1 Base of the Model

The aircraft model is originally shown in the papers of Lewis, et al. [1] and in the NASA report of Mistree, et al. [2]. In this model, there are two subsystem designers: (i) one subsystem designer optimizes the weight of the aircraft and (ii) the other optimizes its aerodynamics. The key equations are the same as described in Lewis and Mistree’s previous work, but one constant \( b_t \) and weighting for objectives are different from their choices.

2 Common Constants

These are common constants shared between two subsystems.

\[
(c_L)_{max} = 2.6
\]

\[ N_e = \text{number of engines} = 3 \]

\[ b_t = \text{TSFC} = 9.7222 \times 10^{-5} \]

\[ R = \text{range} = 2900 \text{ nmi} = 1.7624 \times 10^7 \text{ ft} \]

3 Aerodynamics Player

3.1 Variables

1. \( W_{TO} \): Take Off Weight

2. \( T_i \): Installed Thrust
3. S: Wing Area

4. \( R_{fr} \): Fuel Weight Required

5. \( l \): Fuselage Length

6. \( b \): Wing Span

### 3.2 Constants

\[
\frac{t}{c} = \text{thickness to cord} = 0.12
\]

\( N_p = \text{number of passengers} = 188 \)

\( \rho_{TO-L} = \text{density take off and landing} \ (\text{@ sea level}) = 2.378 \times 10^{-3} \text{slugs/ft}^3 \)

\( \rho_{cr} = \text{density cruise} = 7.37 \times 10^{-4} \text{slugs/ft}^3 \)

\( \nu_{TO-L} = \text{kinematic viscosity take off and landing} \ (\text{@ sea level}) = 1.56 \times 10^{-4} \text{ft}^2/\text{s} \)

\( \nu_{cr} = \text{kinematic viscosity cruise} = 4.06 \times 10^{-4} \text{ft}^2/\text{s} \)

\( V_{TO-L} = \text{Velocity take off and landing} = 220 \text{ ft/s} \)

### 3.3 Equation A: Zero Lift Drag Coefficient

\[
c_D = \frac{(c_{DO})_{wing}}{1.1c_{f,\text{wing}}(1+1.2\left(\frac{t}{c}\right)^4)} + \Delta c_{D0}
\]

\[
\ldots + \frac{(c_{DO})_{body}}{c_{f,\text{body}}(1+0.0025\left(\frac{t}{c}\right)^3)^2} + \Delta c_{D0}
\]

\[
c_f = \text{skin friction coefficient} = 0.455 \left( \log_{10} \frac{V_{ref} \rho_{ref}}{\nu} \right)^{-2.58}
\]

**Note:**

1.

\[
V_{ref} = \begin{cases} 
V_{TO-L} & \text{for take off and landing} \\
V_{cr} & \text{for cruise} 
\end{cases}
\]
2. 

\[ l_{ref} = \begin{cases} 
    l & \text{fuselage length - for body} \\
    c & \text{cord - for wing} 
\end{cases} \]

3. 

\[ S_S = \frac{S_f}{S} = \text{fuselage wetted surface ratio} \]

\[ S_S = \frac{\pi dl}{S} \left( 1 - 2 \left( \frac{d}{l} \right) \right) \left( 1 + \left( \frac{d}{l} \right)^2 \right) \]

4. 

\[ d = \text{fuselage diameter} = 1.83 \left( 1 + 4.325 \frac{N_p}{l} \right) \]

3.4 **Equation B: Lift-to-Drag Ratio**

\[ \frac{L}{D} = \frac{c_L}{c_{D0} + kc_L^2} \]  

(1)

**Note:**

1. \( k = \frac{1}{\pi AR e} \) = Quadratic Drag Polar

2. \( e = 0.96 \left( 1 - \left( \frac{d}{d_b} \right)^2 \right) \) = Oswalt Factor

3. \( AR = \frac{b^2}{S} \)

3.5 **Equation C: Lift Coefficient**

\[ c_L = \begin{cases} 
    \frac{2W_{TO}}{\rho V^2 S} \\
    \frac{2W_{TO} (1 - R_{fr})}{\rho V^2 S} 
\end{cases} \]  

(2)

3.6 **Equation D: Velocities**

\[ V_{cr} = V_{br} = V_{\text{best range}} = \sqrt{\frac{2W_{TO}}{\rho S \sqrt{\frac{2W_{TO}}{\rho S}}}} \]  

(3)

3.7 **Equation E: Landing Field Length**

\[ S_L = \text{Landing Field Length} = 400 + 118 \frac{W_{TO}(1 - R_{fr})}{c_{L,max} S} \]  

(4)
3.8 Equation F: Take-Off Field Length

\[ S_{TO} = \text{Take off Field Length} = \frac{20.9 W_T^2}{c_{L,max} S T_i} + 87 \sqrt{\frac{W_{TO}}{c_{L,max} S}} \]  \hspace{1cm} (5)

where \( T_i \) is the installed thrust.

3.9 Equation G: Approach Climb Gradient

\[ q_L = \text{missed approach climb gradient} = - \left( \frac{L}{D} \right)_L^{-1} + \frac{N_e - 1}{N_e} \frac{T_i}{W_{TO} (1 - R_{fr})} \]  \hspace{1cm} (6)

3.10 Equation H: Take-Off Climb Gradient

\[ q_{TO} = \text{achievable climb gradient at take off} = - \left( \frac{L}{D} \right)_{TO}^{-1} + \frac{N_e - 1}{N_e} \frac{T_i}{W_{TO}} \]  \hspace{1cm} (7)

3.11 Constraints

3.11.1 Continuous

1. \( AR \leq 10.5 \)

2. \( q_L \geq 2.4\% \)

3. \( q_{TO} \geq 2.7\% \)

4. \( S_L \leq 4500 \text{ft} \)

5. \( S_{TO} \leq 6500 \text{ft} \)

6. \( c_{D0,TO - L} \leq 0.02 \)

7. \( c_{D0,CT} \leq 0.02 \)

3.11.2 Discrete

1. \( 1200 \leq S \leq 2500 \)

2. \( 105 \leq l \leq 150 \)

3. \( 85 \leq b \leq 140 \)
4. \( 10 \leq d \leq 20 \)

### 3.12 Target

1. \( \delta_1^A = | \frac{q_L}{1000} - 1 | \)
2. \( \delta_2^A = | \frac{TO}{1000} - 1 | \)
3. \( \delta_3^A = | \frac{S_L}{1000} - 1 | \)
4. \( \delta_4^A = | \frac{AR}{1000} - 1 | \)
5. \( \delta_5^A = | \frac{AR}{1000} - 1 | \)

### 3.13 Weighting

1. \( P_{1}^A = 1 \)
2. \( P_{2}^A = 4 \)
3. \( P_{3}^A = 3 \)
4. \( P_{4}^A = 2 \)
5. \( P_{5}^A = 5 \)

### 3.14 Objective

Find the minimum of:

\[
Z_A = \sum_i P_i^A \delta_i^A
\]

### 4 Weights Player

#### 4.1 Variables

1. \( W_{TO} \): Take Off Weight
2. \( T_i \): Installed Thrust

3. \( S \): Wing Area

4. \( \frac{L}{D}_{TO} \): Lift-to-Drag Ratio at Take-Off

5. \( \frac{L}{D}_{Cr} \): Lift-to-Drag Ratio at Cruise

6. \( \frac{L}{D}_L \): Lift-to-Drag Ratio at Landing

7. \( V_{cr} \): Cruise Speed

### 4.2 Constants

\[
W_{pay} = \text{payload} = 40000 \text{ lbs}
\]
\[
W_{fix} = \text{equipment Weight} = 1100 \text{ lbs}
\]

### 4.3 Equation A: Fuel Available

\[
R_{fa} = \text{fuel available} = 1 - \frac{W_{pay} + W_{fix} + W_{empty}}{W_{TO}}
\]

Note:

1. \[
\frac{W_{empty}}{W_{TO}} = \text{Empty Weight Ratio} = \frac{0.9592}{W_{TO}} + 0.38 \left( 1 - 0.95 \frac{W_L}{W_{TO}} \right)
\]

2. \[
R_{fr} = \text{Fuel Ratio Required} = 1.1 \left( 1 - 0.95 \frac{W_L}{W_{TO}} \right)
\]

3. \[
R_f = \text{Fuel Balance} = \frac{R_{fa}}{R_{fr}}
\]

### 4.4 Equation B: Ratio of Take-Off and Landing Weight

\[
\frac{W_{TO}}{W_L} = \exp \left( \frac{R_{by}}{V_{cr} \left( \frac{L}{D} \right)_{cr}} \right)
\]
4.4.1 Equation C: Landing Field Length

\[ S_L = \text{Landing Field Length} \]
\[ = 400 + \frac{118 W_{TO}(1 - R_{fr})}{c_{L,max} S} \]

4.5 Equation D: Take-Off Field Length

\[ S_{TO} = \text{Take Off Field Length} \]
\[ = 20.9 \frac{W_{TO}^2}{c_{L,max} S T_i} + 87 \sqrt{\frac{W_{TO}}{c_{L,max} S}} \]

4.6 Equation E: Useful Load Fraction

\[ U = \text{Useful Load Fraction} \]
\[ = 1.1 \left( 1 - 0.95 \exp \left( - \frac{b_i R}{(L/D)_{cr} V_{cr}} \right) \right) + \frac{W_{pay}}{W_{TO}} \]

4.7 Equation F: Available Climb Gradient at Landing

\[ q_L = \text{Achievable Climb Gradient @ Landing} \]
\[ = - \left( \frac{L}{D} \right)^{-1}_L + \frac{N_e - 1}{N_e} \frac{T_i}{W_{TO}(1 - R_{fr})} \]

4.8 Equation G: Achievable Climb Gradient at Take-Off

\[ q_{TO} = \text{Achievable Climb Gradient @ Take Off} \]
\[ = - \left( \frac{L}{D} \right)^{-1}_{TO} + \frac{N_e - 1}{N_e} \frac{T_i}{W_{TO}} \]

4.9 Equation H: Productivity Index

\[ PRI = \frac{V_{cr} W_{pay}}{(1 - R_{fa} + R_{fr}) W_{TO} - W_{pay} - W_{fix}} \]

4.10 Constraints

4.10.1 Continuous

1. \( U \geq 0.3 \)

2. \( R_f \geq 1.0 \)

3. \( q_L \geq 2.4 \)
4. \( q_{TO} \geq 2.7 \)

5. \( S_L \leq 4500\text{ft} \)

6. \( S_{TO} \leq 6500\text{ft} \)

4.10.2 Discrete

1. \( 27750 \leq T_i \leq 55000\text{lbs} \)

2. \( 140000 \leq W_{TO} \leq 250000\text{lbs} \)

4.11 Target

1. \( \delta_{W1} = \left| \frac{PRI_{270}}{270} - 1 \right| \)

2. \( \delta_{W2} = \left| \frac{U_0}{U_{0.5}} - 1 \right| \)

3. \( \delta_{W3} = \left| R_f - 1 \right| \)

4. \( \delta_{W4} = \left| \frac{q_L}{0.03} - 1 \right| \)

5. \( \delta_{W5} = \left| \frac{q_{TO}}{0.03} - 1 \right| \)

6. \( \delta_{W6} = \left| \frac{S_L}{4500} - 1 \right| \)

7. \( \delta_{W7} = \left| \frac{S_{TO}}{4500} - 1 \right| \)

4.12 Weighting

1. \( P_{A1} = 1 \)

2. \( P_{A2} = 6 \)

3. \( P_{A3} = 5 \)

4. \( P_{A4} = 4 \)

5. \( P_{A5} = 3 \)
6. \( P_4^A = 2 \)

7. \( P_5^A = 7 \)

### 4.13 Objective

Find the minimum of:

\[
Z_W = \sum_i P_i^W \delta_i^W
\]

### References
