Modeling user adoption of advanced traveler information systems: a control theoretic approach for optimal endogenous growth

Hai Yang a,*, Hai-Jun Huang b

a Department of Civil Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, China
b School of Management, Beijing University of Aeronautics and Astronautics, Beijing 100083, China

Abstract

This study is a subsequent development of the dynamic evolution model of the market penetration of advanced traveler information systems (ATIS) proposed by Yang and Meng [Transport. Res. A 35 (2001) 895]. In previous study we have shown that a benefit-driven, user-optimal ATIS market does not necessarily lead to a socially optimal growth and optimal stationary equilibrium level of market penetration of ATIS products or services. The current study proposes an optimal time-dependent service pricing strategy so as to minimize total system cost throughout the time horizon of growth or optimally reach a socially desirable target level of ATIS market penetration in a final stationary equilibrium. We formulate the problem of interest as an optimal control problem and propose an efficient solution algorithm together with a numerical demonstration of the characteristics of the study problem.

© 2004 Elsevier Ltd. All rights reserved.

Keywords: Advanced traveler information system; Market penetration; Service pricing strategy
1. Introduction

There has been a growing body of development of research on advanced traveler information systems (ATIS) in order to alleviate traffic congestion and enhance the performance of road networks (for a review, see, for example, Yang, 1998). A key issue examined in most studies concerns the effects of market penetration of ATIS defined as the proportion of vehicles equipped with ATIS. Researchers generally recognize the fact that equipped and unequipped drivers have different route choice behavior and different levels of market penetration of ATIS result in different levels of traffic congestion, average network-wide travel time and average travel time saving derived from ATIS. Most existing studies have resorted to parametric sensitivity analyses to estimate ATIS benefits over a wide range of market penetration level. Market penetration is treated as an exogenous variable, and the equipped drivers are spread homogeneously among all origin–destination (OD) pairs. Researchers also found that the benefits through provision of ATIS increases initially with market penetration, and grows up to a maximum value at a certain level of market penetration. Negative benefit (increased total travel time) could be expected in certain situations for very high market penetration due to users’ over-reaction. The general policy implication of the existing findings has been that it is only desirable or necessary to equip a limited percentage of vehicles with ATIS.

Instead of assuming exogenous uniform market penetrations that is unable to be attainable or sustainable in a truly competitive ATIS market, a few attempts are made to predict the final attainable saturation level of ATIS market penetration. Wunderlich (1997) presented an iterative simulation and market acceptance model for forecasting saturation market penetration levels for ATIS user services. The market acceptance model compares the time saved and predicts a market penetration based on the income profile associated with the OD pair and departure time group. Polydoropoulou et al. (1997) developed a model system for assessing market penetration and usage rates of ATIS. The modeling technique permits the incorporation of attitudes and perceptions in choice models, and allows realistic predictions of the effects of ATIS service attributes on its adoption. Yang (1998) developed a theoretical endogenous market penetration model for ATIS services in a road network. Market penetration is determined from the information benefit derived from ATIS using a mixed stochastic and deterministic network equilibrium model. Meanwhile, practical interest in predicting the market potential of ATIS products and services is also growing among ATIS developers and evaluators (Charles River and Associates, 1996). In general these limited existing studies recognize that in a market-driven, user-optimal ATIS market the final saturation level of market penetration of ATIS products or services are governed by a number of factors such as annual average time savings, users’ value of time and ATIS prices.

Together with the prediction of the stationary equilibrium market penetration level of ATIS, Yang and Meng (2001) recently proposed a dynamic evolution model that determines the time line of the growth of ATIS market penetration to reach such a saturation level. User heterogeneity is explicitly taken into account by assuming a continuous distribution of users’ value of time. Furthermore, higher income users are assumed to be more aware of the service and more willing to pay for receiving such services. The market will saturate when the value of the information provided declines to the point that no new users will find it advantageous to purchase that service. The proposed model is meaningful because, in reality, new ATIS technologies are not adopted instantaneously, but that their diffusion takes a few years or even a few decades to reach limits.
in terms of market saturation. From their numerical experiments, they found that the temporal change pattern of total travel time on the network is not monotonically decreasing with time and the total travel time in the stationary status varies with the annual system service charge. Indeed, as recognized in previous studies (Yang, 1999; Yang et al., 1999), there should be an optimal level of market penetration that leads to the minimization of total system travel time. This system-optimal market penetration level is not necessarily consistent with the final market-driven equilibrium saturation level, but might be achieved in an intermediate disequilibrium market penetration level during the growth process.

The observations made by Yang and Meng (2001) raised an interesting and meaningful question: how to choose a service pricing strategy to optimally control the temporal evolution of ATIS adoption toward a final desirable target level of system optimization. Indeed the model proposed by Yang and Meng (2001) offered such an interesting application for the optimal control of the time path of evolution of ATIS market penetration. The model does allow for year-varying annual ATIS service charge and thus has the capability of simulating the dynamic optimal pricing strategy either from the supplier or from the system planner perspectives. From the perspective of a system planner, an optimal dynamic pricing strategy should be sought to influence the temporal evolution of ATIS market penetration so that it will move in a desirable direction and arrive at the final system optimum status in terms of total travel time minimization. From the prospective of the ATIS supplier, the dynamic nature of price elasticity should be of interest to develop optimal pricing strategy for maximum stream of profit. Indeed, the model by Yang and Meng (2001) takes explicit account of the heterogeneity of drivers who choose different adoption time and react to price change differently. This suggests that change in price level will have different impact on ATIS demand at different stages, timing is crucial and should be considered by suppliers in developing such optimal pricing strategy.

This paper develops a control strategy and numerical method for the non-linear optimal endogenous growth of ATIS adoption using the growth model developed by Yang and Meng (2001) in conjunction with network equilibrium theory. The paper is organized below. In the next section, we present the continuous time non-linear growth model for user adoption of ATIS services developed by Yang and Meng (2001). In Section 3, we develop the optimal control model for setting optimal time-varying service pricing for optimal growth. In Section 4, a numerical method is proposed for solving the optimal control problem. In Section 5, numerical simulation experiments are conducted to demonstrate the optimal service pricing strategy. Finally, conclusions are provided in Section 6.

2. Dynamic evolution of ATIS market penetration over time and space

2.1. The modified logistic growth function

Suppose drivers are heterogeneous in terms of their values of time, which are assumed to follow a certain continuous probability distribution. Each driver evaluates the potential net benefits of in-vehicle devices in terms of the monetary value of time saving associated with route choice minus service charge. The time saving derived from ATIS service is determined from a mixed network equilibrium model presented in Appendix A. Drivers decide, buy and use a device in a
certain year if the net potential benefit gained from ATIS crosses a threshold. The cumulated adoption of ATIS devices is described by modified logistic type growth model and the temporal growth rate depends on concurrent information benefit and service charge.

Modeling the disequilibrium dynamic evolution process of ATIS market penetration depends on a valid description of the disequilibrium adjustment process which guides the system of interest from one disequilibrium state to another, eventually setting down to a stationary static equilibrium. Different from the standard diffusion models utilized in the analysis of growth phenomena which normally assume fixed constant influencing the growth rate (Banks, 1994; Parker, 1994). Yang and Meng (2001) use a more sophisticated distribution function to incorporate a combined effect of diffusion and pricing trend of services. This will allow development of guidelines for development of setting price strategies.

\[
\frac{dN_w}{dt} = g(\phi_w^t)N_w \left(1 - \frac{N_w}{N_w^*}\right), \quad w \in W
\]  

(1)

where \(N_w\) is the magnitude of a growing quantity of equipped drivers, \(t\) is the time (in years), and \(N_w^*\) is the number of equipped drivers at saturation market penetration level of ATIS, \(g(\phi_w^t)\) is here referred to as the intrinsic variable growth coefficient as function of the net ATIS benefit in the current year. We assume that \(g'(\phi_w^t) \geq 0\). This means that the more benefit currently gained from ATIS, the higher the concurrent adoption rate of ATIS. Note that we assume that only the current magnitude \(\phi_w^t\) of ATIS benefit will accelerate or decelerate the growth rate because the future stream of benefit might be uncertain to drivers. Furthermore, the differential equation (1) is OD pair specific and thus allows for consideration of the heterogeneity of trip characteristics and the variations of the growth and diffusion over the space of the ATIS market.

If \(g(\phi_w^t) \equiv a = \text{positive constant}\), then we have the standard logistic distribution function:

\[
\frac{dN_w}{dt} = aN_w \left(1 - \frac{N_w}{N_w^*}\right), \quad w \in W
\]  

(2)

to which the solution can be obtained analytically and the final form of the logistic distribution is given below:

\[
N_w(t) = N_w^* \left[1 + \left(\frac{N_w^*}{N_w^0} - 1\right) \exp(-at)\right]^{-1}, \quad w \in W
\]  

(3)

where \(N_w(0) = N_w^0\) when \(t=0\) and \(N_w^0\) is the initial market penetration level. In this case, as time, \(t\), approaches infinity, the second term in the denominator approaches zero and the value of \(N_w(t)\) becomes \(N_w^*\). Thus, \(N_w^*\) is simply the asymptotic value of \(N_w(t)\) for large \(t\).

In general the net ATIS benefit \(\phi_w^t\) is an implicit function of the current level of market penetration \(N_w(t)\) and also dependent upon the service charge in particular year \(t\). As long as \(g(\phi_w^t) \neq \text{constant}\), closed form solution of differential equation (1) is impossible. Nevertheless, we can resort to simple finite difference method in combination with the mixed network equilibrium traffic assignment to find the numerical solution. The expression of the differential equation (1) in discrete time by considering a unit time period is written as

\[
N_w(t + 1) = N_w(t) + g(\phi_w^t)N_w(t) \left[1 - \frac{N_w(t)}{N_w^*}\right], \quad w \in W
\]  

(4)
with \( N_w(0) = N_0^w \) again at \( t=0 \), \( \phi'_w \) is the net information benefit gained in the current year \( t \). We postulate that the variable growth coefficient \( g(\phi'_w) \) takes the following functional form:

\[
g(\phi'_w) = a \exp \left[ b(\phi'_w - \overline{\phi}_w) \right], \quad w \in W, \quad \text{where} \quad a > 0, \quad b > 0, \quad \overline{\phi}_w > 0
\]  

(5)

where \( a \) and \( b \) are two parameters \( (a > 0; b \geq 0) \) and \( \overline{\phi}_w \) is OD pair specific benefit threshold. Clearly, \( g(\phi'_w) > 0, \ g'(\phi'_w) \geq 0 \) and \( g(\phi'_w) = a \) for \( \phi'_w = \overline{\phi}_w; \ g(\phi'_w) > a \) for \( \phi'_w > \overline{\phi}_w; \ g(\phi'_w) < a \) for \( \phi'_w < \overline{\phi}_w \). This means that ATIS adoption will grow according to a basic logistic growth curve when the net ATIS benefit equals a certain threshold \( \overline{\phi}_w \). Otherwise, it will grow either quicker or slower than the basic case if the net information benefit is more or less than the threshold. Therefore, this variable growth coefficient dictates logically the impacts of ATIS benefit on the temporal growth of ATIS market penetration.

2.2. Driver heterogeneity and benefit evaluation

Drivers are heterogeneous in terms of their values of time, which are assumed to follow a certain continuous probability distribution. Each driver evaluates the potential net benefits of in-vehicle devices in terms of the monetary value of time saving minus service charge according to individual value of time. Let \( f(\tau) \) be the probability density function of the distribution of VOTs across the drivers. \( f(\tau) \) can be inferred from income distribution and can be well represented by a log-normal distribution (Yang and Meng, 2001):

\[
f(\tau) = \frac{1}{\beta\sqrt{2\pi}} \tau^{-\frac{1}{2}} \exp \left[ -\frac{1}{2} \left( \frac{\ln \tau - \zeta}{\beta} \right)^2 \right], \quad 0 < \tau < \infty, \quad \beta > 0
\]  

(6)

where \( \zeta \) and \( \beta \) are the mean and the standard deviation of \( \ln \tau \), and are related to the mean \( \mu \) and the standard deviation \( \sigma \) of \( \tau \) by the following equations:

\[
\zeta = \ln \mu - \frac{1}{2} \beta^2, \quad \beta^2 = \ln \left( 1 + \frac{\sigma^2}{\mu^2} \right).
\]  

(7)

Here we make a bold, but rational assumption that higher VOT (income) drivers will adopt the ATIS earlier. In line with this assumption, the expected VOT of the adopters in year \( t \) can be approximately estimated below:

\[
N_w(t-1) = Q_w \int_{\tau'_w-1}^{+\infty} f(x) \, dx = \left[ 1 - F(\tau'_w-1) \right] Q_w, \quad w \in W
\]  

(8)

\[
\tau'_w-1 = F^{-1} \left( 1 - \frac{N_w(t-1)}{Q_w} \right), \quad w \in W
\]  

(9)

where \( N_w(t-1) \) is the cumulative number of ATIS adopters by the end of year \( t-1 \); \( F(\tau) \) is the cumulative probability density function of \( f(\tau) \). \( \tau'_w-1 \) represents the VOT of the last ATIS adopter in year \( t-1 \) and can be used approximately for benefit evaluation of ATIS adopters in year \( t \).

Suppose the annual capital cost to buy and use in-vehicle device in year \( t \) is \( x(t) \) (HK$). Each driver is assumed to make an annual number of trips \( L \) between a specific OD pair. Then the
annual information benefit for a driver between OD pair $w$ to equip with ATIS in year $t$ is estimated to be

$$
\phi'_w = \tau'_w (C^\text{unequ}_w - C^\text{equ}_w) L - x(t), \quad w \in W
$$

(10)

where the term $(C^\text{unequ}_w - C^\text{equ}_w)$ is the travel time saving defined as the difference of expected actual travel time between unequipped and equipped drivers for each OD pair $w \in W$.

2.3. Determination of saturation market penetration level

Suppose that the market for ATIS will grow over time and eventually reach a saturation level of market penetration of the system in a steady equilibrium. The saturation market penetration level is so determined that the net information benefit of the marginal potential adopter vanishes. Namely, $\phi^* = (\ldots, \phi^*_w, \ldots) = 0$. From Eqs. (9) and (10), we have

$$
F^{-1}\left(1 - \frac{N^*_w}{Q_w}\right) (C^\text{unequ}_w(N^*) - C^\text{equ}_w(N^*)) L - x^* = 0, \quad w \in W
$$

(11)

where $C^\text{unequ}_w(N^*)$ and $C^\text{equ}_w(N^*)$ are the expected actual travel times of unequipped and equipped drivers between OD pair $w \in W$, respectively, when the number of drivers equipped with ATIS is $N^*_w$, $w \in W$. $x^*$ is the constant annual service charge fee in the stationary status, $x^* = \lim_{t \to \infty} x(t)$. Therefore, the stationary equilibrium level of ATIS market penetration $N^* = (\ldots, N^*_w, \ldots)$ can be obtained by solving a system of simultaneous non-linear equations (11) for given $x^*$.

3. Optimal control for optimal growth

Note that the total network travel time will change over years as market penetration evolves. The numerical results by Yang and Meng (2001) showed that the temporal change pattern of total travel time is not monotonic and the total travel time in the stationary status also varies with the annual service charge rate. Yang and Meng (2001) also found that the final total network travel time at stationary equilibrium is governed directly by the annual system service charge rate (but irrelevant to parameter $b$ in Eq. (5)) and thus is controllable through appropriate service charge. The temporal evolution pattern of the total network travel time is also dependent upon the annual system service charge as $b \neq 0.0$ and is thus controllable as well by choosing optimal year-varying service charge. Therefore, it is interesting and meaningful to choose a service pricing strategy to optimally control the temporal evolution of ATIS adoption toward the minimization of total system travel time throughout the time horizon or reach the final desirable target level of system optimization.

If the annual service charge is regarded as control variable and the market penetration as state variable, then an optimal control problem can be formulated. The optimal control problem is divided into two separate phases: The first phase is to determine the optimal stationary equilibrium level of ATIS market penetration, and the second phase is to find the optimal time-dependent service pricing strategy so as to minimize total system cost throughout the time horizon or reaches the final desirable target level of system optimization.
3.1. Determination of optimal stationary equilibrium level of ATIS market penetration

The dynamic evolution path of ATIS market penetration depends on its eventual stationary level, and thus the dynamic control problem requires determination of the target or optimal stationary equilibrium level of ATIS market penetration, i.e., \( N^* = (\ldots, N^*_w, \ldots) \) in Eq. (11). The optimal target penetration level is solely dependent upon \( x^* \). Because we assume a spatially uniform service charge fee or \( x^* \) is independent of individual OD pairs, determination of the optimal stationary market penetration level is simply a one-dimensional search problem with respect to the constant annual service charge fee \( x^* \). Let the optimal stationary service charge fee be \( \bar{x}^* \) and the corresponding optimal stationary level of ATIS market penetration be \( \bar{N}^* = (\ldots, N^*_w, \ldots) \). The subsequent phase is how to choose a year-dependent annual charge fee \( x(t) \) to minimize total system cost throughout the time horizon or optimally reach the predetermined target market penetration level \( \bar{N}^* \).

3.2. Optimal control of dynamic evolution path of ATIS market penetration

Note that the total network travel time is solely determined by the current level of ATIS market penetration through the mixed SUE and DUE network equilibrium model (see Appendix A). We thus have the following expression of the total network travel time \( Z(t) \) for a unit modeling time in year \( t \):

\[
Z(t) = \sum_{a \in A} v_a(N(t)) \cdot t_a(v_a(N(t))) = Z(N(t)), \quad t \in [0, T]
\]  

(12)

where \( N(t) = (\ldots, N_w(t), \ldots) \). In the optimal control of the dynamic disequilibrium evolution of ATIS market penetration, we wish to minimize the total system travel time over a fixed planning horizon \([0, T]\). The minimization problem can then be formulated as a discrete time optimal control problem as follows:

\[
J = \sum_{t=0}^{T} \left( Z(N(t)) + \rho \frac{1}{2} \max^2 \{0, -x(t)\} \right)
\]  

(13)

is minimized, subject to

\[
N_w(t + 1) = N_w(t) + g(\phi_w')N_w(t) \left[ 1 - \frac{N_w(t)}{N_w^*} \right], \quad w \in W, \quad t = 0, \ldots, T - 1
\]  

(14)

\[
N_w(0) = N^*_w, \quad w \in W
\]  

(15)

where \( \rho \) is a suitably large positive constant such that the non-negativity of \( x(t) \) is guaranteed by setting a penalty term in Eq. (13), \( g(\phi_w') \) is given by Eq. (5) and \( \phi_w' \) by Eq. (10). In the above, \( N(t) \) is the state variable, \( x(t) \) is the control variable which appears in the definition of \( \phi_w' \).

Note that the terminal boundary conditions

\[
N_w(T) = \bar{N}^*_w, \quad w \in W
\]  

(16)
can also be added into the optimal control problem. However, this optimal stationary equilibrium level of ATIS market penetration is obtained in the first phase with the assumption \( t \to \infty \). Hence, Eq. (16) is hard to be exactly reached in a limited time horizon. Next, in designing solution algorithm and conducting numerical study we do not consider conditions (16) explicitly.

Now we derive the first-order necessary conditions that the solution of the optimal control problem must satisfy. Let \( k(t) \) denote the costate (or adjoint) variables associated with the trajectory equation (14). We define the Hamiltonian sequence as

\[
H[t, N(t), x(t), \lambda(t + 1)] = Z(N(t)) + \sum_{w \in W} \lambda_w(t + 1) \left\{ N_w(t) + g(\phi_w^i)N_w(t) \left[ 1 - \frac{N_w(t)}{N^*_w} \right] \right\}
\]

\[+ \frac{\rho}{2} \max^2 \{0, -x(t)\}, \quad t = 0, \ldots, T - 1.
\] (17)

According to the Pontryagin Minimum Principle suitable to the discrete time version of continuous time optimal control problem (Teo et al., 1991), at a solution point the state variables must satisfy Eqs. (14) and (15) and the costate variables must obey

\[
\lambda_w(t) = \frac{\partial Z(N(t))}{\partial N_w(t)} + \lambda_w(t + 1) \left\{ 1 + \frac{\partial g(\phi_w^i)}{\partial N_w(t)} N_w(t) \left[ 1 - \frac{N_w(t)}{N^*_w} \right] + g(\phi_w^i) \left[ 1 - \frac{2N_w(t)}{N^*_w} \right] \right\}
\] (18)

and

\[
\lambda_w(T) = 0, \quad w \in W
\] (19)

The two systems of difference equations (14), (15) and (18), (19) constitute an important part of the solution algorithm presented in the next section. In Eq. (18), the derivative information can be obtained by sensitivity-based methods, where

\[
\frac{\partial g(\phi_w^i)}{\partial N_w(t)} = ab \exp \left[ b (\phi_w^i - \bar{\phi}_w) \right] \tau_w^{-1} \left( \frac{\partial c^\text{unequ}_w}{\partial N_w} - \frac{\partial c^\text{equ}_w}{\partial N_w} \right) L, \quad w \in W.
\] (20)

4. A solution algorithm

In this section, we propose an approximate iterative algorithm in conjunction with the gradient method for solving the discrete time optimal control problem formulated in the last section. The gradients of the objective function (13) subject to variables \( x(t) \) are given below:

\[
\frac{\partial J}{\partial x(t)} = \sum_{t=0}^{T-1} \frac{\partial H[t, N(t), x(t), \lambda(t + 1)]}{\partial x(t)}
\]

\[= \sum_{w \in W} \lambda_w(t + 1) \frac{\partial g(\phi_w^i)}{\partial x(t)} N_w(t) \left[ 1 - \frac{N_w(t)}{N^*_w} \right] - \rho \max \{0, -x(t)\}, \quad t = 0, \ldots, T - 1
\] (21)

where \( \frac{\partial g(\phi_w^i)}{\partial x(t)} = -ab \exp [b (\phi_w^i - \bar{\phi}_w)] \). The procedure of the algorithm is described as follows (similar algorithms are used in Huang and Yang, 1996; Yang and Huang, 1997):
Step 1. Choose the initial values of the control variables $x^n(t)$ where the iterative index $n=1$ and $t=0,\ldots,T-1$.

Step 2. Compute $N^n_w(t)$ by solving Eqs. (14) and (15) forward from $t=0,\ldots,T$.

Step 3. Compute $Z^n_w(t)$ by solving Eqs. (18) and (19) backward from $t=T,\ldots,1$.

Step 4. Determine the gradient $\partial J/\partial x(t), t=0,\ldots,T$, using Eq. (21).

Step 5. Renew the values of control variable

$$x^{n+1}(t) = x^n(t) - \pi[\partial J/\partial x(t)], \quad t = 0,\ldots,T-1$$

where $\pi$ is a fixed step size. If the changes in the values of $J$ between successive iterations have been sufficiently small, stop; otherwise set $n=n+1$ and go to Step 2.

A fixed step size is employed in carrying on our example studies for the purpose of computational simplicity. However, the steepest descent method as well as other methods can be used to accelerate the rate of convergence (Teo et al., 1991). In addition, in our example studies, all derivative information needed in the algorithm, unless expressed explicitly, is numerically obtained through a sensitivity-based method.

5. Numerical experiments

In this section, we use hypothetical test scenarios to show the effectiveness of the modeling proposed in this paper in finding the optimal stationary equilibrium ATIS market penetration level and the optimal time-dependent service pricing strategy. Sensitivity analysis for examining how various factors such as congestion level and travel time uncertainty affect optimal pricing strategy can also be conducted, but omitted in this paper for short.

Fig. 1 shows the experiment network with the following link travel time function:

$$t_a(v_a) = t^0_a + 0.15 \left( \frac{v_a}{z_a} \right)^4, \quad a \in A$$

where link free-flow travel time $t^0_a$ and link capacity $z_a$ are given in Table 1. There are two OD pairs from node 1 and 6 to node 16 with the total demands $Q_{1\rightarrow 16} = 2000$ (veh/h) and $Q_{6\rightarrow 16} = 2000$ (veh/h), respectively. The feasible paths for the two OD pairs are 63 and 13, respectively. This large difference in the number of paths between the two OD pairs allows for our investigation on the spatial effects of ATIS services (but omitted for short in this paper). The following basic values of the model parameters are used: $L=540$ (trip/year), $a=0.35$ (1/year), $b=2.5 \times 10^{-5}$ (HK$/year)^{-1}$, $\phi_{1\rightarrow 16} = 1.4 \times 10^4$ (HK$/year)$, $\phi_{6\rightarrow 16} = 1.8 \times 10^4$ (HK$/year)$, $\theta=0.006$ (1/min), $\mu=3.0$ (HK$/min)$, $\sigma=0.8$ (HK$/min)$.

Fig. 2 depicts the total system travel time in stationary equilibrium with respect to constant annual service charges ranging from 50 to 6000 (HK$). The total system travel time is computed from the stationary equilibrium level of ATIS market penetration that is the solution of non-linear equations (11) for given constant annual service charge. In our study the equations are actually solved using the MINPACK subroutine HRBRD1 that is based on a variation of Newton’s methods (Press et al., 1992). As shown in Fig. 2, the total system travel time in a stationary state...
reaches its minimum value (i.e., 203,108 veh-min) when the annual fee is 4150 (HK$). The corresponding saturation market penetration level is $N_{16}^* = 1518$ (veh/h) and $N_{6-16}^* = 623$ (veh/h).

Fig. 3 presents the optimal time-dependent service charges during a time horizon of 50 years. This is the solution of the optimal control problem. The curves with fixed fee 4150HK$ for all years and a simple linear fee growth strategy, are also given in this figure for comparison. It
can be seen that in the first several years the annual service charge fees should be very low (even null) for attracting more ATIS users. This can lead to a rapid decline in the total travel times at the early stage of ATIS services (see Fig. 4) because of the rapidly increasing market penetration of ATIS users in these years (see Fig. 5).

Fig. 2. Change of the total travel time with constant annual system service charge in a stationary equilibrium level of ATIS market penetration.

Note that the optimal charge fee gradually increases and moves toward the optimal stationary value. As shown in Fig. 4, the corresponding total system travel time decreases slowly, approaching the minimum value 203,108 (veh-min). Fig. 5 shows the optimal dynamic evolution pattern of ATIS market penetration, reaching 1518 (veh/h) for OD pair (1,16) and 623 (veh/h) for OD pair (6,16) in the 50th year.

Fig. 3. The optimal time-dependent charge rate obtained from the model versus the constant charge rate of 4150HK$ and the linear growth charge rate.
For validating the optimality of the solution, we let the annual service charge fee be a constant 4150HK$ for all years and change in a simple linear form (see Fig. 3), respectively, then investigate the resultant total travel times and dynamic evolution pattern of ATIS users. The results are shown in Figs. 4 and 5. Fig. 4 clearly shows that the total travel times associated with the optimal solution are always less than those associated with the non-optimal solutions (i.e., the constant charge strategy and the linear growth charge strategy). The optimal service charge leads to a 5.05% and 2.33% total travel time savings over the constant and linear growth charge strategies, respectively, in the time horizon of 50 years. Fig. 5 shows that the saturation market penetration levels for both OD pairs are most quickly realized by the optimal charge strategy, then followed by the linear growth charge strategy and lastly by the constant charge strategy.

Fig. 4. Change of the total system travel time over years under alternative annual ATIS service charging strategies.

Fig. 5. Evolution of ATIS market penetration (OD trips with ATIS) under alternative annual ATIS service charging strategies.

For validating the optimality of the solution, we let the annual service charge fee be a constant 4150HK$ for all years and change in a simple linear form (see Fig. 3), respectively, then investigate the resultant total travel times and dynamic evolution pattern of ATIS users. The results are shown in Figs. 4 and 5. Fig. 4 clearly shows that the total travel times associated with the optimal solution are always less than those associated with the non-optimal solutions (i.e., the constant charge strategy and the linear growth charge strategy). The optimal service charge leads to a 5.05% and 2.33% total travel time savings over the constant and linear growth charge strategies, respectively, in the time horizon of 50 years. Fig. 5 shows that the saturation market penetration levels for both OD pairs are most quickly realized by the optimal charge strategy, then followed by the linear growth charge strategy and lastly by the constant charge strategy.
6. Conclusions

In this paper, we first presented a differential equation in a network to characterize the temporal and spatial evolution and the equilibrium saturation level of ATIS market penetration. Net information benefit and driver heterogeneity are incorporated as important factors that affect the growth process. We then developed an optimal control model for setting optimal time-varying service pricing for optimal growth of ATIS market penetration. An approximate iterative algorithm in conjunction with a gradient method have been proposed for solving the optimal control model. Our numerical example demonstrated that the proposed modeling approach is able to find the optimal stationary equilibrium level of ATIS market penetration and the optimal time-dependent service charge rate. Indeed, timing is crucial and must be considered by system planners and ATIS suppliers in developing the optimal pricing strategy.

Acknowledgments

Hai Yang is grateful for financial support for this work from the Hong Kong Research Grants Council (Grant No. HKUST6094/00E). Hai-Jun Huang wants to thank for financial support by the National Natural Science Foundation of China (Grant No. 79825001), he is also supported by MADIS Laboratory of the Chinese Academy of Sciences.

Appendix A. The mixed network equilibrium model

Consider a long time horizon. In a particular year $t$, there is a certain number of drivers equipped or unequipped with ATIS. We apply static, multiple route strategy to model the different routing behaviors and interactions of the two groups of drivers with recurrent congestion because of their differing levels of knowledge of network conditions, (Yang (1998, 1999). Specifically we assume that equipped drivers know route travel time better, while unequipped drivers adhere to a fixed route choice or stochastic traffic assignment. For given numbers of equipped and unequipped drivers in a network, the mixed behavior equilibrium model can be formulated as the following equivalent minimization program (Yang, 1998):

\[
\min F(v, f, h) = \sum_{a \in A} \int_{0}^{c_a} t_a(x) \, dx + \frac{1}{\theta} \sum_{w \in W} \sum_{r \in R_w} h^w_r \ln h^w_r
\]

subject to

\[
\sum_{r \in R_w} f^w_r = M_w, \quad w \in W
\]

\[
\sum_{r \in R_w} h^w_r = N_w, \quad w \in W
\]

\[
f^w_r \geq 0, \quad h^w_r \geq 0, \quad r \in R_w, \quad w \in W
\]
where $A$ is the set of links, $W$ is the set of all OD pairs and $R_w$ is the set of routes between OD pair $w \in W$, $t_a(v_a)$ is the travel time for link $a \in A$ as an increasing and strictly convex function of link flow $v_a$ on that link. Link flow is defined by

$$v_a = \sum_{w \in W} \sum_{r \in R_w} (f_r^w + h_r^w) \delta_{wr}^w, \quad a \in A \quad (A.5)$$

where $\delta_{wr}^w = 1$ if route $r$ between OD pair $w$ uses link $a$, and 0 otherwise. $f_r^w$ and $h_r^w$ are the flows of equipped and unequipped drivers on route $r \in R_w$, $w \in W$, respectively. $M_w$ and $N_w$ are the total number of equipped and unequipped drivers between OD pair $w \in W$,

$$M_w + N_w = Q_w, \quad w \in W \quad (A.6)$$

where $Q_w$ is the total travel number of both the equipped and unequipped drivers over the network between OD pair $w \in W$. The positive value of parameter $\theta$ is related to the variation of travel time perceptions of drivers and is referred to as travel time variability. This variability parameter measures the sensitivity of route choices to travel cost.

The minimization program will lead to the deterministic user-equilibrium assignment model for the equipped drivers, and the logit-based stochastic user equilibrium assignment model for the unequipped drivers.

For equipped drivers

if $f_r^w > 0$, then $c_r^w = c_w$; if $f_r^w = 0$, then $c_r^w \geq c_w, \quad r \in R_w, \quad w \in W \quad (A.7)$

where $c_w$ is the minimum route travel cost between OD pair $w \in W$. The expected travel time $C_{w}^{\text{equ}}$ of equipped drivers is thus assumed to be

$$C_{w}^{\text{equ}} = c_w, \quad w \in W \quad (A.8)$$

For unequipped drivers

$$p_r^w = \frac{\exp(-\theta c_r^w)}{\sum_{k \in R_w} \exp(-\theta c_k^w)}, \quad r \in R_w, \quad w \in W \quad (A.9)$$

where $p_r^w$ is the probability of unequipped drivers choosing route $r \in R_w$, and $c_r^w$ is the travel cost on route $r \in R_w$ between OD pair $w \in W$. The expected travel time $C_{w}^{\text{unequ}}$ of unequipped drivers is thus given by

$$C_{w}^{\text{unequ}} = \sum_{r \in R_w} p_r^w c_r^w, \quad w \in W \quad (A.10)$$

If the information available to equipped drivers is partial or imperfect, then the travel time under ATIS is stochastic as well, but with less variability than traveling without ATIS. In this case the level and amount of information provided by ATIS to the equipped drivers can be incorporated into the model by different values of the variability parameter (Yang, 1998; Yang and Meng, 2001).
References


