Algebraic charge liquids and the underdoped cuprates

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Plan of talk

1. Motivation

2. Algebraic charge liquids in doped Mott insulators

3. Application to underdoped cuprates: some successes and problems

4. Fixing the problems?

5. Lessons and questions

Cuprate materials: Some indisputable basic facts

1. Zero doping : Mott insulator with Neel order (NOT a spin liquid)

2. Superconductivity in doped materials with d-wave symmetry

3. $hc/2e$ flux quantization (cheapest vortex is $hc/2e$)

4. Superfluid density $\rho_s(T=0) \sim x \sim T_c$ (at not too low doping)

Superconductor is gapless

Evidence from various probes

Examples

1. "Metallic" thermal transport in SC

$$\lim_{T \to 0} \kappa / T \to \text{constant}$$

2. Superfluid density $\rho_s(x, T) = \rho_s(x, 0) - A T$

(Note: A independent of x at not too low doping

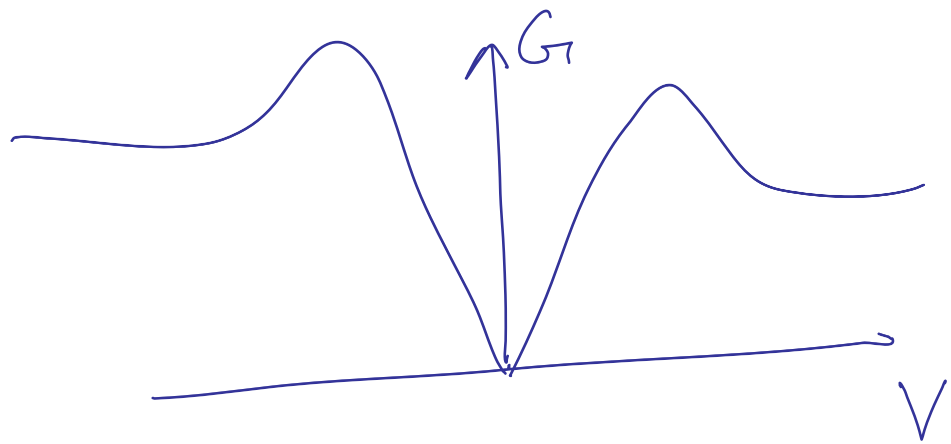
- important implications for T_c (Lee, Wen '98))

Gapless excitations: Bogoliubov quasiparticles?

1. Low energy STM spectrum with

$G(V) \sim V$

down to low T.



2. Nodal quasiparticles in ARPES?

(Caution: Line width too broad ($\sim E$);

resolution not too high ~ few meV)

Mott insulator with Neel order

$d_{x^2-y^2}$ SC with very specific properties (gapless nodal excitations, etc)

A theoretical project

Can we find a few different theoretical routes that naturally incorporate these few basic facts about the undoped & underdoped cuprates ??

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Even this very low ambition project is very hard !

(Existing literature does not seem to have good answers.)

This talk - explore one theoretical route from AF Mott insulator to gapless d-wave SC.

Key feature: "Algebraic charge liquids"

~ charge analog of gapless quantum spin liquid

Non-fermi liquid phases with power law charge correlations.

Doping an antiferromagnetic Mott insulator: Low doping

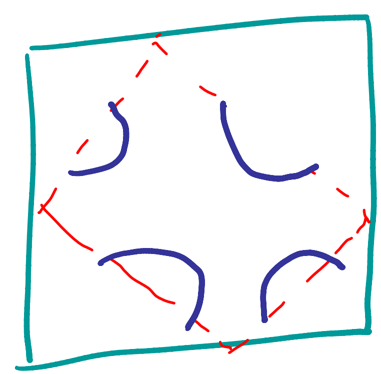
One hole - dispersion minima at $(\pm\pi/2, \pm\pi/2)$

(Kane, Lee, Read '89)

Small density of holes :

Possible state - small hole pockets centered at $(\pm\pi/2, \pm\pi/2)$ + Neel order

(Many papers '88-'91)



A metallic antiferromagnet with hole pockets

Losing the Neel order

Hole motion frustrates Neel order.

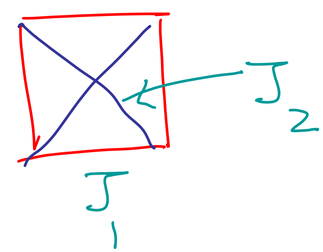
Question: What happens to the antiferromagnetic metal when the Neel order is lost at $T = 0$?

Lessons from insulating quantum magnets

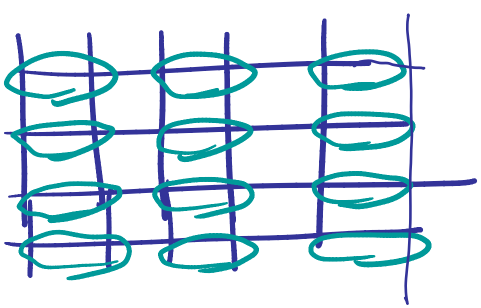
Analagous question well understood in the Mott insulator

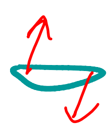
Kill Neel by increasing frustration

(Eg: $J_1 - J_2$ model)



"Natural" result: Valence bond solid paramagnets



 = singlet bond

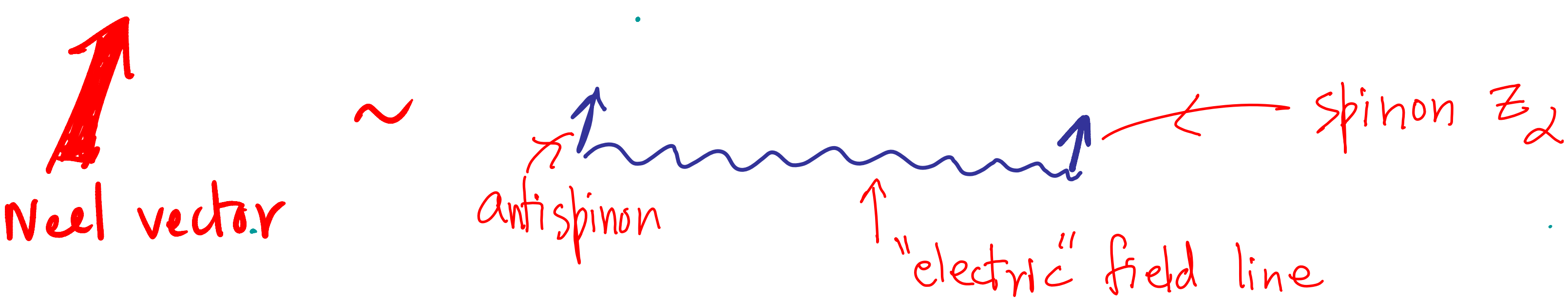
(Read, Sachdev '89)

Deconfined quantum criticality-I

Neel - VBS transition can be 2nd order despite different broken symmetries !

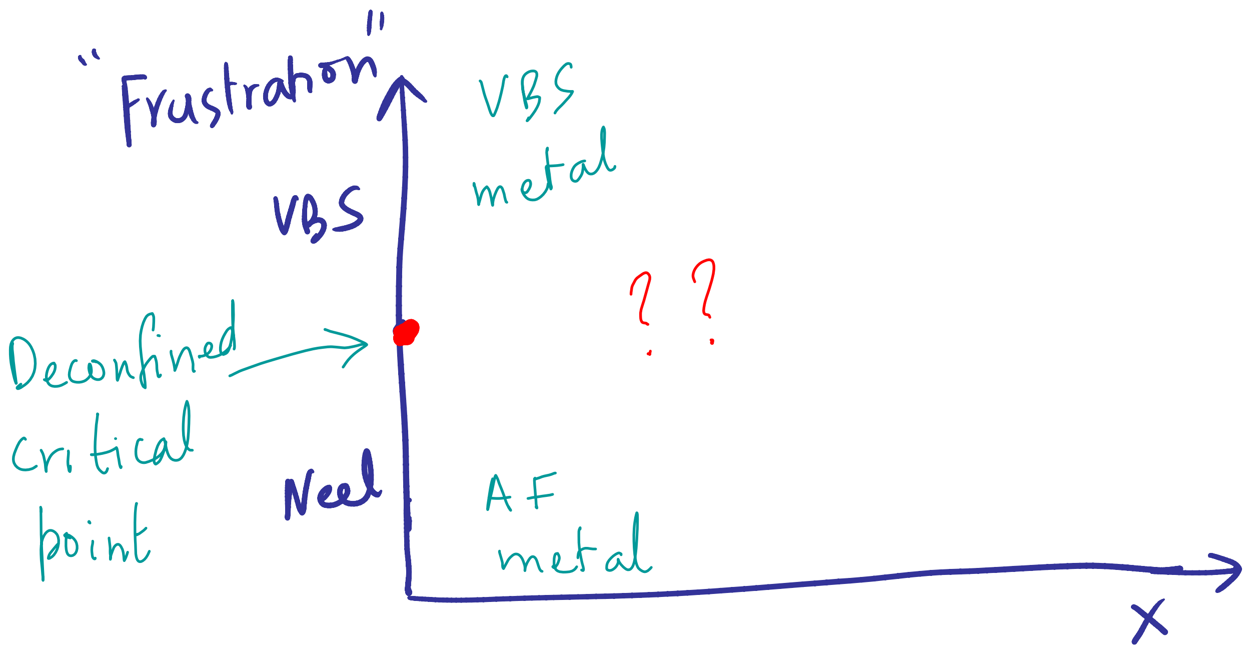
(Senthil, Vishwanath, Balents, Sachdev, Fisher '04)

Critical theory : "deconfined" bosonic spinons z_α + gauge fields



Doping the deconfined critical point-I

Question: Fate of doping an antiferromagnet that is close to this deconfined quantum critical point ??



Doping the deconfined critical point-II

Doped hole : $c_\alpha \nearrow z_\alpha$, $c_\alpha \searrow\ ?$

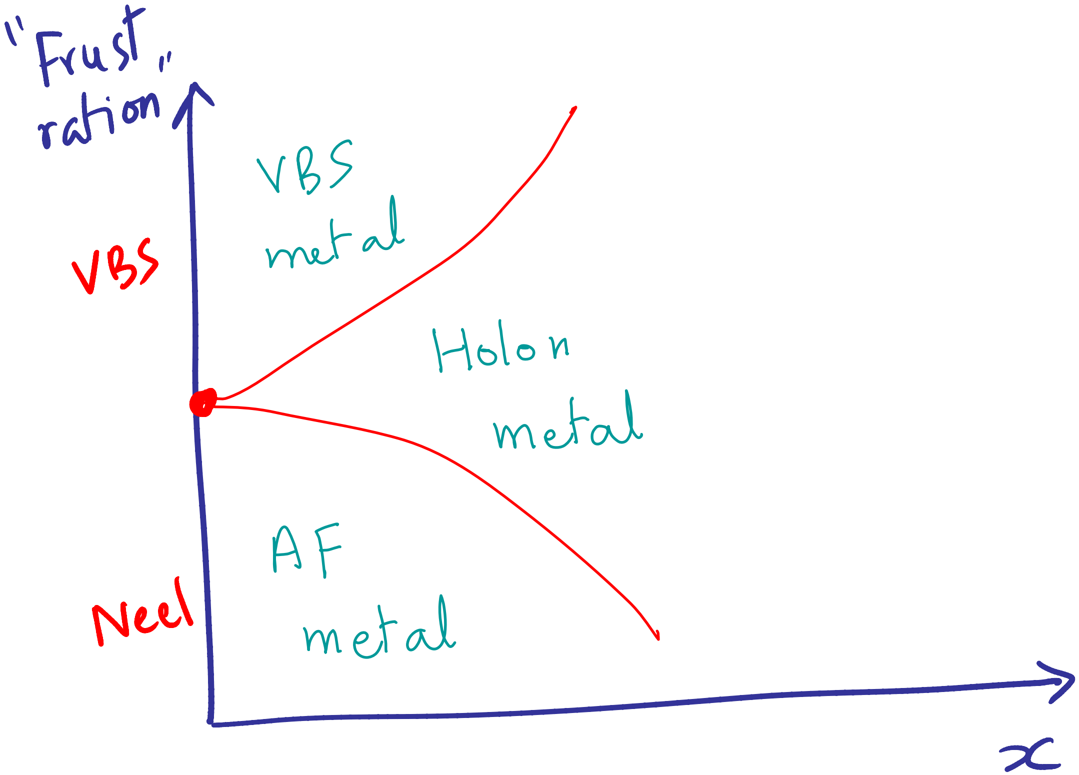
$$? \sim c_\alpha z_\alpha^* \sim f = \text{spinless charge-}e$$

"holon".

Guess : At doping x ; f forms Fermi surface with area $\propto x$.

Confirm : Mean field calculation within "t-J" model .

The holon metal



Key points:

① Fermi surface of holons stabilizes doped metal from VBS order.

② z_α condenses ⇒ Neel order AND holons → holes of doped AF insulator.

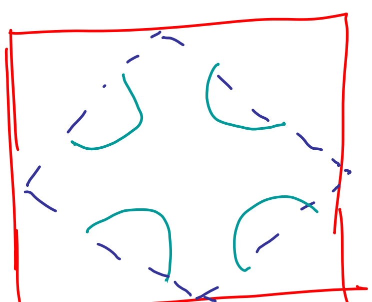
The holon metal: Physical properties-I

Lee '88
Kaul et al. '07

1. Metallic state with no symmetry breaking

2. Two species of holons $f_{\pm}$ that live on opposite sublattices (& carry opposite gauge charge).

3. 2 small Fermi pockets per species with area $\propto x$



The holon metal: Physical properties-II

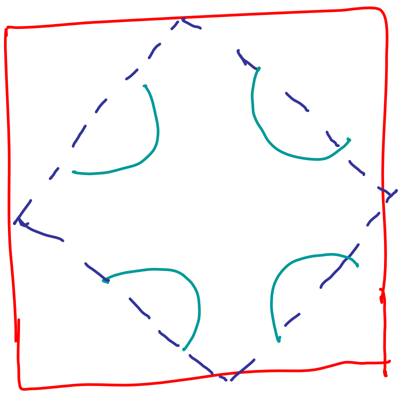
4. Holon Fermi surface ⇒ quantum oscillations in Shubnikov-de Haas, dHvA etc with frequency

$$F/\Phi_0 = \frac{2 A_k}{(2\pi)^2} = \frac{x}{2a^2}$$

5. Spin gap ⇒ No Fermi surface in ARPES!

⇒ Real discrepancy between Fermi surfaces seen in quantum oscillations & ARPES

Superconductivity from the holon metal



Sources of holon pairing interaction

1. Spinon mediated attraction (Sushkov et al. '93-'95)

2. Opposite gauge charges on 2 sublattices

$\Rightarrow$ gauge attraction (Wiegmann '88, Shankar '89, Wen '89, Lee '89)

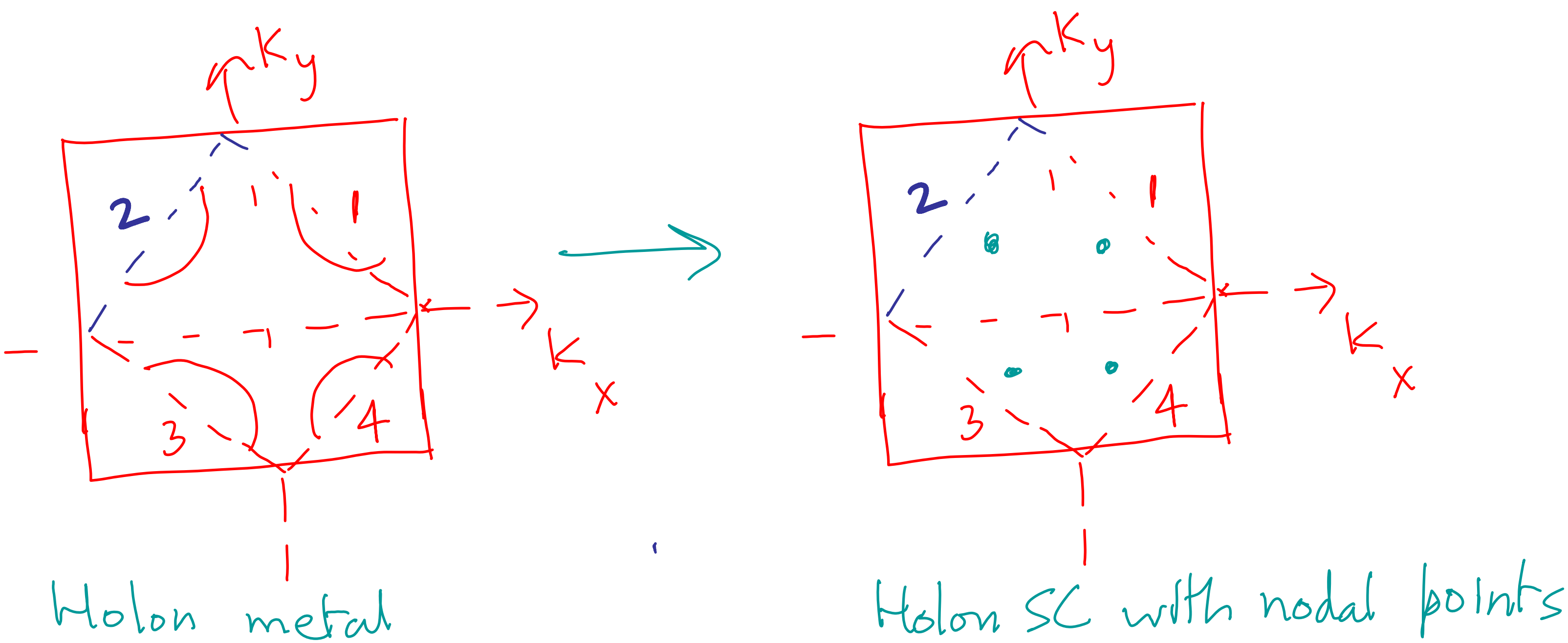
$\Rightarrow$ Holon metal unstable to pairing $\langle f_+ f_- \rangle \neq 0$ at low-T.

$f_+ f_- \sim$ Cooper pair $\Rightarrow$ this is a true superconductor!

Mean field theory for the superconductor

Look for $d_{x^2-y^2}$ pairing symmetry for electrons (Kaul et al. '07)

$\Rightarrow$ Gapless pairing with nodal holons.



Gauge fluctuations: QED_3 theory

$f_+ f_-$ order parameter is "gauge - neutral"

$\Rightarrow$ Nodal holons still coupled to gapless U(1) gauge field a_μ

$\Rightarrow$ Low energy theory of Superconductor is massless QED_3

$$S = \int d^2x\, d\tau\, \bar{\psi}(\not{\partial} - i\not{a})\psi + \frac{1}{2e^2} f_{\mu\nu}^2$$

ψ : Nodal Dirac holons.

Gapless holon superconductor

Massless QED_3 : Strongly interacting scale invariant theory.

Universal power law correlations for various physical quantities

$\Rightarrow$ Superconducting state is exotic (not smoothly connected to d-wave BCS)

(Another algebraic charge liquid)

Properties of the holon superconductor-I
Superfluid density at T = 0

True superconductor $\Rightarrow$ non-zero ρ_s.

Pairing of holons of a holon Fermi surface of area $\sim x$

$$\Rightarrow \rho_s(T=0) \sim x$$

Properties of the holon superconductor-II
Finite T superfluid density

$T \neq 0$: thermal excitation of gapless holons

$$\rho_s(T) - \rho_s(0) \sim \langle J J \rangle$$ with J = electric current of unpaired holons

J : conserved density of massless QED_3

$\Rightarrow$ From scaling $\rho_s(T) = \rho_s(0) - AT$

with A = universal amplitude of massless QED_3

"A" universal $\Rightarrow$ x-independent as well !

$$\Rightarrow \rho_s(x, T) \sim c_1 x - AT$$

for powerful and general reasons.

"A" : estimate in $1/N$ expansion.

Seen in hiTc :: Phenomenologically important
Theory of T_c (Lee, Wen '98)
Mysterious in other theories (see Ioffe-Millis '01)

Comparison to cuprate superconductors

1. $\rho_s(x, T) = c_1 x - AT$
   with A x-independent ✓

2. Metallic low-T thermal transport,
   specific heat $C_v \sim T^2$ at low T ✓

3. Gap in single particle spectrum
   (as measured by STM or ARPES) ✗

Fixing the problem?

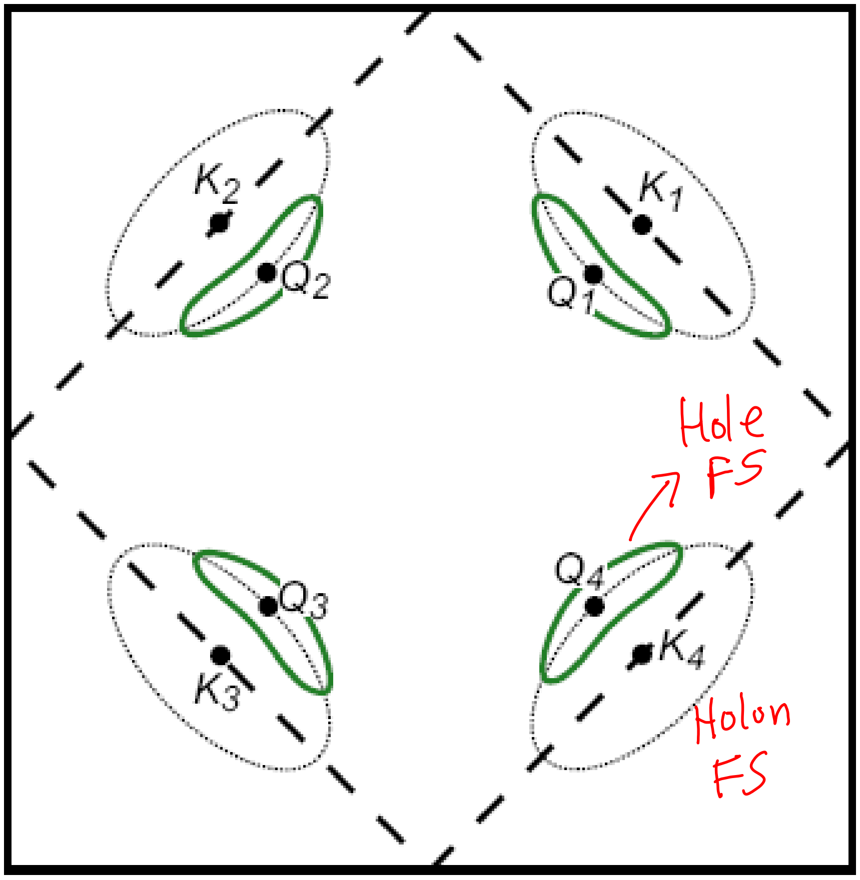
Binding of <u>some</u> holons to spinons due to short range interactions ?

$\Rightarrow$ "Normal" state with coexisting holon and hole Fermi surfaces.

Simplified treatment of binding (Kaul et. al. '08)

- specific results for hole & holon Fermi surfaces

Holon-hole metal



Hole FS has a narrow "banana" shape, and is visible to ARPES/STM.

Both Fermi surfaces will show quantum oscillations.

Superconductor descending from this state has both nodal holes & nodal holons

– mitigate problem of holon superconductor

Summary

1. New route from antiferromagnetic Mott insulator to gapless d-wave SC based on ``algebraic charge liquids"
2. Holon metal: spin gap and a charge fermi surface => sharp fermi surface in quantum oscillations but none in ARPES.

3. Holon superconductor: Scale invariant excitation spectrum with interesting behavior of superfluid density

4. Application to cuprates: Must solve problem of hard spin gap

(holon-hole metal and descendant superconductor?)

Lessons/questions

1. Quantum oscillations do not necessarily imply a Landau Fermi liquid (examples: composite Fermi liquid in a half-filled Landau level, holon metals,………)

$\Rightarrow$ Fermi arcs in ARPES perhaps not incompatible with quantum oscillations in a non-fermi liquid state?

2. Are nodal excitations in the underdoped cuprate SC truly `electron' like quasiparticles with T^3 scattering rate?

3. Are the underdoped cuprate SC smoothly connected to a regular d-wave BCS?

General suggestions/questions-I

1. Quantum oscillations do not imply a Fermi liquid of electron quasiparticles!

Eg: (i) Holon metal : Fermi surface of spinless charge-e fermions

(ii) Composite Fermi Liquid in ½-filled Landau level of Halperin-Lee-Read

Shubnikov-de Haas in experiments but Fermi surface is "composite fermions" not electrons.

General suggestions/questions-II

2. Discrepancy between quantum oscillations & ARPES may be real!

(Not necessarily a technical issue with ARPES resolution)

Eg: Holon metal has "Fermi arcs" at $T \neq 0$ in ARPES, and quantum oscillations in transport.

General suggestions/questions-II

3. Are nodal excitations in the superconductor truly "electron-like quasiparticles" ?

(Any ARPES measurement showing

$$\mathrm{Im}\,\Sigma \sim E^3 \;??)$$

4. IS GROUND STATE OF UNDERDOPED SUPERCONDUCTOR SMOOTHLY CONNECTED TO D-WAVE BCS ??