On the Order Optimality of Large-scale Underwater Networks—Part I: Extended Network Model

**Won-Yong Shin · Daniel E. Lucani ·
Muriel Médard · Milica Stojanovic ·
Vahid Tarokh**

Abstract In this two-part paper, information-theoretic capacity scaling laws are analyzed in an underwater acoustic network with n regularly located nodes on a square, in which both bandwidth and received signal power can be limited significantly. Parts I and II deal with an extended network of unit node density and a dense network of unit area, respectively. A narrow-band model is assumed where the carrier frequency is allowed to scale as a function of n. We first characterize an attenuation parameter that depends on the frequency scaling as well as the transmission distance. A Cut-set upper bound on the throughput scaling is then derived. It is analyzed that under extended networks, the upper bound is inversely proportional to the attenuation parameter, thus resulting in a highly power-limited network. Interestingly, it is seen that the upper bound is intrinsically related to the attenuation parameter but not the spreading factor. Furthermore, we describe an achievable scheme based on the nearest-neighbor multi-hop (MH) transmission, which is suitable due to the low propagation speed of acoustic channel. We show that the MH scheme is order-optimal for all the operating regimes of extended networks.

W.-Y. Shin · V. Tarokh
School of Engineering and Applied Sciences, Harvard University,
Cambridge, MA 02138 USA
E-mail: {wyshin, vahid}@seas.harvard.edu

D. E. Lucani
Instituto de Telecommunicacoes, Universidade do Porto, Porto 4200-465, Portugal
E-mail: dlucani@fe.up.pt

M. Médard
Research Laboratory of Electronics, Massachusetts Institute of Technology,
Cambridge, MA 02139 USA
E-mail: medard@mit.edu

M. Stojanovic
Department of Electrical and Computer Engineering, Northeastern University,
Boston, MA 02115 USA
E-mail: millitsa@mit.edu

Finally, these scaling results are extended to a random network realization. As a result, vital information for the fundamental limits of extended underwater networks is provided by showing capacity scaling laws.

Keywords Achievability · Attenuation parameter · Bandwidth · Capacity scaling law · Carrier frequency · Cut-set upper bound · Extended network · Multi-hop (MH) · Operating regime · Power · Underwater acoustic network

1 Introduction

Gupta and Kumar's pioneering work [1], characterized the connection between the number of nodes, n, and the sum throughput in a large-scale wireless radio network. They showed that the total throughput scales as $\Theta(\sqrt{n/\log n})$ when a multi-hop (MH) routing strategy is used for n source–destination (S–D) pairs randomly distributed in a unit area.[1] MH schemes are then further developed and analyzed in [3–9], while their throughput per S–D pair scales far slower than $\Theta(1)$. Recent results [10,11] have shown that an almost linear throughput in the radio network, i.e. $\Theta(n^{1-\epsilon})$ for an arbitrarily small $\epsilon > 0$, which is the best we can hope for, is achievable by using a hierarchical cooperation (HC) strategy.[2] Besides the schemes in [10, 11], there have been other studies to improve the throughput of wireless radio networks up to a linear scaling in a variety of network scenarios by using novel techniques such as networks with node mobility [12], interference alignment [13], and infrastructure support [14].

Together with the studies in terrestrial radio networks, the interest in study of underwater networks has been growing, due to recent advances in acoustic communication technology [15–18]. In underwater acoustic communication systems, both bandwidth and received signal power are severely limited owing to the exponential (rather than polynomial) path-loss attenuation with long propagation distance and the frequency-dependent attenuation. This is a main feature that distinguishes underwater systems from wireless radio links. Hence, the system throughput is affected by not only the transmission distance but also the useful bandwidth. Based on these characteristics, network coding schemes [17, 19, 20] have been presented for underwater acoustic channels, while network coding showed better performance than MH routing in terms of reducing transmission power. MH networking has further been investigated in other simple but realistic network conditions that take into account the practical issues of coding and delay [21, 22].

One natural question is what are the fundamental capabilities of underwater networks in supporting a multiplicity of nodes that wish to communicate

---

[1] We use the following notation: i) $f(x) = O(g(x))$ means that there exist constants C and c such that $f(x) \leq Cg(x)$ for all $x > c$. ii) $f(x) = o(g(x))$ means that $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$. iii) $f(x) = \Omega(g(x))$ if $g(x) = O(f(x))$. iv) $f(x) = \omega(g(x))$ if $g(x) = o(f(x))$. v) $f(x) = \Theta(g(x))$ if $f(x) = O(g(x))$ and $g(x) = O(f(x))$ [2].

[2] Note that the HC scheme deals with a subtle issue around quantization, which is not our main concern in this work.

concurrently with each other, i.e., multiple S–D pairs, over an acoustic channel. To answer this question, the throughput scaling for underwater networks was first studied [23], where n nodes were arbitrarily located in a planar disk of unit area, as in [1], and the carrier frequency was set to a constant independent of n. That work showed an upper bound on the throughput of each node, based on the physical model [1], which scales as $n^{-1/\alpha}e^{-W_0(\Theta(n^{-1/\alpha}))}$, where α corresponds to the spreading factor of the underwater channel and W_0 represents the branch zero of the Lambert W function [24].[3] Since the spreading factor typically has values in the range $1 \leq \alpha \leq 2$ [23], the throughput per node decreases almost as $O(n^{-1/\alpha})$ for large enough n, which is considerably faster than the $\Theta(\sqrt{n})$ scaling characterized for wireless radio settings [1].

In Part I of this two-part paper, capacity scaling laws for underwater networks are analyzed in an *extended* network [4,5,10,25,26] of unit node density, which is one of fundamentally different network models. Part II [27] shows the analysis for a dense network [1, 6, 10] of unit area used as another extreme network realization.[4] Unlike the work in [23], the *information-theoretic* notion of network capacity is adopted in terms of characterizing the model for successful transmission. Especially, we are interested in the case where the carrier frequency scales as a certain function of n in a narrow-band model. Such an assumption leads to a significant change in the scaling behavior owing to the attenuation characteristics. Recently, the optimal capacity scaling of wireless radio networks has been studied in [29, 30] according to operating regimes that are determined by the relationship between the carrier frequency and the number of nodes, n. The frequency scaling scenario of our study essentially follows the same arguments as those in [29, 30]. We study both an information-theoretic upper bound and an achievable rate scaling while allowing the frequency scaling with n. To the best of our knowledge, such an attempt has never been conducted before in underwater networks.

We explicitly characterize an *attenuation parameter* that depends on the transmission distance and also on the carrier frequency. For extended networks with n regularly distributed nodes on a square, we then derive an upper bound on the total throughput scaling using the cut-set bound. In extended networks, our upper bound is based on the characteristics of power-limited regimes shown in [10]. We show that the upper bound is inversely proportional to the attenuation parameter. This leads to a highly power-limited network for all the operating regimes (i.e., path-loss attenuation regimes), where power consumption is important in determining performance. Interestingly, it is seen that contrary to the case of wireless radio networks, our upper bound heavily depends on the attenuation parameter but not on the spreading factor (corre-

---

[3] The Lambert W function is defined to be the inverse of the function $z = W(z)e^{W(z)}$ and the branch satisfying $W(z) \geq -1$ is denoted by $W_0(z)$.

[4] Since the two networks represent both extreme network realizations we will consider in both Parts I and II, a realistic one would be in-between. In wireless radio networks, the work in [28] generalized the results of [10] to the case where the network area can scale polynomially with the number of nodes, n. In underwater networks, we leave this issue for further study.

sponding to the path-loss exponent in wireless networks). In addition, to show constructively our achievability result, we describe the conventional nearest-neighbor MH transmission [1], which is suitable for underwater networks due to the very long propagation delay of acoustic signal in water [31], and analyze its achievable throughput. We show that under extended regular networks, the achievable rate scaling based on the MH routing exactly matches the upper bound on the capacity scaling for all the operating regimes. Furthermore, a random network scenario is discussed in this work. We show that under extended random networks, the conventional MH-based achievable scheme is not order-optimal for any operating regimes.

The rest of this paper is organized as follows. Section 2 describes our system and channel models. In Section 3, the cut-set upper bound on the throughput is derived. In Section 4, the achievable throughput scaling is analyzed. These results are extended to the random network case in Section 5. Finally, Section 6 summarizes the paper with some concluding remarks.

Throughout this paper, $[\cdot]_{ki}$ denotes the (k, i)-th of a matrix. $\mathbf{I}_n$ is the identity matrix of size $n \times n$, $\det(\cdot)$ is the determinant, $\mathbb{C}$ is the field of complex numbers, and $E[\cdot]$ is the expectation. Unless otherwise stated, all logarithms are assumed to be to the base 2.

2 System and Channel Models

We consider a two-dimensional underwater network that consists of n nodes on a square such that two neighboring nodes are 1 unit of distance apart from each other, i.e., a regular network [25, 26]. This two-dimensional network is usually assumed to be constituted by sensor nodes that are anchored to the bottom of the ocean. We randomly pick a matching of S–D pairs, so that each node is the destination of exactly one source. Each node has an average transmit power constraint P (constant), and we assume that the channel state information (CSI) is available at all receivers, but not at the transmitters. It is assumed that each node transmits at a rate $T(n)/n$, where $T(n)$ denotes the total throughput of the network.

Now let us turn to channel modeling. We assume frequency-flat channel of bandwidth W Hz around carrier frequency f, which satisfies $f \gg W$, i.e., narrow-band model. This is a highly simplified model, but nonetheless one that suffices to demonstrate the fundamental mechanisms that govern capacity scaling. Assuming that all the nodes have perfectly directional transmissions, we also disregard multipath propagation, and simply focus on a line-of-sight channel between each pair of nodes used in [10,11,28]. An underwater acoustic channel is characterized by an attenuation that depends on both the distance r_{ki} between nodes i and k ($i, k \in \{1, \cdots, n\}$) and the signal frequency f, and is given by

$$A(r_{ki}, f) = c_0 r_{ki}^{\alpha} a(f)^{r_{ki}} \tag{1}$$

for some constant $c_0 > 0$ independent of n, where α is the spreading factor and $a(f) > 1$ is the absorption coefficient [16]. For analytical tractability, we

assume that the spreading factor α does not change throughout the network, i.e., that it is the same from short to long range transmissions, as in wireless radio networks [1, 4, 10]. The spreading factor describes the geometry of propagation and is typically $1 \leq \alpha \leq 2$—its commonly used values are $\alpha = 1$ for cylindrical spreading, $\alpha = 2$ for spherical spreading, and $\alpha = 1.5$ for the so-called practical spreading. Note that existing models of wireless networks typically correspond to the case for which $a(f) = 1$ (or a positive constant independent of n) and $\alpha > 2$.[5]

A common empirical model gives $a(f)$ in dB/km for f in kHz as [16, 32]:

$$10 \log a(f) = a_0 + a_1 f^2 + a_2 \frac{f^2}{b_1 + f^2} + a_3 \frac{f^2}{b_2 + f^2}, \tag{2}$$

where $\{a_0, \cdots, a_3, b_1, b_2\}$ are some positive constants independent of n. As stated earlier, we will allow the carrier frequency f to scale with the number of nodes, n. Note that this scaling technique is used to better exploit the nature of the underwater acoustic channel. As a consequence, a wider range of both f and n is covered, similarly as in [28–30]. In particular, we consider the case where the frequency scales at arbitrarily increasing rates relative to n, which enables us to really capture the dependence on the frequency in performance.[6] The absorption $a(f)$ is then an increasing function of f such that

$$a(f) = \Theta \left(e^{c_1 f^2} \right) \tag{3}$$

with respect to f for some constant $c_1 > 0$ independent of n.

The noise n_i at node $i \in \{1, \cdots, n\}$ in an acoustic channel can be modeled through four basic sources: turbulence, shipping, waves, and thermal noise [16]. We assume that n_i is the circularly symmetric complex additive colored Gaussian noise with zero mean and power spectral density (PSD) $N(f)$, and thus the noise is frequency-dependent. The overall PSD of four sources decays linearly on the logarithmic scale in the frequency region 100 Hz – 100 kHz, which is the operating region used by the majority of acoustic systems, and thus is approximately given by [16, 33]

$$\log N(f) = a_4 - a_5 \log f \tag{4}$$

for some positive constants a_4 and a_5 independent of n.[7] This means that $N(f) = O(1)$ since

$$N(f) = \Theta \left(\frac{1}{f^{a_5}} \right) \tag{5}$$

---

[5] The counterpart of α in wireless radio channels is the path-loss exponent.

[6] Otherwise, the attenuation parameter $a(f)$ scales as $\Theta(1)$ from (2), which is not a matter of interest in this work.

[7] Note that in our operating frequencies, $a_5 = 1.8$ is commonly used for the above approximation [16].

in terms of f increasing with n. From (3) and (5), we may then have the following relationship between the absorption $a(f)$ and the noise PSD $N(f)$:

$$N(f) = \Theta\left(\frac{1}{(\log a(f))^{a_5/2}}\right).\tag{6}$$

From the narrow-band assumption, the received signal y_k at node $k \in \{1, \cdots, n\}$ at a given time instance is given by

$$y_k = \sum_{i \in I} h_{ki} x_i + n_k,\tag{7}$$

where

$$h_{ki} = \frac{e^{j\theta_{ki}}}{\sqrt{A(r_{ki}, f)}}\tag{8}$$

represents the complex channel between nodes i and k, $x_i \in \mathbb{C}$ is the signal transmitted by node i, and $I \subset \{1, \cdots, n\}$ is the set of simultaneously transmitting nodes. The random phases θ_{ki} are uniformly distributed over $[0, 2\pi)$ and independent for different i, k, and time. We thus assume a narrow-band time-varying channel, whose gain changes to a new independent value for every symbol. Note that this random phase model is based on a far-field assumption [10, 11, 28],[8] which is valid if the wavelength is sufficiently smaller than the minimum node separation.

Based on the above channel characteristics, operating regimes of the network are identified according to the following physical parameters: the absorption $a(f)$ and the noise PSD $N(f)$ which are exploited here by choosing the frequency f based on the number of nodes, n. In other words, if the relationship between f and n is specified, then $a(f)$ and $N(f)$ can be given by a certain scaling function of n from (3) and (5), respectively.

3 Cut-set Upper Bound

To access the fundamental limits of an extended underwater network, a new cut-set upper bound on the total throughput scaling is analyzed from an information-theoretic perspective [35]. Consider a given cut L dividing the network area into two equal halves, as in [10, 28] (see Fig. 1). Under the cut L, source nodes are on the left, while all nodes on the right are destinations. In this case, we have an $\Theta(n) \times \Theta(n)$ multiple-input multiple-output (MIMO) channel between the two sets of nodes separated by the cut. Specifically, an upper bound based on the power transfer argument [10] is established for extended networks, where the information flow for a given cut L is proportional to the total received signal power from source nodes. Note, however, that the

---

[8] In [34], instead of simply taking the far-field assumption, the physical limit of wireless radio networks has been studied under certain conditions on scattering elements. Further investigation is also required to see whether this assumption is valid for underwater networks of unit node density in the limit of large number of nodes, n.
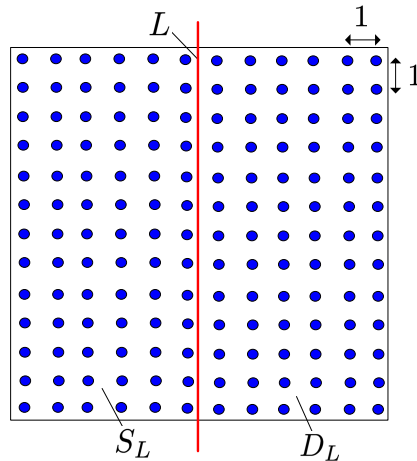
Fig. 1 The cut L in a two-dimensional extended regular network. S_L and D_L represent the sets of source and destination nodes, respectively.

present problem is not equivalent to the conventional extended network framework [10] due to quite different channel characteristics, and the main result is shown here in a somewhat different way by providing a simpler derivation than that of [10].

As illustrated in Fig. 1, let S_L and D_L denote the sets of sources and destinations, respectively, for the cut L in an extended network. We then take into account an approach based on the amount of power transferred across the network according to different operating regimes, i.e., path-loss attenuation regimes. As pointed out in [10], the information transfer from S_L to D_L is highly power-limited since all the nodes in the set D_L are ill-connected to the left-half network in terms of power. This implies that the information transfer is bounded by the total received power transfer, rather than the cardinality of the set D_L. For the cut L, the total throughput $T(n)$ for sources on the left is bounded by the (ergodic) capacity of the MIMO channel between S_L and D_L under time-varying channel assumption, and thus is given by

$$T(n) \leq \max_{\mathbf{Q}_L \geq 0} E\left[\log\det\left(\mathbf{I}_{n/2} + \frac{1}{N(f)}\mathbf{H}_L\mathbf{Q}_L\mathbf{H}_L^H\right)\right], \qquad (9)$$

where $\mathbf{H}_L$ is the matrix with entries $[\mathbf{H}_L]_{ki} = h_{ki}$ for $i \in S_L, k \in D_L$, and $\mathbf{Q}_L \in \mathbb{C}^{\Theta(n) \times \Theta(n)}$ is the positive semi-definite input signal covariance matrix whose k-th diagonal element satisfies $[\mathbf{Q}_L]_{kk} \leq P$ for $k \in S_L$.

The relationship (9) will be further specified in Theorem 1. Before that, we first apply the techniques of [26, 36] to obtain the total power transfer of the set D_L. These techniques involve the design of the optimal input signal covariance matrix $\mathbf{Q}_L$ in terms of maximizing the upper bound (9) on the capacity. If the matrix $\mathbf{H}_L$ has independent entries, each h_{ki} of which is a

proper complex random variable [37], and has the same distribution as $-h_{ki}$ for $i \in S_L, k \in D_L$, then the optimal $\mathbf{Q}_L$ is diagonal, i.e., the maximum in (9) is attained with $[\tilde{\mathbf{Q}}_L]_{kk} = P$ for $k \in S_L$, where $\tilde{\mathbf{Q}}_L$ is the diagonal matrix. We start from the following lemma.

Lemma 1 *Each element h_{ki} of the channel matrix $\mathbf{H}_L$ is a proper complex random variable, where $i \in S_L, k \in D_L$.*

The proof of this lemma is presented in Appendix A.1. It is readily proved that h_{ki} has the same distribution as $-h_{ki}$ for all i and k since the random phases θ_{ki} are uniformly distributed over $[0, 2\pi)$. Thus, using the result of Lemma 1, we obtain the following result.

Lemma 2 *The optimal input signal covariance matrix $\mathbf{Q}_L$ that maximizes the upper bound (9) is unique and is given by the diagonal $\tilde{\mathbf{Q}}_L$ with entries $[\tilde{\mathbf{Q}}_L]_{kk} = P$ for $k \in S_L$.*

We refer to Section III of [36] for the detailed proof. From Lemma 2, the expression (9) is then rewritten as

$$
\begin{aligned}
T(n) &\leq E\left[\log\det\left(\mathbf{I}_{n/2} + \frac{1}{N(f)}\mathbf{H}_L\tilde{\mathbf{Q}}_L\mathbf{H}_L^H\right)\right] \\
&= E\left[\log\det\left(\mathbf{I}_{n/2} + \frac{P}{N(f)}\mathbf{H}_L\mathbf{H}_L^H\right)\right] \\
&\leq E\left[\sum_{k\in D_L}\log\left(1 + \frac{P}{N(f)}\sum_{i\in S_L}|h_{ki}|^2\right)\right] \\
&= \sum_{k\in D_L}\log\left(1 + \frac{P}{N(f)}\sum_{i\in S_L}\frac{1}{A(r_{ki},f)}\right) \\
&\leq \sum_{k\in D_L}\sum_{i\in S_L}\frac{P}{A(r_{ki},f)N(f)},
\end{aligned}
\tag{10}
$$

where the second inequality is obtained by applying generalized Hadamard's inequality [38] as in [10, 26]. The last two steps come from (1) and the fact that $\log(1+x) \leq x$ for any x, which is only tight as x is small. Note that the right-hand side of (10) represents the total amount of received signal-to-noise ratio (SNR) from the set S_L of sources to the set D_L of destinations for a given cut L. To further compute (10), we define the following parameter

$$
P_L^{(k)} = \frac{P}{c_0}\sum_{i\in S_L}r_{ki}^{-\alpha}a(f)^{-r_{ki}}
\tag{11}
$$

for some constant $c_0 > 0$ independent of n, which corresponds to the total power received from the signal sent by all the sources $i \in S_L$ at node k on the right (see (1) and (8)). For convenience, we now index the node positions such that the source and destination nodes under the cut L are located at

positions $(-i_x + 1, i_y)$ and (k_x, k_y), respectively, for $i_x, k_x = 1, \cdots, \sqrt{n}/2$ and $i_y, k_y = 1, \cdots, \sqrt{n}$. The scaling result of $P_L^{(k)}$ defined in (11) can then be derived as follows.

Lemma 3 *In an extended network, the term $P_L^{(k)}$ in (11) is given by*

$$P_L^{(k)} = O\left(k_x^{1-\alpha} a(f)^{-k_x}\right), \tag{12}$$

where k_x represents the horizontal coordinate of node $k \in D_L$ for $k_x = 1, \cdots, \sqrt{n}/2$.

The proof of this lemma is presented in Appendix A.2. We are now ready to show the cut-set upper bound in extended networks.

Theorem 1 *For an underwater regular network of unit node density, the total throughput $T(n)$ is upper-bounded by*

$$T(n) \leq \frac{c_2 \sqrt{n}}{a(f) N(f)}, \tag{13}$$

where $c_2 > 0$ is some constant independent of n.

Proof From (1) and (10)–(12), we obtain the following upper bound on the total throughput $T(n)$:

$$
\begin{aligned}
T(n) &\leq \frac{1}{N(f)} \sum_{k \in D_L} P_L^{(k)} \\
&\leq \frac{1}{N(f)} \sum_{k_x=1}^{\sqrt{n}/2} \sum_{k_y=1}^{\sqrt{n}} P_L^{(k)} \\
&\leq \frac{c_3 P \sqrt{n}}{N(f)} \sum_{k_x=1}^{\sqrt{n}/2} \frac{1}{k_x^{\alpha-1} a(f)^{k_x}} \\
&\leq \frac{c_3 P \sqrt{n}}{N(f)} \sum_{k_x=1}^{\sqrt{n}/2} \frac{1}{a(f)^{k_x}} \\
&\leq \frac{c_3 P \sqrt{n}}{N(f)} \frac{1}{a(f) - 1} \\
&\leq \frac{c_4 P \sqrt{n}}{a(f) N(f)},
\end{aligned}
$$

where c_3 and c_4 are some positive constants independent of n, which is equal to (13). This completes the proof of the theorem.

We remark that this upper bound is expressed as a function of the absorption $a(f)$ and the noise PSD $N(f)$, whereas an upper bound for wireless radio

networks depends only on the constant value α [10]. In addition, using (3), (5), and (6) in (13) results in two other expressions on the total throughput

$$T(n) = O\left(\frac{\sqrt{n}\,(\log a(f))^{a_5/2}}{a(f)}\right) \tag{14}$$

and

$$T(n) = O\left(\frac{\sqrt{n}f^{a_5}}{e^{c_1 f^2}}\right)$$

for some positive constants c_1 and a_5 shown in (3) and (4), respectively. Hence, from (14), it is seen that the upper bound is inversely proportional to the attenuation parameter $a(f)$ and decays fast with increasing $a(f)$, thereby leading to a highly power-limited network irrespective of the parameter $a(f)$.

4 Achievability Result

To show the order optimality of underwater networks, we analyze the achievable throughput scaling with the existing transmission scheme, commonly used in wireless radio networks. In this section, the nearest-neighbor MH routing protocol [1] will be briefly described with a slight modification. The basic procedure of the MH protocol under our extended regular network is as follows:

– Divide the network into square routing cells, each of which has unit area.
– Draw an line connecting a S–D pair. A source transmits a packet to its destination using the nodes in the adjacent cells passing through the line.
– Use the full transmit power at each node, i.e., the transmit power P.

The achievable rate of MH is now shown by quantifying the amount of interference.

Lemma 4 *Consider an extended regular network that uses the nearest-neighbor MH protocol. Then, the total interference power P_I from other simultaneously transmitting nodes, corresponding to the set $I \subset \{1, \cdots, n\}$, is upper-bounded by $\Theta(1/a(f))$, where $a(f)$ denotes the absorption coefficient greater than 1.*

Proof There are $8k$ interfering routing cells, each of which includes one node, in the k-th layer l_k of the network as illustrated in Fig. 2. Then from (1), (7), and (8), the total interference power P_I at each node from simultaneously transmitting nodes is upper-bounded by

$$\begin{aligned}
P_I &= \sum_{k=1}^{\infty}(8k)\frac{P}{c_0 k^\alpha a(f)^k} \\
&= \frac{8P}{c_0}\sum_{k=1}^{\infty}\frac{1}{k^{\alpha-1}a(f)^k} \\
&\leq \frac{8P}{c_0}\sum_{k=1}^{\infty}\frac{1}{a(f)^k} \\
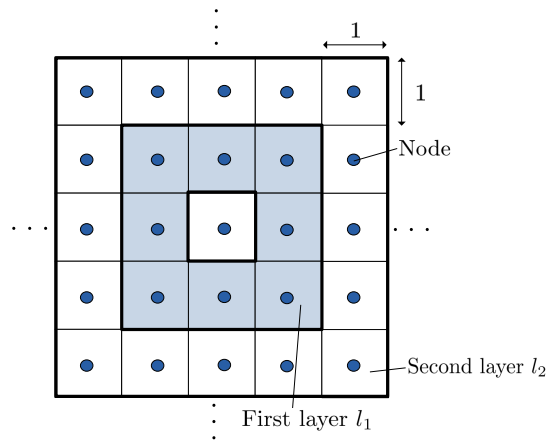&\leq \frac{c_5}{a(f)},
\end{aligned}$$

Fig. 2 Grouping of interference routing cells in extended networks. The first layer l_1 represents the outer 8 shaded cells.

where c_0 and c_5 are some positive constants independent of n, which completes the proof.

Note that the received signal power no longer decays polynomially but rather exponentially with propagation distance in our network. This implies that the absorption term $a(f)$ in (1) will play an important role in determining the performance. It is also seen that the upper bound on P_I does not depend on the spreading factor α. Using Lemma 4, it is now possible to simply obtain a lower bound on the capacity scaling in the network, and hence the following result presents the achievable rates under the MH protocol.

Theorem 2 *In an underwater regular network of unit node density,*

$$T(n) = \Omega \left(\frac{n^{1/2}}{a(f)N(f)} \right) \tag{15}$$

is achievable.

Proof Suppose that the nearest-neighbor MH protocol is used. To get a lower bound on the capacity scaling, the signal-to-interference-and-noise ratio (SINR) seen by receiver $i \in \{1, \cdots, n\}$ is computed as a function of the absorption $a(f)$ and the PSD $N(f)$ of noise n_i. Since the Gaussian is the worst additive noise [39, 40], assuming it lower-bounds the throughput. Hence, by assuming full CSI at the receiver, from (1), (7), and (8), the achievable throughput per

S–D pair is lower-bounded by

$$\log(1 + \text{SINR})$$
$$\geq \log\left(1 + \frac{P/(c_0 a(f))}{N(f) + c_5/a(f)}\right)$$
$$\geq \log\left(1 + \frac{c_6 P}{a(f)N(f)}\right),$$

for some positive constants c_0, c_5, and c_6 independent of n, where the second inequality is obtained from the relationship (6) between $a(f)$ and $N(f)$, resulting in $N(f) = \Omega\left(1/a(f)\right)$. Due to the fact that $\log(1 + x) = (\log e)x + O(x^2)$ for small $x > 0$, the rate of

$$\Omega\left(\frac{1}{a(f)N(f)}\right)$$

is thus provided for each S–D pair. Since the number of hops per S–D pair is given by $O(\sqrt{n})$, there exist $\Omega(\sqrt{n})$ source nodes that can be active simultaneously, and therefore the total throughput is finally given by (15), which completes the proof of the theorem.

Now it is examined how the upper bound shown in Section 3 is close to the achievable throughput scaling.

Remark 1 Based on Theorems 1 and 2, it is easy to see that the achievable rate and the upper bound are of exactly the same order. MH is therefore order-optimal in regular networks with unit node density for all the attenuation regimes.

We also remark that applying the HC strategy [10] may not be helpful to improve the achievable throughput due to long-range MIMO transmissions, which severely degrade performance in highly power-limited networks.[9] To be specific, at the top level of the hierarchy, the transmissions between two clusters having distance $O(\sqrt{n})$ become a bottleneck, and thus cause a significant throughput degradation. It is further seen that even with the random phase model, which may enable us to obtain enough degrees of freedom gain, the benefit of randomness cannot be exploited because of the power limitation.

5 Extension to Random Networks

In this section, we would like to mention a random network configuration, where n S–D pairs are uniformly and independently distributed on a square of unit node density (i.e., an extended random network).

---

[9] In wireless radio networks of unit node density, the HC scheme provides a near-optimal throughput scaling for the operating regimes $2 < \alpha < 3$, where α denotes the path-loss exponent that is greater than 2 [10]. Note that the analysis in [10] is valid under the assumption that α is kept at the same value on all levels of hierarchy.
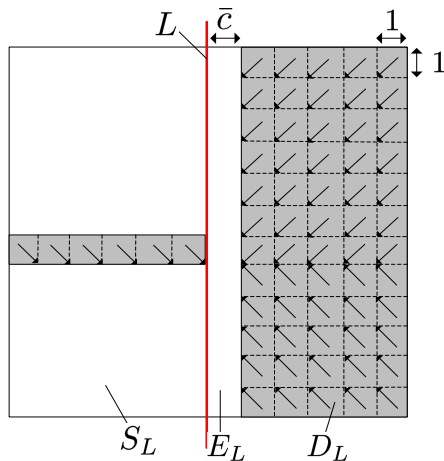
Fig. 3 The node displacement to square vertices, indicated by arrows. The empty zone E_L with width constant $\bar{c}$ is assumed for simplicity.

We first discuss an upper bound for extended random networks. A precise upper bound can be obtained using the binning argument of [10] (refer to Appendix V in [10] for the details). Consider the same cut L, which divides the network area into two halves, as in the regular network case. For analytical convenience, we can artificially assume the empty zone E_L, in which there are no nodes in the network, consisting of a rectangular slab of width $0 < \bar{c} < \frac{1}{\sqrt{7}e^{1/4}}$, independent of n, immediately to the right of the centerline (cut), as done in [28] (see Fig. 3).[10] Let us state the following lemma.

Lemma 5 *Assume a two dimensional extended network where n nodes are uniformly distributed. When the network area is divided into n squares of unit area, there are fewer than $\log n$ nodes in each square with high probability.*

Since the result in Lemma 5 depends on the node distribution but not the channel characteristics, the proof essentially follows that presented in [4]. By Lemma 5, we now take into account the network transformation resulting in a regular network with at most $\log n$ and $2 \log n$ nodes, on the left and right, respectively, at each square vertex except for the empty zone (see Fig. 3). Then, the nodes in each square are moved together onto one vertex of the corresponding square. More specisely, under the cut L, the node displacement

---

[10] Although this assumption does not hold in our random configuration, it is shown in [28] that there exists a vertical cut such that there are no nodes located closer than $0 < \bar{c} < \frac{1}{\sqrt{7}e^{1/4}}$ on both sides of this cut when we allow a cut that is not necessarily linear. Such an existence is proved by using percolation theory [4, 41]. This result can be directly applied to our network model since it only relies on the node distribution but not the channel characteristics. Hence, removing the assumption does not cause any change in performance.

is performed in the sense of decreasing the Euclidean distance between source node $i \in S_L$ and the corresponding destination $k \in D_L$, as shown in Fig. 3, which will provide an upper bound on $P_L^{(k)}$ in (11). It is obviously seen that the amount of power transfer under the transformed regular network is greater than that under another regular network with at most $\log n$ nodes at each vertex, located at integer lattice positions in a square region of area n. Hence, the upper bound for random networks is boosted by at least a logarithmic factor of n compared to that of regular networks discussed in Section 3.

Now we turn our attention to showing an achievable throughput for extended random networks. In this case, the nearest-neighbor MH protocol [1] is also utilized since our network is highly power-limited. Then, the area of each routing cell needs to scale with $2\log n$ to guarantee at least one node in a cell [1,6].[11] Each routing cell operates based on 9-time division multiple access to avoid causing large interference to its neighboring cells [1,6]. For the MH routing, since per-hop distance is given by $\Theta(\sqrt{\log n})$, the received signal power from the intended transmitter and the SINR seen by any receiver are expressed as

$$\frac{c_7 P}{(\log n)^{\alpha/2} a(f)^{\delta\sqrt{\log n}}}$$

and

$$\Omega\left(\frac{1}{(\log n)^{\alpha/2} a(f)^{\delta\sqrt{\log n}} N(f)}\right),$$

respectively, for some constants $c_7 > 0$ and $\delta \geq \sqrt{2}$ independent of n. Since the number of hops per S–D pair is given by $O(\sqrt{n/\log n})$, there exist $\Omega(\sqrt{n/\log n})$ simultaneously active sources, and thus the total achievable throughput $T(n)$ is finally given by

$$T(n) = \Omega\left(\frac{n^{1/2}}{(\log n)^{(\alpha+1)/2} a(f)^{\delta\sqrt{\log n}} N(f)}\right)$$

for some constant $\delta \geq \sqrt{2}$ independent of n (note that this relies on the fact that $\log(1+x)$ can be approximated by x for small $x > 0$). Hence, using the MH protocol results in at least a polynomial decrease in the throughput compared to the regular network case shown in Section 4.[12] This comes from the fact that the received signal power tends to be mainly limited due to exponential attenuation with transmission distance $\Theta(\sqrt{\log n})$. Note that in underwater networks, randomness on the node distribution causes a huge performance

---

[11] When methods from percolation theory are applied to our random network [4, 41], the routing area constructed during the highway phase is a certain positive constant that is less than 1 and independent of n. The distance in the draining and delivery phases, corresponding to the first and last hops of a packet transmission, respectively, is nevertheless given by some constant times $\log n$, thereby limiting performance, especially for the condition $a(f) = \omega(1)$. Hence, using the routing protocol in [4] indeed does not perform better than the conventional MH case [1] in random networks.

[12] In terrestrial radio channels, there is a logarithmic gap in the achievable scaling laws between regular and random networks [1, 25].

degradation on the throughput scaling. Therefore, we may conclude that the existing MH scheme does not satisfy the order optimality under extended random networks regardless of the attenuation parameter $a(f)$.

6 Conclusion

The attenuation parameter and the capacity scaling laws have been characterized in a narrow-band channel of underwater acoustic networks of unit node density. Provided that the frequency f scales relative to the number of nodes, n, the information-theoretic upper bound and the achievable throughput were obtained as functions of the attenuation parameter $a(f)$ in regular networks. In extended networks, based on the power transfer argument, the upper bound was shown to decrease in inverse proportion to $a(f)$. In addition, to show the achievability result, the nearest-neighbor MH protocol was introduced with a simple modification, and its throughput scaling was analyzed. We proved that the MH protocol is order-optimal for all the operating regimes of extended networks. Our scaling results were also extended to the random network scenario, where it was shown that the conventional MH scheme does not satisfy the order optimality for any operating regimes. Although the capacity scaling law was reached for regular network conditions studied here, the exact capacity scaling of random underwater networks remains still open. In Part II of this two-part series, the operating regimes that guarantee the order optimality will be identified in dense underwater networks having unit area.

A Appendix

A.1 Proof of Lemma 1

The following definition is used to simply provide the proof.

Definition 1 [37]: A complex random variable Y is said to be proper if $\tilde{\Sigma}_Y = 0$, where $\tilde{\Sigma}_Y$, called the pseudo-covariance, is given by $E[(Y - E[Y])^2]$.

Since the (k, i)-th element of the channel matrix $\mathbf{H}_L$ is given by (8), it follows that

$$E\left[(h_{ki} - E[h_{ki}])^2\right] = \frac{1}{A(r_{ki}, f)} E\left[\left(e^{j\theta_{ki}} - E\left[e^{j\theta_{ki}}\right]\right)^2\right].$$

From the fact that

$$E\left[e^{j\theta_{ki}}\right] = E\left[\cos(\theta_{ki}) + j\sin(\theta_{ki})\right] = 0$$

due to uniformly distributed θ_{ki} over $[0, 2\pi]$, we thus have

$$\begin{aligned} E\left[(h_{ki} - E[h_{ki}])^2\right] &= \frac{1}{A(r_{ki}, f)} E\left[e^{j2\theta_{ki}}\right] \\ &= \frac{1}{A(r_{ki}, f)} E\left[\cos(2\theta_{ki}) + j\sin(2\theta_{ki})\right] \\ &= 0, \end{aligned} \tag{16}$$

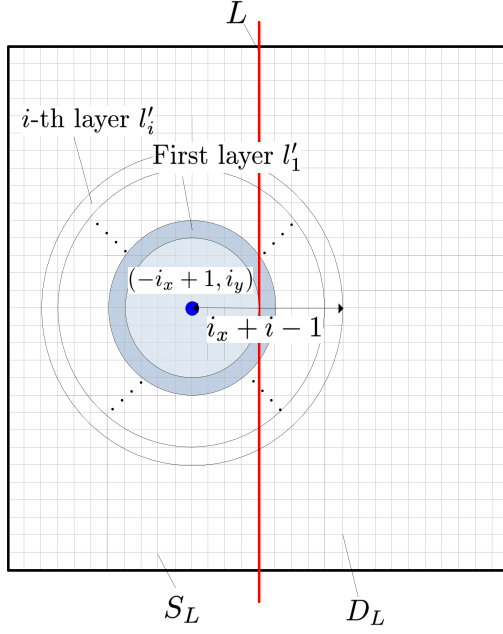which complete the proof, because (16) holds for all $i \in S_L$ and $k \in D_L$.

Fig. 4 Grouping of source nodes in extended networks. There exist $\Theta(k_x)$ nodes in the first layer l'_1. This figure indicates the case where one destination is located at the position (k_x, k_y). The source nodes are regularly placed at spacing 1 on the left half of the cut L.

A.2 Proof of Lemma 3

An upper bound on $P_L^{(k)}$ can be found by using the node-indexing and layering techniques similar to those shown in Section VI of [42]. As illustrated in Fig. 4, layers are introduced, where the i-th layer l'_i of the network represents the ring with width 1 drawn based on a destination node $k \in D_L$, whose coordinate is given by (k_x, k_y), where $i \in \{1, \cdots, \sqrt{n}\}$. More precisely, the ring is enclosed by the circumferences of two circles, each of which has radius $k_x + i$ and $k_x + i - 1$, respectively, at its same center (see Fig. 4). Then from (11), the term $P_L^{(k)}$ is given by

$$P_L^{(k)} = \frac{P}{c_0} \sum_{i_x=1}^{\sqrt{n}/2} \sum_{i_y=1}^{\sqrt{n}} \frac{1}{((i_x + k_x - 1)^2 + (i_y - k_y)^2)^{\alpha/2} \, a(f)^{\sqrt{(i_x+k_x-1)^2+(i_y-k_y)^2}}}.$$

It is further assumed that all the nodes in each layer are moved onto the innermost boundary of the corresponding ring, which provides an upper bound for $P_L^{(k)}$. From the fact that there exist $\Theta(k_x + i)$ nodes in the layer l'_i since the area of l'_i is given by $\pi(2k_x + 2i - 1)$, $P_L^{(k)}$ is

then upper-bounded by

$$
\begin{aligned}
P_L^{(k)} &\leq \frac{P}{c_0} \sum_{i'=k_x}^{\infty} \frac{c_8(i'+1)}{i'^{\alpha} a(f)^{i'}} \\
&\leq \frac{2c_8 P}{c_0 k_x^{\alpha-1}} \sum_{i'=k_x}^{\infty} \frac{1}{a(f)^{i'}} \\
&\leq \frac{2c_8 P}{c_0 k_x^{\alpha-1}} \left(\frac{1}{a(f)^{k_x}} + \int_{k_x}^{\infty} \frac{1}{a(f)^x} dx \right) \\
&\leq \frac{c_9 P}{k_x^{\alpha-1} a(f)^{k_x}}
\end{aligned}
$$

for some positive constants c_0, c_8, and c_9 independent of n, where the fourth inequality holds since $a(f) > 1$, which finally yields (12). This completes the proof.

References

1. Gupta, P. & Kumar, P. R.(2000). The capacity of wireless networks. *IEEE Transactions on Information Theory*, 46(3), 388–404.
2. Knuth, D. E. (1976). Big Omicron and big Omega and big Theta. *ACM SIGACT News*, 8, 18–24.
3. Gupta, P. & Kumar, P. R. (2003). Towards an information theory of large networks: An achievable rate region. *IEEE Transactions on Information Theory*, 49(8), 1877–1894.
4. Franceschetti, M., Dousse, O., Tse, D. N. C., & Thiran, P. (2007). Closing the gap in the capacity of wireless networks via percolation theory. *IEEE Transactions on Information Theory*, 53(3), 1009–1018.
5. Xue, F., Xie, L.-L., & Kumar, P. R. (2005). The transport capacity of wireless networks over fading channels. *IEEE Transactions on Information Theory*, 51(3), 834–847.
6. El Gamal, A., Mammen, J., Prabhakar, B., & Shah, D. (2006). Optimal throughput-delay scaling in wireless networks-Part I: The fluid model. *IEEE Transactions on Information Theory*, 52(6), 2568–2592.
7. El Gamal, A. & Mammen, J. (2006). Optimal hopping in ad hoc wireless networks. In *Proceedings of IEEE INFOCOM* (pp. 1–10).
8. Nebat, Y., Cruz, R. L., & Bhardwaj, S. (2009). The capacity of wireless networks in nonergodic random fading. *IEEE Transactions on Information Theory*, 55(6), 2478–2493.
9. Shin, W.-Y., Chung, S.-Y., & Lee, Y. H. (2011). Parallel opportunistic routing in wireless networks. *IEEE Transactions on Information Theory*, under review for possible publication, [Online]. Available: http://arxiv.org/abs/0907.2455.
10. Özgür, A., Lévêque, O., & Tse, D. N. C. (2007). Hierarchical cooperation achieves optimal capacity scaling in ad hoc networks. *IEEE Transactions on Information Theory*, 53(10), 3549–3572.
11. Niesen, U., Gupta, P., & Shah, D. (2009). On capacity scaling in arbitrary wireless networks. *IEEE Transactions on Information Theory*, 55(9), 3959–3982.

12. Grossglauser, M. & Tse, D. N. C. (2002). Mobility increases the capacity of ad hoc wireless networks. *IEEE/ACM Transactions on Networking*, 10(8), 477–486.
13. Cadambe, V. R. & Jafar, S. A. (2008). Interference alignment and degrees of freedom of the K user interference channel. *IEEE Transactions on Information Theory*, 54(8), 3425–3441.
14. Zemlianov, A. & de Veciana, G. (2005). Capacity of ad hoc wireless networks with infrastructure support. *IEEE Journal on Selected Areas in Communications*, 23(3), 657–667.
15. Partan, J., Kurose, J., & Levine, B. N. (2006). A survey of practical issues in underwater networks. In *Proceedings of International Workshop on Underwater Networks (WUWNet)*.
16. Stojanovic, M. (2007). On the relationship between capacity and distance in an underwater acoustic communication channel. *ACM SIGMOBILE Mobile Computing and Communications Review (MC2R)*, 11(10), 34–43.
17. Lucani, D. E., Médard, M., & Stojanovic, M. (2008). Underwater acoustic networks: Channel models and network coding based lower bound to transmission power for multicast. *IEEE Journal on Selected Areas in Communications*, 26(12), 1708–1719.
18. Lucani, D. E., Stojanovic, M., & Médard, M. (2008). On the relationship between transmission power and capacity of an underwater acoustic communication channel. In *Proceedings of OCEANS'08* (pp. 1–6).
19. Guo, Z., Xie, P., Cui, J. H., & Wang, B. (2006). On applying network coding to underwater sensor networks. In *Proceedings of International Workshop on Underwater Networks (WUWNet)* (pp. 109–112).
20. Lucani, D. E., Médard, M., & Stojanovic, M. (2007). Network coding schemes for underwater networks: The benefits of implicit acknowledgement. In *Proceedings of International Workshop on Underwater Networks (WUWNet)* (pp. 25–32).
21. Carbonelli, C. & Mitra, U. (2006). Cooperative multihop communication for underwater acoustic networks. In *Proceedings of International Workshop on Underwater Networks (WUWNet)* (pp. 97–100).
22. Zhang, W., Stojanovic, M., & Mitra, U. (2010). Analysis of a linear multihop underwater acoustic network. *IEEE Journal of Oceanic Engineering*, 35(10), 961–970.
23. Lucani, D. E., Médard, M., & Stojanovic, M. (2008). Capacity scaling laws for underwater networks. In *Proceedings of Asilomar Conference on Signals, Systems and Computers* (pp. 2125–2129).
24. Chapeau-Blondeau, F. & Monir, A. (2002). Numerical evaluation of the Lambert W function and application to generalization of generalized Gaussian noise with exponent 1/2. *IEEE Transactions on Signal Processing*, 50(9), 2160–2165.
25. Xie, L.-L. & Kumar, P. R. (2004). A network information theory for wireless communication: Scaling laws and optimal operation. *IEEE Transactions on Information Theory*, 50(5), 748–767.
26. Jovicic, A., Viswanath, P., & Kulkarni, S. R. (2004). Upper bounds to transport capacity of wireless networks. *IEEE Transactions on Information Theory*, 50(11), 2555–2565.
27. Shin, W.-Y., Lucani, D. E., Médard, M., Stojanovic, M., and Tarokh, V. (2011). On the order optimality of large-scale underwater networks—Part II: Dense network model. *Springer Wireless Pers. Commun.*, submitted for publication.
28. Özgür, A., Johari, R., Tse, D. N. C., & Lévêque, O. (2010). Information-theoretic operating regimes of large wireless networks. *IEEE Transactions on Information Theory*, 56(1), 427–437.
29. Lee, S.-H. & Chung, S.-Y. (2011). Capacity scaling of wireless ad hoc networks: Effect of finite wavelength. *IEEE Transactions on Information Theory*, under review for possible publication, [Online]. Available: http://arxiv.org/abs/1002.1337.
30. Özgür, A., Lévêque, O., & Tse, D. N. C. (2010). Linear capacity scaling in wireless networks: Beyond physical limits?. In *Proceedings of Information Theory and Applications Workshop (ITA)* (pp. 1–10).
31. Urick, R. J. (1983). *Principles of Underwater Sound*. New York: McGraw-Hill.
32. Berkhovskikh, L. & Lysanov, Y. (1982). *Fundamentals of Ocean Acoustics*. New York: Springer.
33. Coates, R. (1989). *Underwater Acoustic Systems*. New York: Wiley.

34. Franceschetti, M., Migliore, M. D., & Minero, P. (2009). The capacity of wireless networks: Information-theoretic and physical limits. *IEEE Transactions on Information Theory*, 55(8), 3413–3424.

35. Cover, T. M. & Thomas, J. A. (1991). *Elements of Information Theory*. New York: Wiley.

36. Veeravalli, V. V., Liang, Y., & Sayeed, A. M. (2005). Correlated MIMO wireless channels: Capacity, optimal signaling, and asymptotics. *IEEE Transactions on Information Theory*, 51(6), 2058–2072.

37. Neeser, F. D. & Massey, J. L. (1993). Proper complex random processes with applications to information theory. *IEEE Transactions on Information Theory*, 39(7), 1293–1302.

38. Constantinescu, F. & Scharf, G. (1998). Generalized Gram-Hadamard inequality. *Journal of Inequalities and Applications*, 2, 381–386.

39. Médard, M. (2000). The effect upon channel capacity in wireless communications of perfect and imperfect knowledge of the channel. *IEEE Transactions on Information Theory*, 46(5), 933–946.

40. Diggavi, S. N. & Cover, T. M. (2001). The worst additive noise under a covariance constraint. *IEEE Transactions on Information Theory*, 47(11), 3072–3081.

41. Meester, R. & Roy, R. (1996). *Continuum Percolation*. Cambridge, U.K.: Cambridge Univ. Press.

42. Shin, W.-Y., Jeon, S.-W., Devroye, N., Vu, M. H., Chung, S.-Y., Lee, Y. H., & Tarokh, V. (2011). Improved capacity scaling in wireless networks with infrastructure. *IEEE Transactions on Information Theory*, 57(8), 5088–5102.