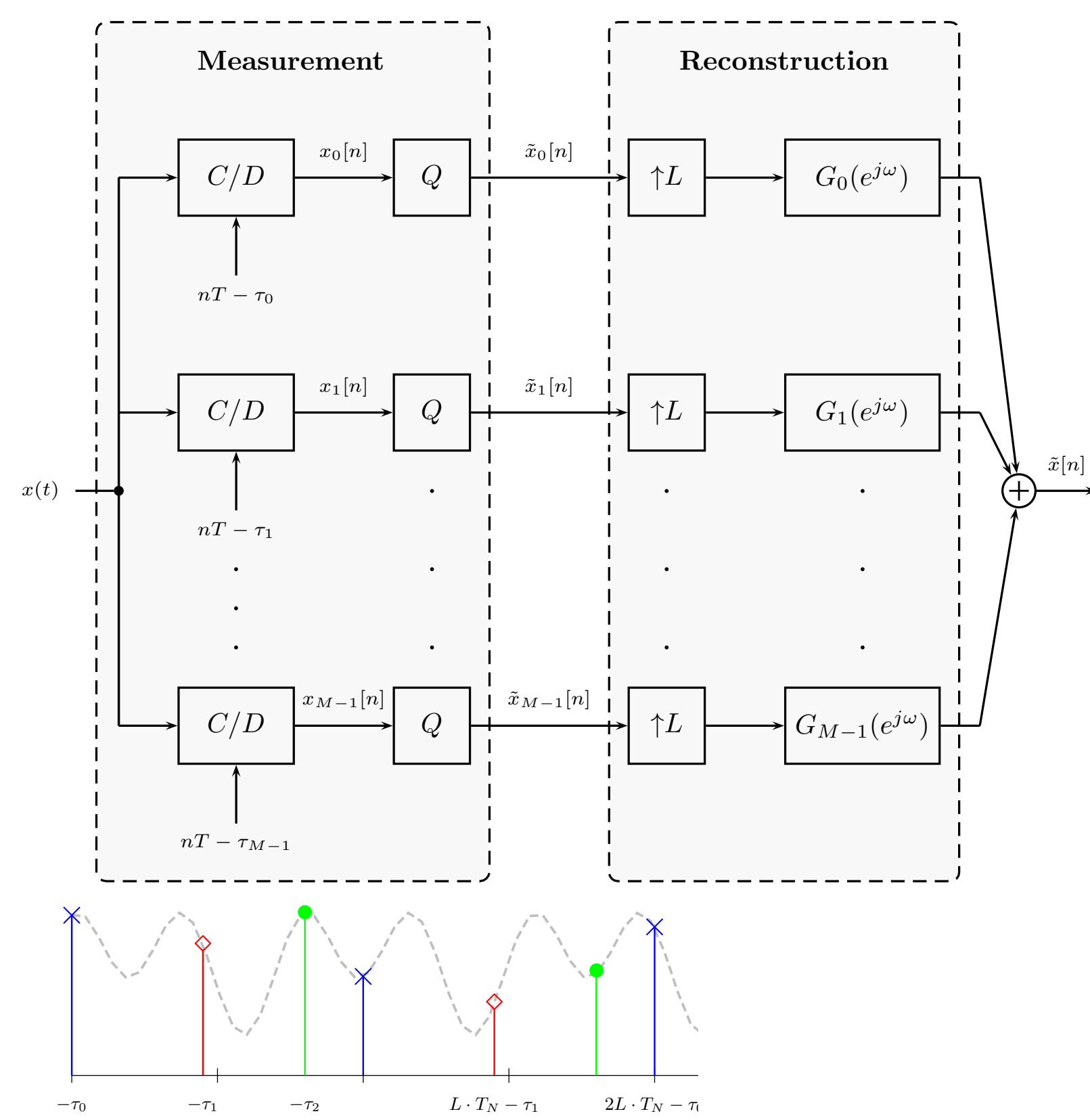
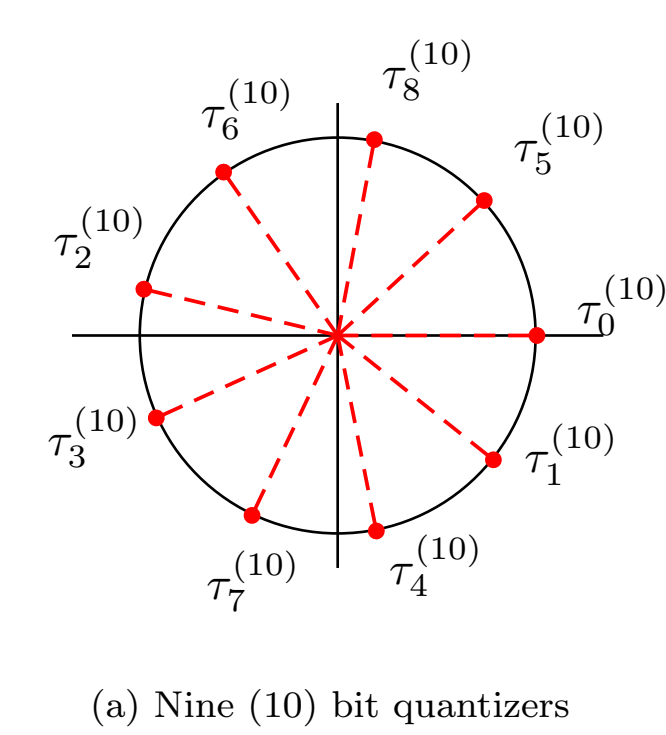


## 1. MOTIVATION

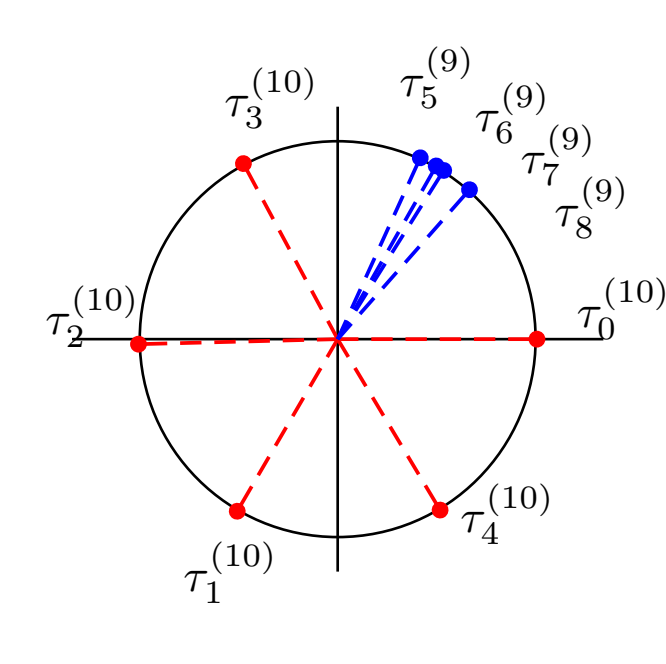
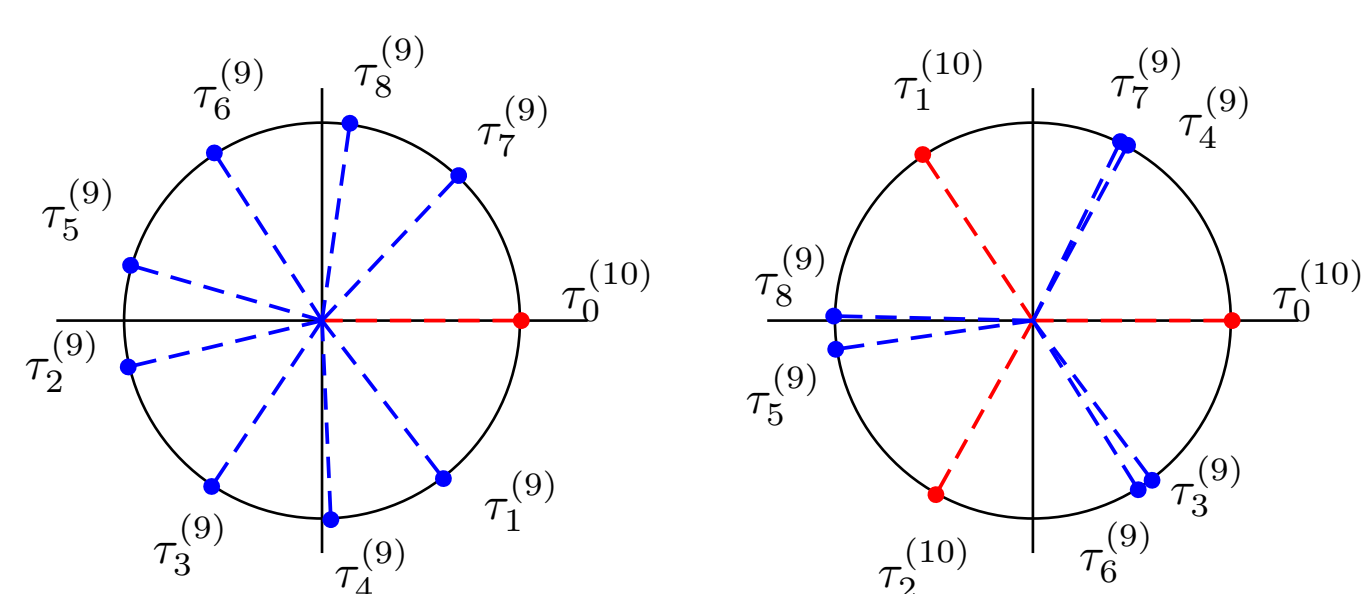
- Time-interleaved ADCs are useful for high bandwidth signals or large oversampling ratios.
- They consist of  $M$  channels that uniformly sample an input at one  $L^{\text{th}}$  of the Nyquist rate.
- DT reconstruction filters derived in [1] and [2] minimize quantization error power  $\sigma_e^2$ .
- Optimize timing offsets  $\tau_m$  and bit allocation  $b_m$  to minimize  $\sigma_e^2$ .



- Assume quantizer additive noise model holds.
- **Homogeneous bit allocation**  $\rightarrow$  uniform offsets



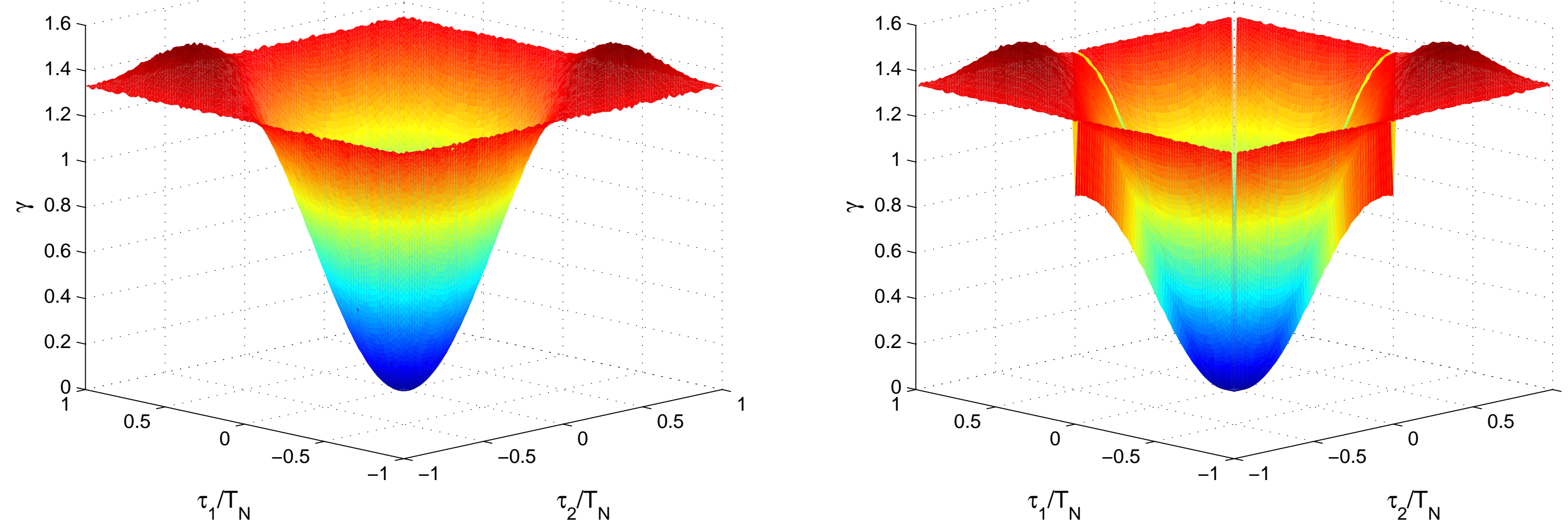
- **Diverse bit allocation**  $\rightarrow$  non-uniform offsets



- High-precision quant.  $\leftrightarrow$  large time separation
- Low-precision quant.  $\leftrightarrow$  small time separation

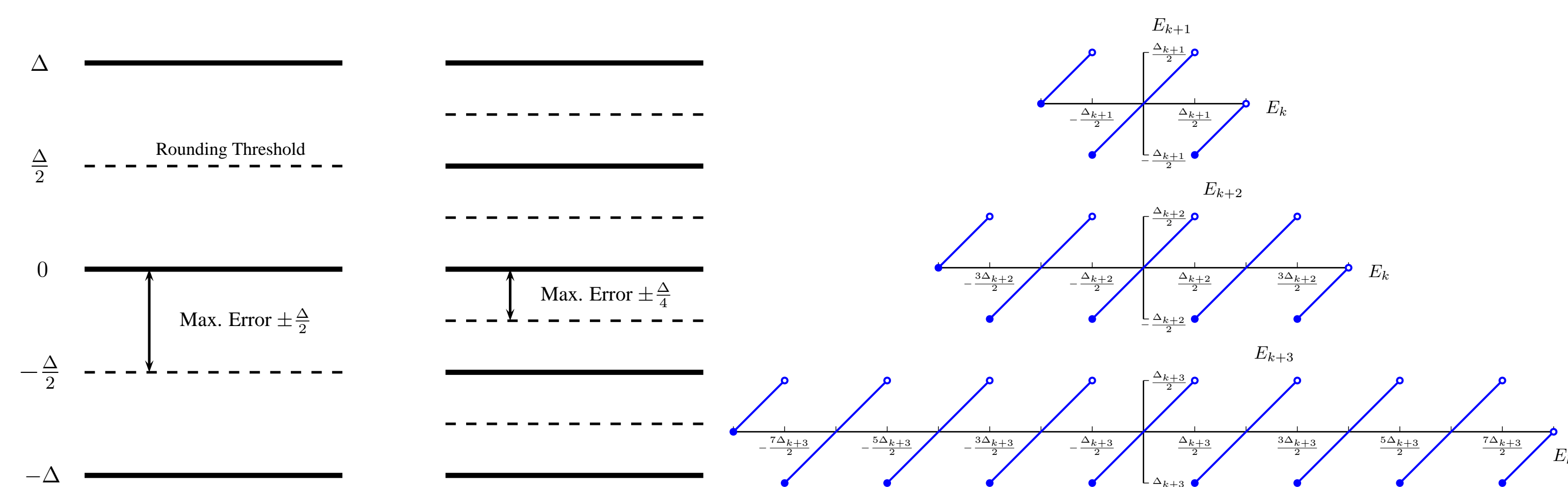
## 3. VALIDITY OF ADDITIVE NOISE MODEL

- In practice, quantization error in different channels may be correlated.
- Close timing offsets for identical quantizers  $\rightarrow$  degraded performance
- Close timing offsets for different quantizers  $\rightarrow$  improved performance



- Simulated reduction factor  $\gamma = \frac{\sigma^2}{\sigma_{e_{min}}^2}$  for noise model (left) and uniform quantizers (right) with  $M = 3$ ,  $L = 2$ , and  $\tau_0 = 0$ .

- Analyze two quantizers with  $k$  and  $k+n$  bits operating on same analog sample.
- Deterministic relationship between quantizer error for different bits.



- Assume additive noise model holds for  $k$  bit quantizer.
- Covariance proportional to area under set of piecewise quadratic curves.

$$\text{cov}(E_k, E_{k+n}) = \frac{1}{2^n \Delta_{k+n}} \int_{-2^{n-1} \Delta_{k+n}}^{2^{n-1} \Delta_{k+n}} e_k E_{k+n}(e_k) de_k \quad (1)$$

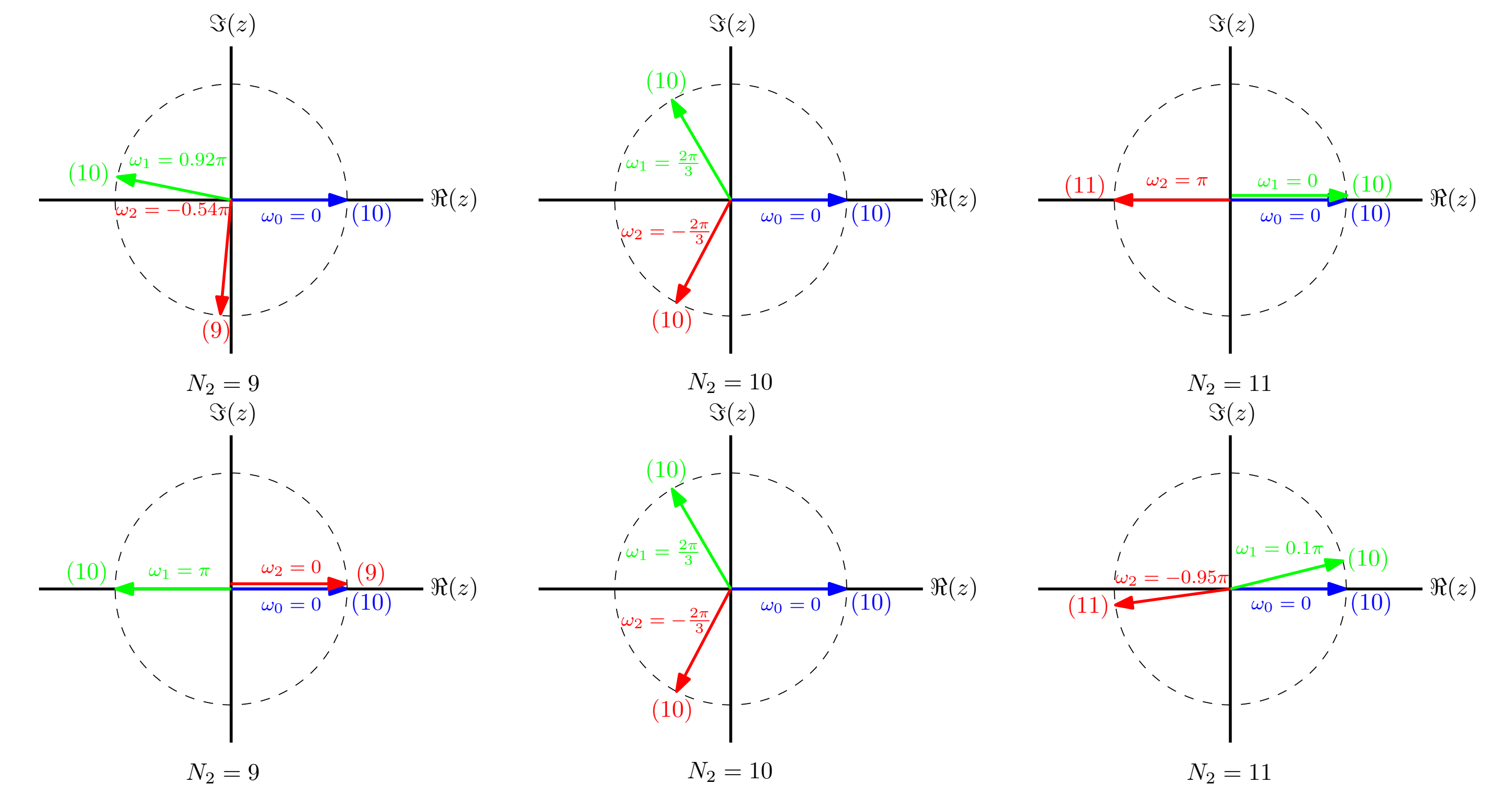
$$= \frac{\Delta_{k+n}^2}{12} \left( \frac{1}{2^n} + \left(1 - \frac{1}{2^{n-1}}\right) + \left(1 - 3 \cdot 2^{n-1}\right) \left(\frac{1}{2^n}\right) \right) \quad (2)$$

$$= (-1/2) \frac{\Delta_{k+n}^2}{12} \quad (3)$$

$$\text{cov}(E_k, E_{k+n}) = (-1/2) \text{var}(E_{k+n})$$

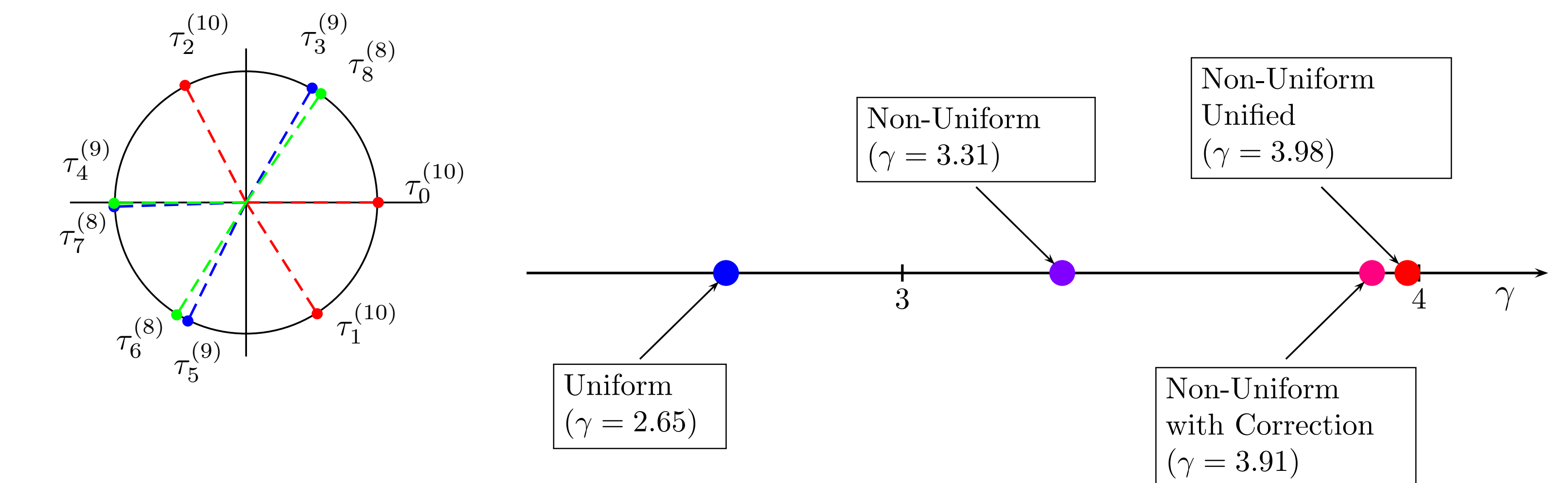
## 5. EFFECT OF CORRELATION

- Consider the case when  $M = 3$ ,  $L = 2$ , and  $\mathbf{b} = [10 \ 10 \ N_2]$ .
- Optimal offsets for noise model (top) and quantizers (bottom) shown below.



1. Optimize predicted timing offsets with additive noise model.
2. Separate identical quantizers in close proximity.
3. Unify different granularity quantizers in close proximity (WLS).

- Consider the case when  $M = 9$ ,  $L = 6$ , and  $\mathbf{b} = [10 \ 10 \ 10 \ 9 \ 9 \ 8 \ 8 \ 8]$ .
- Optimal timing offsets with additive noise model are shown below.



- Improvement of 16.8% in  $\sigma_e^2$  vs. previously best known non-uniform offsets.
- Improvement of 33.4% in  $\sigma_e^2$  vs. previously best known uniform offsets.

- Additive noise model matches quantizer error well in most situations.
- Exploiting cross-channel error correlation can improve performance.

[1] S. Maymon and A.V. Oppenheim. Quantization and compensation in sampled interleaved multi-channel systems. In *ICASSP*, 2010.  
[2] S. Maymon. Sampling and Quantization for Optimal Reconstruction. PhD thesis, Massachusetts Institute of Technology, 2011.