Low Delay Burst Erasure Correcting Codes For Packet Transmission

Emin Martinian
EECS Department, MIT
77 Massachusetts Ave.
Cambridge, MA, 02139
e-mail: emin@allegro.mit.edu

Carl-Erik W. Sundberg
iBiquity Digital Corporation
20 Independence Blvd.
Warren, NJ 07059,USA
e-mail: sundberg@ibiquity.com

Abstract — **We present a bound relating the decoding delay, correctable burst length and rate of any erasure correcting code. Next we describe a new code construction which is optimal in the sense that the proposed codes have the shortest possible decoding delay allowed by our bound. These "Maximally Short" codes provide a useful tool to mitigate packet loss in delay sensitive applications.**

I Introduction

In a variety of networks, packet losses occur in bursts. Conventional erasure correcting codes used for packet recovery often require interleaving and long decoder delay. These long delays are usually unacceptable in real-time multimedia communication such as Voice over IP. Consequently, erasure correction codes with severe limits on delay are desirable.

Consider an application which sends data packets containing k units (e.g. k bits, k bytes) every time step over a channel which suffers occasional burst losses due to congestion, errors, etc. In order to protect against data losses while maintaining the same number of transmitted packets, we encode the information by adding $n-k$ redundant units to each packet.

Specifically a rate $R = k/n$ packet code is a mapping from a sequence of k-unit symbols, $\vec{x}[i] = (x_0[i], x_1[i], ..., x_{k-1}[i])$, to n-unit symbols, $\vec{y}[i] = (y_0[i], y_1[i], ..., y_{n-1}[i])$. A burst of length B starting at time i is defined as the loss of one or more symbols from the set $\{\vec{y}[i], \vec{y}[i+1], ..., \vec{y}[i+B-1]\}$. If the earliest that $\vec{x}[i+j]$ can be recovered due to such a burst is once $\vec{y}[i+1]$, $\vec{y}[i+2]$, ..., $\vec{y}[i+j+T]$ is received then the decoding delay for recovering $\vec{x}[i+j]$ is T.

II A Decoding Delay Bound

Theorem 1 *If a rate R code can correct all erasure bursts of length B with decoding delay at most T, it must satisfy $T/B \geq \max[1, R/(1-R)]$.*

This Theorem can be proved via simple extensions of guard space bounds in [2]. The guard space is the number of erasure free symbols both before and after a burst required to guarantee *eventual* recovery (i.e. with unbounded decoding delay). Hence guard space is a looser requirement in the sense that codes satisfying guard space bounds but not decoding delay bounds exist while the converse is not true. For example, Reed-Solomon block codes, interleaved Reed-Solomon codes [2], as well as other Maximum Distance Separable (MDS) and near MDS [3] block codes meet the strongest possible bounds on guard space, but do not achieve the best decoding delay for a given rate and burst length.

III Code Constructions

Let $P\{u_1, u_2, ..., u_s\}$ be the s parity check symbols for a systematic, $(n, k, d) = (2s + ms, ms + s, s + 1)$ Reed-Solomon block code with input $(u_1, u_2, ..., u_s)$. Then $\mathcal{C}_{m,s}$ is the rate $R = (ms+1)/(ms+1+s)$ code defined by the mapping

$$\vec{y}[i] = (x_0^{ms}[i], P\{x_0[i-1], x_0[i-2], ..., x_0[i-s], x_1^s[i-s-1], x_{s+1}^{2s}[i-2s-1], ..., x_{(m-1)s+1}^{ms}[i-ms-1]\})$$

where $x_a^b[j]$ denotes $(x_a[j], x_{a+1}[j], ..., x_b[j])$. Such codes inherit linearity from the constituent Reed-Solomon codes and are therefore time-invariant convolutional codes. In [1] we show that the decoding delay required by this family of codes to correct a burst of length s is exactly $T = ms + 1$ thus meeting the decoding delay bound with equality. Furthermore, via periodic interleaving of degree λ, the code $\mathcal{C}_{m,s}$ can be transformed into a code capable of correcting a burst of length λs with delay λT (and therefore continuing to meet the delay bound with equality).

To illustrate some properties of the code construction and decoding algorithm, we consider the code $\mathcal{C}_{1,2}$ in Table 1. Imagine that $\vec{y}[i]$ and $\vec{y}[i+1]$ are lost in an erasure burst of length 2. When $\vec{y}[i+2]$ is received, the decoder has $x_1^2[i-1]$ (since it was not lost in the burst) and 2 additional parity check symbols for the $(6,4,3)$ code with input $(x_0[i+1], x_0[i], x_1^2[i-1])$. Therefore the decoder recovers $x_0[i]$ and $x_0[i+1]$ at time $i+2$. When $\vec{y}[i+3]$ is received, the decoder has received $x_0[i+2]$ (since it was not lost in the burst), $x_0[i+1]$ (which was recovered in the previous decoding step), and 2 additional parity check symbols for the $(6,4,3)$ code with input $(x_0[i+2], x_0[i+1], x_1^2[i])$. Therefore the decoder recovers $x_1^2[i]$ at time $i+3$. A similar argument shows that the decoder recovers $x_1^2[i+1]$ at time $i+4$. Hence the code corrects any burst of length 2 with decoding delay 3 meeting the bound in Theorem 1 with equality.

Table 1: A sample encoded sequence for the code $\mathcal{C}_{1,2}$.

$$\vec{y}[i-1] = \left(x_0^2[i-1], P\{x_0[i-2], x_0[i-3], x_1^2[i-4]\}\right)$$
$$\vec{y}[i+0] = \left(x_0^2[i+0], P\{x_0[i-1], x_0[i-2], x_1^2[i-3]\}\right)$$
$$\vec{y}[i+1] = \left(x_0^2[i+1], P\{x_0[i+0], x_0[i-1], x_1^2[i-2]\}\right)$$
$$\vec{y}[i+2] = \left(x_0^2[i+2], P\{x_0[i+1], x_0[i+0], x_1^2[i-1]\}\right)$$
$$\vec{y}[i+3] = \left(x_0^2[i+3], P\{x_0[i+2], x_0[i+1], x_1^2[i+0]\}\right)$$
$$\vec{y}[i+4] = \left(x_0^2[i+4], P\{x_0[i+3], x_0[i+2], x_1^2[i+1]\}\right)$$

References

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