Interacting topological insulators in 3 dimensions: classification and properties

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Electrons: Chong Wang, A. Potter (grad student @ MIT => Berkeley post-doc)

Topological insulators 1.0

Free electron band theory:
two distinct insulating phases of electrons in the presence of time reversal symmetry.

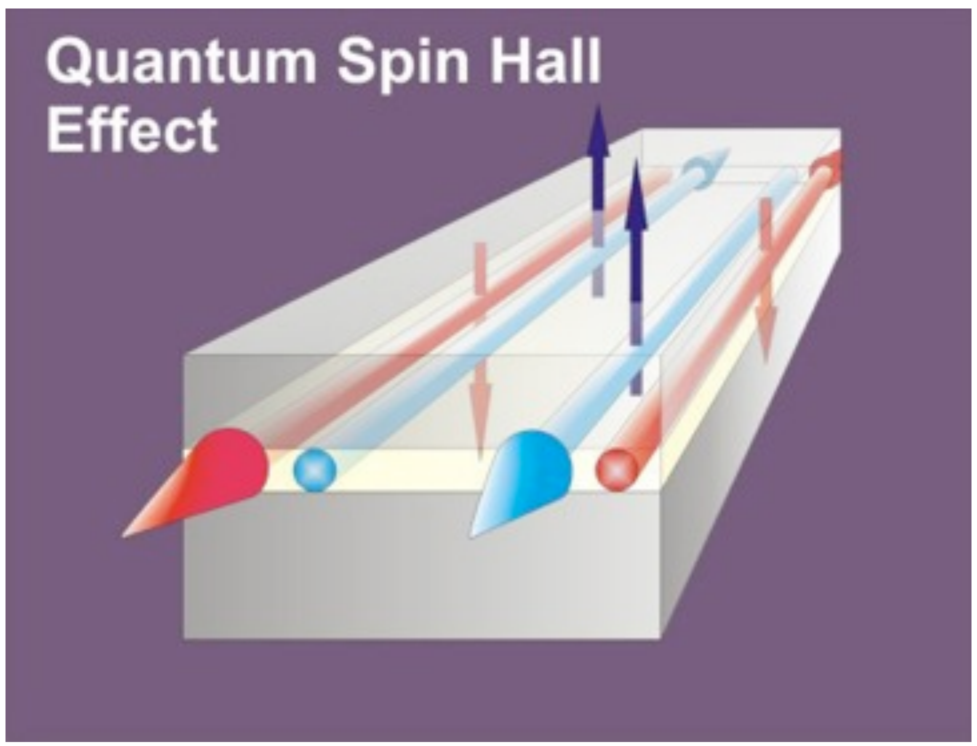
(i) Conventional Band Insulator

(ii) Topological Band Insulator (TBI)

In 3d the TBI requires spin-orbit coupling which destroys conservation of all components of spin (but preserves time reversal).

Properties of topological insulators

Usual characterization: Non-trivial surface states with gapless excitations protected by some symmetry



2d: `helical edge modes'

3d: odd number of Dirac cones

Topological insulators 2.0

Strongly correlated topological insulators

Interaction dominated phases as topological insulators?

Theory:
Move away from the crutch of free fermion Hamiltonians and band topology.

Experiment:
SmB_6 ?? (Fisk et al, Paglione et al, Xia et al, 2012 following theory suggestion of Dzero, Sun,Galitski, Coleman, 2010)

A ``Kondo insulator"

Some questions about interacting topological insulators

1. Are there new phases that have no non-interacting counterpart?

2. Physical properties?

3. Experimental realization?

Some questions about interacting topological insulators

1. Are there new phases that have no non-interacting counterpart?

First refine the question

2. Physical properties?

3. Experimental realization?

How to generalize 3d topological insulators to interacting electrons?

Some simple guidelines.

1. Same realistic symmetry as Topological Band Insulator (TBI): charge conservation, time reversal*

2. TBI phases are ``short range entangled'' (have unique bulk ground state, no `fractional' excitations, etc).

Restrict to interacting ``short range entangled'' phases.

If either of these restrictions is removed there will be a richer set of answers but this is the `minimum' generalization that is also practical.

*Symmetry $U(1) \ltimes Z_2^T$ (U(1): charge conservation; Z_2^T: time reversal)

Aside: short versus long range entangled phases

``Exotic'' gapped Phases of Matter

- phases with ``intrinsic'' topological quantum order, fractional quantum numbers
(eg, fractional quantum Hall state, gapped quantum spin liquids)

Emergent non-local structure in ground state wavefunction:

Characterize as ``long range quantum entanglement''

Topological band insulators do not have ``intrinsic'' topological order and are ``short range entangled''.

Some questions about interacting topological insulators

1. Are there new phases that have no non-interacting counterpart?

First refine the question

2. Physical properties?

3. Experimental realization?

Refined question: Phases and properties of interacting `short range entangled' electronic topological insulators with `realistic' symmetry U(1) $\ltimes Z_2^T$?

Plan of talk

1. Lightning review of free fermion topological insulators

2. A very useful detour: bosonic topological insulators in 3d (review)

3. 3d Electron TIs

- new Z_2^3 classification

- description of the new interacting TIs

Review: free fermion 3d topological insulators

Characterize by

1. presence/absence of non-trivial surface states

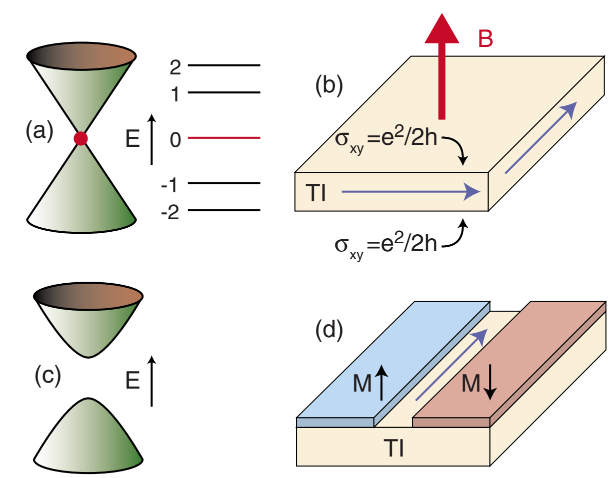
2. EM response

Surface states: Odd number of Dirac cones

Trivial gapped/localized insulator not possible at surface so long as T-reversal is preserved (even with disorder)

Review: free fermion topological insulators

EM response: Surface quantum Hall Effect



If surface gapped by B-field/proximity to magnetic insulator, surface Hall conductance

$$\sigma_{xy} = \left(n + \frac{1}{2}\right)\frac{e^2}{h}$$

Domain wall between opposite T-breaking regions: chiral edge mode of 2d fermion IQHE

Review: Free fermion topological insulators Axion Electrodynamics

Qi, Hughes, Zhang, 09
Essin, Moore, Vanderbilt, 09

EM response of *any* 3d insulator

$$\mathcal{L}_{eff} = \mathcal{L}_{Max} + \mathcal{L}_\theta \quad (1)$$
$$\mathcal{L}_\theta = \frac{\theta}{4\pi^2}\vec{E}.\vec{B}$$

Under $\mathcal{T}$-reversal, $\theta \to -\theta$.
Periodicity $\theta \to \theta + 2\pi$: only $\theta = n\pi$ consistent with $\mathcal{T}$-reversal.
Domain wall with $\theta = 0$ insulator: Surface quantum Hall effect

$$\sigma_{xy} = \frac{\theta}{2\pi}$$

Free fermion TI: $\theta = \pi$.
Interpretation of periodicity:
$\theta \to \theta + 2\pi$: deposit a 2d fermion IQHE at surface.
Not a distinct state.

Consequences of axion response: Witten effect

External magnetic monopole in EM field:

θ term => monopole has electric charge θ/2π.

(``Witten dyons”).

$$
\begin{aligned}
\mathcal{L}_\theta &= \frac{\theta}{4\pi^2}\vec{E}.\vec{B} \\
&= -\frac{\theta}{4\pi^2}\vec{\nabla}A_0.\vec{B} + \ldots . \\
&= \frac{\theta}{4\pi^2}A_0\vec{\nabla}.\vec{B}
\end{aligned}
$$

A very useful detour: Bosonic topological insulators

Why study bosons?

1. Non-interacting bosons necessarily trivial - so must deal with an interacting theory right away

Necessitates thinking more generally about TI phases without the aid of a free fermion model.

2. Natural realizations in quantum spin systems

Is there a spin analog of a topological insulator, i.e a ``Topological Paramagnet'' (as distinct from a ``Quantum spin liquid'')?

(Other realizations: cold atoms)

3. Correlated bosons are stepping stone to correlated fermions.

Focus on some boson TIs with time reversal in 3d.

Physics of 3d boson topological insulators

Vishwanath, TS, 2013

1. Quantized magneto-electric effect (axion angle $\theta = 2\pi$ or even 0)

2. Emergent exotic (fermionic, Kramers or both) vortices at surface, ...

3. Related exotic bulk monopole of external EM field (fermion, Kramers, or both) (Wang, TS, 2013; Metlitski, Kane, Fisher, 2013).

3d boson topological insulators-I: EM response

Vishwanath, TS, 2012

For bosons, $\theta = 2\pi$ is distinct from $\theta = 0$.

Surface of $\theta = 2\pi$ requires the 3d bulk.

$\theta \to \theta + 4\pi$ is trivial.

Related to even integer quantization of σ_{xy} for 2d bosons IQHE
(Y.M. Lu, Vishwanath, 2012;
TS, M. Levin, 2013)

Depending on how T-reversal is realized distinct bosonic TIs exist with $\theta = 2\pi$ or even with $\theta = 0$.

3d boson topological insulators-II: surface physics

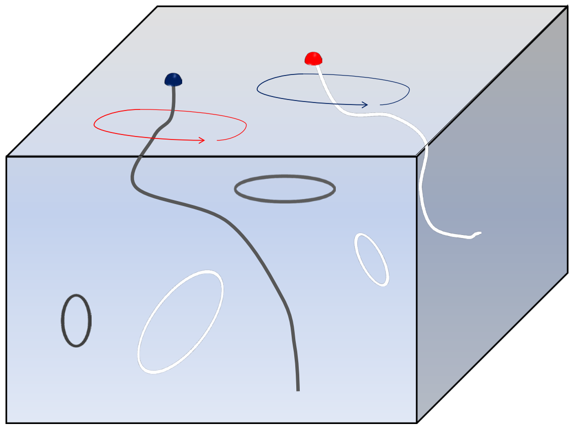
Surface either spontaneously breaks symmetry or, if gapped, has `intrinsic' topological order (even though bulk does not).

Nicely captured in vortex description of surface:

Conventional 2d bosons: charge-vortex duality

Dual Landau-Ginzburg description in terms of bosonic vortex field
- condense vortex to get trivial insulator.

3d boson TI surface: unusual surface vortex (eg: fermionic, Kramers or both) => cannot condense to get trivial gapped insulator.



3d boson topological insulators-III: exotic bulk monopole

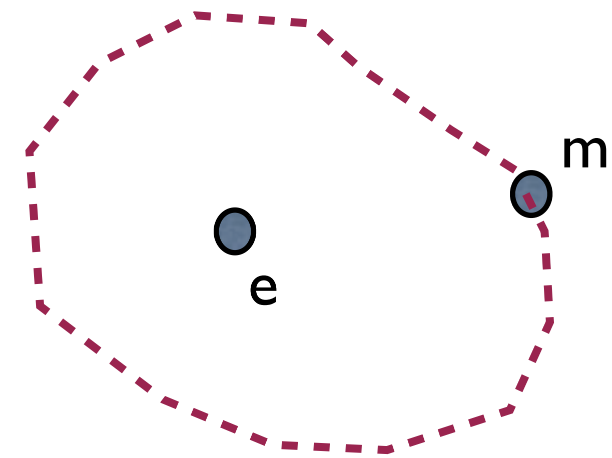
External EM field: what happens to bulk monopole?

`Bulk-edge correspondence':
bulk monopole created at surface by surface vortex of dual LGW theory
=> exotic monopole (fermion, Kramers or both) (Wang, TS, 2013).

See also Metlitski et al 2013: different argument for 3d for $\theta = 2\pi$ TI directly from θ response and a nice use of exotic monopole to constrain surface physics.

Surface topological order of 3d SPTs

3d SPT surface can have intrinsic topological order though bulk does not (Vishwanath, TS, 2012).



Phase of π

Resulting symmetry preserving gapped surface state realizes symmetry `anomalously'.

Targetting such `anamolous' surface topological ordered state has been fruitful in microscopic constructions of boson TIs
(Wang, TS, 2013; Burnell, Chen, Fidkowski, Vishwanath, 2013)

Crucial connection to questions of legitimate symmetry realization in 2d highly entangled states.

Eg: what kinds of Z2 quantum spin liquids with symmetry are legal in strict 2d systems?

Explicit construction of 3d boson SPTs

C. Wang, TS, 2013

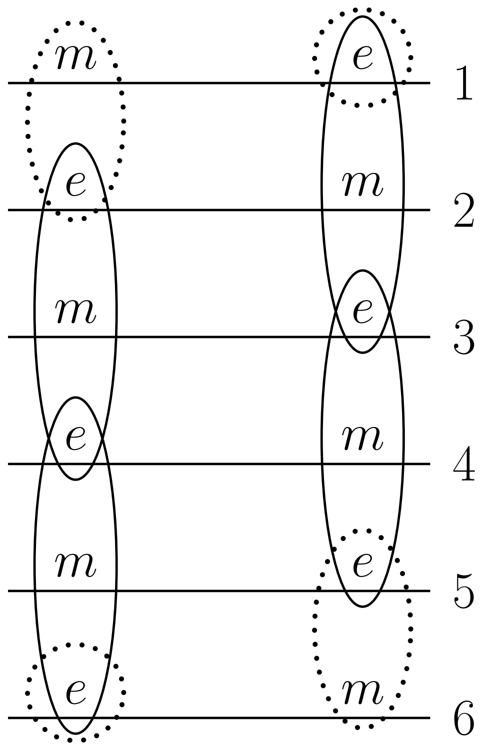
Strategy: stacked layers of Z_2 topological ordered (``toric code") phases with symmetry allowed in strict 2d.

Transition to confine all bulk topological quasiparticles but leave deconfined surface topological order.

Engineer surface topological order specific to SPT surface.

Explicit construction of a 3d boson SPT

C. Wang, TS, 2013



1. Each layer: e is charge 1/2, m is charge 0.

2. Condense $e_i\ m_{i+1}\ e_{i+2}$ (all self and mutual bosons)

=> confine all bulk quasiparticles

3. Surface: e_1, $m_1 e_2$ survive (at top surface)

These are mutual semions, and self-bosons
=> surface Z_2 topological order

Both carry charge-1/2

=> known surface topological order of 3d SPT.

Comments

1. This coupled layer construction can be generalized to all the SPTs with various symmetries studied in Vishwanath, TS, PR X, 2013.

2. Nice example: very simple construction of boson SPT states with only T-reversal including one `beyond cohomology' classification of Chen, Gu, Liu, and Wen (2011).

*Note: SPT: ``Symmetry Protected Topological" (more general name for topological insulators particularly appropriate when there is no conserved charge).

Boson SPT states with only T-reversal

Eg: Spin system with T-reversal and no other symmetries

What are SPT states?
``Time reversal protected topological paramagnets"

Cohomology approach: Z_2 classification => one non-trivial state

But we find the correct classification is Z_2^2.

Extra root state `beyond cohomology'.

Boson SPT states with only T-reversal: ``Within cohomology'' state

AV, TS 12
C. Wang, TS, 13

Surface topological order is best available characterization.

Z_2 gauge theory (``toric code'') where both e and m are Kramers doublets (not possible in 2d with T-reversal).

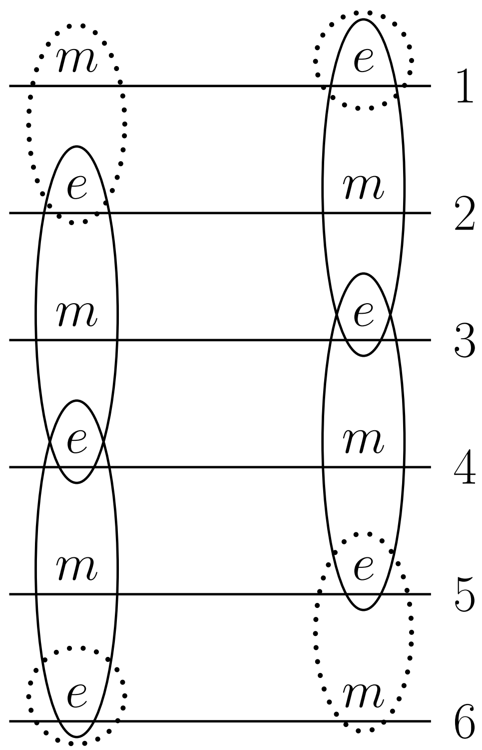
Coupled layer construction:
1. Each layer: conventional 2d Z_2 spin liquid where e_i is Kramers doublet, m_i is singlet.

2. Condense $e_i\ m_{i+1}\ e_{i+2}$ (all self and mutual bosons)

=> confine all bulk quasiparticles

3. Surface: e_1, m_1e_2 survive (at top surface)

These are mutual semions, and self-bosons
=> surface Z_2 topological order, both e_1, m_1e_2 are Kramers.



Beyond cohomology state

AV,TS 12
C. Wang, TS, 13
Burnell, Fidkowski, Chen, AV 13

Surface topological order:
T-reversal invariant `all fermion' Z2 gauge theory where all three topological quasiparticles are fermions
(Not possible in strict 2d with T-reversal).

Coupled layer construction: Start with trivial realization of T-reversal in each 2d layer, condense $\varepsilon_i m_{i+1} \varepsilon_{i+2}$.

T-broken `confined' surface: quantized thermal Hall effect

$$\kappa_{xy} = \pm 4.$$

Domain wall: edge modes of 2d chiral boson integer quantum Hall state.

Summary of boson detour

Many new and useful non-perturbative ideas to discuss topological insulator/SPT states.

Surface topological order of 3d SPT a very useful characterization.

Electronic topological insulators
(Chong Wang, A.Potter, TS 2013)

The problem

Electronic insulators with no bulk topological order/fractionalization (``short range entangled").

Realistic Symmetry: charge conservation, T-reversal (strong spin orbit => spin not conserved).

Band theory: Z_2 classification

Beyond band theory: Strong correlations + strong spin-orbit

How many such phases for there with strong interactions?

What are their physical properties?

The answer

3d electronic insulators with charge conservation/T-reversal classified by Z_2^3 (corresponding to total of 8 distinct phases).

3 `root' phases:
Familiar topological band insulator, two new phases obtained as electron Mott insulators where spins form a spin-SPT (topological paramagnets).

Topological Insulator	**Representative surface state**
Free fermion TI	Single Dirac cone
Topological paramagnet I ($eTmT$)	$\mathbb{Z}_2$ spin liquid with Kramers doublet spinon(e) and vison(m)
Topological paramagnet II ($e_f m_f$)	$\mathbb{Z}_2$ spin liquid with Fermionic spinon(e) and vison(m)

Obtain all 8 phases by taking combinations of root phases.

Comments

1. From electrons can always form bosons and put bosons in SPT.

But not all such boson SPTs stay distinct in an electron system.

2. No `intrinsically' (root) electronic TI other than original topological band insulator.

Proof

Bulk EM response

Start with EM response of *any* 3d insulator

$$\begin{aligned} \mathcal{L}_{eff} &= \mathcal{L}_{Max} + \mathcal{L}_{\theta} \\ \mathcal{L}_{\theta} &= \frac{\theta}{4\pi^2} \vec{E}.\vec{B} \end{aligned} \quad (1)$$

Under $\mathcal{T}$-reversal, $\theta \to -\theta$.
Periodicity $\theta \to \theta + 2\pi$: only $\theta = 0, \pi$ consistent with $\mathcal{T}$-reversal.

If there are 2 distinct insulators with $\theta = \pi$, can combine to make $\theta = 0$ insulator.
=> To look for new insulators, sufficient to restrict to $\theta = 0$.

Bulk magnetic monopole

Witten effect => monopole charge $\frac{\theta}{2\pi}$.
At $\theta = 0$, monopole has charge 0.
Time reversal:

$$\mathcal{T}^{-1} m \mathcal{T} = e^{i\alpha} m^\dagger \quad (1)$$

$$\mathcal{T}^{-1} m^\dagger \mathcal{T} = e^{-i\alpha} m \quad (2)$$

But can combine with (magnetic) gauge transformation to set $\alpha = 0$.
=> Monopole transforms trivially under T-reversal.
Symmetries of monopole fixed.

Remaining possibilities:
Bosonic versus fermionic statistics of the monopole.

Bosonic magnetic monopole: implications for surface effective theory

Tunnel monopole from vacuum into bulk

Tunneling event leaves behind surface excitation which has charge-0 and is a boson.

A convenient surface termination- a surface superconductor*

Monopole tunneling leaves behind hc/e vortex which is a boson (and transforms trivially into a hc/e antivortex under T-reversal).

*Careful proof: allow for surface SC to have coexisting topological order (see Appendix of Wang, Potter, TS 13)

Symmetry preserving surface

Disorder the superconducting surface:

Condense the bosonic hc/e vortex.

Result is a symmetry preserving insulating surface with ``intrinsic" topological order.

hc/e vortex condensate => Charge quantized in units of e.

=> Surface TQFT: every topological sector can be made neutral
(integer charge => bind physical electrons to make neutral).

Surface TQFT = (1, ε, ...) x (1,c) = (Neutral boson TQFT) x (1,c)

=> Bulk SPT order is same as for neutral boson SPT (supplemented by physical electron).

Topological paramagnets

Neutral boson SPTs with time reversal are classified by Z_2^2.

Describe as electron Mott insulators where spins form an SPT.

Strong spin-orbit => spin system only has T-reversal symmetry.

Two root states conveniently characterized by surface topological order.

1. Topological paramagnet-I:

Z_2 topological order where both e and m are Kramers (``within cohomology'' state)

2. Topological paramagnet-II:

``All fermion'' $Z_2$ topological order where all topological particles are fermions (``Beyond cohomology'' state).

Other phases?

Why not first form Cooper pairs and put them in a boson SPT?

Surface topological order of Cooper pair SPT:

Z_2 gauge theory where both e and m carry charge -e (1/2 of Cooper pair charge).

Add electrons to both e and m sectors:

Neutral Kramers fermions in both e and m

=> surface topological order of topological paramagnet

Already included in Z_2^3 classification.

Bulk fermionic monopole?

Can charge and monopole both be fermionic?

Construct such a state:

Bose-Fermi mixture of charge-e fermions and charge-e bosons.

Put fermions in trivial band insulator and bosons in boson SPT.

Take boson gap to ∞ => bulk Hilbert space is that of a pure fermion system.

External monopoles:

Bosons in SPT => neutral monopole is fermion (Wang, TS 13, Metlitski et al, 13).

Bulk fermionic monopole (cont'd) ?

But constructed state is not `edgable' as a pure charge-e fermion system

(`Edgable': Allow physical edge to vacuum).

Symmetry preserving surface termination:
Bosons have intrinsic Z_2 topological order where e and m carry charge e/2, and ε is charge neutral.

Boson charge gap = ∞ => e, m disappear from spectrum but ε survives.

But ε is local with respect to all finite energy excitations
=> surface spectrum admits a local charge neutral fermion.

Not allowed in a pure fermion Hilbert space of charge-e electrons

=> Fermionic monopole state not allowed in strict 3d in an electronic system
(may be allowed at surface of 4+1-D SPT).

Physical characterization of the 8 interacting 3d TIs

4 insulators with $\theta = 0$: Trivial, 3 topological paramagnets

4 insulators with $\theta = \pi$: Topological band insulator and its combinations with the 3 topological paramagnets

How to tell in experiments?

1. Symmetry preserving surface topological order?

Pro: full characterization of surface
Con: Not very practical

2. Alternate: T-breaking surface without topological order (deposit ferromagnet)

Pro: Surface Hall transport as a practical signature
Con: Only a partial characterization.

Hall transport signatures of interacting TIs

Break T explicitly to get a `trivial' surface:

$\theta = \pi$ versus $\theta = 0$ insulators:
Surface electrical Hall conductivity of 1/2 for $\theta = \pi$ (versus 0 for $\theta = 0$)

Surface thermal Hall conductivity:
Two of the $\theta = \pi$ TIs have $\kappa_{xy} = 1/2$ (including usual band TI)
Other two have $\kappa_{xy} = 9/2$.

Similarly two of the $\theta = 0$ TIs have $\kappa_{xy} = 0$ (including trivial insulator)
Other two have $\kappa_{xy} = 4$.

Aside: Role of Kramers

In band theory, topological insulation relies on Kramers structure of electron.

What precisely is role of Kramer's structure of electron in enabling $\theta = \pi$ response in an interacting TI?

Prove that $\theta = \pi$ requires Kramers structure (Wang, Potter, TS 13; Metlitski, Kane, Fisher 13).

Bulk monopoles are ``dyons'' with charge $\pm 1/2$.

Consider bound state of charge-1/2 ``dyon'' with charge-1/2 ``antidyon''.

Bound state carries electric charge-1 and magnetic charge-0 => identify with electron.

What are its time reversal properties?

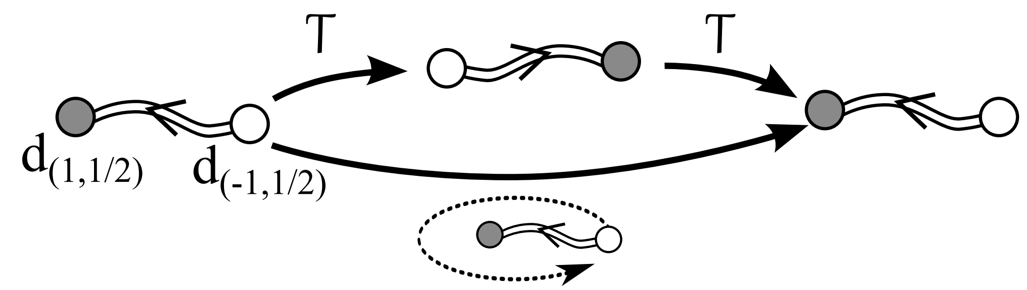
Role of Kramers (cont'd)

``Dyon" and ``Anti-dyon" with same electric charge transform into each other.

$$\mathcal{T}^{-1} d_{(1,1/2)} \mathcal{T} = d_{(-1,1/2)} \quad (1)$$
$$\mathcal{T}^{-1} d_{(-1,1/2)} \mathcal{T} = d_{(1,1/2)}$$

$d_{(q_m,q_e)}$ is dyon operator with magnetic charge q_m and electric charge q_e.

These two see each other as relative monopoles.



=> In bound state, EM field angular momentum is half-integer.

Under T^2 action, this `orbital' part leads to $T^2 = -1$.

=> Bound state (= electron) must be Kramers doublet.

End of proof.

Summary

1. Interacting electron TIs in 3d have a Z_2^3 classification

- apart from trivial and topological band insulators, 6 new TI phases with no non-interacting counterpart.

2. Quantum `anamolous' thermal Hall effect of surface a very useful probe in addition to electrical Hall effect.

Other probes?

3. Most important open question:

what kinds of real electronic insulators may be these new 3d TIs?