## The problem with Elbows：an example

－Pierrehumbert（1980）posits leftward spreading of $\mathrm{L}-$ in $\mathrm{H} * \mathrm{~L}-\mathrm{H} \%$ and $\mathrm{H} * \mathrm{~L}$－ $\mathrm{L} \%$ tunes to explain why F 0 does not interpolate from $\mathrm{H}^{*}$ to the end of the


Two hypotheses concerning the timing of the onset of L－（Pierrehumbert 1980） $>$ L－occurs at a fixed interval after $\mathrm{H}^{*}$
$>$ L－is aligned to the end of the nuclear－accented word．
－To test these hypotheses we have to locate L－
－The correlate of L－is an＇elbow＇or inflection in the F0 trajectory
＇it was very difficult to decide where the L－was located．＇（Pierrehumbert 1980：86）

## Identifying elbows through analysis－by－synthesis

－Analysis－by－synthesis of F0 trajectories
－Rather than identifying elbows using general－purpose algorithms，then modeling the results（e．g．del Giudice et al 2007 ，Reichel \＆Salveste 2015），
－＇Elbow＇targets should be inferred in the process of modeling F0 trajectories．
－Model of $\mathrm{H}^{*} \mathrm{~L}-(\mathrm{T} \%)$ production：
－The transition from $\mathrm{H}^{*}$ to the first L －target is realized as the response of a critically－damped linear second order system（＇spring－mass system＇）to a step input．
－The transition from the first L －target to the second is the response of the same system to an input linear transition between the two targets．
Cf．Fujisaki \＆Hirose（1984），Anderson et al（1984）

－The form of this trajectory if initial velocity $=0$ ：

$$
\log \left(F_{0}\right)=L+(H-L)\left(1+\frac{t}{T}\right) e^{-\frac{t}{T}}+s T\left(\frac{t}{T}-2+\left(2+\frac{t}{T}\right) e^{-\frac{t}{T}}\right)
$$

－Fit this model to the observed F0 trajectories to obtain estimates of F0 targets －Since a critically damped movement strictly never reaches its target，this model does not directly specify the time of the L－target
－The timing of the effective target is specified in multiples of $T$ －e．g． $99 \%$ of the movement is completed in about $5 T$

## Testing hypotheses about the timing of L－

Predictions of the two hypotheses：What should happen to the time constan $T$ when the duration of the interval between $\mathrm{H}^{*}$ and the end of the word
varies？

$T$ is predicted to be a linear function of $\mathrm{H}^{*}$－to－end duration，with intercept $\approx 0$
Slope of the line is $1 / n$ ，where $n$ is the number of time constants to reach the L target．


## Results I－production model

－The critically－damped model does not fit all speakers／utterances wel
－Problem：damped＇spring－mass＇models have peak acceleration at movement onset，bu this is not true of all $\mathrm{H}^{*} \mathrm{~L}$－transitions


－This problem is familiar from the study of other speech movements（e．g．Kröger et al 1995） －Current solution：Model the $\mathrm{H}^{*} \mathrm{~L}$－transition with two step functions，starting the second from the acceleration minimum，with estimated initial velocity

$$
\log \left(F_{0}\right)=L+(H-L)\left(1+\left(\frac{v_{0}}{(H-L)}+\frac{1}{T}\right) t\right) e^{-\frac{t}{T}}+s T\left(\frac{t}{T}-2+\left(2+\frac{t}{T}\right) e^{-\frac{t}{T}}\right)
$$

## Results II－timing of L－


－$T$ tends to increase as duration from $\mathrm{H}^{*}$ to end of word increases $(\beta=0.11, t=6.5)$
－So the interval between $\mathrm{H}^{*}$ and L －is not fixed，but L －does not track word end either
$-\beta=0.11$ would imply that target is achieved at $9 T$ ，also intercept $>0(\beta=0.012, t=3.4)$
－This pattern could represent a compromise between a preferred value for $T$ and a preference to keep L－within the accented word，but there is a lot of variability．

F0 trajectories from two subjects，aligned on $\mathrm{H}^{*}$ peak
－Vical lines mark word en
－Color codes duration from $\mathrm{H}^{*}$－to－word－end

## The Data

－Recordings from Barnes，Veilleux，Brugos \＆Shattuck－Hufnagel（2010） 25 two－word phrases in a context designed to elicit $\mathrm{H}^{*} \mathrm{~L}-\mathrm{H} \%$ melody，with $\mathrm{H}^{*}$ on the first word
First word：vary the number and length of syllables following primary stress
2 álien，lánolin，Líllian，Márilyn，mínimum
3 lúminary，pálimony，céremony，cúlinary，púlmonary
3 críminally，sérially，términally，mínimally，nóminally
（1）Experimenter：George is a thoughtful sort of divorce lawyer－I go to him whenever I need a palimony ruminator．
Subject：A palimony ruminator？？？！！！（ $\mathrm{H}^{*} \mathrm{~L}-\mathrm{H} \%$ ）I thought he wa figuring out your plumbing problems

15 speakers（ 11 female），each produced 4 repetitions of the materials
－ 239 utterances excluded due to errors，disfluencies，pitch tracking problems
Tracked F0 with Praat（Boersma \＆Weenink 2018），segmented the pitch contour from F0 peak $\left(\mathrm{H}^{*}\right)$ to onset of the final rise，and fitted the tone realization model using non－ linear least squares（ nls （ R Core Team 2016））

Barnes．J．N．N．Veilleux，A．．Brupos \＆ S ．Shatuck－－Huffagel． 2010 ．Turning points，tonal targets，and the English L－phrase
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ICPHS XII， $1117-1120$ ．
Fuisaki，H．\＆Hirose，K．1984，Analysis of voice fundamental frequency contoris ger，B．J．，Schröder，G．．．Opgen－Rhein．C．1995．A Aes

Reichel．U．，\＆N．Salveste． 2015 Ponology and Phonetics of English Intonation．PhD Thesis，MIT．


