MULTI-LEVEL APPROACH TO POWER SYSTEM DECISION PROBLEMS -- THE OPTIMIZING LEVEL

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Summary

The multi-level approach to the design of an automatic control system of a large power system or interconnected power systems was described in a previous paper[1] by one of the authors. The multi-level approach breaks down the overall problem into several sub-problems, the solutions of which are then coordinated to achieve the desired overall objective. In the paper referred to, one of the sub-problems identified is that of optimizing control. The present paper presents the formulation of the optimizing problem and, in particular, that of the emergency operating state. The application of mathematical programming techniques which have been found to be highly successful in other fields is described. The optimizing model is seen to be one which may be applied with minor modifications to other optimizing problems in electrical operation and in system planning.

Introduction

The design considerations for a control system whose overall objective is the reliable operation of the generation-transmission system were described in the paper, "The Adaptive Reliability Control System".[1] In order to establish the subject matter of the present paper in the context of the total plan, we will first briefly summarize the salient features of the control structure described in the previous paper.

Electrical operation is viewed as a series of control actions which may be decomposed timewise, into three operating states, designated as: preventive; emergency; and restorative.

In the preventive operating state, the demands of all customers are being satisfied at standard frequency and voltage. The control objective is to continue indefinitely the complete satisfaction of all customer demand at minimum cost.

In the emergency operating state, certain constraints are being violated. The control objective is to take the necessary corrective action so as to meet all the constraints while supplying as much customer demand as possible.

In the restorative operating state, service to some customers has been lost. The control objective is to return to the complete satisfaction of all customer demands in a minimum time.

The mechanism for carrying out the control objective in each state is a hierarchy of three control levels, namely: direct control; optimizing control; and adaptive control.

At the direct control level, high-speed decisions are performed using logic processes and the necessary control actions are carried out directly. The direct control level will be located predominantly at local points within the system with a minimum of simple logic decisions to be done at the control center. Whether done locally or centrally the distinguishing features of direct control are its high-speed and the use of logic programming. The direct control level is, however, subject to instructions from the upper levels.

The optimizing control level solves for the "best" control decisions using a mathematical model of the operating state and an appropriate criterion for optimim performance. All optimizing control decisions would be done on a central computer. Characteristic of the optimizing control is its relatively slower-speed of decision-making and the use of mathematical programming.

The adaptive control level determines and adjusts the settings, parameters, criteria, and logic used in the first and second levels. Whereas both the direct control and optimizing control are automatic, the adaptive control level is a man-machine combination with the system operator playing an active part. The operator would be aided as much as possible by information displays, reports, and off-line calculations.

The various control functions required for each of these levels and for each of the operating states will be coordinated so as to achieve the overall objective of service reliability.

Research and engineering work is continuing at Cleveland Electric Illuminating Company on the development of the control system briefly described above and in more detail in reference 1. The control system is referred to as ARCS.

The present paper is a discussion of the optimizing control in the emergency operating state. The mathematical model is developed in general terms to demonstrate the techniques used. These techniques have been known and tried in optimization problems in other fields but are not too well-known to computer application engineers in the power industry. Thus the purpose of this paper is two-fold:

1. To present in more detail the optimizing control function in the ARCS multi-level structure;
2. To acquaint power engineers working on optimization problems with some of the available techniques which have been tried in connection with the ARCS project.

The Optimizing Problem

An optimizing problem is generally of the form:

find the vector x^* which minimizes $f(x)$ (1)
subject to $g_i(x) \geqslant 0, \ (i = 1,2,\ldots,m)$
$h_i(x) = 0, \ (i = 1,2,\ldots,p)$.

$x = (x_1, x_2, \ldots, x_n)^T$ is the vector of decision variables (the superscript T denoting transposition). $f(x)$ is a measure of performance and will be referred to as the objective function.

For the optimizing problems of the generation-transmission system, the vector, $V = (V_1, V_2, \ldots, V_n)^T$, of the complex voltages at all active busses may be taken as the vector of decision variables.

The inequality and equality constraints result from the requirements of service continuity which all solutions to the optimizing problems should satisfy. These constraints are:

1. Electric network laws
2. Real and reactive power demands at load busses
3. Real and reactive limits at generator busses
4. Real and reactive limits at interconnection busses
5. Voltage limits at generator busses
6. Voltage limits at load busses
7. Thermal ratings of lines and equipment
8. Stability limits
9. System security constraints
10. Set of miscellaneous equality constraints

If it is assumed that the equality constraints, $h_i(V) = 0, \ (i = 1,2,\ldots,p)$, may be satisfied to within a certain tolerance, then these constraints can be expressed as inequalities. The optimizing problem then becomes:

find the vector V^* of complex voltages which minimizes $f(V)$ subject to $g_i(V) \geqslant 0, \ (i = 1,2,\ldots Y)$ where Y is the total number of inequalities. (2)

In the preventive operating state, the objective function, $f(V)$, is the total operating cost. In the emergency operating state, the objective function is the negative of the sum of active powers supplied to the loads. (It should be recalled from reference 1 that the emergency conditions we are concerned with in the optimizing control level is one involving thermal overloads and/or low voltage conditions. Cases of instability or reduced frequency operation would be problems to be resolved at the direct control level.)

It is evident that the objective function and most of the constraints are non-linear functions of V. If there were no constraints, the solution for a minimum of $f(v)$ could be found very efficiently by applying one of several powerful methods[2,3,4] for unconstrained minimization of a non-linear function.

Hence if we transform the original constrained problem to one without constraints we would be able to take advantage of unconstrained minimization of a non-linear function.

Unconstrained Minimization

In the general case, minimization methods for non-linear functions can at best find local minima. To find a local minimum of a function, $f(x)$, a first-order gradient method is generally very slow. Second-order gradient methods, i.e., methods which make use of second derivatives of the function, are far more efficient. Among these are the so-called "conjugate direction" methods[2,3,4]. These methods are noteworthy in that the second derivatives are not explicitly calculated. In fact, one of the methods[4] uses no derivatives at all. The details of these methods are well presented in the reference literature. The method which has been tried on our optimization problem is that due to Fletcher and Powell[2].

The algorithm used in the Fletcher-Powell method for finding a local minimum of $f(x)$ is as follows:

1. Let x_i be the starting point of the $(i + 1)$th step.
2. Calculate the gradient, $\nabla f(x_i)$
3. Calculate direction of descent, $S_i = -H_i \nabla f(x_i)$
   H_o is the unit matrix and
   $H_{i+1} = H_i + A_i + B_i$
   (A_i and B_i are defined below)
4. Find $\alpha_i^* > 0$, such that $f(x_i + \alpha_i S_i)$ is a minimum
5. Repeat 1-4 and stop when $|f(x + 1) - f(x)| < \varepsilon$

Starting with the unit matrix for $i = 0$, H_i remains a symmetric, positive matrix with A_i and B_i given as:

$$A_i = \frac{(\alpha_i^* S_i)(\alpha_i S_i)^T}{(\alpha_i^* S_i)^T Y_i}$$

$$B_i = \frac{-(H_i Y_i)(H_i Y_i)^T}{Y_i^T H_i Y_i}$$

$$Y_i = \nabla f(x_{i+1}) - \nabla f(x_i)$$

Single-Dimensional Minimization

For step 4 of the Fletcher-Powell algorithm, various methods are available for finding the step length along the direction S_i that minimizes the value of $f(x_i + \alpha_i S_i)$. This is a one-dimensional minimization problem and the techniques widely used are either an interpolative procedure or a systematic search such as the Fibonacci method.[5] In our application we use the interpolative technique using quadratic approximation to the function $f(x_i + \alpha_i S_i)$. Fletcher and Powell in their paper[2] used a cubic interpolation.

The Penalty Function Formulation

Let us now consider the transformation of the original problem of minimizing f(V) subject to inequality constraints to an unconstrained minimization. The method that we will use is known as the penalty function technique. Basically the idea is to form a new function:

$$F = f' + p$$

where f' is the original objective function or a variant of it and p is the so-called "penalty function". p is, in general, a linear combination of penalty terms each of which is a function of one of the constraints. Each constraint is thus represented by one term in the penalty function.

Consider the following formulation, which is due to Fiacco and McCormick:[6]

$$F(V,r) = f(V) + r \sum_{i=1}^{Y} \frac{1}{g_i(V)} \qquad (3)$$

where the parameter, $r > 0$.

Here the original function is combined with the penalty function, $p = r \sum_{i=1}^{Y} \frac{1}{g_i(V)}$ to form the new function, F. Suppose we find a point V_o which is feasible, i.e., $g_i(V_o) > 0$ for all i. This means that V_o is strictly in the interior of the constraint space. For some initial value $r_1 > 0$ of the parameter r, and starting with the feasible point V_o, we find the point V_1 which minimizes the function $F(V,r_1)$. The minimization of $F(V,r_1)$ is unconstrained and we can apply the Fletcher-Powell method. It will be noted that in the Fiacco-McCormick formulation, equation (3), one or more of the penalty terms $1/g_i(V)$ will increase very rapidly whenever the point V tends to approach the constraint surface and p becomes infinite at the boundary. Thus in finding the minimizing V_1, the constraints will remain satisfied as the minimum of $F(V,r_1)$ would be expected to stay within the boundary of the constraint space.

For a sequence of values of $r, r_1 > r_2 > \ldots r_x > 0$ the process of unconstrained minimizations is repeated, yielding a sequence of decision vectors $V_1, V_2, \ldots V_x$ each of which is inside the constraint space. Since the value of r is being reduced at each step the penalty is gradually being reduced. As would be expected, the minimum of F(V,r) would approach the minimum of f(V) as r approaches zero. The solution towards the desired minimum via a sequence of unconstrained minimizations is the essence of the Fiacco-McCormick method. In practice the sequence is stopped by suitable stopping criteria.

In general it will be rather unusual to make a choice for the starting point V_o, which is feasible. Fiacco and McCormick point out that any violated constraint can be satisfied by application of the method itself.[6] For instance let $g_k(V) < 0$ be the violated constraint. Apply the Fiacco-McCormick method to the problem where $-g_k(V)$ is the objective function and the penalty terms are made up of all the satisfied constraints. This procedure is applied to each of the violated constraints until all constraints are satisfied.

The process described for finding a feasible point has been applied successfully to a power system problem.

The Emergency State Optimizing Model

Let
G = set of generator busses
I = set of interconnection busses
L = set of load busses
P = set of passive busses
V_k = voltage at bus k
S_k = complex power into bus $k, k \in G, I, L$
A_k = MVA limit at bus $k, k \in G, I$
B_k = voltage limit at bus $k, k \in G, I$
C_k = voltage limit at bus $k, k \in L$
D_k = complex power demand at bus $k, k \in L$
Tij = thermal limit of branch i,j
Re(X) = real part of complex number X
Im(X) = imaginary part of complex number X
Y = admittance matrix of active busses

In the emergency operating state the optimizing problem is:

find the vector V^* which minimizes

$$f(V) = -\sum_{k \in L} Re(S_k)$$

subject to:

$A_k - \lvert S_k \rvert \geq 0$	$\forall\ k \in G, I$
$Re(S_k) - Re(D_k) \geq 0$	$\forall\ k \in L$
$B_k - \lvert V_k \rvert \geq 0$	$\forall\ k \in G, I$
$\lvert V_k \rvert - C_k \geq 0$	$\forall\ k \in L$
$Re(S_k) \geq 0$	$\forall\ k \in G$
$Im(S_k) \geq 0$	$\forall\ k \in G$
$-Re(S_k) \geq 0$	$\forall\ k \in L$
$-Im(S_k) \geq 0$	$\forall\ k \in L$
$T_{ij} - \lvert V_i - V_j \rvert \geq 0$	i,j are extremities of given branch

$$Im(S_k) - Re(S_k) \cdot Im(D_k)/Re(D_k) = 0 \quad k \in L$$

Other constraints may be added as required by the specific network under consideration.

The complex powers appearing in the above equations are expressed in terms of voltages at the active busses according to the fundamental relationship:

$$S_k = V_k \cdot \sum_{i \in G,I,L} \overline{Y_{ki} V_i} \qquad \forall\ k \in G,I,L$$

If the total number of active busses is N then the total number of variables in our problem is 2N since the voltages are complex numbers.

A summary of the computational algorithm following the Fiacco-McCormick technique is as follows:

1. Select a starting point V^o which is feasible. If initial choice is not feasible, go to the feasibility routine which finds a feasible point by repeated application of the method itself.
2. Select initial value of r by the equation:[7]
   $$r_1 = -\nabla f(V^o)^T \cdot \nabla p(V^o) / \|p(V^o)\|^2$$
3. Minimize $F(V, r_i)$ by Fletcher-Powell method for value of $r = r_i$.
4. If $|f(V^{i+1}) - f(V^i)| < \varepsilon_1$ check if $|\nabla f(V^{i+1}) - \nabla f(V^i)| < \varepsilon_2$. If not, or if $|f(V^{i+1}) - f(V^i)| > \varepsilon_1$ reduce r so that $r_{i+1} = C \cdot r_i$ where C is an arbitrary constant, and go back to step 3.

Results and Conclusions

A Fortran IV program has been written for 30 active busses. The number of passive busses may be as high as 75. This program takes up approximately 24,000 words of core memory. This program is made up of several sub-programs, the most important of which are:

1. Routine for finding a feasible starting point
2. Constraint calculations
3. Gradient calculations
4. Fletcher-Powell minimization

The entire program has been tried successfully for a sample problem of seven active busses with an initial non-feasible voltage input. The program is being tested for the Cleveland Electric Illuminating Company system which consists of 24 active and 56 passive busses.

Non-linear programming techniques for power system optimization problems is of great value for electrical operation and system planning. Although the present work is on the emergency model formulation, near-future applications of the method described in this paper are contemplated for the following problems:

1. Economic watt and var dispatch subject to non-linear constraints
2. Ordinary load flow program
3. Load flow for most economic dispatch subject to constraints
4. Optimal load flow where certain parameters are to be minimized or maximized. For example, losses may be minimized or it may be desired to determine the maximum import from the interconnection without violating constraints.

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