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## **A COMPARISON OF INFORMATION PASSING STRATEGIES IN SYSTEM LEVEL MODELING**

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### **ABSTRACT**

Frameworks for modeling the communication and coordination of subsystem stakeholders are valuable for the synthesis of large engineering systems. However, these frameworks can be resource intensive and challenging to implement. This paper compares three frameworks, Multidisciplinary Design Optimization (MDO), traditional Game Theory, and a Modified Game Theoretic approach on the form and flow of information passed between subsystems. This paper considers the impact of "complete" information sharing by determining the effect of merging subsystems. Comparisons are made of convergence time and robustness in a case study of the design of a satellite. Results comparing MDO in two- and three-player scenarios indicate that, when the information passed between subsystems is sufficiently linear, the two scenarios converge in statistically indifferent number of iterations, but additional "complete" information does reduce variability in the number of iterations. The Modified Game Theoretic approach converges to a smaller region of the Pareto set compared to MDO, but does so without a system facilitator. Finally, a traditional Game Theoretic approach converges to a limit cycle rather than a fixed point for the given initial design. There may also be a region of attraction for convergence for a traditional Game Theoretic approach.

### **1. INTRODUCTION**

As engineered systems become increasingly sophisticated, the number of subsystem stakeholders required to bring a system to market will continue to increase. For example, in the early

20<sup>th</sup> century, Ford's Model T car was composed of roughly 700 unique parts. In contrast the modern automobile has more parts in its radio alone and the Boeing 777 has over 3,000,000 unique parts provided by over 300 different suppliers [1]. Similarly, drive-by-wire technology originally developed for the space shuttle has now trickled down to automobiles [2].

The design and development of a large-scale, complex engineering system demands collaboration among stakeholders with expertise in a diverse set of fields. There are a number of approaches that consider collaboration at a system level by modeling stakeholders, the subsystems they are responsible for, and the trade-offs that might be made between these stakeholder subsystems. Key considerations in selecting a particular modeling approach are 1) the way information is shared among subsystems, and 2) the particular form of the information that is passed in a system. This paper examines both of these issues for three strategies for information sharing and system optimization, including Multidisciplinary Design Optimization (MDO), a Game Theoretic approach, and a new, Modified Game Theoretic approach introduced by Honda, et al [3].

MDO is well known in the field of system design [4, 5]. In traditional MDO, a human system-level facilitator allocates resources to each individual subsystem. The facilitator then brokers trade-offs between subsystems so that they may acquire necessary resources. However, no information is directly shared between subsystems themselves. This process of centralized trade-offs continues until the system converges to a solution.

In a traditional Game Theoretic Approach, subsystems are represented by "players" who play a particular "game" that follows specific rules. There is no central, system-level facilitator. Instead, information is shared among subsystems directly. In this paper, it is assumed that the rules of play dictate a sequential and iterative protocol in which subsystem stakeholders each optimize their own subsystem by changing variables they control and holding constant the variables they do not control. This approximates a non-cooperative protocol.

A Modified Game Theoretic approach is similar to a traditional Game Theoretic approach except in the form of information it passes between subsystems. In a traditional approach, information that is passed between subsystems takes the form of a single set of parameters, or a "point design." If rational decisions are made at each iteration, the final design will generally converge to a Nash equilibrium.<sup>1</sup> In contrast, the modified approach uses a gradient form for passed information, and previous work suggests that this will increase the likelihood of convergence to a subset of the Pareto Frontier.

The research questions posed here grow out of an observation that in teams that design large-scale systems, some subsystems may share much of the same information and so may be considered highly coupled [6]. It is then logical to merge such closely coupled subsystems into a single subsystem that can be modeled using a traditional Game Theoretic approach. It also becomes possible to create a "hybrid" approach in which MDO is applied to subsystems that are merged using either a Game Theoretic or Modified Game Theoretic approach. For example, in an MDO approach a system facilitator could allocate resources between subsystem A and a merged subsystem consisting of subsystems B and C. However, subsystems B and C could then follow a Game Theoretic approach through a cooperative game to sub-allocate resources.

The questions posed here include:

- How does a two subsystem, hybrid MDO approach compare with a three subsystem MDO approach in terms of performance?
- How do traditional Game Theoretic and Modified Game Theoretic Approaches compare?
- How does a hybrid MDO approach compare with a Modified Game Theoretic Approach?

## 2. RELATED WORK

In order to facilitate communication between subsystem stakeholders, a variety of frameworks have been created to model system and subsystem level communication and coordination. While MDO models include an all-at-once approach [4], we focus in this work on models that operate upon some sort of system decomposition structure.

Although centralization of decisions and models has distinct advantages, it is more common place in complex systems design to utilize a decomposition structure – hierarchical and

nonhierarchical being two of the primary structures, to centralize the design of complex systems. There are various approaches to determining the decomposition structure including object decomposition, aspect decomposition, sequential decomposition, and model based decomposition [7]. Our approach follows traditional space mission design approach and decomposes system by the disciplines. i.e. each subsystem requires different expertise to design.

Once a decomposition structure is determined, then a communication and coordination model is necessary. This model will provide protocols and formulations for critical system solution mechanics including objective function formulation, intra- and inter-subsystem communication protocol, design variable control, and convergence conditions. There are a number of protocol models including Analytic Target Cascading (ATC) [8], Concurrent Subspace Optimization (CSSO) [9, 10], Bilevel Integrated System Synthesis (BLISS) [11] and Collaborative Optimization (CO) [12].

Analytic Target Cascading has been proven to guarantee that a distributed system converges and that the converged value is a globally optimal solution [8]. Additionally, its hierarchy allows for traceability of the design process and provides for integration of marketing and business systems while establishing clear relationships between design subsystems [13]. The main advantage of CO is that it does not require system analysis, but multidisciplinary feasibility may not be satisfied. Thus, some intermediate designs could be infeasible. CSSO guarantees both individual and multidisciplinary feasibility at each iteration, but requires all disciplines to indirectly share of all constraints. Unlike other MDO formulation, BLISS keeps common variables as constants at a lower level, and optimizes only common variables at an upper level. This BLISS formulation is similar to how NASA/Jet Propulsion Laboratory's Advanced Projects Design Team (Team X) [14, 15] designs aerospace mission optimization.

While system models based on MDO principles provide effective frameworks to handle large-scale systems, they typically require significant coordination, consensus, and communication. Without high levels of these formalities, a complex design problem could simply become a distributed design problem where each subsystem stakeholder would solve their own optimization problem while making certain assumptions about other subsystems. Game theoretical system models lend themselves well to modeling these distributed design problems where no formal coordination mechanism or communication infrastructure exists.

The simplest game theoretic model is the non-cooperative model, which assumes that subsystems operate in semi-isolation, only exchanging their optimized design solutions. Variable control is uniquely allocated to the subsystems – there is no design variable sharing. Convergence and solution quality become the primary issues in this type of model.

While an equilibrium for this model is guaranteed under mixed solutions [16], the solution process might not always

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<sup>1</sup> In this case, the Nash Equilibrium is a design solution in which no designer can improve his subsystem unless other subsystems also change their designs.

converge to it. Vincent [17] studied this issue using simple two designer - two design variable problems. Other researchers then developed a number of game theoretical constructs to study these types of decentralized design problems and their resulting solution quality [18-20]. Studies on system convergence include the development of conditions for convergence or divergence in simple two subsystem problems [21], and systems when there are more than two subsystems controlling multiple design variables [22]. In addition, the level of nonlinearity has been extended [23] using nonlinear control theory concepts to predict relative domains of attraction for each Nash equilibrium solution. While most of this foundation work dealt with sequential models, work by Smith and Eppinger demonstrated a similar principle for models with simultaneous execution.

Despite this extensive body of work in system-level modeling, some research gaps remain. In particular, limited work has been conducted to compare MDO with Game Theoretic approaches, or to assess the role of the form of information passed between subsystems. This work seeks to fill that gap in research.

### 3. METHODS AND CASE STUDY

This study investigated four scenarios which are summarized in Table 1.

**TABLE 1. SYSTEM-LEVEL MODELING SCENARIOS INVESTIGATED IN THIS STUDY**

Scenario	Modeling approach	Representation
1	MDO	
2	"Hybrid" MDO-Game theoretic	
3	Traditional Game Theoretic	
4	Modified Game Theoretic	

MDO, a traditional Game Theoretic Approach, and a modified Game Theoretic Approach are applied to various scenarios drawn from the same case study of a satellite design problem. This satellite design problem and subsystem models are the *Firesat* satellite example given in Wertz and Larson's Space Mission Analysis and Design (SMAD) [24] but have been adapted to include a higher fidelity power subsystem model.

Rather than implementing approximately 16 subsystems required for full scale Aerospace Mission Design as in Team X, only 4 subsystems (Orbital, Payload, Power, and Propulsion) were implemented and it was assumed that the other subsystems are parametric functions of those 4 subsystems. Parameters such as inclination angle, initial altitude, and mission duration are treated as constants.

### 3.1 Individual Subsystem Models

#### 3.1.1 Orbital Subsystem

The orbital subsystem determines changes in velocity ( $\Delta V$ ) as function of the operating altitude ( $h$ ).

$$\Delta V = \phi_{orb}(h) \tag{1}$$

The model assumes that this particular satellite uses coplanar orbital transfer with no orbit plane change. Finally, this  $\Delta V$  includes orbital transfer from initial orbit to operating orbit, altitude maintenance, and deorbit transfer. The objective of this subsystem is to determine an appropriate altitude that minimizes  $\Delta V$  given a particular satellite's image goals.

#### 3.1.2 Payload Subsystem

The payload of this *Firesat* satellite design captures infrared images of the Earth in order to determine locations of forest fires. Thus, the main objective for this design problem is to minimize the ground resolution of given a certain payload mass and power. The basic functionality of payload is:

$$[M_{pl}, P_{pl}] = \phi_{pl}(GR, h) \tag{2}$$

where  $M_{pl}$  is mass of payload,  $P_{pl}$  is power of payload,  $GR$  is ground resolution, and  $h$  is the operating altitude. This model assumes that the operating wavelength and the width of the square detector is kept constant. Note that ground resolution is typically a design variable as well as a design objective for typical payload formulations. In other words, a typical payload designer optimizes the ground resolution or other image quality index while keeping mass and power within a certain design budget.

#### 3.1.3 Power Subsystem

The power subsystem is responsible for designing solar panels and the secondary battery in this example. It is assumed that the power required by payload already includes a certain power margin for the payload. The power subsystem's objective is to minimize the mass of the subsystem while meeting a required power output and an eclipse condition:

$$M_{pow} = \phi_{pow}(P_{pl}, h) \tag{3}$$

Where  $M_{pow}$  is the mass of the power subsystem. The power subsystem requires the operating altitude information to determine the average daylight and the maximum eclipse duration.

### 3.1.4 Propulsion Subsystem

The propulsion subsystem determines the required propellant and thruster mass as a function of payload mass, power subsystem mass, and required  $\Delta V$ . The propulsion function is:

$$[M_{prop}, M_{thrust}] = \phi_{prop}(M_{pl}, M_{pow}, \Delta V) \quad (4)$$

where  $M_{prop}$  is the mass of the propellant and  $M_{thrust}$  is the mass of the thruster. Most initial satellite designs allocate a given mass for each subsystem as a percentage of initial payload mass. This model utilizes that factor and assumes that the mass of other subsystems are 128.6% of payload mass. A mass margin of 25% for the dry mass (mass excluding propellant) and 15% design margin for propellant mass have been included. The objective of the propulsion subsystem is to minimize the total mass of the system including these margin values.

Because the Propulsion subsystem requires mass data from all other subsystems, the output from this subsystem could also be the total system mass. For design optimization, this output removes the need to have a system engineer as an integration facilitator. Thus, a more appropriate Propulsion subsystem functionality can be given by:

$$M_{tot} = \tilde{\phi}_{prop}(M_{pl}, M_{pow}, \Delta V) \quad (5)$$

## 3.2 MDO, Traditional Game Theoretic, and Modified Game Theoretic Formulations

The traditional approach for aerospace mission design involves the following steps [25]:

1. Determine orbital design (usually based on human expertise);
2. Design a payload given orbital choice;
3. Using payload and orbital design, optimize spacecraft bus;
4. If design is not satisfactory, return to step 1 or 2 and change orbital or payload design.

Given that the Orbital and Payload subsystems are highly coupled, they have been combined into single subsystem for this case study.

The most critical system attributes for this satellite design are image quality (ground resolution), total system mass (loaded mass), and cost. The system mass is critical in that the value of total system mass is directly related to cost. In this study, the cost of the satellite is not considered because cost and reliability

models for most subsystems were not available. Furthermore, cost is considered during the last phase of design at Mission Design Laboratory (MDL) at NASA Goddard Space Flight Center [6]. Thus, cost is not a traded parameter during the early engineering design phase, though it is generally traded later on. Therefore the two objectives of this case study are the minimization of the ground resolution and total system mass.

### Scenario 1: MDO with three subsystems

There are many possible MDO formulations for this satellite design problem. One key decision for implementing MDO is determining appropriately shared design variables. There are at least two logical choices for shared design variables for this case study. One possible set of design variables are  $[M_{pl}, P_{pl}, \Delta V]$  and another possible set is  $[GR, h]$  and  $[M_{pl}, P_{pl}, \Delta V, GR, h]$ . Because a payload designer tries to optimize ground resolution while keeping the spacecraft bus feasible,  $[M_{pl}, P_{pl}, \Delta V]$  is a reasonable choice for the system design variables.

To fit into this particular design formulation, the Payload and Orbital subsystems are combined and redefined as follows:

$$[GR, h] = f_1(M_{pl}, P_{pl}, \Delta V) \quad (6)$$

To convert from  $\phi_{orb}$  and  $\phi_{pl}$  into  $f_1$ , the subsystem will solve the following optimization problem.

Find  $GR$  and  $h$  that maximizes  $GR$  and subject to the following constraints:

$$\begin{aligned} [\tilde{M}_{pl}, \tilde{P}_{pl}] &= \phi_{pl}(GR, h) \\ \tilde{M}_{pl} &\leq M_{pl} \\ \tilde{P}_{pl} &\leq P_{pl} \\ \phi_{orb}(h) &\leq \Delta V \end{aligned} \quad (7)$$

Note that this  $f_1$  is a pseudo-inverse of  $\phi_{orb}$  and  $\phi_{pl}$ . Also note that  $f_1$  is not a bijection because the optimization problem is infeasible for many  $[M_{pl}, P_{pl}, \Delta V]$  combinations.

However, this formulation does mimic the role of payload designers during the Satellite design. Also, unlike some of the traditional MDO formulations, only information regarding coupled variables is passed between the system and the subsystems. In other words, the subsystems are responsible for the determination of best design choices in the cases of uncoupled design variables such as propellant type, solar cell type, and second battery materials.

Given  $f_1, \phi_{pow}$ , and  $\phi_{prop}$ , we can formulate MDO as below:

Find  $[M_{pl}, P_{pl}, \Delta V]$  that minimizes

$$f(GR, M_{tot}) = \gamma \frac{GR}{GR_o} + (1-\gamma) \frac{M_{tot}}{(M_{tot})_o} \quad (8)$$

subject to

$$\begin{aligned} [GR, h] &= f_1(M_{pl}, P_{pl}, \Delta V) \\ M_{pow} &= \phi_{pow}(P_{pl}, (h)_s) \\ M_{tot} &= \tilde{\phi}_{prop}(M_{pl}, \Delta V, (M_{pow})_s) \\ [M_{pl}, P_{pl}, \Delta V, GR, h, M_{pow}, M_{tot}] &\geq 0 \\ h &= h_s \\ M_{pow} &= (M_{pow})_s \end{aligned} \quad (9)$$

where  $GR_o$  is initial ground resolution,  $(M_{tot})_o$  is initial total system mass, and  $(\cdot)_s$  represents slack variable to minimize direct communication between subsystems.

The above optimization can be solved using an iterative linearized optimization scheme to find local optima. The linearized optimization problem for each iteration can be written as follows.

Find  $[M_{pl}, P_{pl}, \Delta V]$  that minimizes

$$\Delta f = \frac{\gamma}{GR_o} \Delta GR + \frac{(1-\gamma)}{(M_{tot})_o} \Delta M_{tot} \quad (10)$$

subject to

$$\begin{aligned} \Delta GR &= \frac{\partial(f_1)_1}{\partial M_{pl}} \Delta M_{pl} + \frac{\partial(f_1)_1}{\partial P_{pl}} \Delta P_{pl} + \frac{\partial(f_1)_1}{\partial(\Delta V)} \Delta(\Delta V) \\ \Delta h &= \frac{\partial(f_1)_2}{\partial M_{pl}} \Delta M_{pl} + \frac{\partial(f_1)_2}{\partial P_{pl}} \Delta P_{pl} + \frac{\partial(f_1)_2}{\partial(\Delta V)} \Delta(\Delta V) \\ \Delta M_{pow} &= \frac{\partial \phi_{pow}}{\partial P_{pl}} \Delta P_{pl} + \frac{\partial \phi_{pow}}{\partial(h)_s} \Delta(h)_s \\ \Delta M_{tot} &= \frac{\partial f_3}{\partial M_{pl}} \Delta M_{pl} + \frac{\partial f_3}{\partial(\Delta V)} \Delta(\Delta V) + \frac{\partial f_3}{\partial(M_{pow})_s} \Delta(M_{pow})_s \\ h + \Delta h &= (h)_s + \Delta(h)_s \\ M_{pow} + \Delta M_{pow} &= (M_{pow})_s + \Delta(M_{pow})_s \\ |\Delta M_{pl}| &\leq \eta M_{pl} \\ |\Delta P_{pl}| &\leq \eta P_{pl} \\ |\Delta(\Delta V)| &\leq \eta \Delta V \end{aligned} \quad (11)$$

where  $i = 1, 2, 3$  and  $\eta$  is the dynamic step size.

To achieve system optimality, the value of  $f$  is calculated after each linearized optimization and whenever  $f$  increases, the value of  $\eta$  is halved. The information flow between subsystems for this formulation is shown in Figure 1.

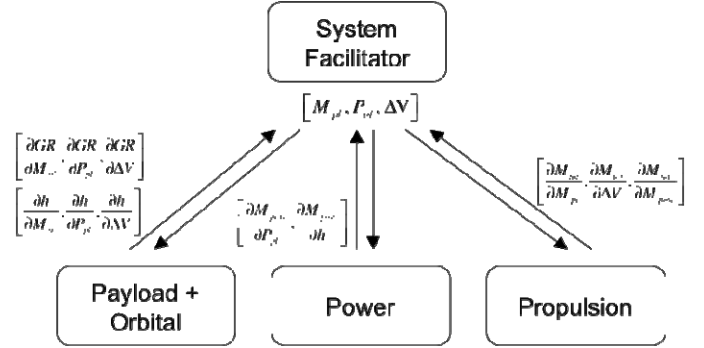


FIGURE 1: INFORMATION FLOW BETWEEN SYSTEM DESIGNER AND SUBSYSTEM DESIGNERS FOR 3 SUBSYSTEM MDO DESIGN.

### Scenario 2: MDO with two subsystems

In the three subsystem MDO formulation, a combined Payload and Orbital subsystem is required to perform the optimization. This optimization is computationally expensive and it takes an order of magnitude longer to compute than the Power and Propulsion subsystems. Thus, it is logical to explore the effect of complete information sharing between the Power and Propulsion subsystems. In particular we should ask “is there any benefit for this cooperative protocol for information sharing?”

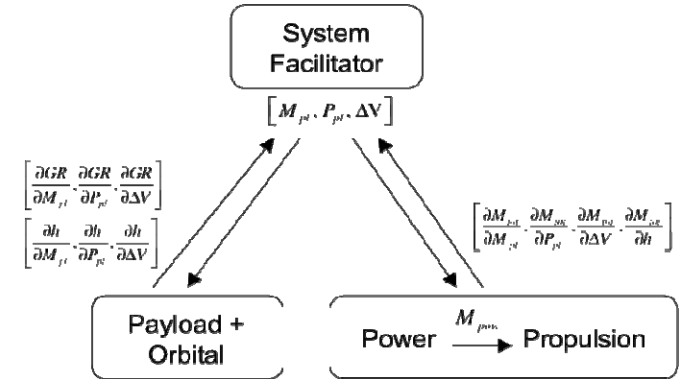


FIGURE 2: INFORMATION FLOW BETWEEN SYSTEM DESIGNER AND SUBSYSTEM DESIGNERS FOR A 2 SUBSYSTEM MDO DESIGN

To combine the Power and Propulsion subsystems together coherently, this formulation allows the Power subsystem to directly pass the coupled variable,  $M_{pow}$ , to the Propulsion subsystem. The information flow for this

formulation is represented in Figure 2. Note that as a system design becomes more complex, the information passing required to combine subsystems will become more complex as well.

Mathematically, the two subsystem MDO formulation becomes:

Find  $[M_{pl}, P_{pl}, \Delta V]$  that minimizes

$$f(GR, M_{tot}) = \gamma \frac{GR}{GR_o} + (1 - \gamma) \frac{M_{tot}}{(M_{tot})_o} \quad (12)$$

subject to

$$\begin{aligned} [GR, h] &= f_1(M_{pl}, P_{pl}, \Delta V) \\ M_{tot} &= \tilde{\phi}_{prop}(M_{pl}, \Delta V, \phi_{pow}(P_{pl}, (h)_s)) \\ [M_{pl}, P_{pl}, \Delta V, GR, h, M_{tot}] &\geq 0 \\ h &= (h)_s \end{aligned} \quad (13)$$

where all of variables are kept the same as for the three subsystem formulation. The main benefit for the cooperative protocol is that the slack variable for  $M_{pow}$  disappears and reduces the possible sources of error for convergence. The above optimization problem is solved by linearization in a similar manner to the three subsystem MDO problem.

### Scenario 3: Two Player Traditional Game Theoretic Approach

The game theoretical formulation is depicted in Figure 3. It is assumed that there are two players and that the first designer controls the Payload and the Orbital (P+O) subsystems and the second designer controls the Power and the Propulsion subsystems (P+P). The two designers have two competing objectives: while P+O will aim at minimizing the ground resolution, leading to an increase in the total system mass, the P+P designer will seek to minimize the total system mass, hereby increasing the ground resolution. The design variables are as before:  $(M_{pl}, P_{pl}, \Delta V)$ . In order to be able to implement a feasible design P+O will need to control the two variables:  $(M_{pl}, P_{pl})$ . P+P will only control the remaining change in velocity,  $\Delta V$ . The non-cooperative game theoretical protocol is carried out using a sequential iterative process, as in .

Starting from the initial design based on the SMAD example for *Firesat*, the goal of P+O is to minimize the ground resolution while keeping the value of  $\Delta V$  constants. P+O works as follows:

The orbital subsystem will find  $h$  from  $\Delta V$ , which determines a set of feasible values for  $(M_{pl}, P_{pl})$ .

In the set of feasible designs, P+O chooses the one that minimizes the ground resolution.

The two values  $(M_{pl}, P_{pl})$  along with the value of  $h$  is sent to the P+P designer who will select the  $\Delta V$  that minimizes the total mass.

This process is looped until convergence is reached. One requirement for convergence is that that the  $h$  and  $\Delta V$  respectively from P+O and P+P are consistent.

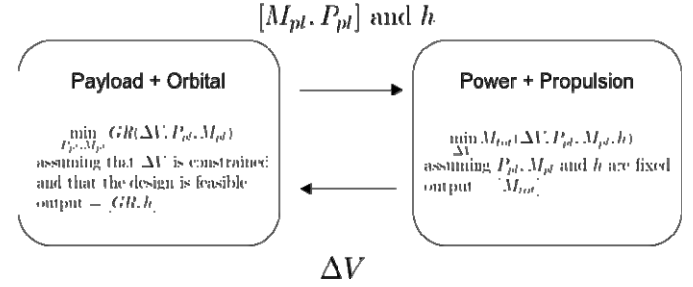


FIGURE 3: INFORMATION FLOW BETWEEN SUBSYSTEM DESIGNERS FOR A 2 PLAYER TRADITIONAL GAME THEORETIC APPROACH

### Scenario 4: Two Player Modified Game Theoretic Approach

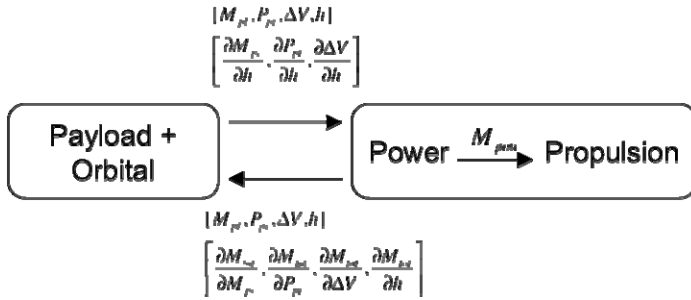
In the Modified Game Theoretic approach, the optimization problem is reformulated by removing the system engineer. Rather than sharing only design variable values as in a traditional non-cooperative protocol, subsystems are allowed to pass gradient information as well (see Figure 4). This additional information allows each subsystem to attempt to optimize their objectives without hindering other subsystems' objectives. The optimization problem for each subsystem is described as follows.

#### a. Payload + Orbital

Find  $[\Delta GR, \Delta h]$  that minimizes  $\Delta GR$

Subject to:

$$\begin{aligned} \Delta M_{tot} &= \frac{\partial M_{tot}}{\partial M_{pl}} \Delta M_{pl} + \frac{\partial M_{tot}}{\partial P_{pl}} \Delta P_{pl} + \frac{\partial M_{tot}}{\partial (\Delta V)} \Delta (\Delta V) \\ &= \left( \frac{\partial M_{tot}}{\partial M_{pl}} \frac{\partial M_{pl}}{\partial GR} + \frac{\partial M_{tot}}{\partial P_{pl}} \frac{\partial P_{pl}}{\partial GR} + \frac{\partial M_{tot}}{\partial (\Delta V)} \frac{\partial \Delta V}{\partial GR} \right) \Delta GR \\ &\quad + \left( \frac{\partial M_{tot}}{\partial M_{pl}} \frac{\partial M_{pl}}{\partial h} + \frac{\partial M_{tot}}{\partial P_{pl}} \frac{\partial P_{pl}}{\partial h} + \frac{\partial M_{tot}}{\partial (\Delta V)} \frac{\partial \Delta V}{\partial h} \right) \Delta h \quad (14) \\ &= 0 \\ |\Delta h| &\leq \eta \end{aligned}$$



**FIGURE 4: INFORMATION FLOW BETWEEN SUBSYSTEM DESIGNERS FOR A 2 PLAYER MODIFIED GAME THEORETIC APPROACH**

*b. Power + Propulsion*

Find  $\Delta h$  that minimizes  $\Delta M_{tot}$   
 Subject to:

$$\Delta M_{tot} = \left( \frac{\partial M_{tot}}{\partial M_{pl}} \frac{\partial M_{pl}}{\partial h} + \frac{\partial M_{tot}}{\partial P_{pl}} \frac{\partial P_{pl}}{\partial h} + \frac{\partial M_{tot}}{\partial \Delta V} \frac{\partial \Delta V}{\partial h} + \frac{\partial M_{tot}}{\partial h} \right) \Delta h \quad (15)$$

$$\Delta h \leq \bar{\eta}$$

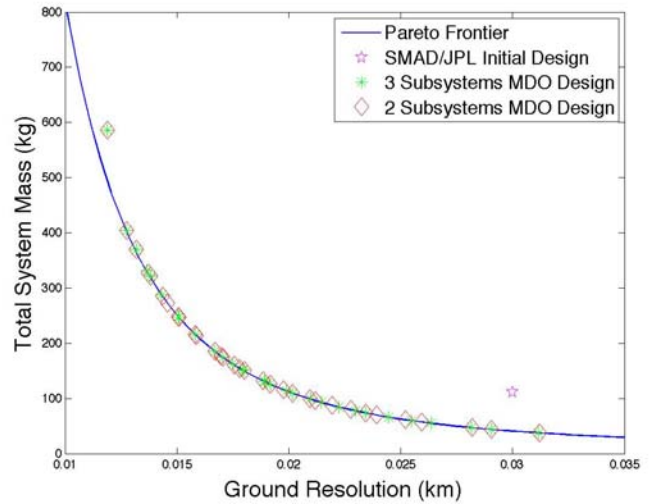
Note that aim of the Power + Propulsion subsystem is to minimize  $M_{tot}$  without increasing  $GR$ . This constraint forces  $[M_{pl}, P_{pl}, \Delta V, h]$  to move in particular direction. At each iteration, each subsystem takes their turn solving the optimization problem. Whenever  $M_{tot}$  and  $GR$  increases after a full iteration, the value of  $\bar{\eta}$  will be halved. While this insures convergence of this algorithm, it does not guarantee the optimality.

**4. RESULTS AND DISCUSSION**

A Pareto frontier between the total system mass and ground resolution is calculated to serve as a baseline comparison for the four scenarios. To determine this Pareto set, ground resolution is fixed and optimized for total system mass as a function of altitude assuming complete information sharing between all of the subsystems. This frontier is shown as a solid line in Figure 5.

**4.1 Comparison of MDO formulations**

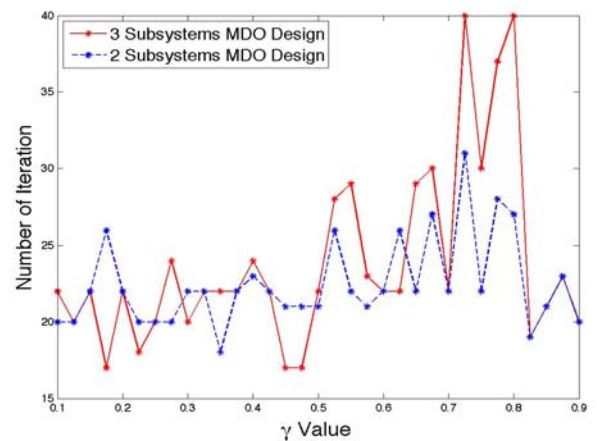
The first research question posed in Section 1 asked how a two subsystem, hybrid MDO approach might compare with a three subsystem MDO approach. To explore convergence of these two different MDO formulations, the initial values of  $\eta$  and  $\gamma$  values while the initial design is fixed. The initial satellite design consists of ground resolution of 0.03 km and an altitude of 700 km to match the Firesat example in *Space Mission Analysis and Design*. There should be an ideal  $\eta$  value for the number of iterations necessary for convergence. (Note



**FIGURE 5: CONVERGENCE RESULT FOR MDO FORMULATIONS.**

that when  $\eta$  is too high, errors caused by linearization will drive the design toward an infeasible design and waste resources by analyzing subsystem infeasibilities.) However this ideal  $\eta$  value is difficult to determine *a priori*. There are a few possible approaches to determine an optimal  $\eta$  value by calculating the Lipschitz constant, but it may be not be practical for a design problem. Figure 5 shows that both MDO formulations do converge to a subset of the Pareto frontier. For some  $\gamma$  values, there was not convergence to a Pareto solution due to numerical error for calculating derivatives.

Figure 6 shows the number of iterations required for convergence vs.  $\gamma$  values when the initial  $\eta$  value is fixed at



**FIGURE 6: NUMBER OF ITERATIONS VS.  $\gamma$  WHEN THE INITIAL VALUE FOR  $\eta$  IS FIXED AT 0.15**

0.15. Notice that the number of iterations is highly dependent on the value of  $\gamma$  and the 2 subsystem MDO scenario does not necessary converge more quickly than the 3 subsystem MDO for particular  $\gamma$  values. Furthermore, Table 2 shows descriptive statistics and hypothesis testing results for number of iterations needed to converge for the two MDO formulations. To obtain this statistical analysis  $\gamma$  values are varied from [0.1, 0.125, ..., 0.9] for  $\eta$  values which were fixed to 4 different values. To determine if the mean and variance for the two MDO formulations are statistical different, Welch's t test and F test were applied. It is shown that for most values of  $\eta$ , the difference in the number of average iterations is not statistically significant if using 5% confidence, while for all values of  $\eta$ , the variances were statistically smaller for the two subsystem MDO case. Thus, for this particular case study, complete information sharing between Power and Propulsion helped reduce the variance in the rate of convergence, but did not necessarily reduce the average number of design iterations.

#### 4.2 Comparison of Traditional and Modified Game Theoretic Approaches

The second question asked in this paper was: How do traditional Game Theoretic and Modified Game Theoretic approaches compare? Figure 7 shows convergence regions for a Modified Game Theoretic approach as well as the limit cycle for a traditional Game Theoretic approach. Note that by passing gradient information between two subsystems and linearly constraining the other objective, the modified game theoretic approach does converge to a Pareto set from the same initial design as the MDO formulation. Also, the convergence region is not a point, but a small region in the Pareto set depending on the initial step size for  $\Delta h$  as well as initial order of optimization. (i.e. if Orbital + Payload starts the optimization or if Power + Propulsion starts the optimization.)

A traditional Game Theoretic approach converges to a limit cycle. However, this limit cycle includes solutions with much greater total system mass compared to ground resolution. Thus, if human designers were responsible for designing at each iteration rather than an automated computer system, it is likely they would stop the optimization routine at a point near the Pareto Set.

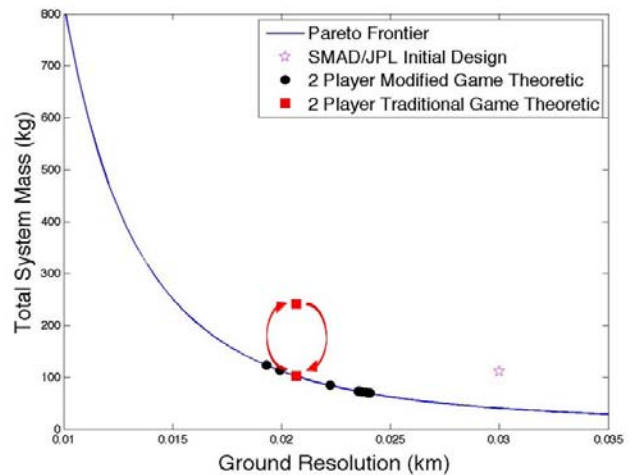
#### 4.3 Comparison of a MDO approaches with a Modified

**TABLE 2: DESCRIPTIVE STATISTICS AND STATISTICAL TEST RESULTS FOR CONVERGENCE RATES FOR TWO MDO FORMULATIONS**

	Number of Iteration for MDO				Statistical Tests		
	3 Subsystems		2 Subsystems		p value		
	mean	Std	mean	std	mean	variance	
$\eta$	0.05	33.06	8.28	31.61	3.86	0.366	4.18E-05
	0.15	23.88	5.98	22.45	2.87	0.224	7.71E-05
	0.25	23.12	9.19	19.85	3.05	0.059	1.26E-08
	0.40	23.94	5.70	21.58	3.24	0.044	1.99E-03

#### Game Theoretic Approach

The final question considered in this paper asked how an MDO formulation and a Modified Game Theoretic Approach compared. In this paper, it is found that the number of iterations required is comparable with that found for the MDO formulation. For initial  $\Delta h = [5, 10, 25, 50, 75]$ , the average number of iterations was 23.4 with a standard deviation of 8.96. Thus, the number of iterations required for a Modified Game Theoretic approach is similar to both MDO formulations, but with slightly higher variance than the 3 subsystem MDO formulation. Furthermore, the Modified Game Theoretic approach converges to a smaller subset of the Pareto Frontier compared to the MDO formulations. Because the Modified Game Theoretic approach converged to the Pareto Frontier near the initial design, this result most likely shows that the convergence region for a Modified Game Theoretic approach is sensitive to the initial design.



**FIGURE 7: COMPARISON OF MODIFIED AND TRADITIONAL GAME THEORETIC APPROACHES**

#### 5. CONCLUSIONS AND FUTURE WORK

This paper examines the impact of "complete" information sharing by determining the effect of merging subsystems into a "hybrid" subsystem, and introduced a Modified Game Theoretic Approach as a way of testing the impact of such a merging. A satellite design case study is used as a test case. A summary of responses to the original research questions follows:

- How does a two subsystem, hybrid MDO approach compare with a three subsystem MDO approach in terms of performance?

In this case study, when the information passed between subsystems is sufficiently linear, two- and three-subsystem MDO scenarios converge in a statistically indifferent number of iterations, but additional "complete" information does reduce the variability in the number of iterations.



- *How do traditional Game Theoretic and Modified Game Theoretic Approaches compare?*

A traditional Game Theoretic approach is found to converge to a limit cycle rather than a fixed point for the given initial design. There may be region of attraction for convergence for the traditional Game Theoretic approach, but it is not located in this study. In comparison, the modified approach converges to a subset of the Pareto set.

- *How does a hybrid MDO approach compare with a Modified Game Theoretic Approach?*

The Modified Game Theoretic approach does converge to a smaller region of Pareto set compare to MDO formulations, but removes the necessity for a system facilitator.

There are a number of areas of further study. While a modified Game Theoretic approach is used, the underlying process structure for the game theory-based formulations is non-cooperative in nature. Leader/follower and augmented cooperative formulations may lend additional insight into computational and information passing requirements. Also, there are 16 subsystems in the full satellite model. Studying more players in this design process would create additional challenges and would more accurately reflect design and engineering practice. Lastly, while we focused on optimizing one satellite in this study, future work could include optimizing an entire set of distributed satellites using concepts from coalition, bargaining, and grey game theory.

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