SPT: a window into highly entangled phases

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Why study SPT?

1. Because it may be there.......

Focus on electronic systems with realistic symmetries in d = 3, 2, 1.

Eg: Symmetries of usual topological insulator (charge conservation, time reversal).

Previous week talk: new Z_2^3 classification of interacting topological insulators in 3d (Wang, Potter, TS, 2013).

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1. d = 2: Duality between SPT and Symmetry Enriched Topological (SET) ordered phases (Levin-Gu 2012).

2. d = 2: Surface topological order of 3d SPT not realizable in strict 2d with same symmetry (Vishwanath, TS, 2012; Wang, TS 2013)

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3. Interesting implications for gapless quantum spin/non-fermi liquids in 2d or 3d. (Wang, TS 2013)

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SPT: a window into gapless quantum matter

Three examples:

1. Gapless spin liquids in quantum spin ice

2. Absence of certain gapless 2d quantum vortex liquids

eg: `algebraic vortex liquids' proposed as theory of Kagome magnets like Herbertsmithite.

3. New insights into deconfined quantum criticality

Gapless quantum matter

Gapless excitations in many body systems:

1. Old story: Goldstone modes related to broken symmetry

2. `New' story: Gapless excitations protected by long range entanglement + unbroken symmetry

Familiar examples: Fermi liquids, nodal superconductors
Gapless fermionic quasiparticles

Modern examples: Quantum critical points, gapless quantum spin liquids, non-fermi liquids
Often have no quasiparticles of any kind

Symmetry and gapless quantum matter

Symmetry obviously plays crucial role in a gapless state.

1. Explicitly breaking symmetry may lead to relevant perturbations at IR fixed point.

2. More general question: Can such a fixed point exist at all in the first place?

Is symmetry realized consistently at the fixed point?

Can fixed point theory be lattice regularized while preserving symmetry in same dimension?

Alternate: Fixed points with `anomalous' symmetry

Famous example

Gapless odd number of Dirac cones:

Impossible in `acceptable' strict 2d with T-reversal symmetry.

Can occur in 2d if we give up T-reversal.

Odd Dirac cone: Gapless 2d IR fixed point forbidden in strict 2d by symmetry.

Allowed at surface of a 3d bulk with SPT order.

SPT and gapless quantum matter

1. Gapless 2d surface states of interacting 3d SPT phases
=> strongly interacting IR fixed points prohibited in strictly 2d with same symmetry.

SPT generates useful no-go constraint on acceptable gapless states with symmetry.

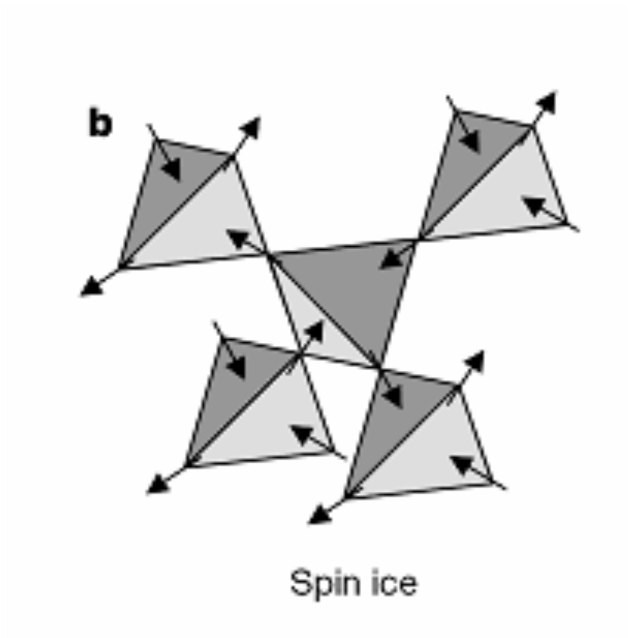
2. Gapless 3d phases with emergent photons - gauged versions of 3d SPT phases (illustrate with quantum spin ice example).

Quantum spin ice, quantum spin liquids, and symmetry

Next few slides: stolen and adapted from L. Balents

Classical spin ice

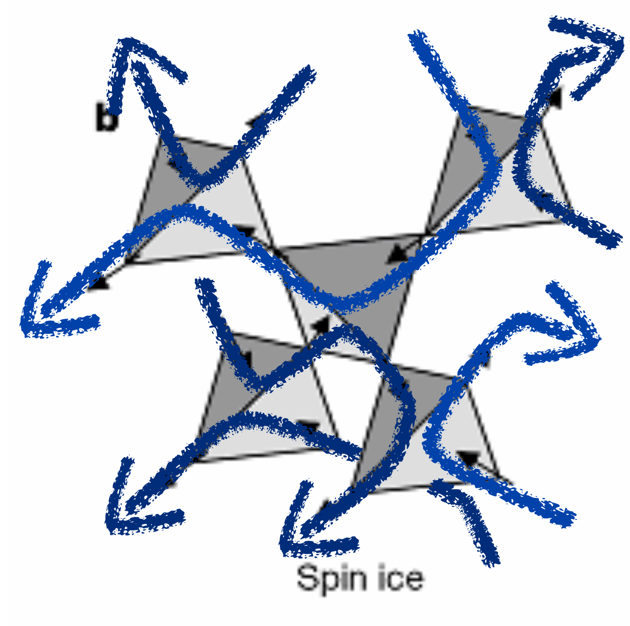
Spin ice: Ising spins on 3d pyrochlore lattice with interactions enforcing 2 in - 2 out \`ice rule'



$$H \approx J_{zz} \sum_{\langle ij \rangle} S_i^z S_j^z$$

Materials: $Ho_2Ti_2O_7$, $Dy_2Ti_2O_7$

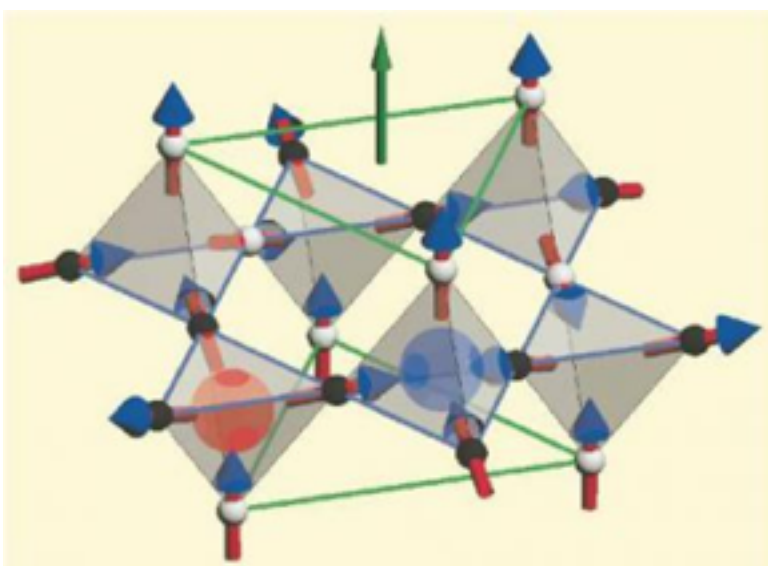
Classical spin ice: `artificial magnetostatics'



$$\vec{S} \sim \vec{b}$$

$$\vec{\nabla} \cdot \vec{b} = 0$$

Castelnovo *et al*, 2008



Defect tetrahedra (3 in - 1 out or 3 out- 1 in) in spin ice manifold: `magnetic monopoles'

Quantum spin ice

New spin ice materials where quantum effects on the Ising spins are clearly important.

Eg: $Yb_2Ti_2O_7$, Pr2Zr2O7, Pr2Sn2O7...?

Experiment: ($Yb_2Ti_2O_7$ Gaulin et al , Pr2Zr2O7 Nakatsuji, Broholm et al) : Many deviations from classical spin ice behavior at low-T.

Eg: Large weight at $\omega >> T$ in inelastic neutron scattering in Pr2Zr2O7(Nakatsuji, Broholm et al, Nat. Comm. 2013).

S. Curnoe, 2008
S. Onoda, 2010

Hamiltonian

$$\begin{aligned} H \;=\; & J_{zz} \sum_{\langle i,j \rangle} S_i^z S_j^z \\ & -J_{\pm} \sum_{\langle i,j \rangle} \left(S_i^+ S_j^- + S_i^- S_j^+ \right) \\ & + J_{z\pm} \sum_{\langle i,j \rangle} \left[S_i^z \left(\zeta_{ij} S_j^+ + \zeta_{ij}^* S_j^- \right) + i \leftrightarrow j \right] \\ & + J_{\pm\pm} \sum_{\langle i,j \rangle} \left(\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^- \right) \end{aligned}$$

classical NN spin ice

+ quantum fluctuations

= "quantum spin ice"

+ dipolar

Quantum spin ice Hamiltonian for Yb2Ti2O7

Ross, Savary, Gaulin, Balents, 2011

$\boxed{J_{zz} = 0.17\pm0.04 \text{ meV}}$

$J_{\pm} = 0.05\pm0.01$ meV $J_{z\pm} = 0.14\pm0.01$ meV $J_{\pm\pm} = 0.05\pm0.01$ meV

Reliably extracted from fitting spin wave dispersion in high field state.

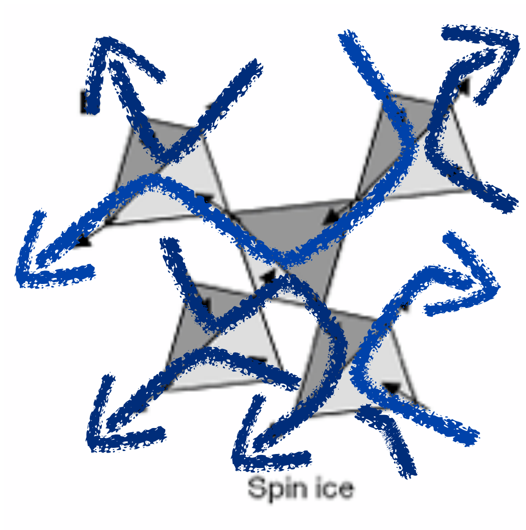
Parameters => appropriate to call this quantum spin ice.

Quantum spin ice and quantum spin liquids

Quantum fluctuations in spin ice manifold:

Magnetic field lines quantum fluctuate.

If field lines have zero tension => quantum spin liquid with an emergent U(1) gauge field.



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3d ``U(1) quantum spin liquid''

Excitations: 1. Gapless artificial photon

2. Gapped `magnetic monopole' (3 in - 1 out defect tetrahedra)

3. Other gapped point particles carrying internal `electric' charge

Many ongoing experiments to look for this!

Quantum spin ice, quantum spin liquids, and symmetry

Some crucial questions for theory:

1. What distinct kinds of quantum spin liquids with symmetry are possible for this Hamiltonian?

Note: Only physical symmetry - Time reversal x space group.

2. How to theoretically access these distinct quantum spin liquids?

3. How to distinguish in experiments?

SPT as a tool

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Insights from understanding interacting SPT insulators enable us to answer these questions!!

Aside: Role of Kramers

In band theory, topological insulation relies on Kramers structure of electron.

What precisely is role of Kramer's structure of electron in enabling $\theta = \pi$ response in an interacting TI?

Prove that $\theta = \pi$ requires Kramers structure (Wang, Potter, TS 13; Metlitski, Kane, Fisher 13).

Bulk monopoles are ``dyons'' with charge $\pm 1/2$.

Consider bound state of charge-1/2 ``dyon'' with charge-1/2 ``antidyon''.

Bound state carries electric charge-1 and magnetic charge-0 => identify with electron.

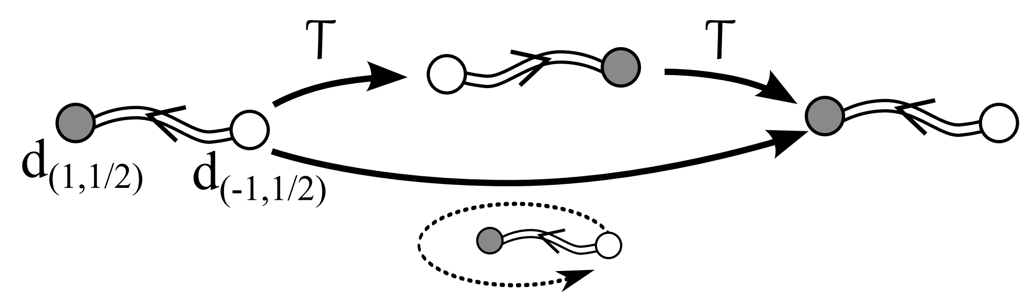
What are its time reversal properties?

Role of Kramers (cont'd)

\`\`Dyon" and \`\`Anti-dyon" with same electric charge transform into each other.

$$\begin{aligned} \mathcal{T}^{-1} d_{(1,1/2)} \mathcal{T} &= d_{(-1,1/2)} \\ \mathcal{T}^{-1} d_{(-1,1/2)} \mathcal{T} &= d_{(1,1/2)} \end{aligned} \tag{1}$$

$d_{(q_m,q_e)}$ is dyon operator with magnetic charge q_m and electric charge q_e.



These two see each other as relative monopoles.

=> In bound state, EM field angular momentum is half-integer.

Under T^2 action, this \`orbital' part leads to $T^2 = -1$.

=> Bound state (= electron) must be Kramers doublet.

End of proof.