# Half-filled Landau level and topological insulator surfaces

T. Senthil (MIT)


With Chong Wang,
grad student @ MIT →Harvard

C. Wang, and TS, arXiv:1505.03520 (PR X 2016), arXiv:1505.05141 (PR X 2015)

*C. Wang and TS, arXiv:1507.08290 (PR B 2016): Synthesis and physical pictures - this talk*

**C. Wang and TS, to appear**

Related work:
Son, arXiv:1502.03446 (PR X 2015); M. Metlitski and A. Vishwanath, arXiv:1505.05142.

Many other important contributions: (Experiments) Shayegan group; Barkeshli, Mulligan, Fisher 15; Geraedts, Zaletel, Mong, et al, 15; Murthy, Shankar, 15; Mross Alicea, Motrunich 15; Potter, Serbyn, Vishwanath, 15.

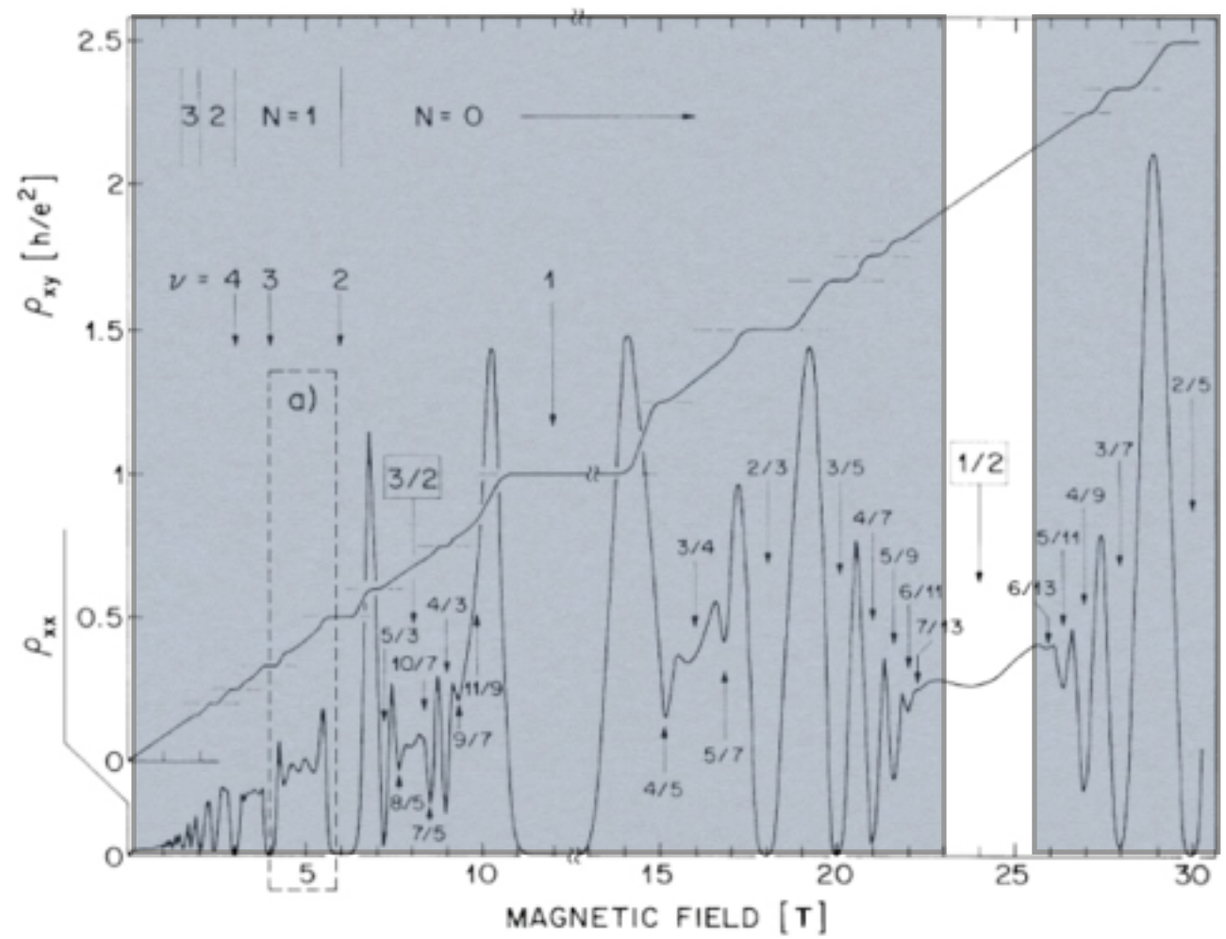
# The half-filled Landau level

2d electron gas in a strong magnetic field in the ``quantum Hall regime'' at filling factor $\nu = 1/2$

Experiment: Metal with $\rho_{xx} \neq 0$, $\rho_{xy} \neq 0$, but $\rho_{xx} \ll \rho_{xy}$.

Theory (Halperin, Lee, Read'93): ``Composite Fermi Liquid''

At $\nu = 1/2$, Jain's composite fermions move in zero effective magnetic field and can form a Fermi surface.



# Composite fermi liquid theory (Halperin, Lee, Read (HLR)

Effective theory:

$$\mathcal{L} = \bar{\psi}_{CF}\left(i\partial_t - a_0 - iA_0^{ext} + \frac{(\vec{\nabla} - i(\vec{a} + \vec{A}^{ext})^2}{2m}\right)\psi_{CF} + \frac{1}{4\pi}a_\mu\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda \quad (1)$$

Many aspects of theory confirmed in experiment (eg, existence of Fermi surface)

Jain sequence: $\nu = n/(2n+1)$

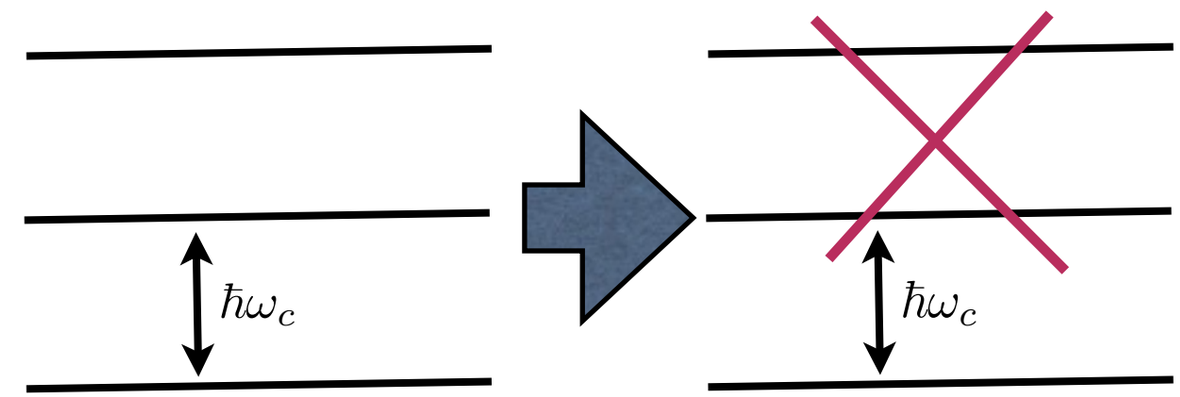
Understand as filling composite fermion Landau levels (Jain 89)

HLR: energy gap of Jain states = spacing of effective Landau levels of composite fermions

# Unfinished business in theory

1. Theory should be defineable within the Lowest Landau Level (LLL) but HLR is not in the LLL.

Many refinements in the late 90s (Shankar, Murthy; Read; Halperin, Stern, Simon, van Oppen; D.-H. Lee, Pasquier, Haldane,.....) but dust never settled.




$V_{int} \ll \hbar\omega_c$: project to LLL $\qquad$ $V_{int} \ll \hbar\omega_c$: project to LLL

2. Particle-hole symmetry

A symmetry of the LLL Hamiltonian (with eg, 2-body interactions) but not manifest in HLR.

Issue identified in the 90s (Grotov, Gan, Lee, Kivelson, 96; Lee 98) but no resolution.

# Particle-hole symmetry in LLL

At $\nu = 1/2$, regard LLL as either ``half-empty or half-full'':

Start from empty level, fill half the LLL

or start from filled LL and remove half the electrons

# Particle-hole symmetry: formal implementation

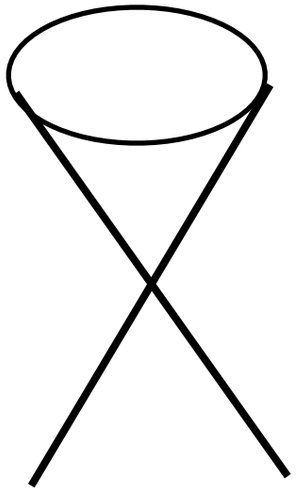
Electron operator $\psi(x,y) \simeq \sum_m \phi_m(x,y) c_m$ after restriction to LLL.
($\phi_m(x,y)$: various single particle wave functions in LLL).

Particle-hole: **Antiunitary symmetry** $C$
$C\psi C^{-1} = \psi^\dagger = \sum_m \phi_m^*(x,y) c_m^\dagger$

Symmetry of 1/2-LLL with, eg, 2-body interaction is (at least) U(1) X C

# Particle-hole symmetric CFL: a proposal

Is the Composite Fermion a Dirac Particle? (Son , 2015)



Composite fermion $\psi_v$ forms a single Dirac cone tuned away from neutrality:

Anti-unitary p/h (C) acts in same way as time reversal usually does on Dirac fermion:

$$C\psi_v C^{-1} = i\sigma_y \psi_v \Rightarrow C^2 \psi_v C^{-2} = -\psi_v$$

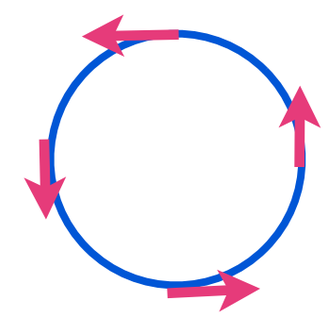
Composite fermion is Kramers doublet under $C$.

# Particle-hole symmetric CFL: a proposal (cont'd) (Son, 2015)

Effective Lagrangian

$$\mathcal{L} = i\bar{\psi}_v \left(\not{\partial} + i\not{a}\right) \psi_v - \mu_v \bar{\psi}_v \gamma_0 \psi_v + \frac{1}{4\pi} \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda \; + \mathcal{L}_{bg}[A_\mu]$$

Low energy theory: focus on states near Fermi surface.



Meaning of Dirac?

As CF goes around FS, pick up π Berry phase.

# Particle-hole symmetric CFL: a proposal (cont'd) (Son, 2015)

Effective internal magnetic field $B^* = \vec{\nabla} \times \vec{a} = B - 4\pi\rho$.
Composite fermion density $n_v = \frac{B}{4\pi}$

B* same as in HLR but composite fermion density $n_{CF}$ is different.
In HLR $n_{CF} = \rho$ (at any filling)

At $\nu = 1/2$, $n_v = n_{CF}$ but they are different away from it.
This slight difference is crucial!

# Plan of talk

***A. Understanding the p/h symmetric composite fermi liquid***

1.. Connection to surface of 3d topological insulators - field theoretic justification
(C. Wang, TS, arXiv:1505.05141; M. Metlitski, A. Vishwanath, 1505.05142.)

2. Simple physical picture of the particle-hole symmetric composite fermion
(C. Wang, TS, arXiv:1507.08290)

3. Numerical calculations
(Scott D. Geraedts, Michael P. Zaletel, Roger S. K. Mong, Max A. Metlitski, Ashvin Vishwanath, Olexei I. Motrunich, arXiv: 1508.04140)

***B. Composite Fermi liquids in LLL at generic ν:***
***Quantum vortex liquids with Fermi surface Berry phases***
*(Wang, TS, to appear)*

# Plan of talk

Focus in this talk;

***A. Understanding the p/h symmetric composite fermi liquid***

1.. Connection to surface of 3d topological insulators - field theoretic justification
(C. Wang, TS, arXiv:1505.05141; M. Metlitski, A. Vishwanath, 1505.05142.)

But first I will quickly describe this!

2. Simple physical picture of the particle-hole symmetric composite fermion
(C. Wang, TS, arXiv:1507.08290)

3. Numerical calculations
(Scott D. Geraedts, Michael P. Zaletel, Roger S. K. Mong, Max A. Metlitski, Ashvin Vishwanath, Olexei I. Motrunich, arXiv: 1508.04140)

Time permitting

***B. Composite Fermi liquids in LLL at generic ν:***
***Quantum vortex liquids with Fermi surface Berry phases***
*(Wang, TS, to appear)*

Particle-hole symmetric composite fermions: a physical picture (Wang, TS, 2015)

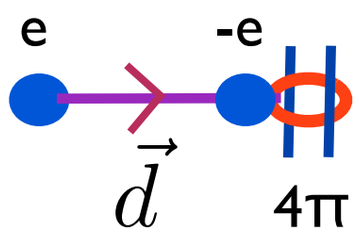
# Old physical picture of Composite Fermion (CF) in LLL

## *Neutral dipolar fermions*

**N. Read, 1994**; many subsequent papers in late 90s (Shankar, Murthy; Haperin, Stern; D.-H. Lee; Pasquier-Haldane,…..)

LLL wave function (Rezayi-Read 94)

$$\psi(z_1, ....., z_N) = P_{LLL} det(e^{i\vec{k}_i \cdot \vec{r}_j}) \prod_{i<j} (z_i - z_j)^2$$




CF: electron bound to 4π-vortex: Vortex has depletion of charge -e => CF is neutral.

CF has dipole moment perpendicular to it's momentum. $\vec{d} \perp \vec{k}$

Heuristic wave function argument: Plane wave factors push vortex away from electron

However this picture misses some physics and further is not p/h symmetric (charge changes sign but vorticity does not).

Wang, TS 15

# New picture of composite fermion in LLL

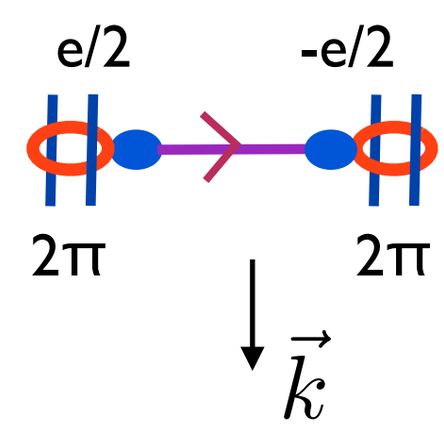
Fermion wavefunctions in LLL $\psi(z_1, z_2, ....., z_N) = \prod_{i<j}(z_i - z_j) f(z_1, ....., z_N)$

$f(z_1, \ldots.., z_N)$: a symmetric polynomial.

=> one vortex is exactly on electron due to Pauli.

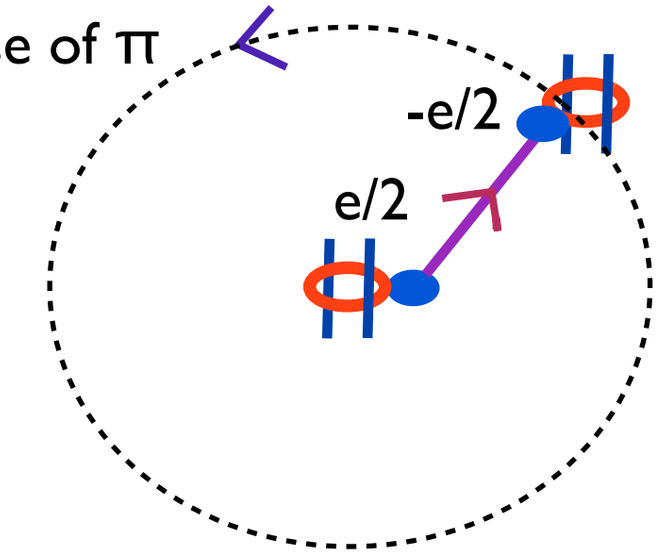
Only second vortex is displaced from first in direction perpendicular to CF momentum.

Each vortex has charge -e/2 => single vortex exactly on electron has charge +e/2 and the displaced vortex has charge -e/2.

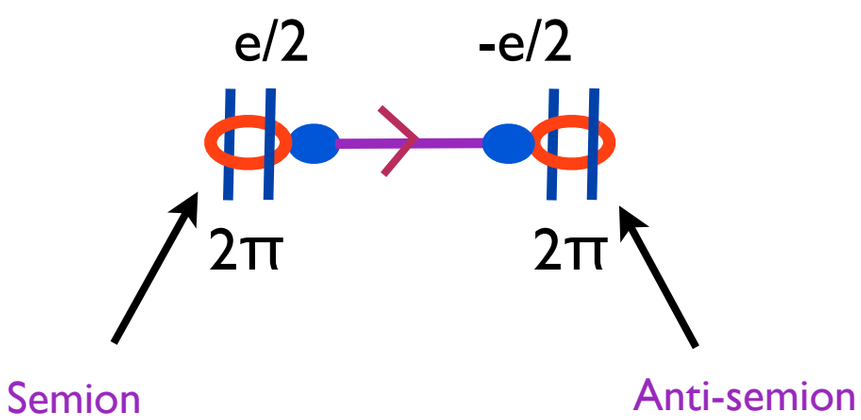


# Internal structure of composite fermion(*) in LLL

Phase of π




Two ends have mutual statistics of π

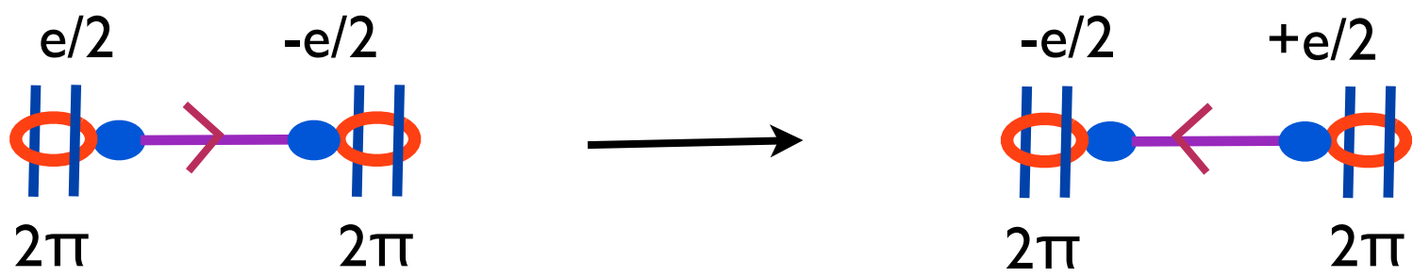



Bound state (of course) is a fermion.

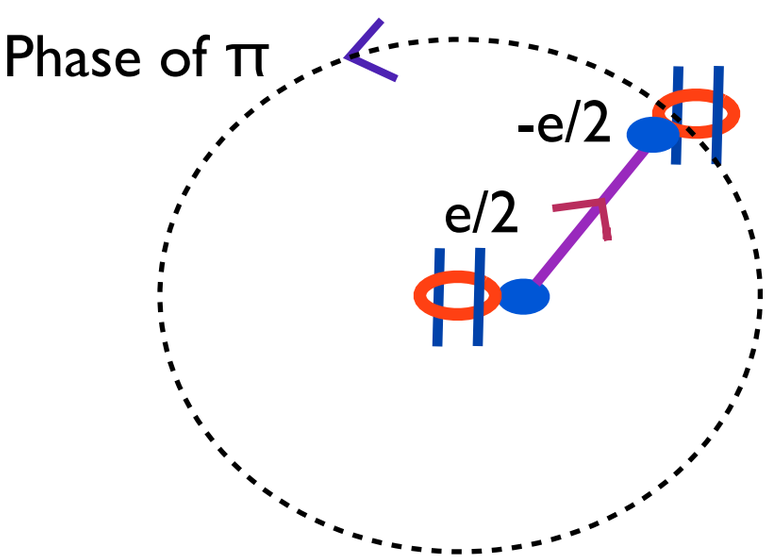
* Assume ends are well separated compared to vortex size.

# New picture of composite fermion in LLL (cont'd)

C:




Anti-unitary C interchanges relative coordinates of the two ends.




Solve QM of relative motion =>

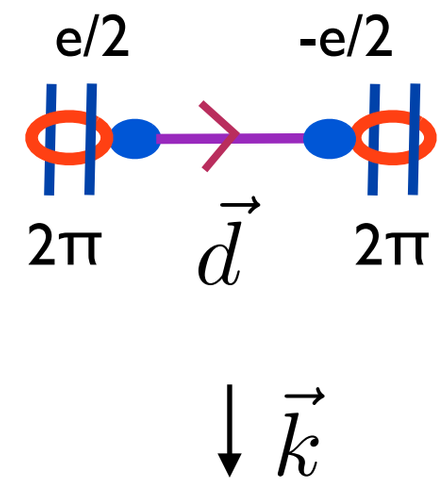
ground state ``spin"-1/2 doublet which is Kramers under C.

# New picture of composite fermion in LLL (cont'd)

Non-zero CF momentum => non-zero dipole moment

=> ``spin" of composite fermion polarized perpendicular to it's momentum

(spin-momentum locking expected of Dirac fermion).




$$\vec{d} \perp \vec{k}$$

$$=> \langle \vec{\sigma} \rangle \perp \vec{k}$$

Composite fermion goes around FS => momentum rotates by 2π => spin rotates by 2π => Berry phase of π as expected for a Dirac fermion.

# 1/2-filled LL and correlated TI surfaces

Derivation of and more insight into p/h symmetric composite fermi liquid theory

(C. Wang, TS, 15; M. Metlitski, A. Vishwanath, 15)

# p/h symmetric LL as a surface of a 3d fermionic topological insulator: Preliminaries

Consider (initially free) fermions with ``weird" action of time-reversal (denote C):

$C \rho C^{-1} = - \rho$

$\rho$ = conserved ``charge" density.

Full symmetry = U(1) x C
(called class AIII in Topological Insulator/Superconductor literature*)

Eg: Triplet time reversal-invariant superconductor where physical $S_z$ is conserved and plays the role of such a $\rho$.

*distinct symmetry from usual spin-orbit coupled insulators which have U(1) $\ltimes$ C symmetry (i.e, $\rho$ is usually even under time reversal).

# p/h symmetric LL as a surface of 3d fermion SPT (cont'd)

Surface: Single massless Dirac fermion

C symmetry guarantees that surface Dirac cone is exactly at neutrality.

$$\mathcal{L} = \bar{\psi}\left(-i\partial\!\!\!/ + A\!\!\!/\right)\psi + \ldots.$$

external probe gauge field

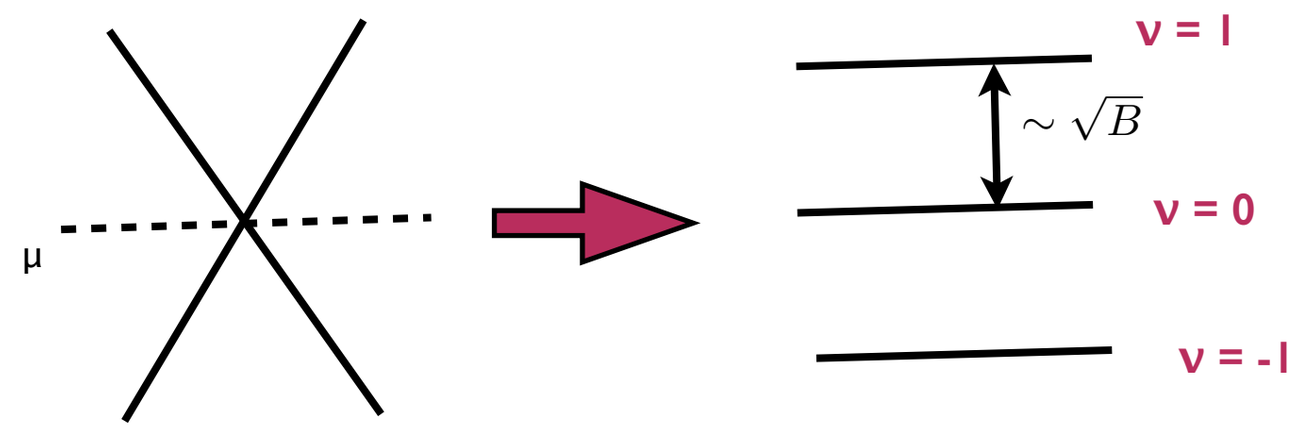
2-component fermion

# p/h symmetric LL as a surface of 3d fermion SPT (cont'd)

ρ is odd under C => `electric current' is even.

External E-fields are odd but external B-fields are even.

=> Can perturb surface Dirac cone with external B-field.



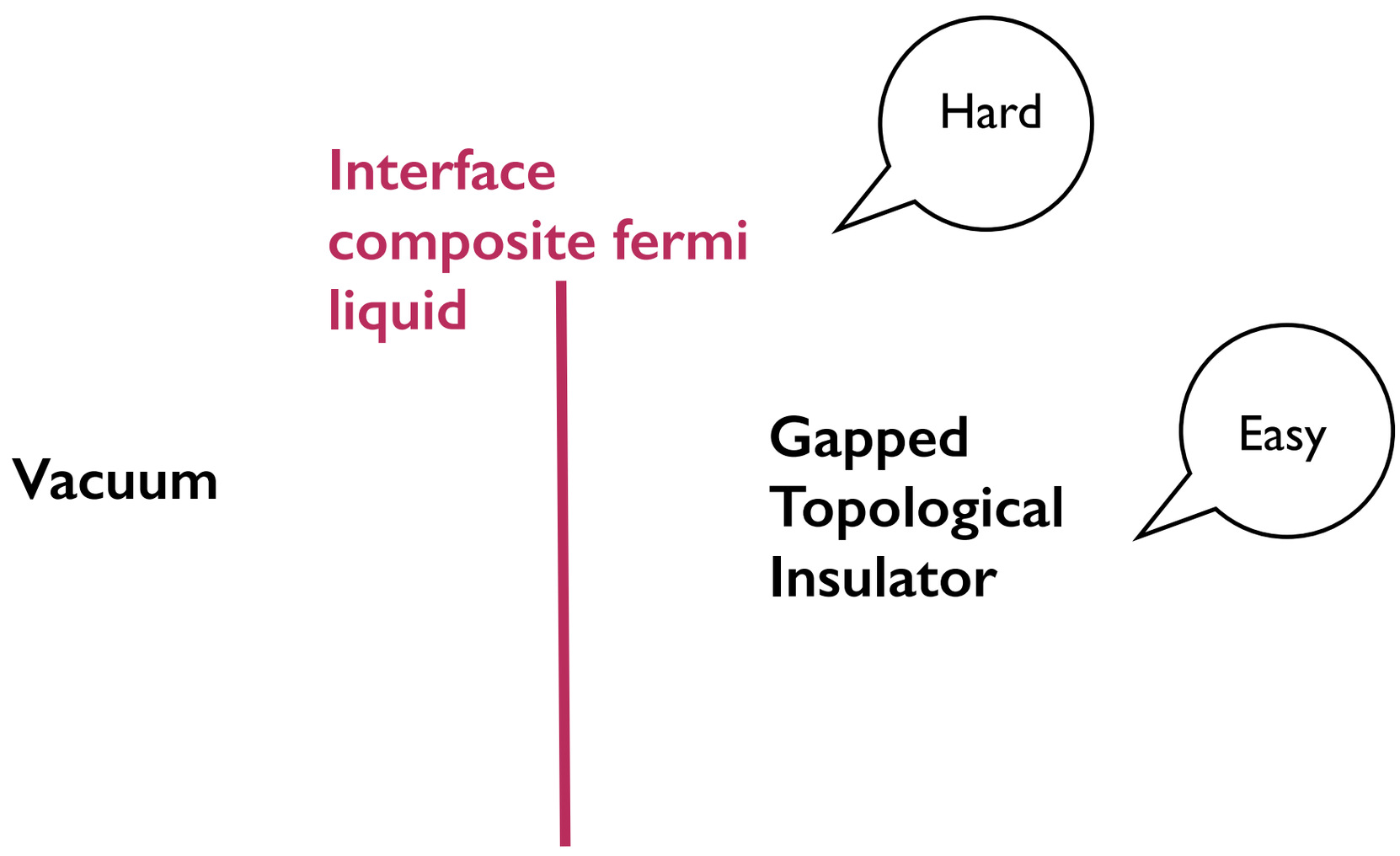

C-symmetry: ν = 0 LL is exactly half-filled.

Low energy physics: project to 0LL

With interactions => map to usual half-filled LL

# Comments

*Implication: Study p/h symmetric half-filled LL level by studying correlated surface states of such 3d fermion topological insulators.*



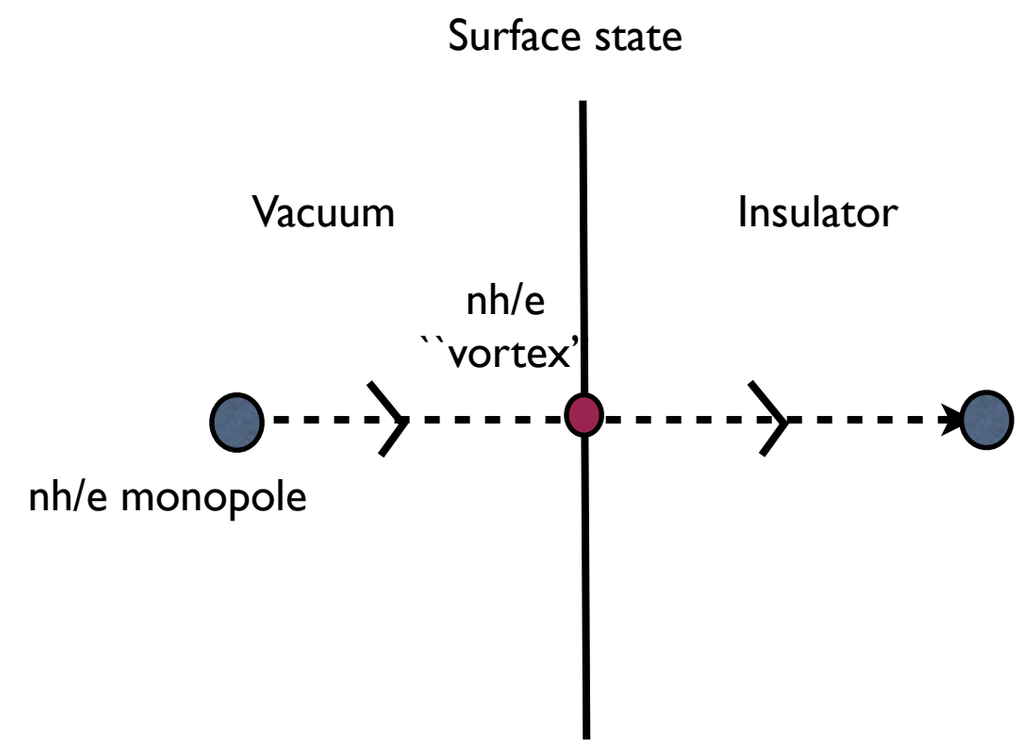

Exploit understanding of relatively trivial bulk TI to learn about non-trivial correlated surface state.

# Surface magnetic field and bulk magnetic monopoles

Build up magnetic field B at surface by moving magnetic monopoles from vacuum to inside of TI.

Moving strength-n monopole => surface magnetic flux increases by 2πn.

Monopoles inside TI are non-trivial => monopole insertion leaves behind non-trivial surface excitation.



# Magnetic monopoles inside bulk TI

Elementary monopole (flux $2\pi$) also has electric charge $\pm\, e/2$
(related to magneto-electric response $\theta = \pi$, Qi, Hughes, Zhang 09).

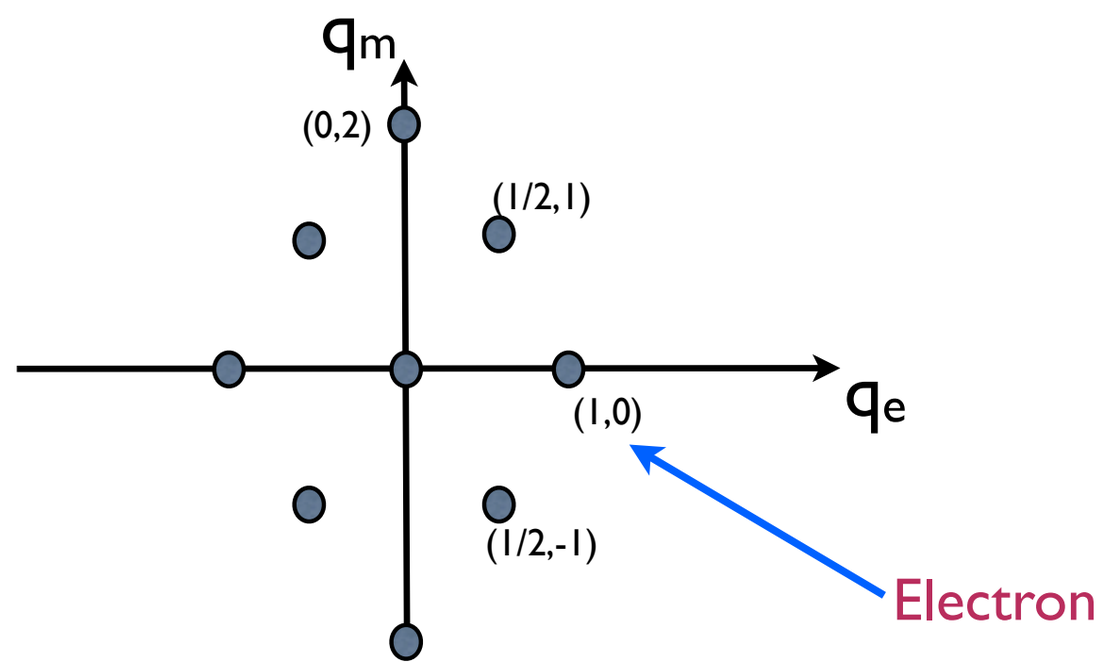
Useful to plot spectrum ($q_e$, $q_m$) of allowed combinations of electric and magnetic charges
(``charge-monopole" spectrum).

Examples:

Electron has ($q_e = 1$, $q_m = 0$)

Elementary monopoles ($q_e = \pm\, 1/2$, $q_m = 1$)

# Bulk charge-monopole spectrum

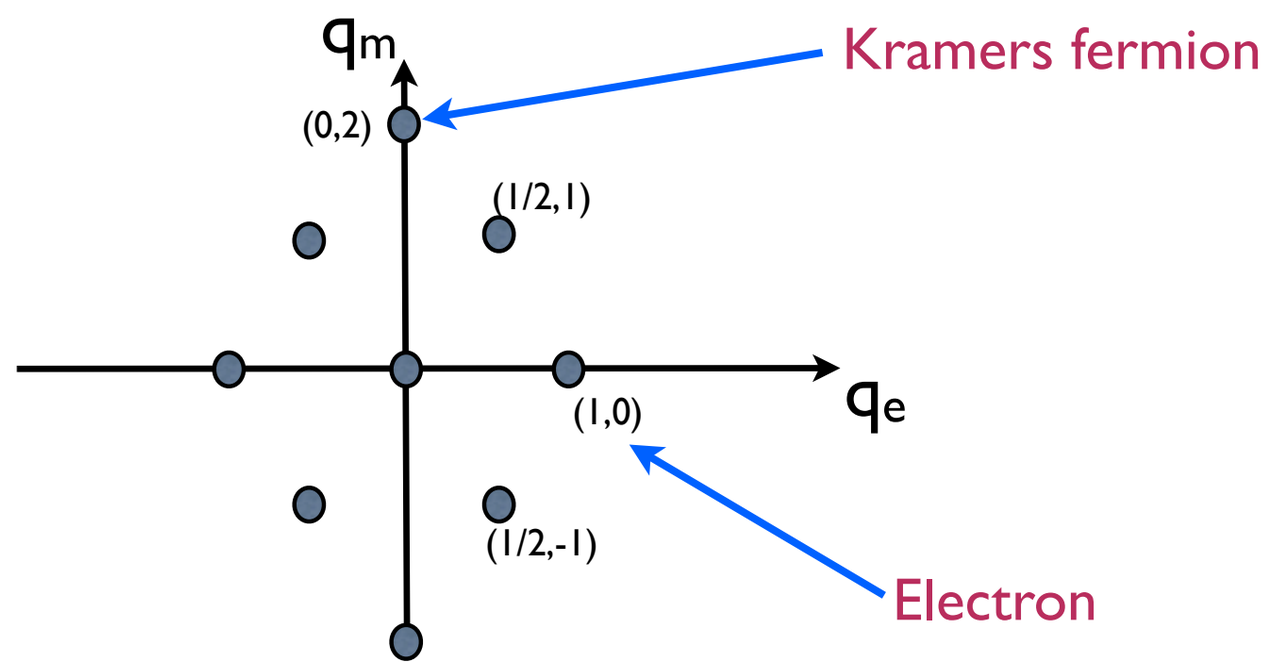



1. (1/2, 1) and (-1/2, 1) particles are both bosons which are interchanged under C.

2. Their bound state (0,2) is a Kramers fermion (Wang, Potter, TS, 13; Metlitski, Kane, Fisher, 13) (simple calculation)

- identify with composite fermion

# Bulk charge-monopole spectrum




1. (1/2, 1) and (-1/2, 1) particles are both bosons which are interchanged under C.

2. Their bound state (0,2) is a Kramers fermion (Wang, Potter, TS, 13; Metlitski, Kane, Fisher, 13) (simple calculation)

- identify with composite fermion

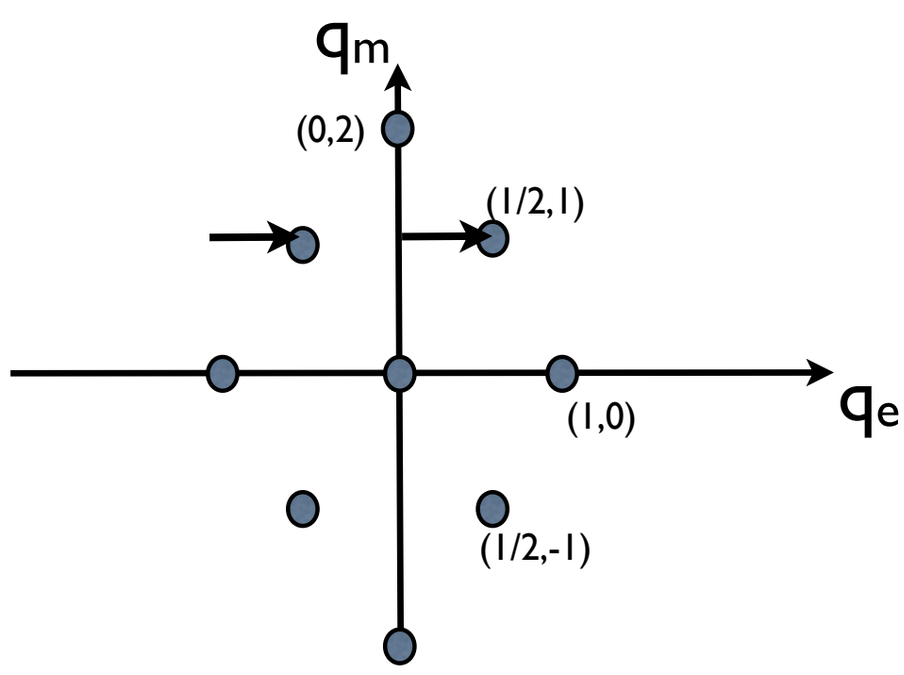
# A remarkable duality

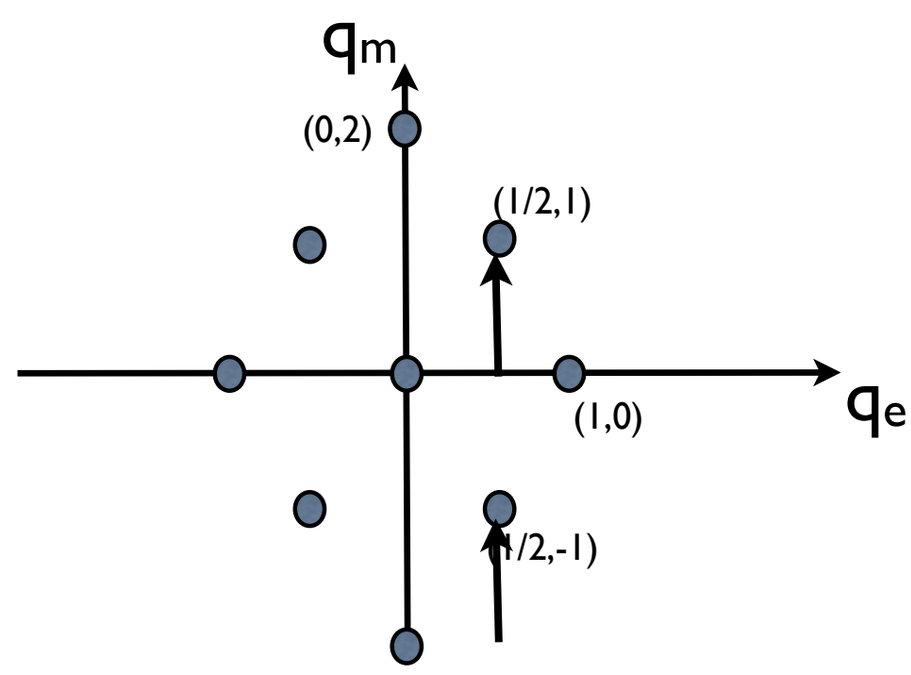
Can view charge-monopole lattice in two equivalent ``dual" ways.

1. Topological insulator of electric fermion (1,0)

or

2. Topological insulator of composite fermion (0,2)



# Consequences for surface: Justification of Dirac CF theory

Each time surface flux increases by $4\pi$, we add a neutral Kramers doublet fermion to surface - identify with composite fermion.

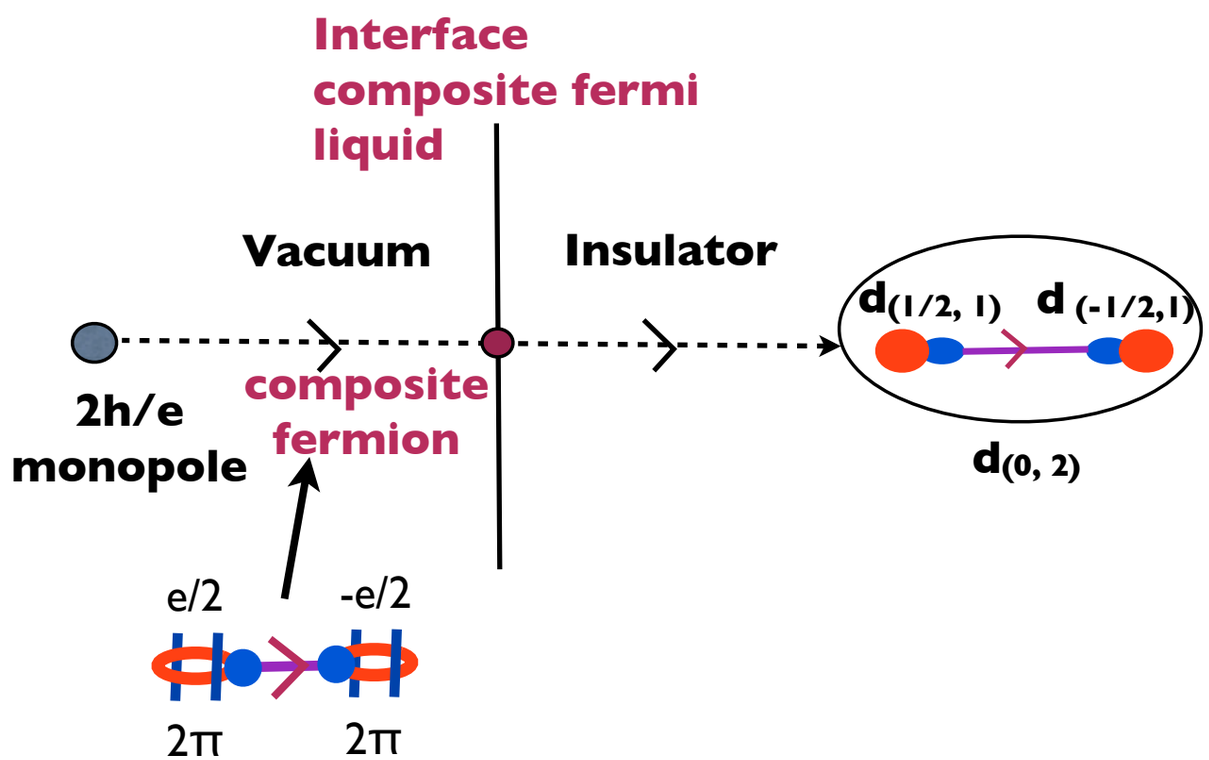
Surface magnetic field B => density $B/4\pi$ of neutral composite fermions.

Duality of Surface theory (fermionic version of charge-vortex duality):

`Electric' picture: Dirac cone of electric fermions in B-field (= Half-filled LL problem)

Dual `magnetic' theory: Dirac cone of neutral composite fermions at finite density + U(1) gauge field (= proposed Dirac composite fermion theory)

# Physical picture of the composite fermion: further justification




CFL as a surface state of the 3d fermion SPT:

Boundary composite fermion ~ bulk (0,2) monopole = bound state of two dyons which are bulk avatars of the two ends of the dipole forming the composite fermion.

# Composite fermi liquids as vortex metals

HLR/Jain composite fermion: Charge - flux composites

Particle-hole symmetric composite fermion: Neutral vortex

Describe CFL as a vortex liquid metal formed by neutral fermionic vortices.

Vortex metal description:

- Simple understanding of transport
(similar to other 2d quantum vortex metals, eg, in Galitski, Refael, Fisher, TS, 06)

- Extensions to CFLs away from $\nu = 1/2$

# Transport in the CFL

1. Longitudinal electrical conductivity $\propto$ composite fermion resistivity (natural from vortex liquid point of view)

   Hall conductivity $= \frac{e^2}{2h}$ (exactly)

2. Longitudinal thermal conductivity = composite fermion thermal conductivity

   Wiedemann-Franz violation (Wang, TS, 15)

$$\frac{\kappa_{xx}}{L_0 T \sigma_{xx}} = \left(\frac{\rho_{xy}}{\rho_{xx}}\right)^2 > 10^3$$

   $L_0$: free electron Lorenz number

   (Also actually in HLR)

3. Thermoelectric transport:

   Vortex metal: Nernst effect from mobile vortices unlike HLR (Potter et al, 15)

# Composite Fermi liquids in LLL at generic ν: Quantum vortex liquids with Fermi surface Berry phases

General $\nu = 1/(2q)$ but with LLL restriction:

Attach 2q vortices to electron.

LLL => neutral fermionic vortex distinct from Jain/HLR composite fermion (many people in late 1990s)

Effective theory must have no Chern-Simons term for internal gauge field to ensure this.

Trade Chern-Simons term for Fermi surface Berry phase $\phi_B = -2\pi\nu$

Effective Lagrangian

$$\mathcal{L} = \mathcal{L}_{\phi_B}[\psi_v, a_\mu] - \frac{1}{4q\pi}\epsilon_{\mu\nu\lambda} a_\mu \partial_\nu A_\lambda + \frac{1}{8q\pi}\epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda .$$

Unified viewpoint on CFLs of fermions and of bosons at $\nu = 1$ (Read 98; Alicea et al 05)

# Comments/summary

1. Old issue of p/h symmetry in half-filled Landau level: simple, elegant answer

General viewpoint: Regard LLL composite fermi liquid as a quantum metal of neutral fermionic vortices.

2. Surprising, powerful connection to correlated 3d TI surfaces

3. Many other related results

- clarification of many aspects of correlated surfaces of 3d TIs

- particle-vortex duality for 2+1-d massless Dirac fermions

- classification of time reversal invariant 3d spin liquids with emergent gapless photon

.....