W06D1
Current, Current Density, Resistance and Ohm's Law, Magnetic Field, Magnetic Force

Today's Reading Assignment: Current, Current Density, and Resistance and Ohm's Law, Magnetic Fields and Forces Course Notes: Sections 6.1-6.5, 8.1-8.3

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Announcements

Week 6 Problem Solving and Math Review Tuesday from 9-11 pm in 26-152

PS 5 due Week 6 Tuesday at 9 pm in boxes outside 32-082 or 26-152

W06D2 Reading Assignment Course Notes: Magnetic Forces, Currents & Dipoles; Sections 8.3, 9.1-9.2

Exam 2 Thursday March 21 7:30 - 9:30 pm

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Outline

Current and Current Density

Resistance and Ohm's Law

Magnetic Field

Magnetic Forces

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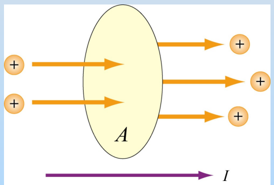
Current: Flow Of Charge

Average current I_{av}: Charge ΔQ flowing across area A in time Δt

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

Instantaneous current: differential limit of I_{av}

$$I = \frac{dQ}{dt}$$



Units of Current: Coulomb/second = Ampere

How Big is an Ampere?

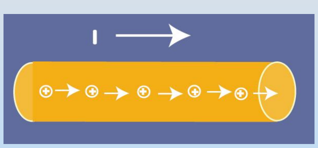
- Household Electronics ~1 A
- Battery Powered ~100 mA (1-10 A-Hr)
- Household Service 100 A
- Lightning Bolt 10 to 100 kA

- To hurt you 40 (5) mA DC(AC)
- To throw you 60 (15) mA DC(AC)
- To kill you 0.5 (0.1) A DC(AC)

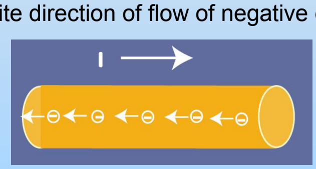
- Fuse/Circuit Breaker 15-30 A

Direction of the Current

Direction of current is direction of flow of pos. charge

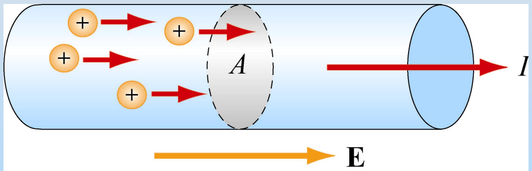


or, opposite direction of flow of negative charge



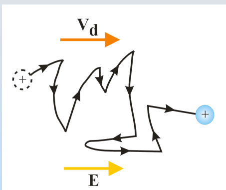
Why Does Current Flow?

If an electric field is set up in a conductor, charge will move (making a current in direction of E)



Note that when current is flowing, the conductor is not an equipotential surface (and $E_{inside} \neq 0$)!

Microscopic Picture



Drift velocity is the average velocity forced by applied electric field in the presence of collisions.

Magnitude is typically 4×10^{-5} m/sec, or 0.04 mm/second!

To go one meter at this speed takes about 10 hours!

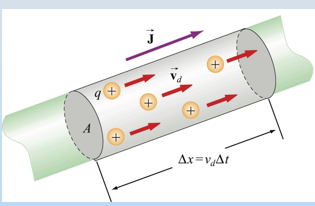
Summary

Current: I

Charge $\Delta Q = q(nA\Delta x)$

Displacement $\Delta x = v_d \Delta t$

Drift speed $I_{avg} = \dfrac{\Delta Q}{\Delta t} = nqv_d A$



Current Density J

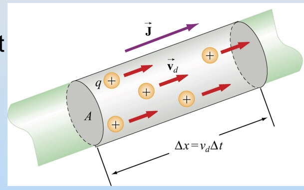
Let n = number of charged objects per unit volume

q = charge of object

$\vec{\mathbf{v}}_q$ = drift velocity of object

The current density is current per unit area

$$\vec{\mathbf{J}} \equiv nq\vec{\mathbf{v}}_q \Rightarrow \sum_i n_i q_i \vec{\mathbf{v}}_{q_i}$$



Generalization for many charged moving objects

$$\vec{\mathbf{J}} \equiv \sum_i n_i q_i \vec{\mathbf{v}}_{q_i}$$

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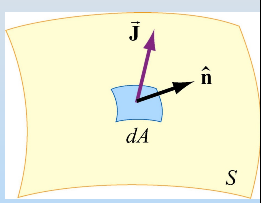
Current and Current Density J

$$\vec{\mathbf{J}} \equiv nq\vec{\mathbf{v}}_q \Rightarrow \sum_i n_i q_i \vec{\mathbf{v}}_{q_i}$$

Current is the flow (flux) of current density through an open surface

$$I = \int_S \vec{\mathbf{J}} \cdot \hat{\mathbf{n}}\, dA = \int_S \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$$



Special case: $\vec{\mathbf{J}}$ uniform and perpendicular to surface

$$I = JA$$

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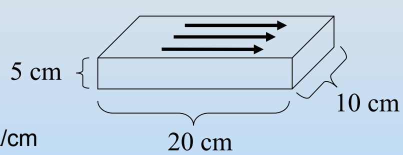
Concept Question: Current Density

A current I = 200 mA flows in the wire below. What is the magnitude of the current density J?



1. J = 40 mA/cm
2. J = 20 mA/cm
3. J = 10 mA/cm
4. J = 1 mA/cm^2
5. J = 2 mA/cm^2
6. J = 4 mA/cm^2

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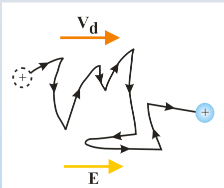
Conductivity and Resistivity



Ability of current to flow depends on density of charges & rate of scattering

Two quantities summarize this:

σ_c: conductivity

ρ_r: resistivity

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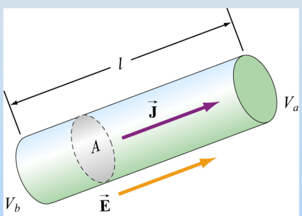
Microscopic Ohm's Law

$$\boxed{\vec{\mathbf{E}} = \rho_r \vec{\mathbf{J}} \quad ; \quad \vec{\mathbf{J}} = \sigma_c \vec{\mathbf{E}}}$$

$$\boxed{\rho_r \equiv \frac{1}{\sigma_c}}$$



ρ_r and σ_c depend only on the microscopic properties of the material, not on its shape

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Demonstrations:

Temperature Effects on Resistance F4

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=F%204&show=0

Conducting Glass F1

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=F%201&show=0

Conductivity of Ionizing Water F5

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=F%205&show=0

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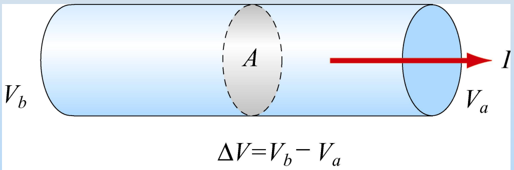
Why Does Current Flow?

Instead of thinking of Electric Field, think of potential difference across the conductor



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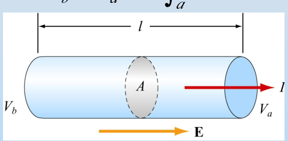
Ohm's Law

What is relationship between electric potential difference and current?

$$\Delta V = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E\ell$$



$$\left.\begin{array}{l} J = \dfrac{E}{\rho} = \dfrac{\Delta V / \ell}{\rho} \\ J = \dfrac{I}{A} \end{array}\right\} \Rightarrow \Delta V = I\left(\frac{\rho\ell}{A}\right) \equiv IR$$

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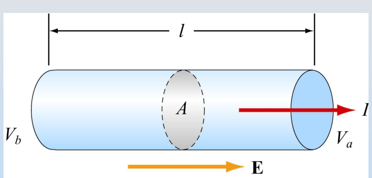
Ohm's Law



$$\boxed{\Delta V = IR} \qquad \boxed{R = \frac{\rho\ell}{A}}$$

R has units of Ohms (W) = Volts/Amp

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How Big is an Ohm?

- Short Copper Wire milliohms (mΩ)
- Notebook paper (thru) ~1 G Ω
- Typical resistors Ω to 100 M Ω

- You (when dry) 100 k Ω
- You (when wet) 1 k Ω
- Internally (hand to foot) 500 Ω

Stick your wet fingers in an electrical socket:

$I = V / R \sim 120\,\mathrm{V} / 1\mathrm{k}\Omega \sim 0.1\,\mathrm{A}$ You're dead!

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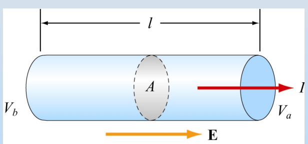
Concept Question: Resistance

When a current flows in a wire of length *L* and cross sectional area *A*, the resistance of the wire is

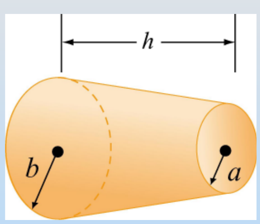


1. Proportional to *A*; inversely proportional to *L*.
2. Proportional to both *A* and *L*.
3. Proportional to *L*; inversely proportional to *A*.
4. Inversely proportional to both *L* and *A*

P18-20

Group Problem: Calculating Resistance

Consider a material of resistivity ρ_r in a shape of a truncated cone of altitude *h*, and radii *a* and *b*, for the right and the left ends, respectively, as shown in the figure. Assuming that the current is distributed uniformly throughout the cross-section of the cone, what is the resistance between the two ends? You may find the following integral useful (where and are constants).



$$\int \frac{du}{(\alpha u + \beta)^2} = -\frac{1}{\alpha(\alpha u + \beta)}$$

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Magnetic Fields

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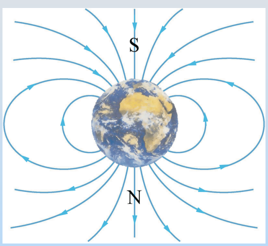
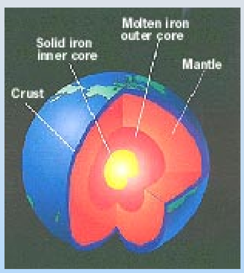
Magnetic Field of the Earth





North magnetic pole
located in southern hemisphere

http://www.youtube.com/watch?v=AtDAOxaJ4Ms

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Demonstrations:

Magnetic Field Lines of a Bar Magnet G2

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=G%202&show=0

Magnetic Field Lines of a Single Wire G12

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=G%2012&show=0

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How Big is a Tesla?

- Earth's Field — 5×10^{-5} T = 0.5 Gauss
- Brain (at scalp) — ~1 fT
- Refrigerator Magnet — 1 mT
- Inside MRI — 3 T
- Good NMR Magnet — 18 T
- Biggest in Lab — 150 T (pulsed)
- Biggest in Pulsars — 10^8 T

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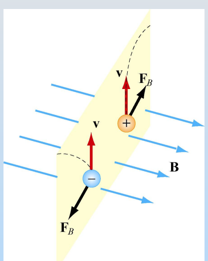
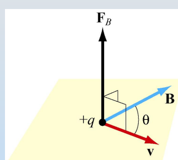
Moving Charges Feel Magnetic Force





$$\boxed{\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}}$$

Magnetic force perpendicular both to:
Velocity **v** of charge and magnetic field **B**

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Concept Question: Units Magnetic Field

What are the correct SI units for the magnetic field?

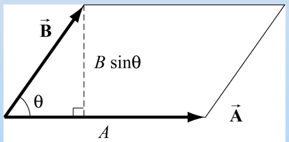
1. C/N-m-s
2. N-m-s/C
3. N/C
4. N-s/C-m
5. C-m/N-s

P18-27

Cross Product: Magnitude

Computing magnitude of cross product A x B:

$$\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}} \qquad \left|\vec{\mathbf{C}}\right| = \left|\vec{\mathbf{A}}\right|\left|\vec{\mathbf{B}}\right| \sin\theta$$



$|\vec{\mathbf{C}}|$: area of parallelogram

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Cross Product: Direction

Right Hand Rule #1:

$$\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}}$$

For this method, keep your hand flat!

1) Put Thumb (of right hand) along **A**
2) Rotate hand so fingers point along **B**
3) Palm will point along **C**

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Cross Product: Signs

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \qquad \hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}$$

$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}} \qquad \hat{\mathbf{k}} \times \hat{\mathbf{j}} = -\hat{\mathbf{i}}$$

$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \qquad \hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$$

Cross Product is Cyclic (left column)
Anti-commutative:
Switching **vectors** changes sign (right column)

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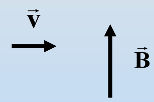
Concept Question: Cross Product

An electron is traveling to the right with speed v in a magnetic field that points up. What is the direction of the force on the electron



1. up
2. down
3. left
4. right
5. into page
6. out of page

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Demonstration: Magnetic Deflection of TV Image G6

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=G%206&show=0

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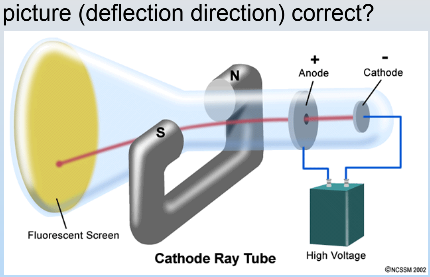
Concept Question: Force Direction

Is this picture (deflection direction) correct?



1. Yes
2. No
3. I don't know

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Putting it Together: Lorentz Force

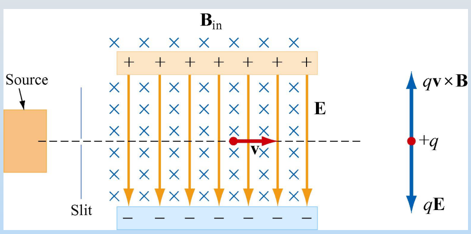
Force on charged particles in electric and magnetic fields

$$\vec{\mathbf{F}}_{elec} = q\vec{\mathbf{E}} \qquad \vec{\mathbf{F}}_{mag} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

Electric Force   Magnetic Force

$$\boxed{\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)}$$

Velocity Selector

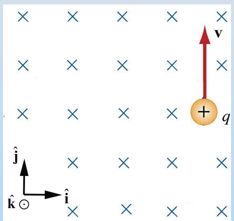


Particle moves in a straight line when

$$\vec{\mathbf{F}}_{net} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}) = 0 \;\Rightarrow\; v = \frac{E}{B}$$

What Kind of Motion in Uniform B Field?

Group Problem: Cyclotron Motion



A charged particle with charge q is moving with speed v in a uniform magnetic field B as shown in the figure.

(1) Draw a sketch of the orbit

Find

(2) R : radius of the circle

(3) T : period of the motion

(4) ω : cyclotron angular frequency

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