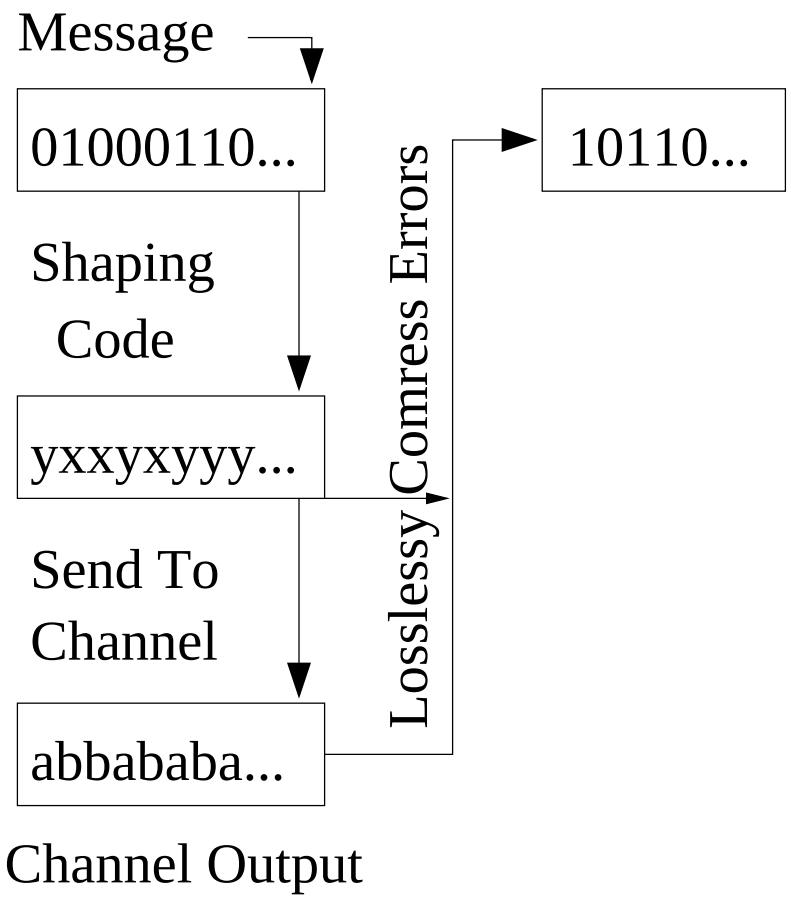

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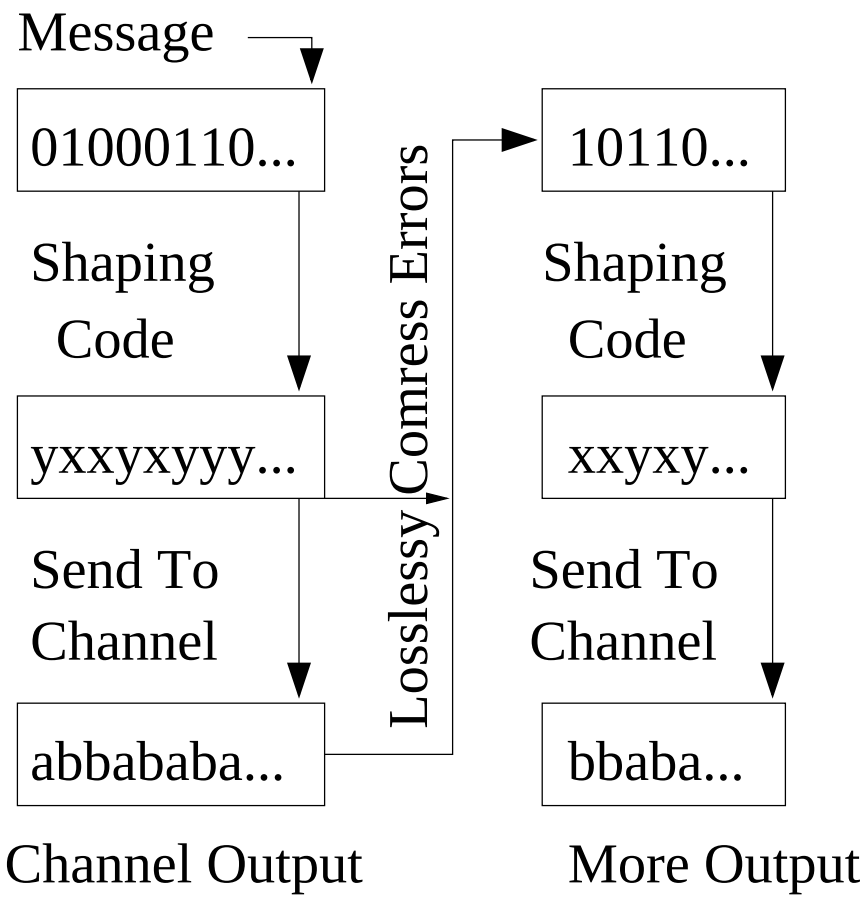

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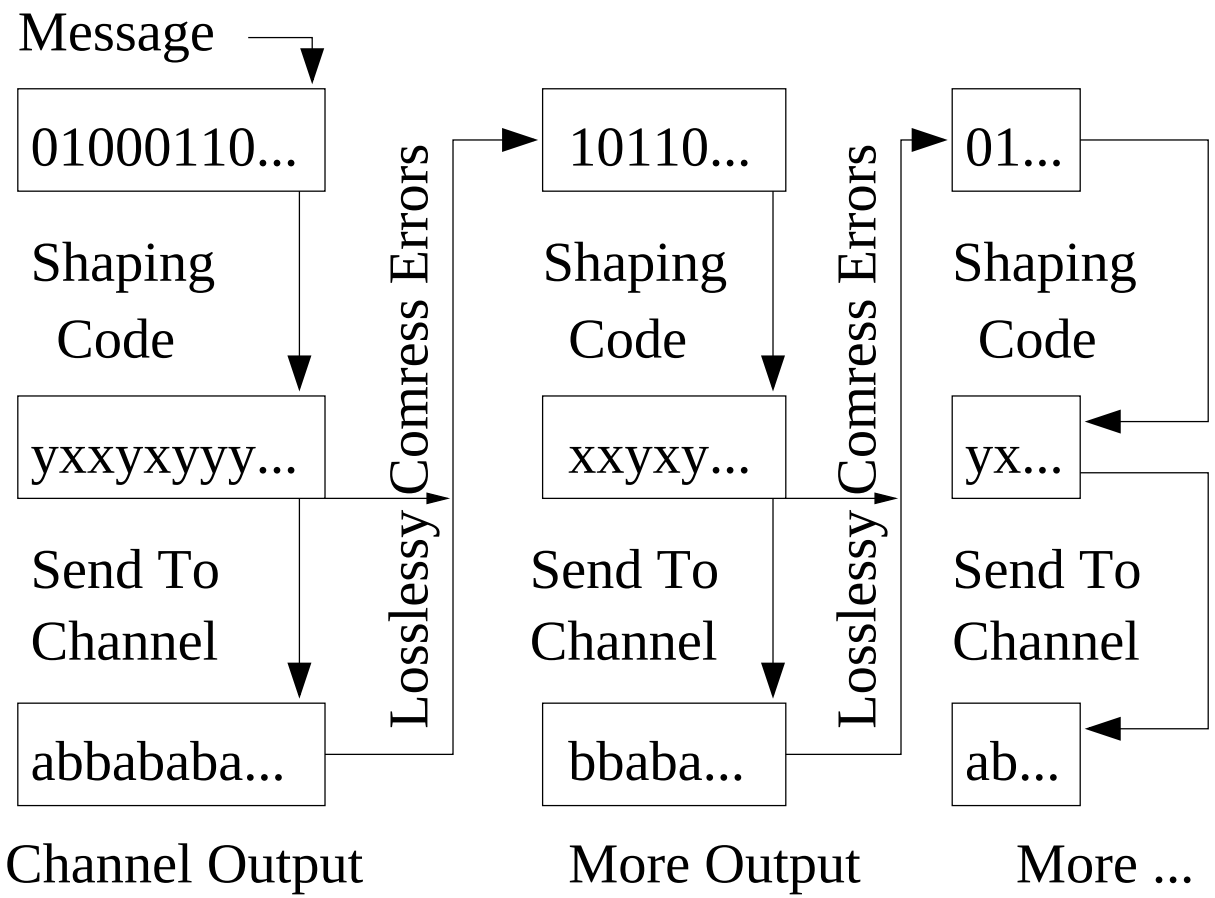
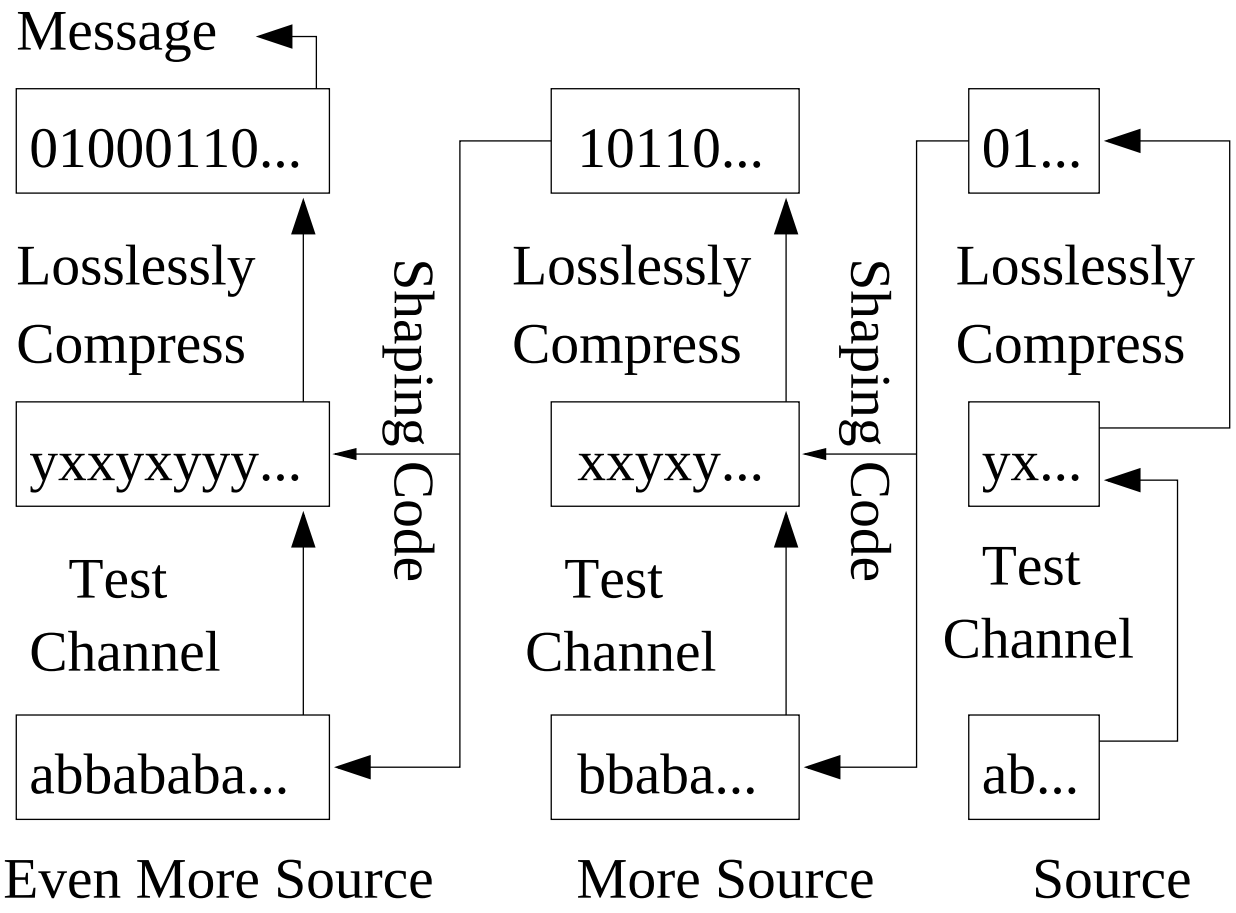


Dual Source Coding w/ Feedforward



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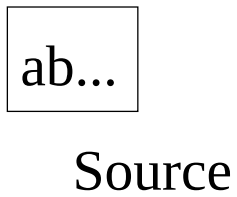

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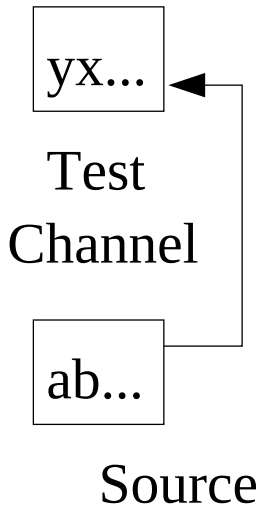

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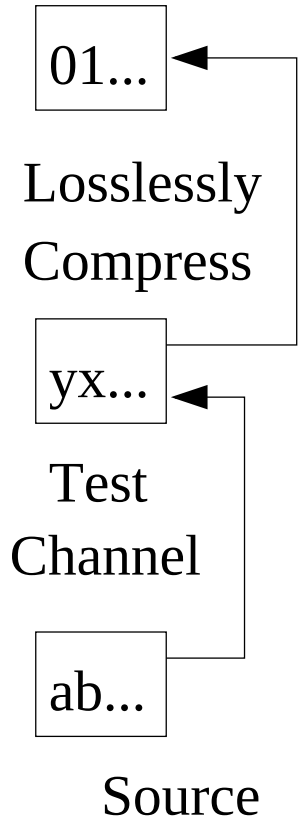

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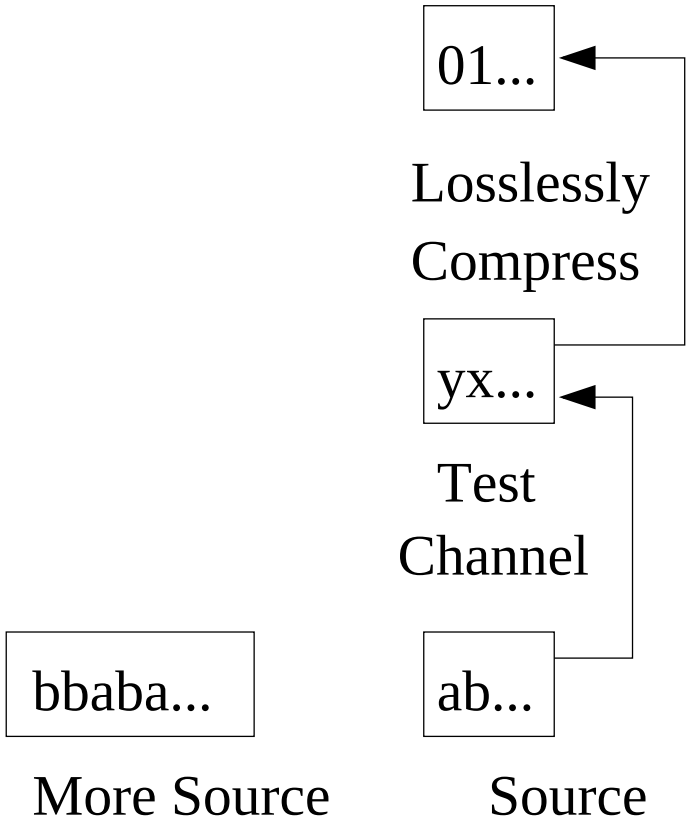

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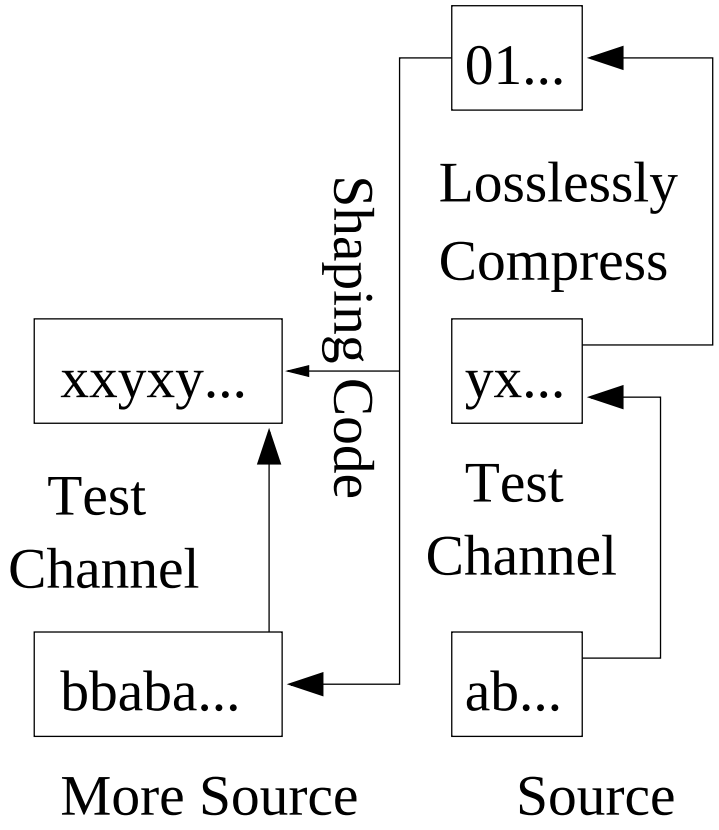

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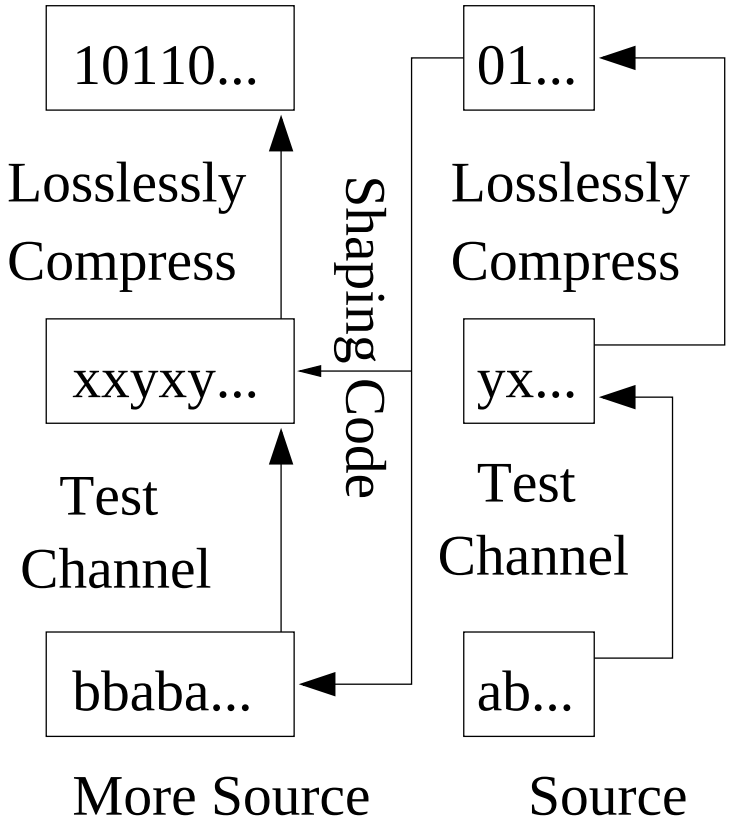

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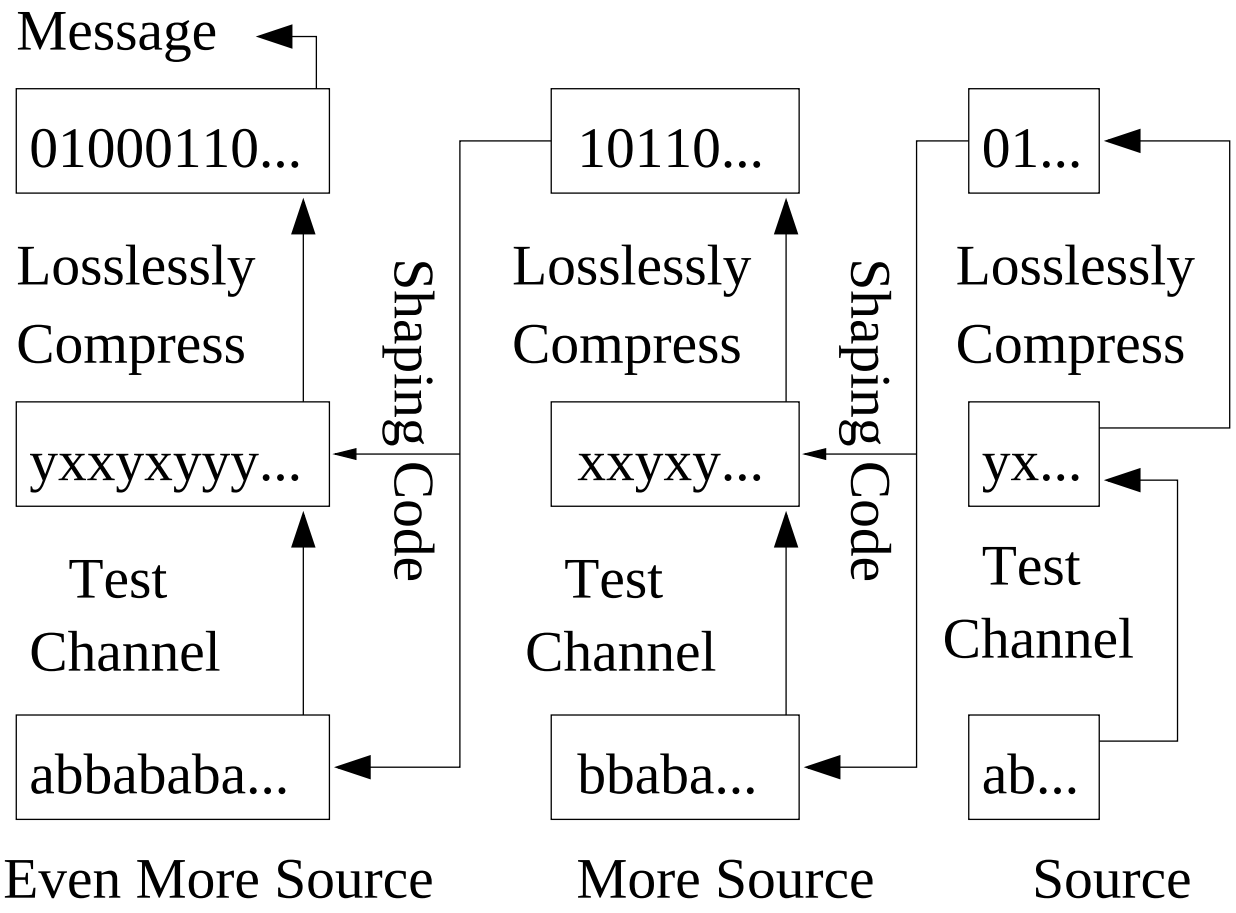
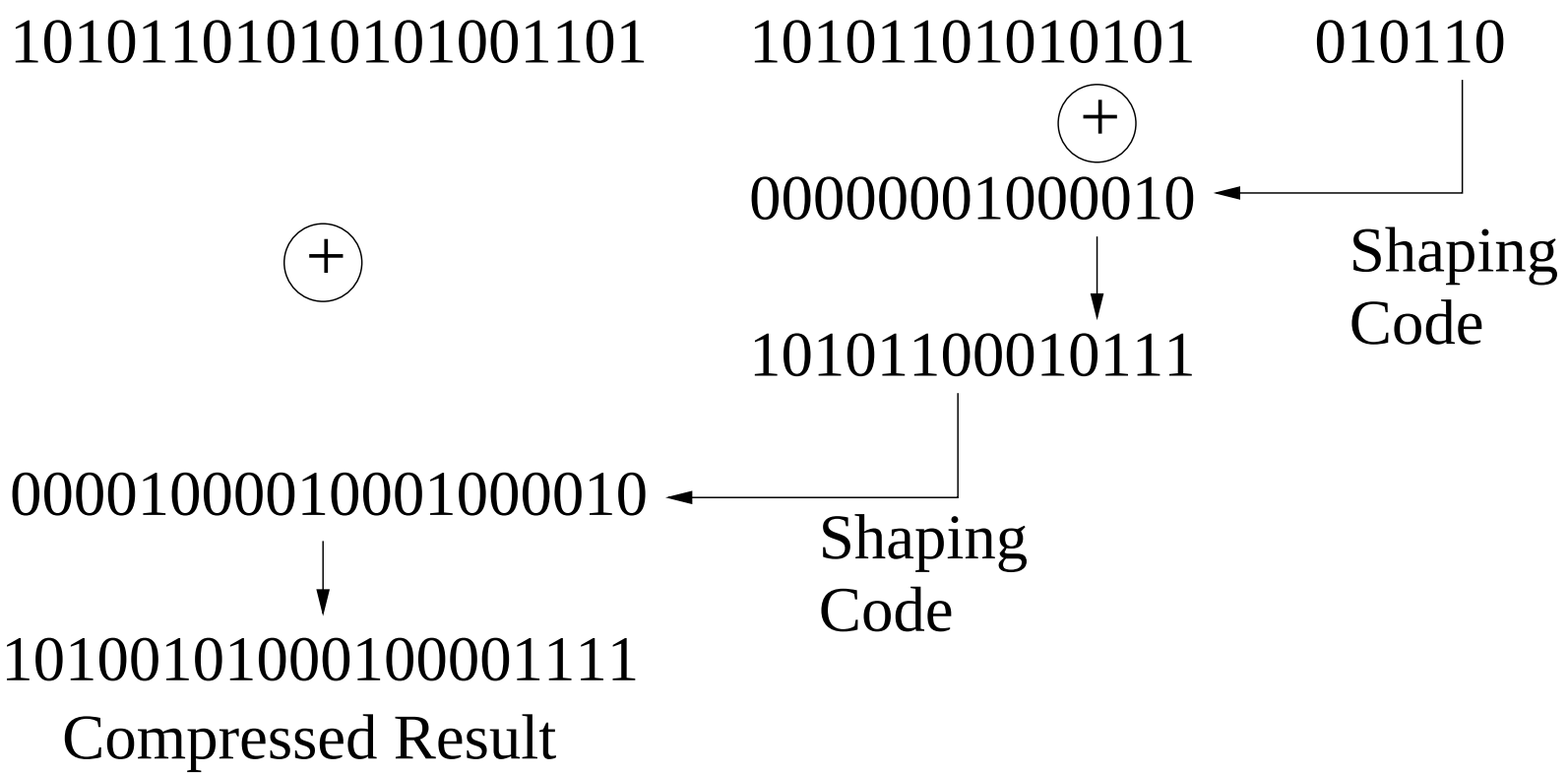


Binary-Hamming Example

Binary Symmetric Source, Hamming Distortion $R(D = .11) = 1/2$

Compression:



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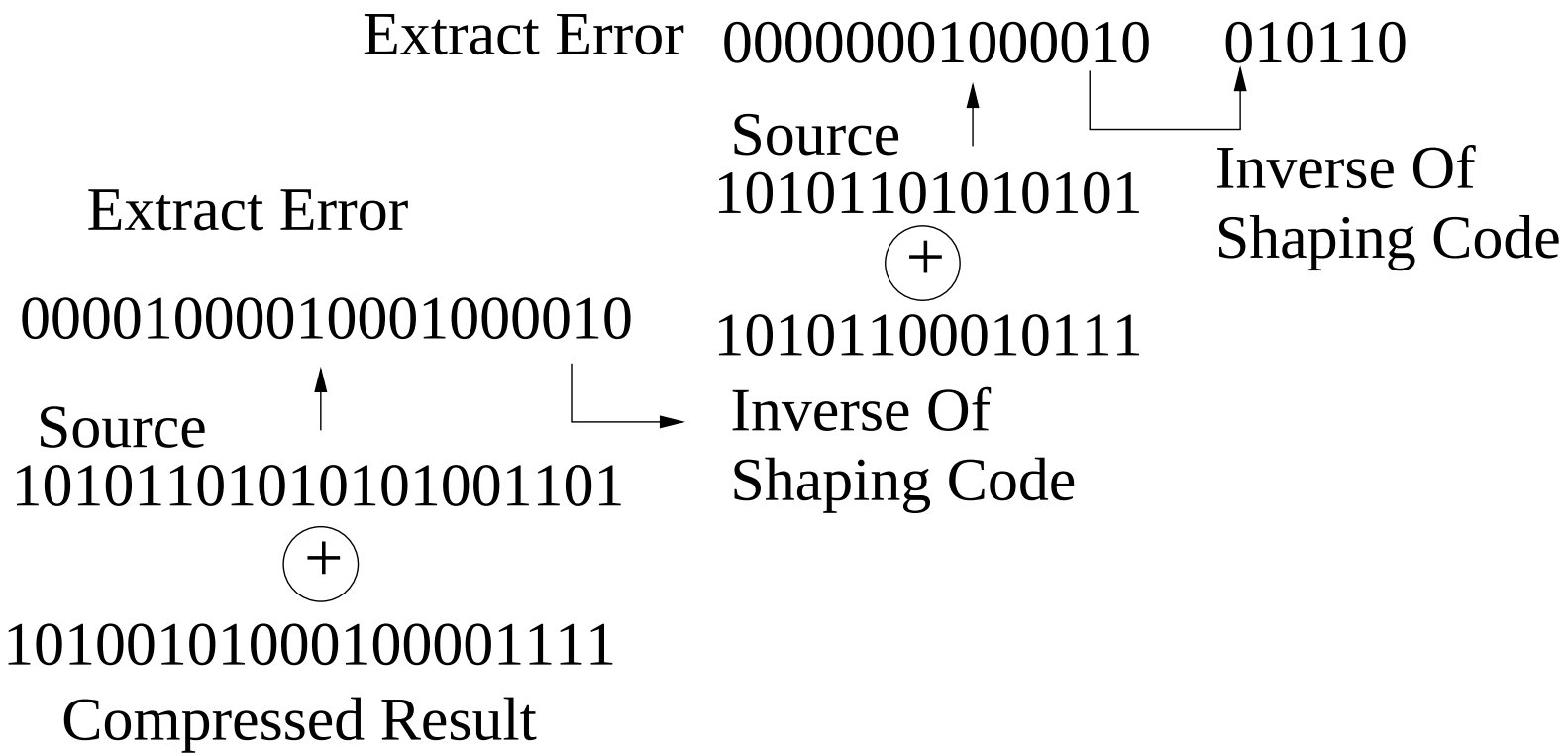

Concluding Remarks

- Fixed lag side information (feedforward) can be useful:
  - Constructively achieve $R(D)$ with linear complexity!
- Another illustration of duality
- Compression for control provides new challenges/opportunities