Coordination of Multiple Autonomous Vehicles

Herbert G. Tanner, Ali Jadbabaie and George J. Pappas

Department of Electrical and Systems Engineering
200 South 33rd Street
University of Pennsylvania
Philadelphia, PA 19104

***Abstract*—We analyze the coordinated motion of a group of nonholonomic vehicles that are controlled in a distributed fashion exhibiting flocking behavior. The coordinated group motion is a result of the aggregated effect of the control actions of the members of the group and not imposed by some centralized scheme. Each vehicle is independently control by a linear combination of a potential field force that ensures collision avoidance and attraction towards the group, and an alignment force that steers the vehicle to obtain an orientation equal to the average heading of all other vehicles in the group. The result of this control policy is that eventually all vehicles attain a common heading and move close to each other while avoiding collisions.**

I. Introduction

Recent technological advances offered more efficient computation and less expensive communication. The ability to compute locally and share information has facilitated the development of new multi-agent systems. Such type of systems promise increased performance, efficiency and robustness, at a fraction of the cost compared to their centralized counterparts, utilizing distributed coordination sensing and actuation. The question that now arises is how to achieve the desired level of coordination in multi-agent systems.

Nature is abundant in marvelous examples of coordinated behavior. Across the scale, from biochemical cellular networks, up to ant colonies, schools of fish, flocks of birds and herds of land animals, one can find systems that exhibit astonishingly efficient and robust coordination schemes [1, 21, 34, 13, 9]. At the same time, several researchers in the area of statistical physics and complexity theory have addressed flocking and schooling behavior in the context of non-equilibrium phenomena in many-degree-of-freedom dynamical systems and self organization in systems of self-propelled particles [33, 32, 31, 19, 17, 28, 6, 15]. Similar problems have become a major thrust in systems and control theory, in the context of cooperative control, distributed control of multiple vehicles and formation control; see for example [16, 22, 23, 7, 18, 10, 29, 14, 20]. The main goal of the above papers is to develop a decentralized control strategy, such that a global objective, such as a tight formation with fixed pairwise inter vehicle distances is achieved.

In 1986 Craig Reynolds [25] made a computer model of coordinated animal motion such as bird flocks and fish schools. He called the generic simulated flocking creatures “boids”. The basic flocking model consists of three simple steering behaviors which describe how an individual boid maneuvers based on the positions and velocities its nearby flockmates:separation, alignment, and cohesion. In 1995, a similar model was proposed by Vicsek *et al.* [33]. Under an alignment rule, a spontaneous development of coherent collective motion is observed, resulting in the headings of all agents to converge to a common value. A proof of convergence for Vicsek's model (in the noise-free case) was given in [14].

In this paper provide a system theoretic justification for the flocking phenomenon in [25]. In our flock model, we consider dynamic nonholonomic systems steered in a decentralized fashion. We show that all agents headings converge to the same value, velocities will eventually become the same and pairwise distances will converge, resulting in a flocking behavior. Our analysis makes use of Lyapunov stability and algebraic graph theory. While the proof techniques are totally different from those in [14], the end result is similar, suggesting that addition of cohesion and separation forces in addition to alignment as well as addition of dynamics, does not affect the stability of the flocking motion.

The paper is organized as follows: In Section II we describe the dynamics of each vehicle in the group and introduce its control law. Section III shows how this control law gives rise to flocking behavior for the group with simultaneous collision avoidance. Numerical simulations verifying the stability results are presented in Section IV and the paper concludes with a summary of its contributions in Section V.

II. Flocking Control

Consider a group of N vehicles, moving on the plane according to the following dynamics:

$$\dot{x}_i = v_i \cos\theta_i \tag{1a}$$
$$\dot{y}_i = v_i \sin\theta_i \tag{1b}$$
$$\dot{\theta}_i = \omega_i \tag{1c}$$
$$\dot{v}_i = a_i \quad i = 1, \dots, N\,, \tag{1d}$$

where $r_i = (x_i, y_i)^T$ is the position vector of vehicle i, θ_i its orientation (Figure II), v_i its translational speed and a_i, and ω_i its control inputs. The relative positions between the vehicles are denoted $r_{ij} = r_i - r_j$.

For every pair of vehicles i and j consider an artificial potential function V_{ij} that depends on the distance between them. We do not require a particular structure for V_{ij}; any such function will do as long as it depends only on $\|r_{ij}\|$. Since $r_{ij} = r_i - r_j$, this implies that V_{ij} will be symmetric with respect to r_i and r_j. As an example

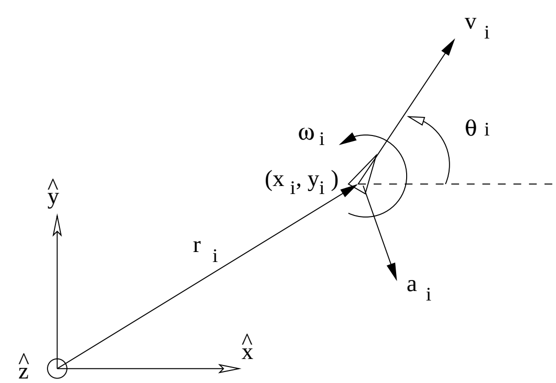

Fig. 1. Control forces acting on vehicle i.

of such a function, consider the following:

$$V_{ij}(\|r_{ij}\|) = \frac{1}{\|r_{ij}\|^2} + \log\|r_{ij}\|^2\,,$$

which is depicted in Figure 2. Then, we define the potential energy for vehicle i as:

$$V_i \triangleq \sum_{\substack{j=1\\ j\neq i}}^{N} V_{ij}(\|r_{ij}\|).$$

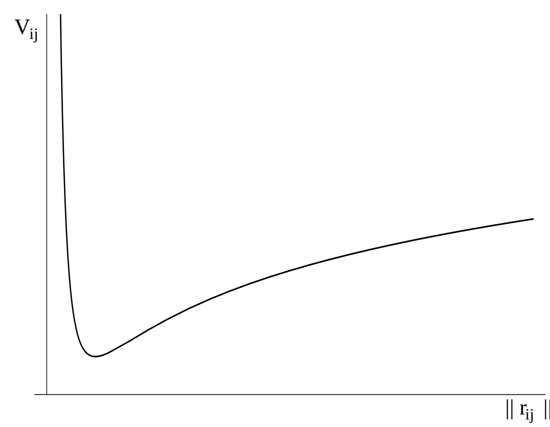

Fig. 2. The artificial potential between two vehicles.

Let the control law for vehicle i be given as:

$$a_i = -(\nabla_{r_i} V_i)_x \cos\theta_i - (\nabla_{r_i} V_i)_y \sin\theta_i \tag{2a}$$

$$\omega_i = -k \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{\theta_i - \theta_j}{\|r_{ij}\|^2} + (\nabla_{r_i} V_i)_x \sin\theta_i - (\nabla_{r_i} V_i)_y \cos\theta_i. \tag{2b}$$

with k a constant positive parameter.

III. Closed Loop Stability

Let the neighboring relations be represented by edges in the following graph:

Definition III.1 *The neighboring graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{W}\}$ is a weighted graph consisting of:*

- *a set of vertices $\mathcal{V}$ indexed by the set of mobile agents;*
- *a set of pairs $\mathcal{E} = \{\varepsilon_{ij} = (v_i, v_j) \mid v_i, v_j \in \mathcal{V},$ and i neighbor of $j\}$;*
- *a set of weights $\mathcal{W}$, over the set of edges, where each edge e_{ij} is labeled by the inverse squared distance between agents i and j, $\|r_{ij}\|^{-1}$.*

Assume an arbitrary orientation σ on $\mathcal{G}$. Let the oriented neighboring graph be denoted $\mathcal{G}^\sigma$ and consider its incidence matrix, B. Then the agent heading differences $e_{ij} \triangleq \theta_i - \theta_j$ can be expressed as:

$$e = B^T \theta \tag{3}$$

The dynamics of the (3) can be derived from (2b):

$$\dot{e} = -B^T \dot{\theta} = -B^T B W e + B^T F \tag{4}$$

where W is the diagonal matrix indexed by the edges in $\mathcal{G}$, containing the weights in $\mathcal{W}$ along its diagonal, and F is given as

$$F = \begin{bmatrix} \vdots \\ (\nabla_{r_{ij}} V_{ij})_x (\sin\theta_i - \sin\theta_j) - (\nabla_{r_{ij}} V_{ij})_y (\cos\theta_i - \cos\theta_j) \\ \vdots \end{bmatrix}.$$

Although $B^T B$ is positive semi-definite, the quadratic form $e^T B^T B e$ is strictly positive definite. This is because $e^T B^T B e = 0 \Leftrightarrow \|Be\| = 0 \Leftrightarrow e \in \mathcal{N}(B) \Leftrightarrow e \in \mathcal{R}(B^T)^\perp$, which, due to (3), only happens if $e = 0$. On the other hand, with V_{ij} Lipschitz in Ω, F will be sector bounded:

$$\|F\| \leq 2L_v \|e\|,$$

where L_v is the Lipschitz constant of V_{ij}. Therefore, for a sufficiently large k, (4) will be absolutely stable.

Consider the positive semi-definite function:

$$V_t = \frac{1}{2}\sum_{i=1}^N \Big(\sum_{\substack{j=1 \\ j\neq i}}^N V_{ij} + v_i^T v_i\Big) + \frac{1}{2} e^T e.$$

Due to V_i being symmetric with respect to r_{ij} and the fact that $r_{ij} = -r_{ji}$

$$\frac{\partial V_{ij}}{\partial r_{ij}} = \frac{\partial V_{ij}}{\partial r_i} = -\frac{\partial V_{ij}}{\partial r_j}, \tag{5}$$

which implies: $\frac{d}{dt}\sum_{i=1}^N \frac{1}{2} V_i = \sum_{i=1}^N \nabla_{r_i} V_i \cdot v_i$.

Let Ω be the set defined as:

$$\Omega \triangleq \{(r_{ij}, v_i, \theta_i) \mid V_t \leq c,\ i, j = 1, \ldots, N\},$$

which is nonempty for a sufficiently large choice of c, and closed by continuity of V_t. It is also bounded, because boundedness of V_t implies boundedness of all V_{ij}, which in turns implies the boundedness of every r_{ij}, since V_{ij} increases monotonically with r_{ij}. Bounds for v_i follow trivially and θ_i is always bounded in $[-\pi, \pi]$.

Proposition III.2 *The system of N boids with dynamics* (1) *steered by control laws* (2a)-(2b)*, with initial conditions in Ω, converges to one of the minima of V_t and a common orientation.*

Proof: Taking the time derivative of V_t:

$$\dot{V}_t = \sum_{i=1}^N \nabla_{r_i} V_i \cdot \dot{r}_i + \sum_{i=1}^N v_i a_i + e^T \dot{e} = e^T \dot{e} \tag{6}$$

Since (4) is LTI and absolutely stable, it means that there exists a sufficiently small $\epsilon > 0$ such that: $e\dot{e} \leq -\epsilon \|e\|^2$. Since $\dot{V}_t \leq 0$, Ω is positively invariant. Applying LaSalle's invariant principle on (1) in Ω we conclude that all trajectories converge to the largest invariant set in $\{(r_{ij}, v_i, \theta_i) \mid \dot{V}_t = 0,\ i, j = 1, \ldots, N\}$. Equality $\dot{V}_t = 0$ holds only at configurations where all agents have the same constant heading, $\theta_1 = \cdots = \theta_N = \bar{\theta}$. Let S_θ be the set where all orientations are the same:

$$S_\theta \triangleq \{(r_1, \theta_1, \ldots, r_N, \theta_N) \mid \theta_1 = \cdots = \theta_N = \bar{\theta}\}.$$

In this set, we have: $\tan\bar{\theta} = k = \frac{\dot{y}_i}{\dot{x}_i}$, $i = 1, \ldots, N$. Differentiating $\frac{\dot{y}_i}{\dot{x}_i} = k$ we get:

$$\frac{d}{dt}\left(\frac{\dot{y}_i}{\dot{x}_i}\right) = 0 \Rightarrow \frac{a_{y_i}}{a_{x_i}} = \frac{(\nabla_{r_i} V_i)_y}{(\nabla_{r_i} V_i)_x} = k.$$

This means that the potential force applied on i is aligned with its velocity, $\nabla_{r_i} V_i \cdot \dot{r}_i = \pm \|\nabla_{r_i} V_i\| \, |v_i|$. If $-\nabla_{r_i} V_i \cdot \dot{r}_i \leq 0$, then consider the function $V_{v_i} = \frac{1}{2} v_i^2$. Note that $\dot{V}_{v_i} = -\|\nabla_{r_i} V_i\| \, |v_i| \leq 0$, establishing the convergence of v_i to the largest invariant set in $S_{v_i} = \{(r_i, v_i) \mid \dot{V}_{v_i} = 0\}$. S_{v_i} contains configurations where either $\nabla_{r_i} V_i = 0$ or $v_i = 0$. The latter configurations are not invariant, unless $\nabla_{r_i} V_i = 0$, because in S_{v_i}, $\dot{v}_i = -\|\nabla_{r_i} V_i\|$. The system will thus converge to configurations where $-\nabla_{r_i} V_i = 0$, which are minima of V_i. If $-\nabla_{v_i} V_i \cdot \dot{r}_i > 0$, then $\dot{V}_i < 0$, so V_i will be monotonically decreasing, eventually reaching one of its minima. In any case, the system converges to a minimum of $\sum_i^N V_i$. ∎

Collision avoidance is guaranteed since in all configurations where $r_{ij} = 0$ for some $i, j \in \{1, \ldots, N\}$, the function V_t tends to infinity implying that these configurations lay in the exterior of any Ω with c bounded.

IV. Simulations

In this section we verify numerically the stability results obtained in the previous sections. The group is consisted of five autonomous agents with dynamics described by (1). The initial conditions (positions and velocities) were generated randomly within a ball of radius $R_0 = 10$[m]. Figures 3-8 show snapshots of the group motion during a simulation time period of 10 seconds. In these Figures the position of the agents are represented by (red) dots, connected to each other by (blue) line segments which represent the neighboring relations. The (green) trails left behind by the dots correspond to the agents paths. Figures 3-8 show how the headings of all agents asymptotically approach a common direction. The asymptotic convergent behavior of the relative heading differences is depicted in Figure 9. Figure 10 gives an enlarged picture of the shape of the group after the period of 10 simulation seconds. The agents tend to attain positions on the vertices of an inscribed polygon with 5 faces, which corresponds to a local minimum of the group potential function.

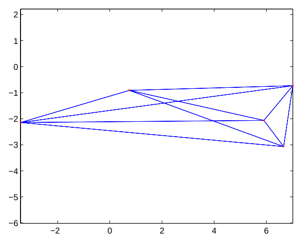

Fig. 3. Initial state.

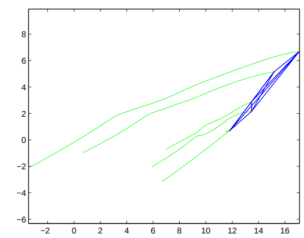

Fig. 4. Sim. time $t = 2$ sec.

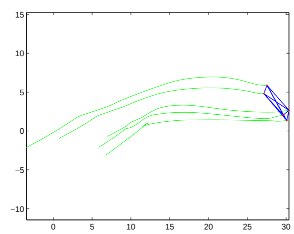

Fig. 5. Sim. time $t = 4$ sec.

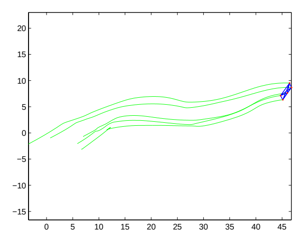

Fig. 6. Sim. time $t = 6$ sec

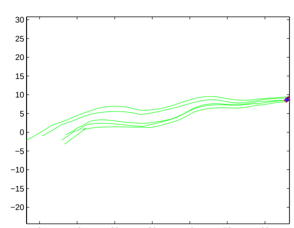

Fig. 7. Sim. time $t = 8$ sec

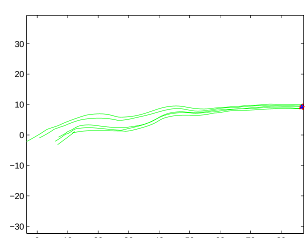

Fig. 8. Sim. time $t = 10$ sec

V. Conclusions

In this paper we proposed a set of control laws that give rise to stable flocking motion for a group of nonholonomic vehicles capable of sharing state information. In this way, we theoretically explained the flocking behavior observed in the animation models of [25]. The proof is based on the mechanics of a system of particles and the connectivity properties of the graph of intervehicle interconnections. The absolute stability properties of the heading dynamics indicate that the control scheme will be robust with respect

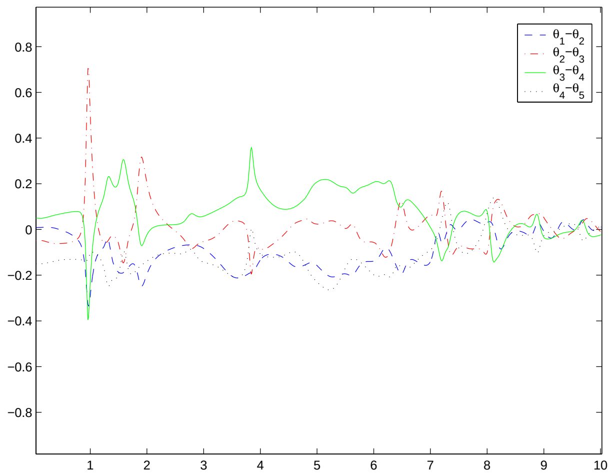

Fig. 9. Relative heading trajectories.

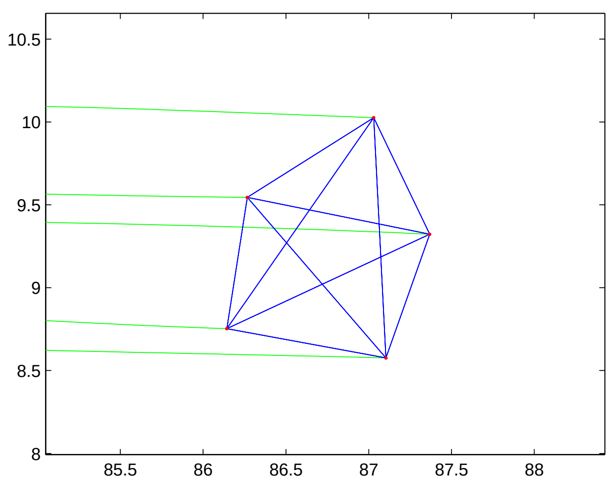

Fig. 10. Convergence of inter-agent distances.

to measurement or estimation errors. Research efforts are directed to quantification of such robustness properties and decentralization of the control laws to address the case where vehicles can share information with only a subset of their group.

References

[1] C. M. Breder. Equations descriptive of fish schools and other animal aggergations. *Ecology*, 35:361–370, 1954.

[2] D. E. Chang, A. M. Bloch, N. E. Leonard, J. E. Marsden, and C. A. Woosley. The equivalence of controlled lagrangian and controlled hamiltonian systems. *ESAIM: Control, Optimization and Calculus of Variations*, 8:393–422, 2002.

[3] F.H. Clarke, Yu.S. Ledyaev, R.J. Stern, and P.R. Wolenski. *Nonsmooth Analysis and Control Theory*. Graduate Texts in Mathematics; 178. Springer, New York, 1998.

[4] Frank H. Clarke. *Optimization and Nonsmooth Analysis*. Classics in applied mathematics; 5. SIAM, Philadelphia, 1990.

[5] A. Czirok, A. L. Barabasi, and T. Vicsek. Collective motion of self-propelled particles:kinetic phase transition in one dimension. *Physical Review Letters*, 82:209–212, 1999.

[6] M.R. Evans. Spontaneous symmetry breaking in a one dimensional driven diffusive system. *Physical Review Letters*, 74:208–211, 1995.

[7] J. Alexander Fax and Richard M. Murray. Graph Laplacians and stabilization of vehicle formations. *15th IFAC Congress, Barcelona, Spain*, 2002.

[8] A. F. Filippov. *Differential equations with discontinuous right-hand side*. Mathematics and Its Applications (Soviet Series). Kluwer Academic Publishers, The Netherlands, 1988.

[9] G. Flierl, D. Grunbaum, S. Levin, and D. Olson. From individuals to aggregations: the interplay between behavior and physics. *Journal of Theoretical Biology*, 196:397–454, 1999.

[10] V. Gazi and K. M. Polycarpou. Stability analysis of swarms. In *American Control Conference, Anchorage, Alaska*, 2002.

[11] C. Godsil and G. Royle. *Algebraic Graph Theory*. Springer Graduate Texts in Mathematics # 207, New York, 2001.

[12] Herbert Goldstein, Charles Poole, and John Safko. *Classical Mechanics*. Adisson Wesley, San Francisco, third edition edition, 2002.

[13] D. Grunbaum and A. Okubo. Modeling social animal aggregations. *Frontiers in Theoretical Biology*, 100 of Lecture Notes in Biomathematics:296–325, 1994.

[14] A. Jadbabaie, J. Lin, and A. S. Morse. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *Accepted for publication in IEEE Transactions on Automatic Control*, July 2002.

[15] M. Kardar, G. Parisi, and Y.-C. Zhang. Dynamic scaling of growing interfaces. *Physical Review Letters*, 56:889–892, 1986.

[16] N. Leonard and E. Friorelli. Virtual leaders, artificial potentials and coordinated control of groups. *IEEE Conference on Decision and Control*.

[17] H. Levine and W. J. Rappel. Self organization in systems of self-propelled particles. *Physical Review E*, 63:208–211, 2001.

[18] Y. Liu, K.M. Passino, and M. Polycarpou. Stability analysis of one-dimensional asynchronous swarms. In *American Control Conference, Arlington, VA*, pages 716–721, June, 2001.

[19] A. S. Mikhailov and D. Zannette. Noise induced breakdown of collective coherent motion in swarms. *Physical Review E*, 60:4571–4575, 1999.

[20] P. Ögren, M. Egerstedt, and X. Hu. A control Lyapunov function approach to multiagent coordination. *IEEE Transactions on Robotics and Automation*, 18, October 2002.

[21] A Okubo. Dynamical aspects of animal grouping: swarms, schools, flocks, and herds. *Advances in Biophysics*, 22:1–94, 1986.

[22] R. Olfati and R. M. Murray. Distributed cooperative control of multiple vehicle formations using structural potential functions. In *15th IFAC World Congress*, Barcelona, Spain, 2002.

[23] R. Olfati and R. M. Murray. Distributed structural stabilization and tracking for formations of dynamic multi-agents. In *IEEE Conference on Decision and Control*, Las Vegas, NV, 2002.

[24] Brad Paden and Shankar Sastry. A calculus for computing filipov's differential inclusion with application to the variable structure control of robot manipulators. *IEEE Transactions on Circuits and Systems*, CAS-34(1):73–82, 1987.

[25] C. Reynolds. Flocks, birds, and schools: a distributed behavioral model. *Computer Graphics*, 21:25–34, 1987.

[26] C. P. Schenk, P Schutz, M. Bode, and H.G. Purwins. Interaction of self organized quasi particles in a two dimensional reaction-diffusion system: The formation of molecules. *Physical Review E*, 58:6480–6486, 1998.

[27] Daniel Shevitz and Brad Paden. Lyapunov stability theory of nonsmooth systems. *IEEE Transactions on Automatic Control*, 39(9):1910–1914, 1994.

[28] N. Shimoyama, K. Sugawa, T. Mizuguchi, Y. Hayakawa, and M. Sano. Collective motion in a system of motile elements. *Physical Review Letters*, 76:3879–3873, 1996.

[29] P. Tabuada, G. J. Pappas, and P. Lima. Feasible formations of multi-agent systems. In *Proceedings of the American Control Conference*, Arlington, VA, 2001.

[30] D. Terzopoulos. Artificial life for computer graphics. *Communications of teh ACM*, 42(8):32–42, 1999.

[31] J. Toner and Y. Tu. Long range order in a two dimensional xy model: How birds fly together. *Physical Review Letters*, 75:4326–4329, 1995.

[32] J. Toner and Y. Tu. Flocks, herds, and schools: A quantitative theory of flocking. *Physical Review E.*, 58:4828–4858, 1998.

[33] T. Vicsek, A. Czirok, E. Ben Jacob, I. Cohen, and O. Schochet. Novel type of phase transitions in a system of self-driven particles. *Physical Review Letters*, 75:1226–1229, 1995.

[34] K. Warburton and J. Lazarus. Tendency-distance models of social cohesion in animal groups. *Journal of Theoretical Biology*, 150:473–488, 1991.