Half-filled Landau level and topological insulator surfaces

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C. Wang and TS, to appear

Related work:

Many other important contributions: (Experiments) Shayegan group; Barkeshli, Mulligan, Fisher 15; Geraedt, Zaletel, Mong, et al, 15; Murthy, Shankar, 15; Mross Alicea, Motrunich 15; Potter, Serbyn, Vishwanath, 15.
The half-filled Landau level

2d electron gas in a strong magnetic field in the "quantum Hall regime" at filling factor \( \nu = 1/2 \)

Experiment: Metal with \( \rho_{xx} \neq 0, \rho_{xy} \neq 0 \), but \( \rho_{xx} \ll \rho_{xy} \).

Theory (Halperin, Lee, Read’93): "Composite Fermi Liquid"

At \( \nu = 1/2 \), Jain’s composite fermions move in zero effective magnetic field and can form a Fermi surface.
Composite fermi liquid theory
(Halperin, Lee, Read (HLR))

Effective theory:

\[
\mathcal{L} = \bar{\psi}_C F \left( i \partial_t - a_0 - i A_{0}^{ext} + \frac{(\vec{\nabla} - i(\vec{a} + \vec{A}^{ext}))^2}{2m} \right) \psi_C F + \frac{1}{4\pi} a_\mu \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda \tag{1}
\]

Many aspects of theory confirmed in experiment (eg, existence of Fermi surface)

Jain sequence: \( \nu = n/(2n+1) \)

Understand as filling composite fermion Landau levels (Jain 89)

HLR: energy gap of Jain states = spacing of effective Landau levels of composite fermions
Unfinished business in theory

1. Theory should be defineable within the Lowest Landau Level (LLL) but HLR is not in the LLL.

Many refinements in the late 90s (Shankar, Murthy; Read; Halperin, Stern, Simon, van Oppen; D.-H. Lee, Pasquier, Haldane, .......) but dust never settled.

2. Particle-hole symmetry

A symmetry of the LLL Hamiltonian (with eg, 2-body interactions) but not manifest in HLR.

Issue identified in the 90s (Grotov, Gan, Lee, Kivelson, 96; Lee 98) but no resolution.
Particle-hole symmetry in LLL

At $\nu = 1/2$, regard LLL as either "half-empty or half-full":

Start from empty level, fill half the LLL

or start from filled LL and remove half the electrons
Particle-hole symmetry: formal implementation

Electron operator \( \psi(x, y) \approx \sum_m \phi_m(x, y)c_m \) after restriction to LLL.
(\( \phi_m(x, y) \): various single particle wave functions in LLL).

Particle-hole: **Antiunitary symmetry** \( C \)
\[
C\psi C^{-1} = \psi^\dagger = \sum_m \phi_m^*(x, y)c_m^\dagger
\]

Symmetry of 1/2-LLL with, eg, 2-body interaction is (at least) \( U(1) \times C \)
Is the Composite Fermion a Dirac Particle? (Son, 2015)

Composite fermion $\psi_v$ forms a single Dirac cone tuned away from neutrality:

Anti-unitary p/h (C) acts in same way as time reversal usually does on Dirac fermion:

$$C \psi_v C^{-1} = i \sigma_y \psi_v \Rightarrow C^2 \psi_v C^{-2} = -\psi_v$$

Composite fermion is Kramers doublet under $C$. 
Particle-hole symmetric CFL: a proposal (cont’d)  
(Son, 2015)

Effective Lagrangian

\[
\mathcal{L} = i \bar{\psi}_v \left( \partial^\mu + iA^\mu \right) \psi_v - \mu_v \bar{\psi}_v \gamma_0 \psi_v + \frac{1}{4\pi} \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda + \mathcal{L}_{bg}[A_\mu]
\]

Low energy theory: focus on states near Fermi surface.

Meaning of Dirac?

As CF goes around FS, pick up \( \pi \) Berry phase.
Effective internal magnetic field $B^* = \vec{\nabla} \times \vec{a} = B - 4\pi \rho$.

Composite fermion density $n_v = \frac{B}{4\pi}$

$B^*$ same as in HLR but composite fermion density $n_{CF}$ is different.
In HLR $n_{CF} = \rho$ (at any filling)

At $\nu = 1/2$, $n_v = n_{CF}$ but they are different away from it.
This slight difference is crucial!
Plan of talk

A. Understanding the p/h symmetric composite fermi liquid

1. Connection to surface of 3d topological insulators - field theoretic justification
   (C. Wang, TS, arXiv:1505.05141; M. Metlitski, A. Vishwanath, 1505.05142.)

2. Simple physical picture of the particle-hole symmetric composite fermion
   (C. Wang, TS, arXiv:1507.08290)

3. Numerical calculations
   (Scott D. Geraedts, Michael P. Zaletel, Roger S. K. Mong, Max A. Metlitski, Ashvin Vishwanath, Olexei I. Motrunich, arXiv: 1508.04140)

B. Composite Fermi liquids in LLL at generic ν:
Quantum vortex liquids with Fermi surface Berry phases
(Wang, TS, to appear)
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B. Composite Fermi liquids in LLL at generic ν:
   Quantum vortex liquids with Fermi surface Berry phases
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Particle-hole symmetric composite fermions: a physical picture (Wang, TS, 2015)
Old physical picture of Composite Fermion (CF) in LLL

Neutral dipolar fermions

N. Read, 1994; many subsequent papers in late 90s
(Shankar; Murthy; Haperin, Stern; D.-H. Lee;
Pasquier-Haldane,.....)

LLL wave function (Rezayi-Read 94)

\[
\psi(z_1, \ldots, z_N) = P_{LLL} \det(e^{i\vec{k}_i \cdot \vec{r}_j}) \prod_{i<j} (z_i - z_j)^2
\]

CF: electron bound to 4\(\pi\)-vortex: Vortex has depletion of charge -e
\(\Rightarrow\) CF is neutral.

CF has dipole moment perpendicular to it’s momentum.

Heuristic wave function argument: Plane wave factors push vortex away from electron

However this picture misses some physics and further is not p/h symmetric (charge changes sign but vorticity does not).
New picture of composite fermion in LLL

Fermion wavefunctions in LLL

\[ \psi(z_1, z_2, \ldots, z_N) = \prod_{i<j}(z_i - z_j)f(z_1, \ldots, z_N) \]

\[ f(z_1, \ldots, z_N): \text{a symmetric polynomial.} \]

\[ \Rightarrow \text{one vortex is exactly on electron due to Pauli.} \]

Only second vortex is displaced from first in direction perpendicular to CF momentum.

Each vortex has charge \(-e/2\) \(\Rightarrow\) single vortex exactly on electron has charge \(+e/2\) and the displaced vortex has charge \(-e/2\).
Internal structure of composite fermion(*) in LLL

Two ends have mutual statistics of $\pi$

Bound state (of course) is a fermion.

Assume ends are well separated compared to vortex size.
New picture of composite fermion in LLL (cont’d)

Anti-unitary $C$ interchanges relative coordinates of the two ends.

Solve QM of relative motion $\Rightarrow$

ground state ``spin''-1/2 doublet which is Kramers under $C$. 

Phase of $\pi$
New picture of composite fermion in LLL (cont’d)

Non-zero CF momentum => non-zero dipole moment

=> "spin" of composite fermion polarized perpendicular to it’s momentum

(spìn-momentum locking expected of Dirac fermion).

Composite fermion goes around FS => momentum rotates by $2\pi$

=> spin rotates by $2\pi$ => Berry phase of $\pi$ as expected for a Dirac fermion.
1/2-filled LL and correlated TI surfaces

Derivation of and more insight into p/h symmetric composite fermi liquid theory

(C. Wang, TS, 15; M. Metlitski, A. Vishwanath, 15)
Consider (initially free) fermions with \textit{``weird''} action of time-reversal (denote $C$):

$$C \rho C^{-1} = -\rho$$

$\rho = \text{conserved \textit{``charge''} density.}$

Full symmetry = $U(1) \times C$

(called class AIII in Topological Insulator/Superconductor literature*)

Eg: Triplet time reversal-invariant superconductor where physical $S_z$ is conserved and plays the role of such a $\rho$.

*distinct symmetry from usual spin-orbit coupled insulators which have $U(1) \rtimes C$ symmetry (i.e, $\rho$ is usually even under time reversal).
Surface: Single massless Dirac fermion

$C$ symmetry guarantees that surface Dirac cone is exactly at neutrality.

$$\mathcal{L} = \bar{\psi} \left( -i \theta + \mathcal{A} \right) \psi + \ldots$$

2-component fermion

external probe  gauge field
$p/h$ symmetric LL as a surface of 3d fermion SPT (cont’d)

$\rho$ is odd under C $\Rightarrow$ `electric current’ is even.

External E-fields are odd but external B-fields are even.

$\Rightarrow$ Can perturb surface Dirac cone with external B-field.

C-symmetry: $\nu = 0$ LL is exactly half-filled.

Low energy physics: project to 0LL

With interactions $\Rightarrow$ map to usual half-filled LL
Implication: Study p/h symmetric half-filled LL level by studying correlated surface states of such 3d fermion topological insulators.

Exploit understanding of relatively trivial bulk TI to learn about non-trivial correlated surface state.
Surface magnetic field and bulk magnetic monopoles

Build up magnetic field $B$ at surface by moving magnetic monopoles from vacuum to inside of TI.

Moving strength-$n$ monopole $\Rightarrow$ surface magnetic flux increases by $2\pi n$.

Monopoles inside TI are non-trivial $\Rightarrow$ monopole insertion leaves behind non-trivial surface excitation.
Magnetic monopoles inside bulk Tl

Elementary monopole (flux $2\pi$) also has electric charge $\pm e/2$ (related to magneto-electric response $\theta = \pi$, Qi, Hughes, Zhang 09).

Useful to plot spectrum $(q_e, q_m)$ of allowed combinations of electric and magnetic charges (``charge-monopole” spectrum).

Examples:

Electron has $(q_e = 1, q_m = 0)$

Elementary monopoles $(q_e = \pm 1/2, q_m = 1)$
1. (1/2, 1) and (-1/2, 1) particles are both bosons which are interchanged under C.

2. Their bound state (0,2) is a Kramers fermion (Wang, Potter, TS, 13; Metlitski, Kane, Fisher, 13) (simple calculation)

   - identify with composite fermion
1. $(1/2, 1)$ and $(-1/2, 1)$ particles are both bosons which are interchanged under $C$.

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A remarkable duality

Can view charge-monopole lattice in two equivalent "dual" ways.

1. Topological insulator of electric fermion (1,0)

or

2. Topological insulator of composite fermion (0,2)
Consequences for surface: Justification of Dirac CF theory

Each time surface flux increases by $4\pi$, we add a neutral Kramers doublet fermion to surface - identify with composite fermion.

Surface magnetic field $B \Rightarrow$ density $B/4\pi$ of neutral composite fermions.

Duality of Surface theory (fermionic version of charge-vortex duality):

`Electric’ picture: Dirac cone of electric fermions in B-field (= Half-filled LL problem)

Dual `magnetic’ theory: Dirac cone of neutral composite fermions at finite density + U(1) gauge field (= proposed Dirac composite fermion theory)
Physical picture of the composite fermion: further justification

CFL as a surface state of the 3d fermion SPT:

Boundary composite fermion ~ bulk (0,2) monopole = bound state of two dyons which are bulk avatars of the two ends of the dipole forming the composite fermion.
Composite fermi liquids as vortex metals

HLR/Jain composite fermion: Charge - flux composites

Particle-hole symmetric composite fermion: Neutral vortex

Describe CFL as a vortex liquid metal formed by neutral fermionic vortices.

Vortex metal description:

- Simple understanding of transport
  (similar to other 2d quantum vortex metals, eg, in Galitski, Refael, Fisher, TS, 06)

- Extensions to CFLs away from $\nu = 1/2$
Transport in the CFL

1. Longitudinal electrical conductivity \( \propto \) composite fermion resistivity (natural from vortex liquid point of view)
   
   Hall conductivity = \( \frac{e^2}{2\hbar} \) (exactly)

2. Longitudinal thermal conductivity = composite fermion thermal conductivity
   
   Wiedemann-Franz violation (Wang, TS, 15)
   
   \[
   \frac{\kappa_{xx}}{L_0 T \sigma_{xx}} = \left( \frac{\rho_{xy}}{\rho_{xx}} \right)^2 > 10^3
   \]

   \( L_0 \): free electron Lorenz number

   (Also actually in HLR)

3. Thermoelectric transport:
   
   Vortex metal: Nernst effect from mobile vortices unlike HLR (Potter et al, 15)
Composite Fermi liquids in LLL at generic $\nu$:
Quantum vortex liquids with Fermi surface Berry phases

General $\nu = 1/(2q)$ but with LLL restriction:

Attach 2q vortices to electron.

LLL => neutral fermionic vortex distinct from Jain/HLR composite fermion (many people in late 1990s)

Effective theory must have no Chern-Simons term for internal gauge field to ensure this.

Trade Chern-Simons term for Fermi surface Berry phase $\phi_B = -2\pi\nu$

Effective Lagrangian

$$\mathcal{L} = \mathcal{L}_{\phi_B}[\psi, a_\mu] - \frac{1}{4q\pi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu A_\lambda + \frac{1}{8q\pi} \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda.$$

Unified viewpoint on CFLs of fermions and of bosons at $\nu = 1$ (Read 98; Alicea et al 05)
Comments/summary

1. Old issue of p/h symmetry in half-filled Landau level: simple, elegant answer

General viewpoint: Regard LLL composite fermi liquid as a quantum metal of neutral fermionic vortices.

2. Surprising, powerful connection to correlated 3d TI surfaces

3. Many other related results
   - clarification of many aspects of correlated surfaces of 3d TIs
   - particle-vortex duality for 2+1-d massless Dirac fermions
   - classification of time reversal invariant 3d spin liquids with emergent gapless photon

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