Doped Mott insulators and high Tc superconductivity
Lecture 2

T. Senthil and M. Randeria
Plan

0. Unfinished business from Lecture 1
   - Quasi-2d organic materials

1. Cuprate phenomenology
How many ways does Nature have to deal with doping a Mott insulator?

Electron doped.

AF with localized carriers.

$\text{La}_{2-x}\text{Sr}_x\text{NiO}_{4+\delta}$

Micro phase separation: stripes

$\text{Sr}_{1-x}\text{La}_x\text{TiO}_3$


Organic ET salts.

Metal-insulator transition by tuning U/t.

Possibility of a “spin liquid”.

Doping yields a superconductor.

A second family of HiTc superconductors!
Q2D organics $\kappa$-(ET)$_2$X

$X = \text{Cu(NCS)}_2, \text{Cu[N(CN)}_2\text{]Br, Cu}_2\text{(CN)}_3$.....

t'/t = 0.5 ~ 1.1

Mott insulator

anisotropic triangular lattice
Pressure tuned superconductivity in the organics

Pressure decreases U/t.

Mott transition is induced by tuning U/t at fixed density of one electron per site.

\[ \kappa\text{-Cu[N(CN)_2]Cl} \]
\[ t'/t = 0.75 \]
Metal-insulator transition by tuning $U/t$. 

AF Mott insulator

Cuprate superconductor

metal

$U/t$
Metal-insulator transition by tuning $U/t$. 

- AF Mott insulator
- Cuprate superconductor
- Organic superconductor
- Metal
Metal-insulator transition by tuning $U/t$.

- **AF Mott insulator**
- **Cuprate superconductor**
  - $T_c = 100K$, $t = 0.4eV$, $T_c/t = 1/40$.
- **Organic superconductor**
  - $T_c = 12K$, $t = 0.05eV$, $T_c/t = 1/40$. 

$U/t$
Doping of an organic Mott insulator.

Superconductivity in doped ET, \((\text{ET})_4\text{Hg}_{2.89}\text{Br}_8\), was first discovered Lyubovskaya et al in 1987. Pressure data from Taniguchi et al, J. Phys soc Japan, 76, 113709 (2007).
Brief aside: Quantum spin liquids in the organics

Same family of organics also provide fascinating examples of quantum spin liquid Mott insulators.

Many interesting phenomena:

Insulator with specific heat, spin susceptibility like a metal.

Most dramatic: Metallic thermal transport in an insulator!
No magnetic order

(Expt: Kanoda, 2002–present) \( \kappa-(ET)_2Cu_2(CN)_3 \)

Weak Mott insulator close to Mott transition

\( \sim \) Isotropic \( \Delta \) lattice

No ordering to 32 mK

\( \ll J \approx 250 \text{ K} \)
A gapless spin liquid

\[
\chi(T \to 0) \to \text{const.} \\
\frac{C}{T}(T \to 0) \to \text{const.} \quad \left\{ \begin{array}{l}
\text{Wilson ratio } \frac{\chi T}{C} = \text{const. } \sim O(1)
\end{array} \right.
\]
Another candidate spin liquid on a triangular lattice

\[ EtMe_3Sb[Pd(dmit)_2]_2 \]

**Phenomenology broadly similar to kappa-ET spin liquid.**

Weak Mott insulator - close to pressure driven Mott transition.

No magnetic ordering to \( T \ll J \) but gapless spin excitations (NMR, specific heat).
Metallic thermal transport in a Mott insulator

Gapless excitations are mobile in dmit spin liquid!
End of digression

Quantum spin liquids near the Mott transition:

Growing number of experimental candidates - many dramatic phenomena.

A new chapter in condensed matter physics

Back to cuprates......................
High Tc Phase diagram

Plan

1. Overdoped – is it `conventional’?

2. What is strange about the strange metal?

3. Theory interlude
**Tl-2201**: Clean single layer cuprate, no complications from CuO chains; go from optimal to extreme overdoped

**Pros**: Hard to grow, Tl toxic
A preliminary look: transport

\[ \rho(T) \propto T \quad \text{(optimal)} \]

Overdoped looks more conventional.
Overdoped metal

• Does it have a Fermi surface? Size and shape?

Methods to detect – ARPES, deHaas–van Alphen and related quantum oscillations, other….. eg Angle Dependant Magneto–Resistance (ADMR)

• Is it really a Fermi liquid with Landau quasiparticles?
Overdoped metal: Is there a Fermi surface?

**ARPES**: Yes!

![Graph showing Fermi surface and Tl-2201 properties](image)

\[ T_c = 30 \text{ K} \]

Plate, ..., Damascelli, PRL 2005

"Large" Fermi surface with area \( \propto (1-x) \)

First measurement: "Angle dependant magneto resistance" (Hussey et. al., 2003)
deHaas van Alphen, other `quantum oscillations': classic Fermi surface determination methods

Oscillations of "everything" periodic in $\frac{1}{B}$ due to Landau level formation

$E_{\text{g}}: \quad M_{osc} = B \frac{R_T R_D}{\pi} \sin \left( \frac{2\pi F}{B} + \phi \right)$

Frequency $F = \frac{hc}{4\pi^2 e} A \quad \text{"extremal" area of Fermi Surface}$

"Lifshitz-Kosevich": $R_T = \frac{X}{\sinh X} \quad X = \frac{2\pi^2 k_B T}{\hbar \omega_c} \quad (\omega_c = \frac{eB}{mc})$ (thermal suppression)

"Dingle": $R_D = e^{-\frac{2\pi \hbar}{\omega_c \text{imp}}} \quad \text{(disorder suppression)}$
Remarks on quantum oscillations

1. Need very clean materials \( \omega_c T_{\text{imp}} \geq 1 \)
   (exponential suppression in Dingle factor)

2. Low-\( T \), high \( B \)-fields: \( k_B T \leq \frac{\hbar \omega_c}{2 \pi^2} \)

3. Oscillation frequencies \( \leftrightarrow \) area of Fermi surface
   
   \( T \)-dependence of amplitude \( \leftrightarrow \) effective mass \( m^* \)
   
   \( B \)-dependence of \( \leftrightarrow \) impurity scattering time \( T_{\text{imp}} \)

4. Some non-Fermi liquids may also show quantum oscillations
   - amplitude possibly not described by Lifshitz-Kosevich
Quantum oscillations in Tl–2201

$T_c = 10 \text{ K}$, $B$ upto 60 T; oscillations in both $M$ and in $c$-axis $\rho$

$F = 18\,100 \pm 50 \text{T}$ consistent with “large Fermi surface”

$m^*/m_e = 4$ consistent with other probes
Thermal conductivity: Wiedemann–Franz law

Usual Fermi liquid: \( k = AT + \beta T^3 \) as \( T \to 0 \)

Electrons

Phonons

Wiedemann–Franz: \( \lim_{T \to 0} \frac{k}{4\sigma} = \frac{\pi^2}{3} \left( \frac{k_B e}{\hbar} \right)^2 = L_0 \)

OD \( \text{Tl}_2\text{Ba}_2\text{O}_6 \): \( \frac{k}{4\sigma} = 0.99 L_0 \) \( \checkmark \)
Is the OD state really a Fermi liquid?

\[ S(T \gg T_c, B) = a_1 T + a_2 T^2 \]  (Julian, Mackenzie, 1996)

Similar in extensive study of OD LSCO

Hussey et al., Science 2009

Other: Possibility of anisotropic quasiparticle scattering along Fermi surface

\[ \frac{1}{\tau} = A + BT^2 + CT \cos^2 2\phi \]

(detailed fits to magneto-transport data - Hussey et al., 2007)
High Tc Phase diagram

Plan

1. Overdoped – is it `conventional’?

2. What is strange about the strange metal?

3. Theory interlude
What is strange about the strange metal?

Mohit Randeria Lec 1

1. Photoemission

Sharp (upto thermal smearing) large Fermi surface but no Landau quasiparticles

2. Transport - linear T resistivity

This lecture - more discussion of transport and other anomalies in strange metal.
The strange metal: electrical transport

Linear-T resistivity near optimal doping with nearly zero intercept.

Slope of resistivity/layer roughly the same (1.5 \( \mu \Omega \) cm/K) for all materials.

Sheet resistance = \( \rho / d \sim (h/e^2) \) T/J

"Bad metal": \( k_f l \leq 2\pi \) at high T; no sign of phonons

Bi-2201  Martin et al '90

Mackenzie '97
Linear resistivity at very low-T

Tied to "quantum criticality"?

Quantum critical point: second order phase transition at T = 0

Daou, ... Taillefer, Nat. Phys. 2008
Magnetotransport: Hall effect

$R_H \uparrow$ as $T \downarrow$

For Drude metal $\sigma_{xy} = (\omega_c T) \sigma_{xx}$

$\Rightarrow \cot \Theta_H = \frac{\sigma_{xx}}{\sigma_{xy}} = \frac{i}{\omega_c T}$

"Hall angles"

$\Theta_H \rightarrow E$

Here $\cot \Theta_H \propto T^2$

but $f \propto T$

Mackenzie '97
Optical transport: high frequency tail

Power law tail at high frequency ($70 \text{ meV} \leq \omega \leq 1 \text{ eV}$)

$$\sigma(\omega) = C (-i\omega)^{\gamma - 2}$$

$$\gamma - 2 \approx 0.65$$

$$\Rightarrow |\sigma(\omega)| \sim \frac{1}{\omega^{0.65}}$$

and phase angle \(\tan^{-1}(\frac{\sigma_2}{\sigma_1}) = \frac{\pi}{2} (2 - \gamma)\) independent of \(\omega\)

Van der Marel et al., Science '03
(similar early data, Bontemps '94)
Optical transport: low frequency peak

Roughly described by Drude form

$$\sigma(\omega, T) = \frac{\sigma_{DC}(T)}{1 + (\omega \tau(T))^2}$$

with "scattering time"

$$\tau(T) \propto \frac{\hbar}{k_B T}$$
Spin physics: spin susceptibility and NMR relaxation

\[ \chi = \text{const. for } T > T_c \]  (consistent with Knight shift)

Striking difference between \(^{17}\text{O}\) and \(^{63}\text{Cu}\) in \(1/T_1\), "Korringa" \[ \frac{1}{T_1} K_s^2 = \text{const. violated} \] at Cu site

\[ \Rightarrow \text{slow growth of AF correlations} \]
Dynamic spin correlations: neutron scattering in LSCO

Aeppli et al., Science '97

Nearly singular scaling of spin fluctuations at incommensurate wave vector near optimal doping. However, similar data apparently does not exist in YBCO.
Transition to SC: onset of coherence

ARPES results

Sharp quasiparticles emerge for $T < T_c$. 
Onset of coherence in transport

- Microwave
  - Bonn, Hardy, et al. '93, '99

- Thermal Hall effect
  - (Separate electron transport from phonon)

- Collapse of scattering rate of nodal excitations for $T < T_c$.

- Mean free paths $\sim 1$ micron

N. P. Ong '99

$\partial_j q \rightarrow \Theta_B \nabla T$
Neutron resonance

Sharp magnetic excitation with gap of 41 meV (in YBCO) visible only for $T < T_c$.

Seen in almost all cuprates at various doping levels

Keimer et al. '95
Summary on strange metal

Strange metal: Power laws in many physical quantities;

Large Fermi surface but no Landau-like quasiparticles

Slow growth of antiferromagnetic spin correlations

Transition to superconductivity accompanied by appearance of coherent quasiparticles and a sharp spin triplet `resonance’ mode.
Some basic questions

1. How does a metal emerge from a Mott insulator?

2. Why superconductivity?


Doped Mott insulator: Hole motion in background of valence bonds.

\[ \sigma = (\langle \uparrow \downarrow \rangle - \langle \uparrow \uparrow \rangle)/\sqrt{2} \]
Large doping: Hubbard-U not very effective in blocking charge motion

Expect `large Fermi surface’ with area set by 1-x.

What happens as doping is reduced to approach Mott insulator?
Low doping: Most of the time most electrons unable to hop to neighboring sites due to Mott-blocking.

If electrons stay localized next to each other long enough, will develop superexchange which will lock their spins into singlets.

Electron configuration changes at long times – conveniently view as motion of holes in sea of singlets.

Resulting state: metallic but with a spin gap due to valence bond formation => `pseudogap metal".
Why superconductivity?

Crucial Anderson insight:

Singlet valence bond between localized spins: A localized Cooper pair.

`Pairing’ comes from superexchange due to a repulsive Hubbard interaction.

If spins were truly localized, Cooper pairs do not move => no superconductivity.

Nonzero doping: allow room for motion of valence bonds => superconductivity!

Hole picture: Coherent hole motion in valence bond sea
Fate of collection of valence bonds

Two general possibilities:
Valence bonds can crystallize to form a solid
(`Valence Bond Solid'')
OR
Stay liquid to form a `Resonating Valence Bond'

Ongoing debates on which one is more relevant but very formation of valence bond crucial ingredient in much thinking about cuprates.

VBS state (with doping a `bond centered' stripe)

RVB state = quantum spin liquid
Doped Mott insulators and high Tc superconductivity
T. Senthil and Mohit Randeria

Lecture 3
Cartoon understanding of phase diagram

Formation of singlet valence bond

Coherence of hole motion
Does valence bond formation provide a legitimate theoretical route for superconductivity in a repulsive doped Mott insulator?

Many different kinds of studies: (work of large # of people over 20 yrs)

1. 1d doped spin ladder:
   Zero doping – spin gapped insulator due to valence bond formation.
   Dope – (power law) superconductor.

2. Quasi-1d: Weakly coupled ladders

3. Inhomogenous 2d: Checkerboard Hubbard model


5. Superconductivity in doped spin liquid Mott insulators (i.e insulators with one electron per site)
Superconductivity in doped spin liquids: mean field

Incorporate no double occupancy constraint of t-J model in approximate "mean field"

$f.c. \text{ Zhang, Gros, Rice, Shiba '88}$

"Gutzwiller" mean field

"Non-Fermi liquid"

"Fermi liquid"

Spin gap formation

Pseudo gap

Coherence of hole motion

Kotliar, Liu '88

"Slave boson" mean field

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Superconductivity in doped spin liquids: variational wavefunctions

\[ |\Psi_{gd}\rangle = P_{\text{no double occupancy}} |\text{dBCS}\rangle \]

Gap parameter of |dBCS\rangle in variational wave function

Paramekanti, Randena, Trivedi
2001
Common features of superconductivity in doped (paramagnetic) Mott insulators

Generic phase diagram

"Low" $x$: $T_c$ controlled by phase stiffness $S_\parallel \rightarrow 0$ as $x \rightarrow 0$

Spins start gapping at higher temperature $T^*$

"High" $x$: $T_c$ controlled by pairing gap $\Delta \rightarrow 0$
High Tc Phase diagram

Plan

1. Phenomenology of the pseudogap – review, quantum oscillations, and competing orders

2. Some theory

3. Refined basic questions

Pseudogap: loss of ``density of states” in spin, single particle properties
Some real high Tc phase diagrams

Bi2212

ARPES data from U. Chatterjee et al (2010)
Courtesy: J.C. Campuzano
K. Ohishi et al., cond-mat/0412313

Courtesy: J. C. Davis
Summary of ARPES in underdoped pseudogap regime

1. Big antinodal gap – 50 meV or bigger

2. Gapless Fermi arcs near node that shrink as $T$ is reduced; possibly even extrapolate to 0 at $T = 0$.

3. Gap is apparently centered on large Fermi surface
New mystery: quantum oscillations in a magnetic field at low $T$

- Delta Haas-van Alfen, Shubnikov-de Haas oscillations in ultra-pure $YBCO_{6+x}$ ($x \approx 0.5$) and $YBa_2Cu_3O_8$ in $B \approx 60T$

  Dominant frequency $530T$

  $\Rightarrow$ small pocket.

  (Proust, Taillefer, ... '07)

Other frequencies with lower amplitude

  Eg: $1650T$ (Sebastian et al. '08)
High field ground state: contrast between under and over-doped

OD Tl2201: \( F = 18\,100 \, T \)
\( \iff \) 63\% of Brillouin Zone

UD YBCO_{6.8}: \( F = 530 \, T \)
\( \iff \) 3\% of Brillouin Zone
How do all this fit together?

\[ T^* > T > T_c \text{, low } H : \text{“gapless Fermi arcs” that shrink as } T \downarrow \text{, antinodal gap } \approx 50 \text{ meV} \]

Low \( T \), high \( H \): closed Fermi pocket, possibly near antinode.

\[ T > T_c \]

\[ T \to 0, H \approx 50T \]
How to fit together?

1. How can a closed Fermi surface emerge at low $T$?

2. Can a 50T field really close the antinodal gap $\Delta p_n \approx 50$ meV?

$H \sim 50T$ is actually a small field.

Eg: If $\Delta p_n$ = “pairing” gap, $H_{c2} \sim \frac{\Phi_0}{(\hbar v_F/\Delta)^2} \gg 50T$
Arcs versus pockets

Could it be that the arcs are really just one side of a closed pocket near the nodal region?

1 quadrant of BZ

Fermi arc

OR

Small hole pocket

(Back side invisible to ARPES for some reason?)

To be clarified in future experiments...
Competing order and fluctuations

Apart from superconductivity, many other ordered or nearly ordered (i.e. short range ordered) states have been reported in the underdoped cuprates.

Some prominent examples:
1. Antiferromagnetism/SDW/spin stripes
2. Charge order – charge stripes/CDW/checkerboard
4. Others – broken T-reversal, circulating currents,......

Implication/importance of these for pseudogap/SC/ strange metal not currently understood.
Phase fluctuations above $T_c$: Nernst/diamagnetism

If $T_c$ controlled by phase stiffness, might expect region with enhanced superconducting phase fluctuations in the `normal’ state above $T_c$.

Experiment: Microwave conductivity (Corson, .....Orenstein)

Nernst effect and diamagnetism (Wang, Li,....... Ong) (next few slides courtesy of Lu Li)

This fluctuations regime surely exists but does not extend all the way to $T^*$. 
Vortex Nernst effect

Vortices move in a temperature gradient
Phase slip generates Josephson voltage

\[ 2eV_J = h\phi = 2\pi h n_v \]
\[ E_J = B \times v \]

Nernst signal: \[ e_y = \frac{E_y}{|\nabla T|} \]

Wang et al. PRB(2001)
Diamagnetic signal: need high-resolution magnetometry!

Magnetization curve of type-II superconductors

Magnetization study:

**Advantage**
- Clear determination of $H_c2$ and $H_c1$
- Area = Condensation energy $U$

**Difficulty**
- In cuprates, $H_c2 \sim 50-150$ T
- $M < 1000$ A/m ( ~ 12 G)
- **HARD** to resolve with commercial SQUID magnetometers
  ($H_{max} = 5$ T or 7 T)

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Torque on moment: \( \tau = m \times B \)

Deflection of cantilever: \( \tau = k \phi \)

\[ M_{\text{eff}} = \frac{\tau}{\mu_0 H V \sin(\phi)} \]
Torque magnetometry

Torque on moment: \( \tau = m \times B \)

Deflection of cantilever: \( \tau = k \varphi \)

\[ M_{\text{eff}} = \frac{\tau}{\mu_0 HV \sin(\varphi)} \]
Examples of Magnetization curves (I): M vs. T and the onset temperature $T_{onset}$

Enhanced diamagnetic signals above $T_C$

$$M_{eff} = \frac{\tau}{\mu_0 H \text{sin(} \theta \text{)}}$$

Weakly linear orbital background $\Delta \chi_{orb} H$

Lu Li, Thesis
Conclusion

Diamagnetism up to 130 K

Non-linear M-H

A universal SC fluctuation onset temperature vs x?

Bi2212
Other order and fluctuations: Antiferromagnetism

AF LRO disappears at very low doping but some soft spin fluctuations persist to high doping.
Eg: Neutron resonance in SC state seen in most cuprates.

Resonance frequency decreases with Tc in underdoped.

Soft mode of AF LRO?
`Universal’ spin fluctuation spectrum of superconducting cuprates

``Hieroglyph``

Yamada plot for LSCO

Low frequency dynamic incommensurate spin fluctuations in YBCO but apparently not in (dynamic “spin stripes”) Yamada plot
Broken translation symmetry I: charge stripes

Static charge stripes seen in Nd-doped La$_{2-x}$Sr$_x$CuO$_4$, and La$_{1.5/8}$Ba$_{1.5/8}$CuO$_4$
Broken translation symmetry in STM: methods

Tunnel current

\[ I \approx \frac{\Delta \pi e}{\hbar} e^{-s \sqrt{\frac{\hbar m}{\pi^2 e V}}} \rho_t(0) \int_{-eV}^{0} \rho_s(\varepsilon) d\varepsilon \]

unknown prefactor

Exploit asymmetry of spectrum

(i) R-map: \( R(\vec{r}, V) = \frac{I(\vec{r}, s, +V)}{I(\vec{r}, s, -V)} \)

(ii) Z-map: \( Z(\vec{r}, V) = \frac{dI}{dV}(\vec{r}, s, +V) \frac{dI}{dV}(\vec{r}, s, -V) \)

Alternate:

Fourier transform real space \( \frac{dI}{dV} \), look for \( V \) independent Fourier peak.
Broken translational symmetry in STM: bond-centered `glass’

“R-map”

Kohsaka, ..., Davis
Science'07

\[ \text{Ca}_{1.88} \text{Na}_{0.12} \text{CuO}_2 \text{Cl}_2 \]

Similar data in Bi-2212

12 nm

Apparently seen only at very low doping $\leq 12\%$. 

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"R-map"

Broken translational symmetry in STM: bond-centered `glass'

Kohsaka, ... Davis Science'07

\( \text{Ca}_{1.88} \text{Na}_{0.12} \text{CuO}_2 \text{Cl}_2 \)

Similar data in Bi-2212

Apparent seen only at very low doping ≤ 12 %
Broken translational symmetry in STM: bond-centered ‘glass’

“R-map”

Kohsaka, ... D avis
Science '07

\[
\text{Ca}_{1.88}\text{Na}_{0.12}\text{CuO}_2\text{Cl}_2
\]

Similar data in Bi-2212

12 nm

4a_0

Apparently seen only at very low doping \( \leq 12\% \).
Broken translational symmetry in STM: bond-centered `glass'

"R-map"

Kohsaka, ... Davis Science'07

\[ \text{Ca}_{1.88} \text{Na}_{0.12} \text{CuO}_2 \text{Cl}_2 \]

Similar data in Bi-2212

Apparently seen only at very low doping \( \leq 12\% \).
Broken translational symmetry in STM: bond-centered `glass'

"R-map"

Kohsaka, ... Davis, Science '07

\[ \text{\textit{Ce}}_{1.88} \text{\textit{Na}}_{0.12} \text{\textit{CuO}}_{2} \text{\textit{Cl}}_{2} \]

Similar data in Bi-2212

Apparentley seen only at very low doping \( \leq 12 \% \).
Electronic nematics

Break lattice rotation symmetry without breaking translation symmetry

YBCO has no static stripes but seems to develop enhanced "nematic" order below $T^*$

Spin fluctuation spectrum YBCO$_{6.45}$ (Hinkov,..., Keimer, Science 2008)

Resistivity anisotropy

Ando et al., 2002

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Correlation with $T^*$

Nematic onset roughly at $T^*$

$D(T) = \frac{\nu_a}{T} - \frac{\nu_b}{T}$

$D(T) = \text{anisotropy in } \frac{\nu}{T}$
Field induced magnetic ordering at low-T

Magnetic field stabilizes SDW order in favor of superconductivity.

LSCO
Lake et al. '02
(Similar data: Khaykovich et al. '02)

YBCO 645
Haug, ... Keimer, PRL 2009.
Should the pseudogap be understood as valence bond formation of nearly localized spins with many competing orders as consequence?

Or

Is some competing (fluctuating) order the cause of pseudogap?
Summary of some important underdoped phenomena

1. Antinodal gap ($\geq 50$ meV), gapless. $T$-dependent Fermi arcs near node

2. "Landau" quasi-particles emerge only below a low "coherence" scale $T_{coh} \approx T_c$

3. Persistence of SC amplitude without phase coherence above $T_c$ (microwave, Ong Nernst/magnetization)

4. Quantum oscillations in a field

5. Other competing order (eg: SDW, CDW, ...)
   Eg: At low-$T$ SDW can be stabilized by magnetic field
A phenomenological synthesis

Coherent picture of results of ARPES, Nernst/magnetization, and quantum oscillations (TS, P.A.L., 2009)
Ong `high' field phase diagram

Resistive transition
~ melting of vortex solid

Lose pair amplitude

Quantum oscillation

 Resistive

H_{res} \approx 40T

\geq 150T (conclusion of Ong et al)

Must understand within framework of "vortex liquid"
Key assumption: Electron coherence in a field

\[ H \sim 0 \left( H_{\text{res}} \right) \text{ does not suppress } T_{\text{coh}} \text{ to 0 but only } T_c. \]

(Not valid in simplest slave boson theory; need better justification)

Exptl support: STM tunneling into vortex core (Hudson, Davis 2008)

Easier to weaken SC than to kill coherence peak
Low $T$: kill SC by phase fluctuations
- emergence of large FS with quasiparticles with small spectral weight.

Effect of $B$-field: induce SDW order to reconstruct Fermi surface

quantum oscillations
Two things happen upon crossing $T_c$ (at $H = 0$).

(i) Lose phase coherence of pair order parameter

BUT ALSO

(ii) Lose single particle coherence (as $T_{coh} \approx T_c$)

$T_c$ - not just a phase disordering transition of SC but also a "coherence" transition for electrons.
Modeling single particle incoherence

Simplified model: take single particle scattering rate $\gamma = \text{large, } \propto T$
for $T > T_{\text{coh}}$

$\gamma \approx T_c$ for $T < T_c$

Model SC phase disordering as before with a phase decay rate $\Gamma \ll \Delta_0$
Pseudogap and Fermi arcs

For $\gamma \ll D_0 K$

```
    \gamma
  \hline
  2D_0 K
```

For $\gamma \gg D_0 K$

```
    \gamma
  \hline
```

"Pseudogap" like

$\gamma \gg D_0 K$ always satisfied near nodal $K \Rightarrow$ get Fermi arcs AND pseudogap

Arc length set by $\gamma \approx D_0 K \Rightarrow$ decrease as $T \Downarrow$

(Norman et. al. '98, '07; Chubukov et. al. '08)
Summary of ``synthesis”

1. Quantum oscillations in $T=0$ vortex liquid
   - emergence of large FS
   - reconstruction by field induced SDW

2. Pseudogap / Fermi arcs at $T > T_{coh} \approx T_c$:
   Incoherent single particle excitations + pairing / other order
   fluctuations

KEY issue for microscopic theory: single particle
(in)coherence & interplay with ordering
Back to basic theory questions

**IS SUPERCONDUCTIVITY POSSIBLE IN A DOPED MOTT INSULATOR?**

Many different kinds of studies:

1. 1d doped spin ladder:
   Zero doping – spin gapped insulator due to valence bond formation.
   Dope – (power law) superconductor.

2. Quasi-1d: Weakly coupled ladders

3. Inhomogenous 2d: Checkerboard Hubbard model

4. Superconductivity in doped VBS Mott insulators (`large-N' methods): spontaneously generate weakly coupled ladders.

5. Superconductivity in doped spin liquid Mott insulators (i.e. insulators with one electron per site)

   ONLY THIS GIVES A ROUTE TO A GAPLESS SUPERCONDUCTOR
Refined basic theory questions

Is superconductivity with gapless nodal excitations possible in a doped Mott insulator?

Only currently known route is by doping a gapless spin liquid Mott insulator.
Does this force us to a spin liquid based approach to cuprates?
More questions

More generally, large Fermi surface visible (at least at short time scales) already in underdoped.

How should we understand the emergence of the large Fermi surface in a doped Mott insulator?

Theory: How does the Fermi surface die?
Even more questions

What about antiferromagnetism?

Doped systems have short ranged AF which grows as $x \downarrow$

$\Rightarrow$ Did not completely forget origin as doped AF Mott insulator

Likely possibility: $\xrightarrow{x \downarrow} \text{AF Mott} \xrightarrow{1\text{st order}} \text{Metal/SC} \xrightarrow{\mu = \text{chem. potential}}$ hole rich

$\Rightarrow$ As function of $x$, phase separation (stripes)
Last question

AF Mott $\rightarrow$ Metal/dSC $\rightarrow$

Phase coexistence

$\frac{1}{r}$ + Coulomb $\frac{1}{r}$

stripes

Are stripes essential to the story or are they a complication?