

# AN EXPLORATION OF THE ROLE OF SYSTEM LEVEL VARIABLE CHOICE IN MULTIDISCIPLINARY DESIGN

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The process of designing large engineering systems can involve extensive sharing of resources among many competing subsystems. These resources may be represented as design parameters that are shared between the system level and each subsystem. Frameworks for understanding the role of the choice of system level design variables on overall system design may be valuable for the synthesis of complex engineering systems. This paper examines the outcome of Multidisciplinary Design Optimization (MDO) using three distinct combinations of system level variables on a satellite design example. The results of these sets of system level variables are compared on their convergence time and robustness.

## Nomenclature

|               |   |  |
|---------------|---|--|
| $GR$          | = | Ground Resolution  |
| $h$           | = | Altitude   |
| $\Delta V$    | = | Total Change in Velocity Required by Satellite   |
| $M_{pl}$      | = | Mass of Payload  |
| $P_{pl}$      | = | Power Required by Payload  |
| $M_{pow}$     | = | Mass of Power Subsystem  |
| $M_{prop}$    | = | Mass of Propellant   |
| $M_{thrust}$  | = | Mass of Thruster   |
| $M_{tot}$     | = | Total System Mass (Loaded Mass)  |
| $\phi_\alpha$ | = | Objective Function for Subsystem $\alpha$ where $\alpha \in \{orbital, payload, power, propulsion\}$ |

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|-------------------------------|--|
| $f_1, \tilde{f}_1, \hat{f}_1$ | = Objective Functions for Combined Orbital and Payload Subsystem <sup>††</sup> |
| $bi, \tilde{bi}, \hat{bi}$    | = System Level Design Variable for MDO Formulations <sup>††</sup>              |
| $zi, \tilde{zi}, \hat{zi}$    | = Uncoupled Subsystem Design Objectives for MDO Formulations <sup>††</sup>     |
| $zic, \tilde{zic}, \hat{zic}$ | = Coupled Subsystem Design Objectives for MDO Formulations <sup>††</sup>       |
| $sic, \tilde{sic}, \hat{sic}$ | = Slack Variables for MDO Formulations <sup>††</sup>                           |
| $\eta$                        | = Dynamic Step Size for MDO  |
| $\gamma$                      | = Parameter to Trade System Level Design Objectives.                           |

## I. Introduction

The design and development of large-scale, complex engineering systems demands thoughtful approaches to balancing trade-offs among subsystems. Subsystems are often in competition with each other for a limited set of important system level resources, such as cost, power, and mass. In a large engineering system, there can be a staggering number of variables under consideration because of the range of subsystem variables that must also be included. In modeling and optimizing such systems, it becomes the case that there are multiple possible variables that can be selected to be system level variables. The question this paper investigates is: Do different combinations of system level variables product different outcomes in system simulations?

This paper considers this problem from the point of the view of a design team with the goal of facilitating the process of design. This work utilizes a bilevel formulation for Multidisciplinary Design Optimization (MDO) to simulate ideal trade-off scenarios for the system designer. MDO represents a scenario that a systems level facilitator analyzes for potential trade-offs between subsystems and then determines how resources should be allocated. Typically, this process begins when a system designer allocates resources to all subsystems. Then each subsystem is optimized using the allocated resources, or else trade-offs are made to acquire the necessary resources. This process continues iteratively until it converges to a satisfactory solution.

## II. RELATED WORK

In order to facilitate communication between subsystem stakeholders, multiple frameworks have been created to model system and subsystem level communication and coordination. While MDO models include an all-at-once approach [1], we focus in this work on a model that operates upon a system decomposition structure.

Although centralization of decisions and models has distinct advantages, it is more commonplace in complex systems design to utilize a decomposition structure to centralize the design of complex systems. There are various approaches to determining the decomposition structure including object decomposition, aspect decomposition, sequential decomposition and model based decomposition [2]. Our approach follows a traditional space mission design approach and decomposes a system by discipline (i.e. each subsystem requires different expertise to design).

Once a decomposition structure is determined, then a communication and coordination model is necessary. This model will provide protocols and formulations for critical system solution mechanics including objective function formulation, intra- and inter-subsystem communication protocol, design variable control, and convergence conditions. There are a number of protocol models including Analytic Target Cascading [3], Concurrent Subspace Optimization (CSSO) [4], Bilevel Integrated System Synthesis (BLISS) [5] and Collaborative Optimization (CO) [6].

Analytic Target Cascading has been proven to guarantee that the distributed system converges and that the converged value is a globally optimal solution [3]. Additionally, its hierarchy allows for traceability of the design process and provides for integration of marketing and business systems while establishing clear relationships between design subsystems [7]. The main advantage of Collaborative Optimization is that it does not require system analysis, but multidisciplinary feasibility may not be satisfied. Thus, some intermediate designs could be infeasible. CSSO guarantees both individual and multidisciplinary feasibility at each iteration, but requires all disciplines to indirectly share all constraints. Unlike other MDO formulation, BLISS keeps common variables as constants at a lower level, while optimizes only common variables at an upper level. This BLISS formulation is most similar to how NASA/Jet Propulsion Laboratory's Team X [8, 9] designs aerospace mission optimization.

Other research [10] has examined optimal system structure from a system design perspective using socio-technical analysis by examining the modularity of systems using Design Structure Matrices [11] to determine the

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<sup>††</sup> Note that three different notations signify the three formulations studied in the paper.

"best" system structure. Hoyle, et al. [12] have studied variables in complex systems primarily at the subsystem level. Qiu, et al. [13, 14] have investigated system coordination from the stakeholder level through the lens of concurrent engineering. This paper seeks to fill a gap in understanding the role of system level variables. It examines variables at the system level, in between the stakeholder and subsystem levels of previous work. This work is less concerned with the modularity of variable sets than the impact of system variables on performance.

### III. METHODS AND A CASE STUDY

Three different MDO formulations are applied to a case study of a satellite design problem by varying the choices of system level design variables. This satellite design problem and subsystem models are adapted from the *Firesat* satellite example given in Wertz and Larson's Space Mission Analysis and Design (SMAD) [15], with the exception of a higher fidelity power subsystem model. Rather than implementing all 16 subsystems required for full scale Aerospace Mission Design as in Team X, only 4 key subsystems (Orbital, Payload, Power, and Propulsion) are implemented. It is assumed that other subsystems are parametric function of those 4 subsystems. Parameters such as inclination angle, initial altitude, and mission duration are treated as constants.

#### A. Individual Subsystem Models

##### 1. Orbital Subsystem

The orbital subsystem determines changes in velocity ( $\Delta V$ ) as a function of the operating altitude ( $h$ ).

$$\Delta V = \phi_{orb}(h) \quad (1)$$

The model assumes that this particular satellite uses coplanar orbital transfer with no orbit plane change. Finally, this  $\Delta V$  includes orbital transfer from initial orbit to operating orbit, altitude maintenance, and deorbit transfer. The objective of this subsystem is to determine an appropriate altitude that minimizes  $\Delta V$  given a particular satellite's image goals.

##### 2. Payload Subsystem

The payload of this *Firesat* satellite design captures infrared images of the Earth in order to determine locations of forest fires. Thus, the main objective for this design problem is to minimize the ground resolution of given a certain payload mass and power. The basic functionality of payload is:

$$[M_{pl}, P_{pl}] = \phi_{pl}(GR, h) \quad (2)$$

where  $M_{pl}$  is mass of payload,  $P_{pl}$  is power of payload,  $GR$  is ground resolution, and  $h$  is the operating altitude. This model assumes that the operating wavelength and the width of square detector are kept constant. Note that ground resolution is typically a design variable as well as a design objective for typical payload formulations. In other words, a typical payload designer optimizes the ground resolution or other image quality index while keeping mass and power within a certain design budget.

##### 3. Power Subsystem

In this example, the power subsystem is responsible for designing solar panels and the secondary battery. It is assumed that the power required by payload already includes a certain power margin for the payload. The power subsystem's objective is to minimize the mass of the subsystem while meeting a required power output and an eclipse condition:

$$M_{pow} = \phi_{pow}(P_{pl}, h) \quad (3)$$

where  $M_{pow}$  is mass of power subsystem. It may be surprising that the power subsystem requires operating altitude information. However, the altitude is needed to determine the average daylight and the maximum eclipse duration.

#### 4. Propulsion Subsystem

The propulsion subsystem determines the required propellant and thruster mass as a function of payload mass, power subsystem mass, and required  $\Delta V$ . The propulsion function is:

$$[M_{prop}, M_{thrust}] = \phi_{prop}(M_{pl}, M_{pow}, \Delta V) \quad (4)$$

where  $M_{prop}$  is the mass of the propellant and  $M_{thrust}$  is the mass of the thruster. Most initial satellite designs allocate a given mass for each subsystem as a percentage of initial payload mass. This model utilizes that factor and assumes that the mass of other subsystems are 128.6% of payload mass. A mass margin of 25% for the dry mass (mass excluding propellant) and 15% design margin for propellant mass have been included. The objective of the propulsion subsystem is to minimize the total mass of the system including these margin values.

Because the Propulsion subsystem requires mass data from all other subsystems, the output from this subsystem could also be the total system mass ( $M_{tot}$ ). For design optimization, this output removes the need to have a system engineer as an integration facilitator. Thus, a more appropriate Propulsion subsystem functionality can be given by:

$$M_{tot} = \tilde{\phi}_{prop}(M_{pl}, M_{pow}, \Delta V) \quad (5)$$

### B. Three Sets of System Level Variables.

The traditional approach for aerospace mission design involves the following steps [15]:

1. Determine orbital design (usually based on previous designs);
2. Design a payload given orbital choice;
3. Use payload and orbital design, optimize spacecraft bus;
4. If design is not satisfactory, revert back to step 1 or 2 and change orbital or payload design.

Given that the orbital and payload design are highly coupled, they have been combined into a single subsystem for this case study.

The most critical system attributes for this satellite design are image quality (ground resolution), total system mass (loaded mass), and cost. The system mass is critical in that the value of the total system mass is directly related to cost. In this study, the cost of the satellite is not considered because cost and reliability models for most subsystems were not available. Furthermore, cost is considered at the last phase of design at the Mission Design Laboratory (MDL) at NASA Goddard Space Flight Center [10]. Thus, cost is not traded during the typical engineering design phase. Therefore, the two objectives of this case study are the minimization of ground resolution and total system mass.

There are many possible MDO formulations for this satellite design problem. One key decision for implementing MDO is determining which design variables are most appropriate for sharing between a subsystem and the system designer. There are at least three logical choices for shared design variables for this case study that are both realistic and computationally tractable. One possible set of system level design variables are mass of payload, power required by payload, and total change in velocity ( $[M_{pl}, P_{pl}, \Delta V]$ ) and the other possible sets are ground resolution and altitude ( $[GR, h]$ ) and mass of payload, power required by payload, and altitude ( $[M_{pl}, P_{pl}, h]$ ). The effects of these three sets of possible system level design variables are explored in this case study.

#### 1. Case 1: MDO using Payload Mass, Payload Power, and Total Change in Velocity as System Design Variables.

For this particular system design variable choice, the Payload and Orbital subsystems are combined and redefined as follows:

$$[GR, h] = f_1(M_{pl}, P_{pl}, \Delta V) \quad (6)$$

To convert from  $\phi_{orb}$  and  $\phi_{pl}$  into  $f_1$ , the subsystem solves the following optimization problem.

Find  $GR$  and  $h$  that maximizes  $GR$  subject to following constraints:

$$\begin{aligned} [\tilde{M}_{pl}, \tilde{P}_{pl}] &= \phi_{pl}(GR, h) \\ \tilde{M}_{pl} &\leq M_{pl} \\ \tilde{P}_{pl} &\leq P_{pl} \\ \phi_{orb}(h) &\leq \Delta V \end{aligned} \tag{7}$$

Note that this  $f_1$  is a pseudo-inverse of  $\phi_{orb}$  and  $\phi_{pl}$  rather than inverse because there is non-unique set of  $[GR, h]$  for a  $[M_{pl}, P_{pl}, \Delta V]$  combination. The designer is picking the best  $GR$  out of the set of possible  $[GR, h]$  for a given  $[M_{pl}, P_{pl}, \Delta V]$ . Also note that  $f_1$  is not a bijection because the optimization problem is infeasible for many  $[M_{pl}, P_{pl}, \Delta V]$  combinations. However, this formulation does mimic the role of payload designers during the Satellite design. Also, unlike some of its traditional MDO formulations, only information regarding coupled variables is passed between the system and its subsystems. In other words, the subsystems are responsible for the determination of best design choices in the cases of uncoupled design variables such as propellant type, solar cell type, and second battery materials.

Given  $f_1$ ,  $\phi_{orb}$ , and  $\phi_{pl}$ , we can formulate the MDO problem using Park's notation [16] as below:

Find  $[b1, b2, b3]$  (where  $b1 = M_{pl}$ ,  $b2 = P_{pl}$ , and  $b3 = \Delta V$ ) that minimizes

$$f(z_1, z_3) = \gamma \frac{z_1}{z1_o} + (1 - \gamma) \frac{z_3}{z3_o} \tag{8}$$

subject to

$$\begin{aligned} [z1, z1c] &= f_1(b1, b2, b3) \\ z2c &= f_2(b2, s1c) = \phi_{pow}(b2, s1c) \\ z3 &= f_3(b1, b3, s2c) = \tilde{\phi}_{pl}(b1, b3, s2c) \\ [b1, b2, b3] &\geq 0 \\ [z1, z1c, z2c, z3] &\geq 0 \\ z1c &= s1c \\ z2c &= s2c \end{aligned} \tag{9}$$

where  $z1 = GR$ ,  $z1c = h$ ,  $z2c = M_{pow}$ ,  $z3 = M_{tot}$ ,  $z1_o$  is initial ground resolution, and  $z3_o$  is initial total system mass. Also,  $\gamma \in [0, 1]$  is a trade-off parameter between ground resolution and total system mass. (Note that  $z1$  represents an uncoupled objective and  $z1c$  represents a coupled objective). Finally,  $s1c$  and  $s2c$  represent slack variables to minimize direct communication between subsystems.

The above optimization can be solved using an iterative linearized optimization scheme to find a local optima. The linearized optimization problem for each iteration can be written as follows.

Find  $[\Delta b1, \Delta b2, \Delta b3]$  that minimizes

$$\Delta f = \frac{\gamma}{z1_o} \Delta z1 + \frac{(1 - \gamma)}{z3_o} \Delta z3 \tag{10}$$

subject to

$$\begin{aligned}
\Delta z1 &= \frac{\partial(f_1)_1}{\partial b1} \Delta b1 + \frac{\partial(f_1)_1}{\partial b2} \Delta b2 + \frac{\partial(f_1)_1}{\partial b3} \Delta b3 \\
\Delta z1c &= \frac{\partial(f_1)_2}{\partial b1} \Delta b1 + \frac{\partial(f_1)_2}{\partial b2} \Delta b2 + \frac{\partial(f_1)_2}{\partial b3} \Delta b3 \\
\Delta z2c &= \frac{\partial f_2}{\partial b2} \Delta b2 + \frac{\partial f_2}{\partial s1c} \Delta s1c \\
\Delta z3 &= \frac{\partial f_3}{\partial b1} \Delta b1 + \frac{\partial f_3}{\partial b3} \Delta b3 + \frac{\partial f_3}{\partial s2c} \Delta s2c \\
z1c + \Delta z1c &= s1c + \Delta s1c \\
z2c + \Delta z2c &= s2c + \Delta s2c \\
|\Delta bi| &\leq \eta bi
\end{aligned} \tag{11}$$

where  $i = 1, 2, 3$  and  $\eta$  is the dynamic step size.

To achieve the system optimality, the value of  $f$  is calculated after each linearized optimization and whenever  $f$  increases,  $\eta$  is halved. The information flow between subsystems for this formulation is shown in Fig. 1.

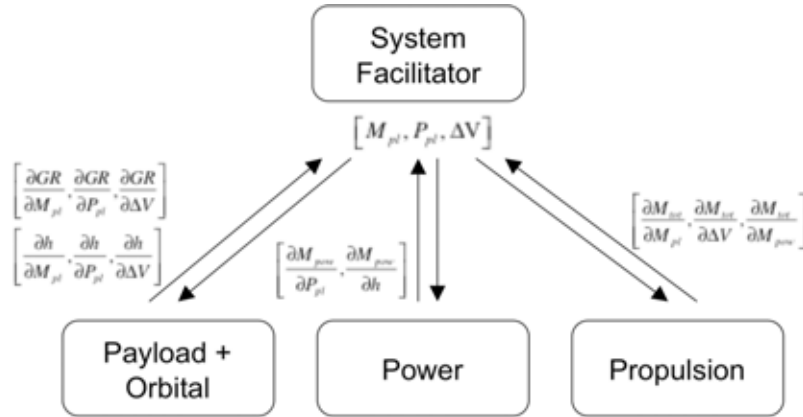


Figure 1. Information Flow between system designer and subsystem designers for 1<sup>st</sup> Case

## 2. Case 2: MDO using Ground Resolution and Altitude as System Design Variables.

In this formulation, ground resolution is considered as a system design variable as well as a design objective. One mathematical way to treat this phenomenon is to create an identity mapping between the design variable and the design objective for ground resolution. Then, the combined Payload and Orbital subsystem function becomes:

$$[GR, M_{pl}, P_{pl}, \Delta V] = \tilde{f}_1(GR, h) \tag{12}$$

where  $\tilde{f}_1$  utilizes  $\phi_{orb}$  and  $\phi_{pl}$  as well as an identity function that maps  $GR$  from the design variable space to the objective space to obtain all of the outputs. The MDO formulation for this system design variable set becomes:

Find  $[\tilde{b}1, \tilde{b}2]$  (where  $\tilde{b}1 = GR$  and  $\tilde{b}2 = h$ ) that minimizes

$$\tilde{f}(z_1, z_3) = \gamma \frac{\tilde{z}1}{z1_o} + (1-\gamma) \frac{\tilde{z}3}{z3_o} \quad (13)$$

subject to

$$\begin{aligned} [\tilde{z}1, \tilde{z}1c^1, \tilde{z}1c^2, \tilde{z}1c^3] &= \tilde{f}_1(\tilde{b}1, \tilde{b}2) \\ \tilde{z}2c &= \tilde{f}_2(\tilde{b}2, \tilde{s}1c^2) = \phi_{pow}(\tilde{s}1c^2, \tilde{b}2) \\ \tilde{z}3 &= \tilde{f}_3(\tilde{s}1c^1, \tilde{s}1c^3, \tilde{s}2c) = \phi_{pl}(\tilde{s}1c^1, \tilde{s}1c^3, \tilde{s}2c) \\ [\tilde{b}1, \tilde{b}2] &\geq 0 \\ [\tilde{z}1, \tilde{z}1c^1, \tilde{z}1c^2, \tilde{z}1c^3, \tilde{z}2c, \tilde{z}3] &\geq 0 \\ \tilde{z}1c^1 &= \tilde{s}1c^1 \\ \tilde{z}1c^2 &= \tilde{s}1c^2 \\ \tilde{z}1c^3 &= \tilde{s}1c^3 \\ \tilde{z}2c &= \tilde{s}2c \end{aligned} \quad (14)$$

where  $\tilde{z}1 = GR$ ,  $\tilde{z}1c^1 = M_{pl}$ ,  $\tilde{z}1c^2 = P_{pl}$ ,  $\tilde{z}1c^3 = \Delta V$ ,  $\tilde{z}3 = M_{tot}$ , and  $\tilde{s}ic$  are slack variables for their respective coupled subsystem objectives. Note that because there is more coupling between subsystems, this formulation may require more iterations to converge. This MDO formulation will be solved by linearizing in a similar manner as the first formulation. See Fig. 2 for the information flow between system facilitator and subsystem designers.

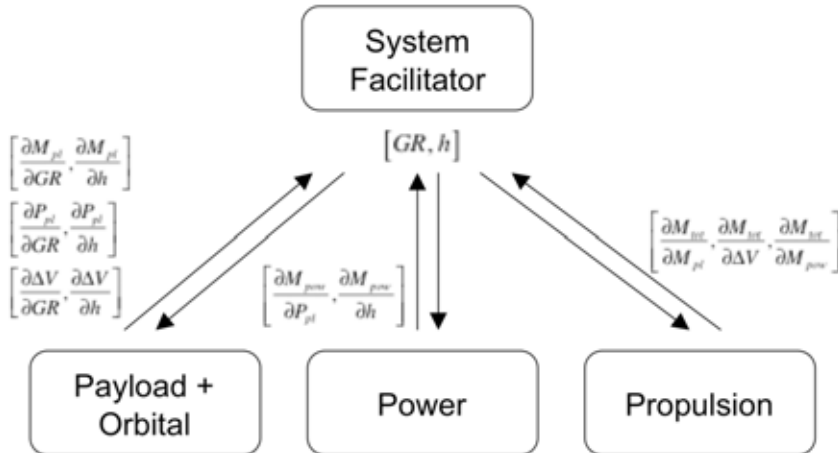


Figure 2. Information Flow between system designer and subsystem designers for 2<sup>nd</sup> Case

### 3. Case 3: MDO using Payload Mass, Payload Power, and Altitude as System Design Variables.

This formulation replaces  $\Delta V$  with altitude from the first MDO formulation. By changing from  $\Delta V$  to altitude  $h$ , only the propulsion subsystem will require slack variables. Thus, there may be less propagation of error due to linearization errors. In this formulation, the combined Payload and Orbital ( $\tilde{f}_1$ ) function will be modified to solve the following optimization problem and also to obtain  $\Delta V$  using the  $\phi_{orb}$  function. The information flow for this particular formulation is shown in Fig. 3.

Given  $[M_{pl}, P_{pl}, h]$ , find maximum  $GR$  that satisfies following constraints:

$$\begin{aligned}
[\tilde{M}_{pl}, \tilde{P}_{pl}] &= \phi_{pl}(GR, h) \\
\tilde{M}_{pl} &\leq M_{pl} \\
\tilde{P}_{pl} &\leq P_{pl}
\end{aligned} \tag{15}$$

Finally, the MDO formulation for this allocation of system variables becomes:

Find  $[\hat{b}1, \hat{b}2, \hat{b}3]$  (where  $\hat{b}1 = M_{pl}$ ,  $\hat{b}2 = P_{pl}$ , and  $\hat{b}3 = h$ ) that minimizes

$$\hat{f}(z_1, z_3) = \gamma \frac{\hat{z}1}{z1_o} + (1-\gamma) \frac{\hat{z}3}{z3_o} \tag{16}$$

subject to

$$\begin{aligned}
[\hat{z}1, \hat{z}1c] &= f_1(\hat{b}1, \hat{b}2, \hat{b}3) \\
\hat{z}2c &= f_2(\hat{b}2, \hat{b}3) = \phi_{pow}(\hat{b}2, \hat{b}3) \\
\hat{z}3 &= f_3(\hat{b}1, \hat{s}1c, \hat{s}2c) = \tilde{\phi}_{pl}(\hat{b}1, \hat{s}1c, \hat{s}2c) \\
[\hat{b}1, \hat{b}2, \hat{b}3] &\geq 0 \\
[\hat{z}1, \hat{z}1c, \hat{z}2c, \hat{z}3] &\geq 0 \\
\hat{z}1c &= \hat{s}1c \\
\hat{z}2c &= \hat{s}2c
\end{aligned} \tag{17}$$

where  $\hat{z}1 = GR$ ,  $\hat{z}1c = \Delta V$ ,  $\hat{z}2c = M_{pow}$ ,  $\hat{z}3 = M_{tot}$  and  $\hat{s}ic$  are slack variables for respective coupled subsystem objectives.

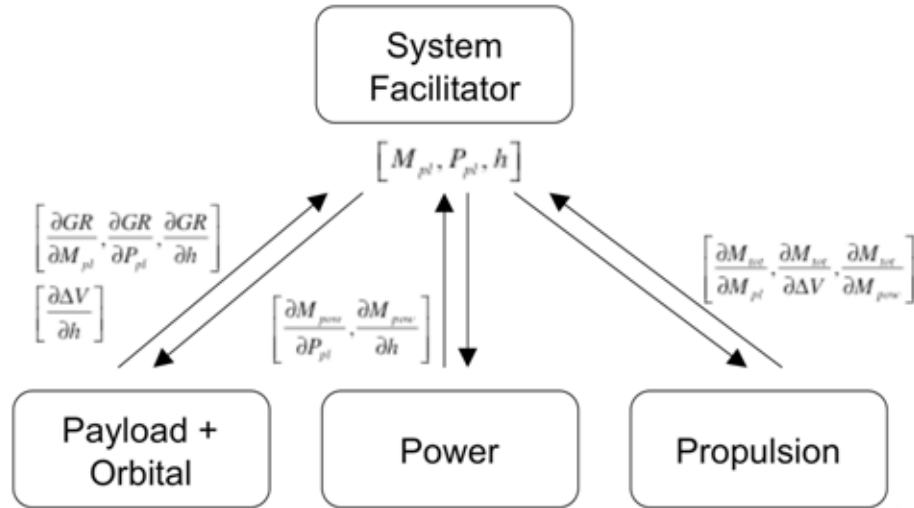


Figure 3. Information Flow between system designer and subsystem designers for 3<sup>rd</sup> Case

### C. MDO Implementation

There are many implementation schemes for MDO, especially for treating temporary infeasible solutions. In this particular case study, the convergence rate and region depends on the particular implementation scheme. The MDO implementation scheme is shown in Fig. 4. Note that the implementation of the MDO algorithm can be modified to improve convergence results for each case, but the aim of this study is to understand the impact of design variable



choices on the system design. Thus, the exploration of the effect of the particular choice of MDO implementation is outside the scope of this paper.

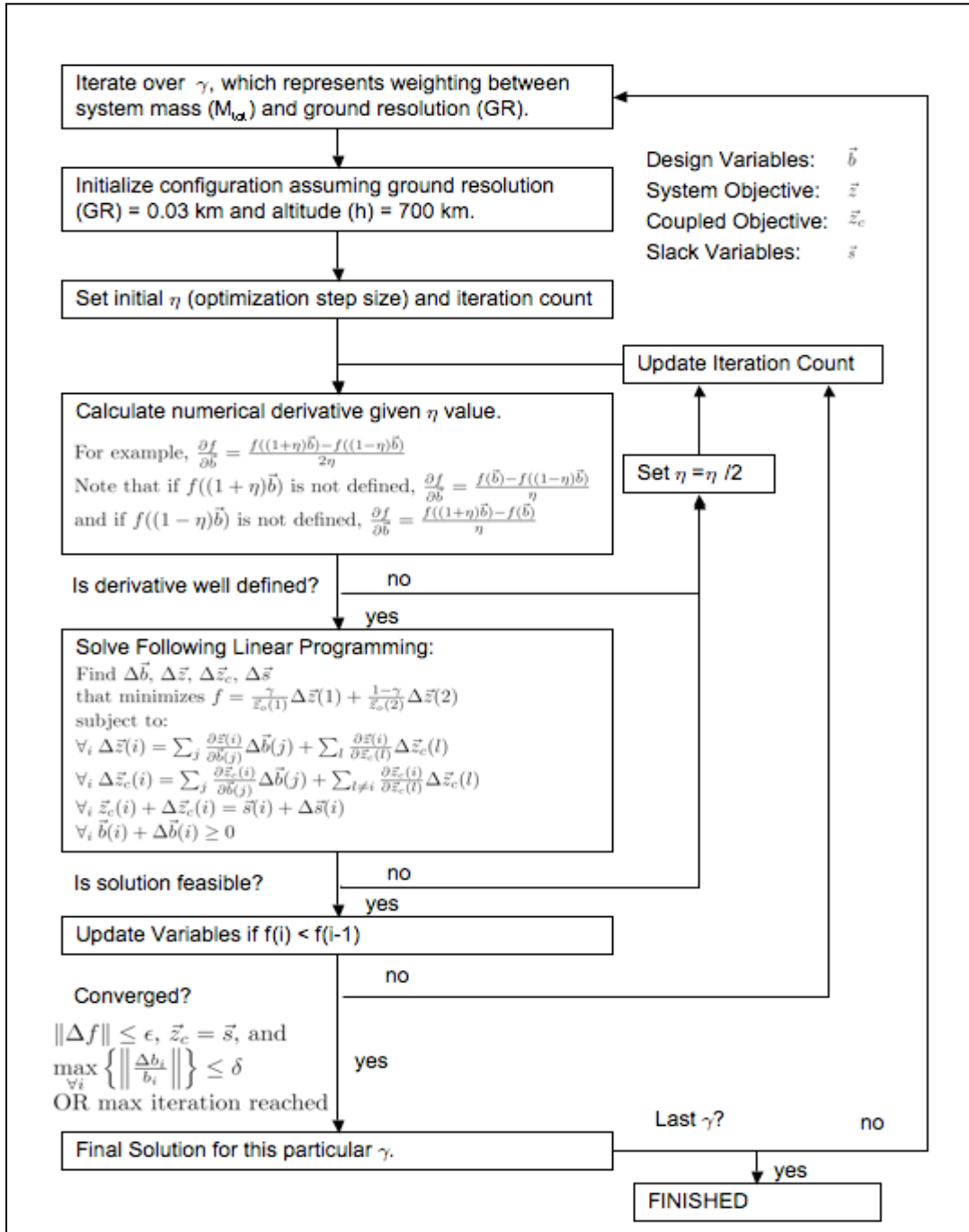


Figure 4. The MDO algorithm used in this case study.

## D. Robustness Criteria

To evaluate the design variable choices, the convergent design is tested to assess the robustness of the design. Our goal in robust optimization is to maximize the objective functions with respect to design variables  $x$  while considering interval uncertainties in respective design variables. This robustness criterion has been adapted from Azarm's work [17, 18]. The method checks to see if the current design point, obtained from the MDO solution process, is robust from an objective and constraint perspective. Objective robustness means that the objective function values do not vary beyond an allowable region from their optimal values. Feasibility robustness implies that constraints should not be violated beyond an allowable region for the design point in consideration.

The designers specify the an allowable objective variation range for all objectives:  $\Delta f_0 = (\Delta f_{0,1} \dots \Delta f_{0,M})$ , which determines an acceptable objective variation range (AOVR). Similarly, an acceptable constraint variation range (ACVR) is defined as:  $\Delta g_0 = (\Delta g_{0,1} \dots \Delta g_{0,J})$ . Here, M and J are the number of objective and constraint functions, respectively. The effects of design variable uncertainties on objective values of a design  $x_0$  can be represented by a mapping from  $x_0$ 's tolerance region to a corresponding tolerance region in the  $\Delta f$  space. This region obtained by the mapping is called the Objective Sensitivity Region (OSR). The design  $x_0$  is objectively robust if the worst case estimate of OSR, or the Worst Case Objective Sensitivity Region (WCOSR), lies inside AOVR. To calculate the worst case effects of  $x_0$ 's tolerance on  $\Delta f$  space, the objective sensitivity region is maximized using:

$$\max_{\Delta x} WCOSR(\Delta x) = \left[ \sum_{m=1}^M |\Delta f_m(\Delta x)|^2 \right]^{1/2} \quad (18)$$

Subject to:  $\Delta x_{lower} \leq \Delta x \leq \Delta x_{upper}$

Similarly, the uncertainties in  $x_0$  are mapped to the  $\Delta g$  space to examine their effects on constraint function values. This region is called the Constraint Sensitivity Region (CSR). The design  $x_0$  is feasibly robust if the worst case estimate of CSR, known as Worst Case Constraint Sensitivity Region (WCCSR), lies inside ACVR. To calculate WCCSR, we solve the following optimization problem:

$$\max_{\Delta x} WCCSR(\Delta x) = \left[ \sum_{j=1}^J |\Delta g_j(\Delta x)|^2 \right]^{1/2} \quad (19)$$

Subject to:  $\Delta x_{lower} \leq \Delta x \leq \Delta x_{upper}$

As this is a multidisciplinary problem, slack variables ( $s_{ic}$ ) are used to decouple the subsystems from each other by minimizing the direct communication between them. In order to obtain system optimality, these slack variables are forced to be strictly equal to their respective objective function values ( $s_{ic} = z_{ic}$ ) as stated in equation 9. However, when there is uncertainty in design variables, it becomes necessary to allow a small positive acceptable tolerance between the slack variables and their respective objective function values. For robust designs, the slack variable constraints for all three cases become:

$$|s_{ic} - z_{ic}| \leq \epsilon \quad (20)$$

The tolerance region ( $\epsilon$ ), for all three case studies in this research is assumed to be 5%.

The design variables that we assume have uncertainties are:  $\Delta V$ ,  $Ppl$  and mission duration (MD).  $\Delta V$  and  $Ppl$  have a tolerance region of ( $\pm 5\%$ ) and mission duration has a tolerance of ( $+2400\%$ ). These values came from discussions with engineers at the Jet Propulsion Laboratory and are representative of typically values they operate with. These values for  $\Delta V$  and  $Ppl$  are chosen from the fact that payload requires more power as electric circuits degrade over time.  $\Delta V$  is also used for correcting any significant error in inclination angle and operating altitude. Also, due to the uncertainty in each thrust, the error in  $\Delta V$  accumulates significantly over time. The fact that space missions tend to last much longer than their intended duration contributes to the high uncertainty value of MD. The robust design procedure used in this research is shown in Figure 5.

The robust design approach used in this research differs from the one originally developed in [17, 18] as our approach checks for robustness after a solution is found while the original approach searches for robust designs while solving the MDO problem. Also, the original approach has some inherent limitations which lead to the

rejection of some designs as non-robust that are actually robust. We overcome this limitation by making a simple modification to the approach.

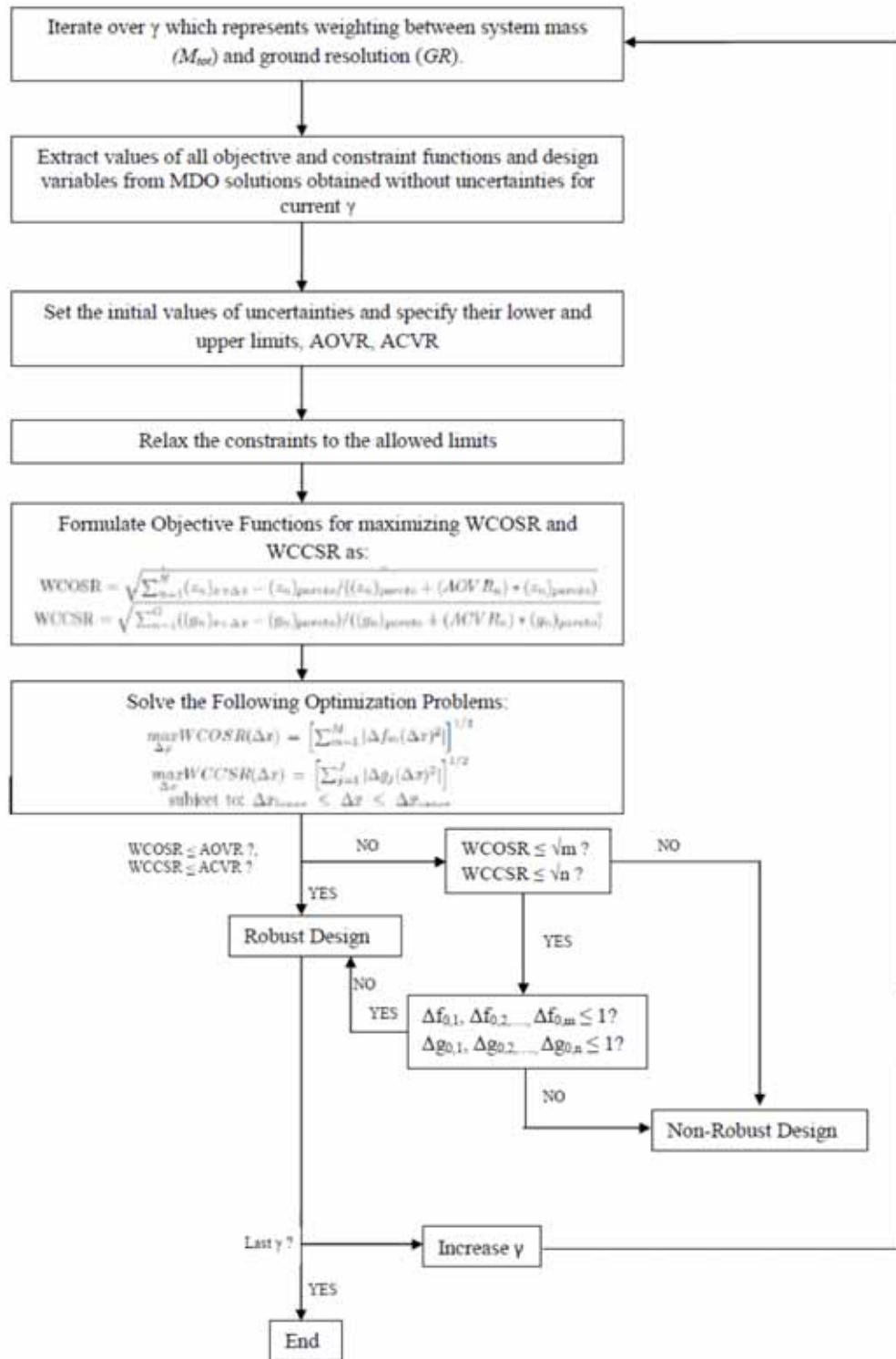


Figure 5. Robust Design Algorithm

The modification comes after evaluating if  $WCOSR \leq AOV R$  and  $WCCSR \leq ACVR$ . To evaluate objective robustness in the original approach, the AOV R is first normalized in order to avoid a scaling effect. In the normalized space, AOV R becomes a hyper-cube. The hyper-sphere inscribed in the hyper-cube, tangent to its sides is used as the worst case estimate of AOV R in the original approach. Hence, if the design lies inside the hyper-sphere, it is termed as objectively robust. However, there is some region of the hyper-cube which is not covered by the interior hyper-sphere. Hence, if a design lies inside the hyper-cube but outside the hyper-sphere, it is rejected as non-robust while it actually is robust. To overcome this limitation, if a design lies outside the interior hyper-sphere, we check if it lies inside the hyper-sphere circumscribing the hyper-cube. If the design lies inside the outer hyper-sphere, it is then checked if the violation in each objective function is inside its allowed variation. If this criterion is satisfied, then the design is termed as objectively robust. In a similar way we check for feasibility robustness. If a design satisfies both objective and feasibility robustness criteria, it is marked as a robust design.

#### IV. RESULTS

A Pareto frontier between the total system mass and ground resolution is calculated to serve as a baseline comparison for the three different MDO formulations. To determine this Pareto set, the ground resolution is fixed and optimized for total system mass as a function of altitude assuming complete information sharing between all of the subsystems.

To explore the convergence of three different MDO formulations, initial  $\eta$  values are varied as well as  $\gamma$  values while the initial design remains fixed. The initial satellite design consists of a ground resolution of 0.03 km and an altitude of 700 km to match the *Firesat* example in SMAD [15]. There should be ideal values of initial  $\eta$  value for the number of iterations necessary for convergence. (Note that when the initial  $\eta$  value is too high, errors caused by linearization will drive the design toward an infeasible design and waste resources analyzing subsystem infeasibilities.) However, this ideal initial  $\eta$  value is difficult to determine *a priori*. There are a few possible approaches to determine the optimal  $\eta$  value by calculating the Lipschitz constant, but it may be not be computationally practical for this type of design problem.

Figure 6 shows the convergence region for all 3 cases. This graph shows that Case 1 (payload mass, payload power, and total change in velocity) converges to the Pareto frontier for all but one  $\gamma$  value and both initial  $\eta$  values. In contrast, Case 2 (ground resolution and altitude) converges erratically when the initial  $\eta$  value is equal to 0.05. It may converge to the Pareto frontier, or it could converge significantly away from the frontier. Furthermore, there is no visible pattern that predicts a convergence location as a function of  $\gamma$  values. Thus, it demonstrates highly unpredictable convergence behavior. This result suggests that the designer should explore the sensitivity of the design with respect to  $\gamma$  to determine the stability of the convergent design. When the initial  $\eta$  is equal to 0.25, Case 2 converges consistently to two distinct designs, one that is Pareto optimal and another design that is suboptimal. Finally, Case 3 (payload mass, payload power, and altitude) converges systematically close to the Pareto frontier, but not necessary on top of it.

The next criterion used to compare the design variables choices is the number of iterations required for convergence. Table 1 and Fig. 7 shows that Case 2 requires significantly fewer iterations on average. Because Case 2 converges prematurely to suboptimal solutions, this result seems intuitive. However, a less intuitive finding is that the variation in the number of iterations is very small for Case 2. Figure 6 shows that about half of the design points converge to a suboptimal solution and other half converge to a Pareto solution for Case 2 with an initial  $\eta$  of 0.05. However, it is not obvious from Fig. 7 which  $\gamma$  values lead to suboptimal designs. Thus, this result shows that premature convergence to a suboptimal solution cannot be detected by iteration counts. Additionally, Cases 1 and 3 demonstrate the trade-off between the number of iterations and optimality. The average number of iterations required by Case 3 is less than Case 1 by 4 to 10 iterations for all three initial  $\eta$  values. However, Case 3 converges slightly away from the Pareto frontier, while Case 1 almost always achieves Pareto optimality.

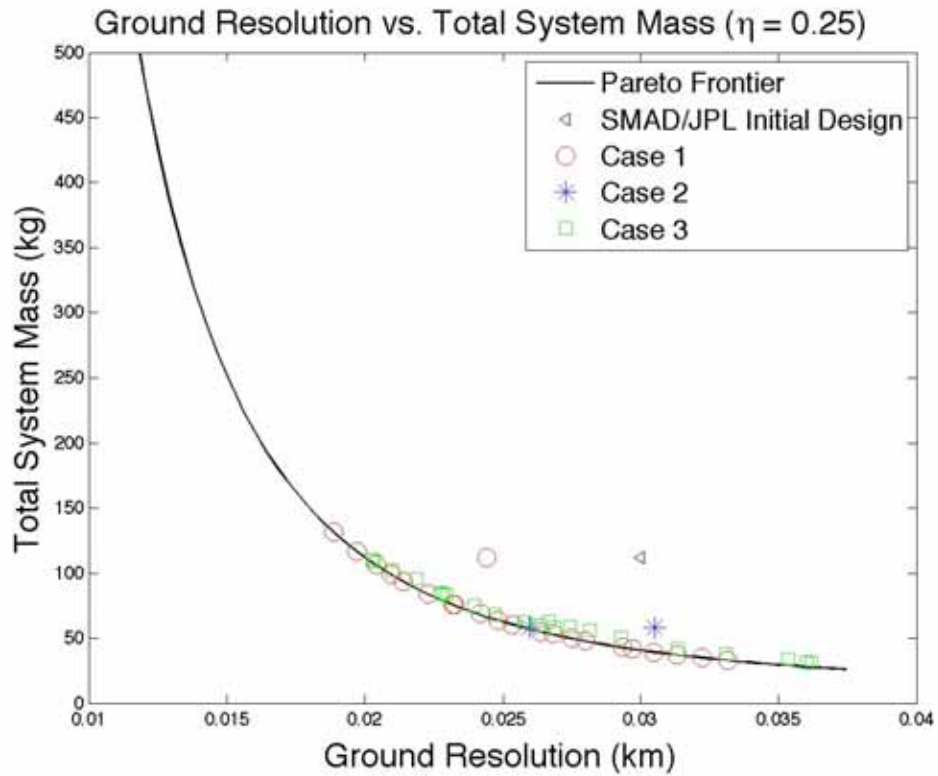
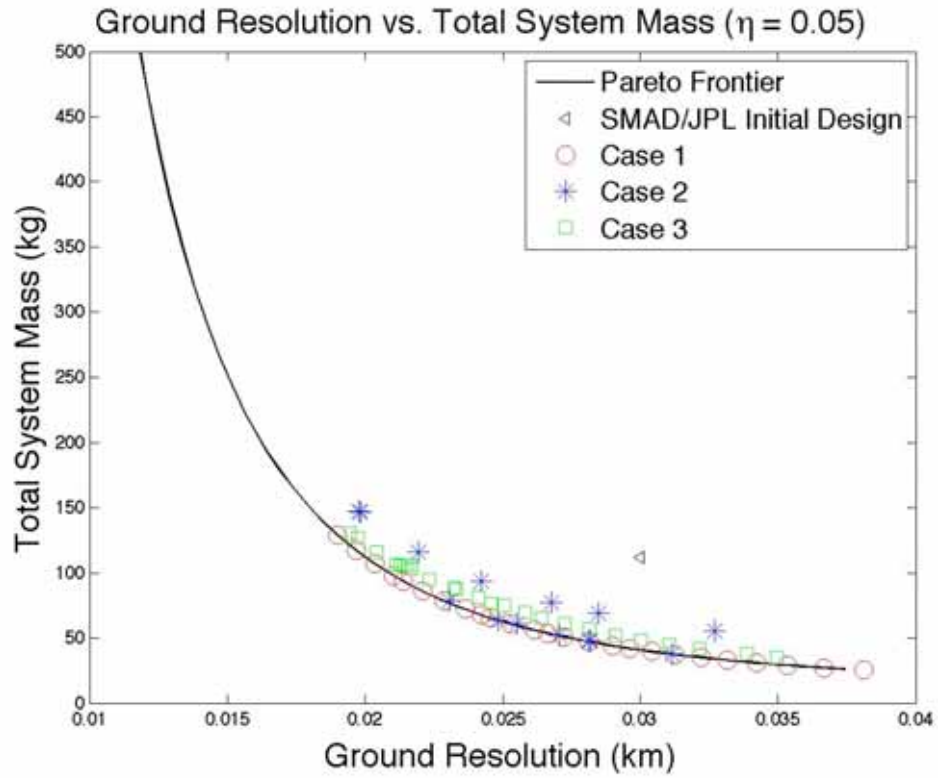
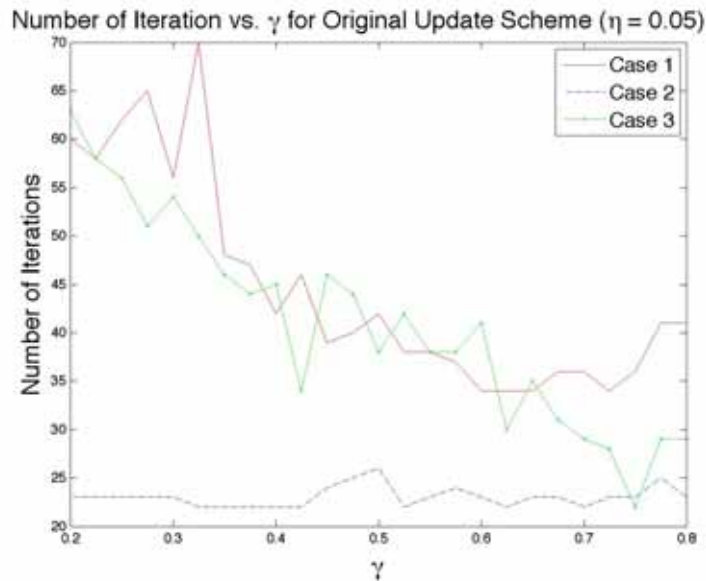


Figure 6. Convergence Result for initial  $\eta = 0.05$  (top) and initial  $\eta = 0.25$  (bottom)

**Table 1. Statistics related to the number of iterations required for convergence for three different  $\eta$  values.**

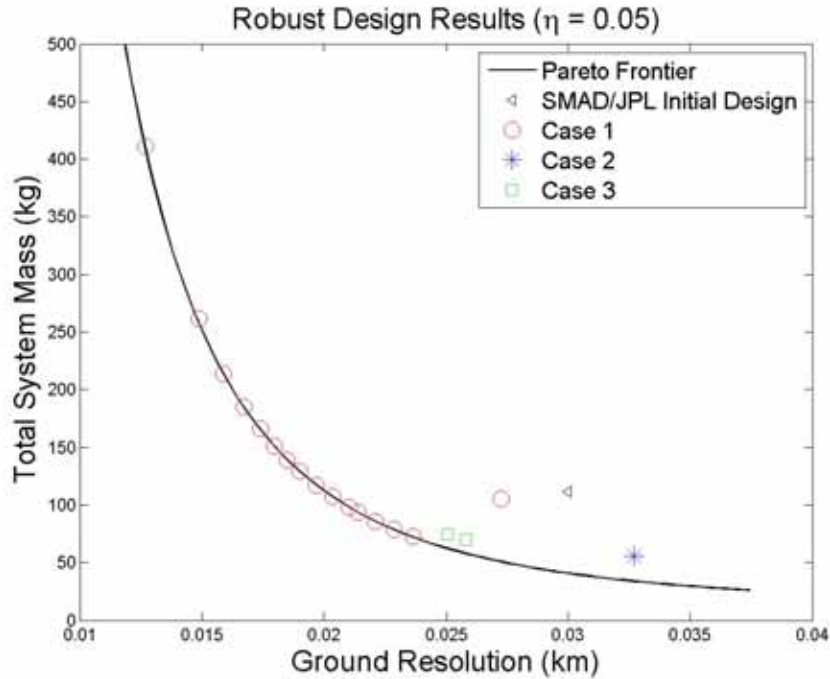
|        | $\eta = 0.05$ |                    | $\eta = 0.10$ |                    | $\eta = 0.25$ |                    |
|--------|---------------|--------------------|---------------|--------------------|---------------|--------------------|
|        | Average       | Standard Deviation | Average       | Standard Deviation | Average       | Standard Deviation |
| Case 1 | 44.6          | 10.9               | 36.4          | 11.5               | 29.3          | 10.4               |
| Case 2 | 23.0          | 1.1                | 20.6          | 1.1                | 15.0          | 8.3                |
| Case 3 | 40.8          | 10.7               | 26.3          | 5.5                | 24.9          | 3.7                |



**Figure 7. Number of Iteration vs.  $\gamma$  value for all cases when initial  $\eta$  value is fixed at 0.05**

Fig. 8 shows the robust design results for all three cases. In case 1, both  $\Delta V$  and  $Ppl$  are design variables. MD is a design variable in all three cases. However, in case 2, both  $\Delta V$  and  $Ppl$  are also modeled as objective functions.  $\Delta V$  is an objective function in case 3 as well. For these cases, the variations in  $\Delta V$  and  $Ppl$  are modeled as the limits of their respective AOVR. This means for case 2, the design is robust against a 2400% variation in MD while not allowing the  $\Delta V$  and  $Ppl$  more than 5% variation. Similarly, for case 3, MD is allowed to vary 2400% while the uncertainties in  $\Delta V$  are restricted by using its AOVR.

It can be seen in Fig. 8, that case 1 has the highest number of robust design solutions. It should also be noted that the AOVR and ACVR are different in all three cases. For case 1, the AOVR and ACVR are 5% for all objective and constraint functions. In case 2, the AOVR for  $M_{tot}$  and  $M_{pow}$  had to be as large as 3500% to obtain any robust design points. Similarly, in case 3, robust designs are obtained with a minimum constraint violation (ACVR) of 170%.



**Figure 8. Robust Design Results for all three cases.**

## V. CONCLUSION

This case study demonstrates the importance of choosing set of system level design variables during the design of large scale systems in certain MDO formulations. The main results of this particular study are that:

1. The choice of system design variables can influence the optimality of a design significantly. Case 2 demonstrates that suboptimal convergence may be detected using sensitivity analysis, suggesting that the designer should explore the sensitivity to MDO optimization parameters such as  $\gamma$  and initial  $\eta$  values.
2. The number of iterations required for convergence can depend strongly on the choice of system design variable. Counterintuitively, a lower number of iterations does not necessarily imply a suboptimal design. However, there is classic trade-off between convergence iteration and optimality in terms of the average number of iterations.
3. The robustness of designs is greatly affected by the choice of system design variables. Case 1 provides the most number of robust design points with minimum objective and constraint function violations. Cases 2 and 3 yield significantly fewer robust design points with large violations in either objective or constraint functions.

It is theorized that the choices for system design variables influence the degree of nonlinearity of a subsystem. This nonlinearity in a subsystem is the most likely cause of the differences. Additionally, because a subsystem is limited to communicating only with the system facilitator, slack variables are introduced to capture discrepancies between subsystems for coupled subsystem design objectives. The choice of the system design variables also influences the required number of slack variables and degree of error introduced by lack of communications. Further studies are needed to verify these claims in other scenarios. Future work should further examine the ramifications of system level design variables on performance, and also consider strategies for selection of system level design variables.

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