Duality in condensed matter physics

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Duality in condensed matter physics/field theory

Two equivalent descriptions of the same theory but from different points of view.

Very powerful *non-perturbative* insights into strongly interacting theories.

Origins: classical statistical mechanics of 2d Ising model (Kramers, Wannier 1941)

Many profound developments since in both (quantum) condensed matter and in quantum field theory.

Classic example I: the 2d Ising model

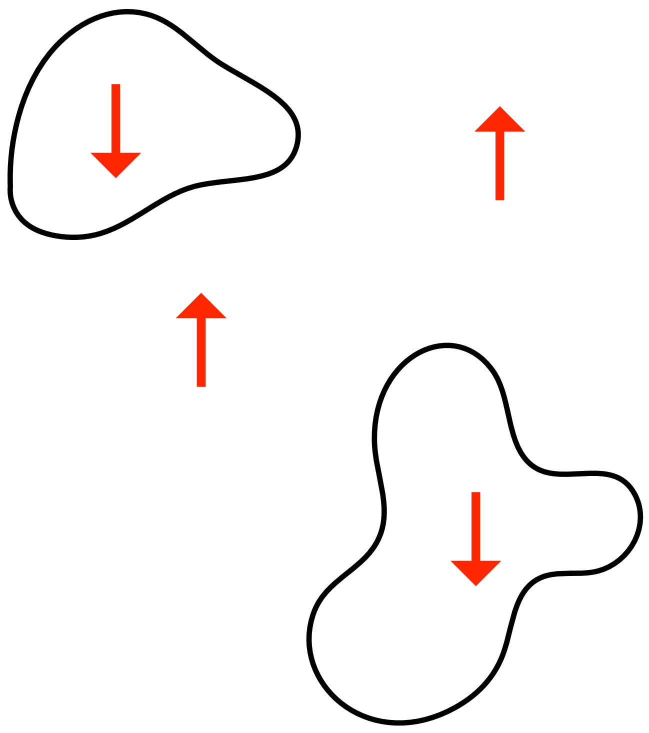
2d Ising model

Low T phase $\Longleftrightarrow$ High T phase

Physics: Describe either in terms of Ising spins or in terms of domain walls (= topological defects of Ising ferromagnet).

Low-T phase: Spins ordered but domain walls costly

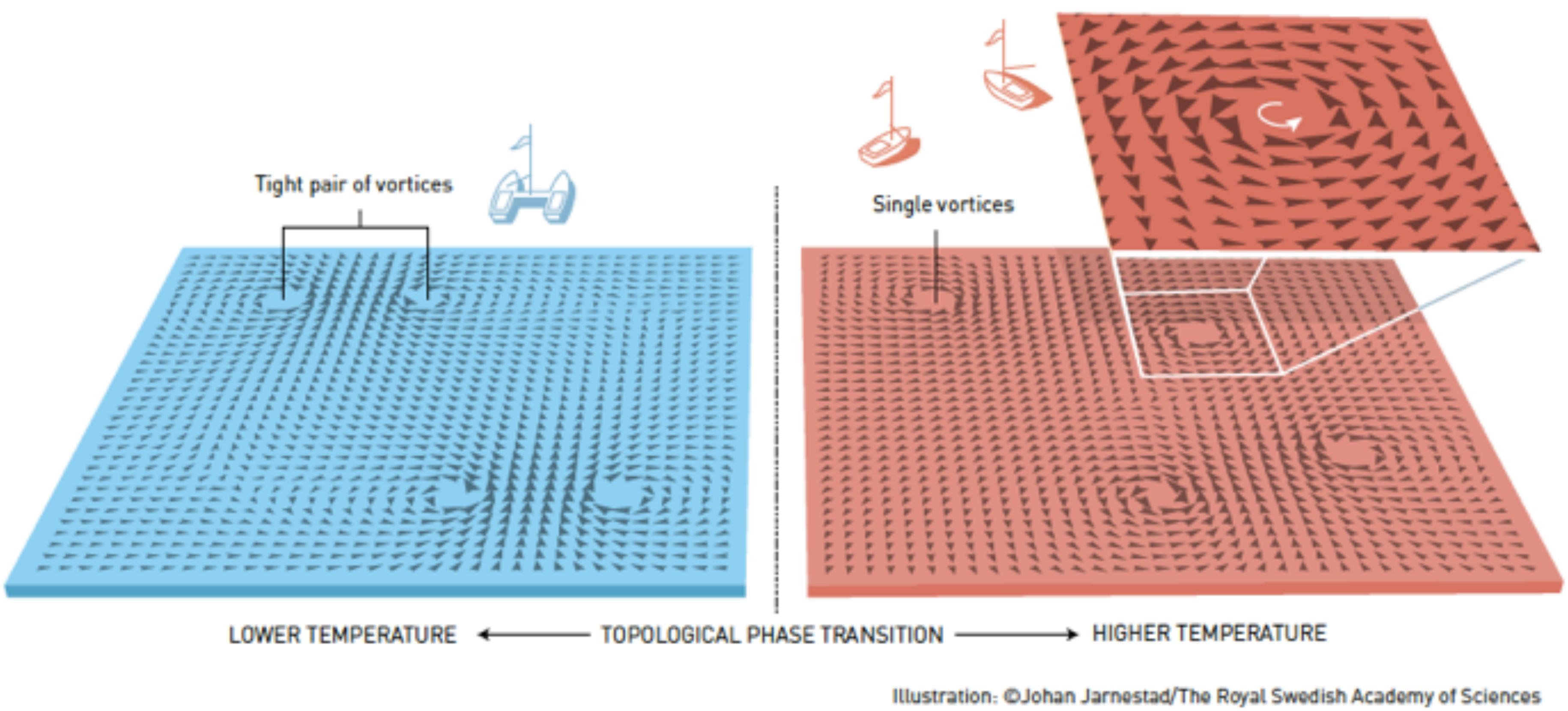
High-T phase: Spins disordered; domain walls have proliferated.



Classic example II: Berezinsky-Kosterlitz-Thouless theory of 2d XY systems

Physics Nobel 2016
to K and T

Phase transition driven by vortices in XY order parameter



Duality of 2d XY model:
Reformulate as gas of vortex/antivortices interacting through 2d Coulomb potential.

In the context of ***quantum*** many body physics in 1+1 - D, these dualities are tremendously powerful and are part of the standard theoretical toolbox.

Is there a generalization to 2d quantum matter?

An old answer: yes (for strongly correlated bosons in 2d).

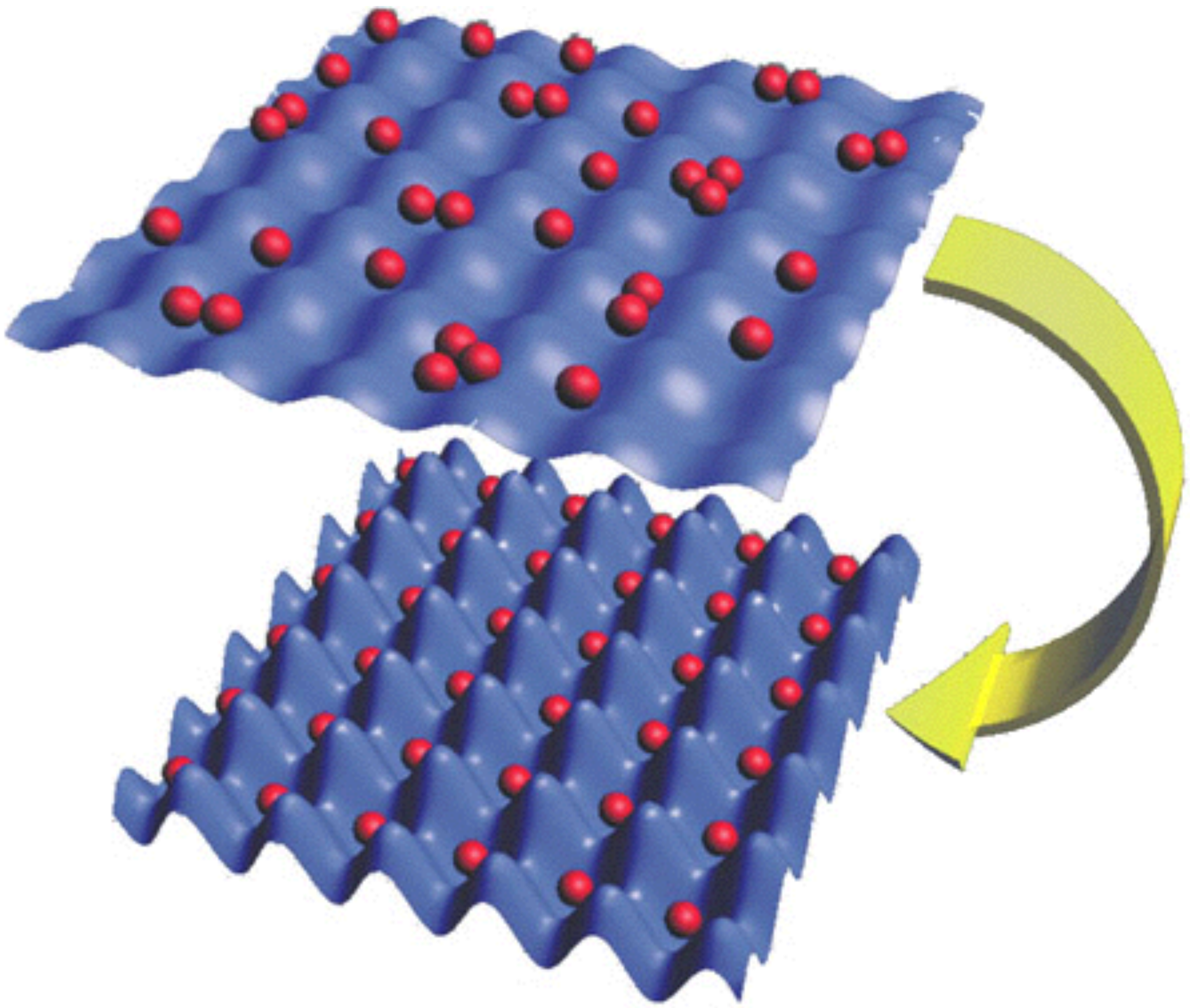
``Charge-vortex duality" for bosons

Dasgupta, Halperin 1981;
Peskin 1978;
Fisher, Lee, 1989

(Also: Wegner, 1971 for the quantum Ising model in 2+1-D)

Strongly correlated boson systems in 2d

Paradigmatic model: Boson Hubbard model
Specialize to integer filling/site.



$$H = -t \sum_{<ij>} \phi_i^\dagger \phi_j + h.c + U \sum_i n_i(n_i - 1)$$

t >> U: superfluid
U >> t: Boson Mott insulator

Superfluid-insulator transition: Continuum effective theory

$$\mathcal{L} = |\partial_\mu \phi|^2 + r|\phi|^2 + u|\phi|^4$$

Global $U(1)$ symmetry: $\phi(x) \to e^{i\alpha}\phi(x)$.

Two phases:
$\langle \phi \rangle \neq 0$: superfluid order

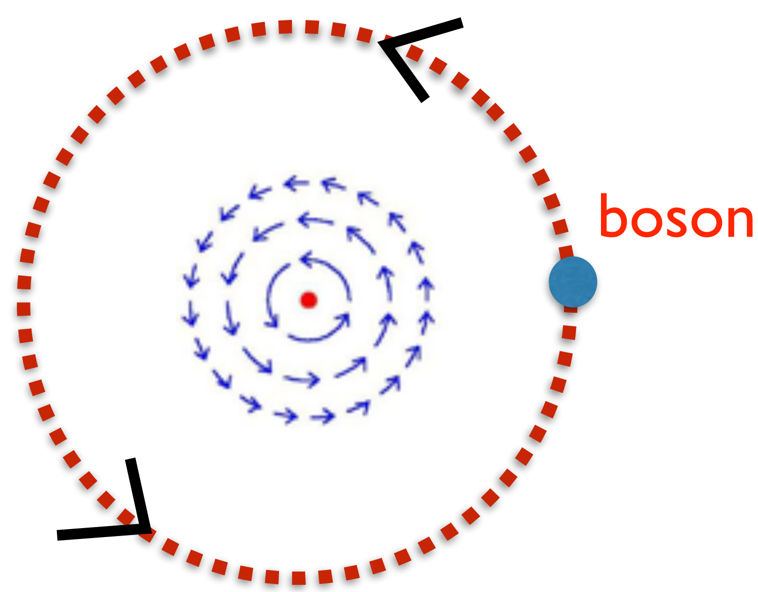
$\langle \phi \rangle = 0$ Mott insulator

Quantum critical point: extremely well understood, see, eg, Sachdev book

Charge-vortex duality for bosons

Alternate description of phases/phase transitions in terms of vortices of XY order parameter.

In 2d: vortices are point defects.



2π phase

Charge-vortex duality for bosons

A vortex sees

(i) particle density as effective magnetic field $b = 2\pi\rho$

(ii) particle current as effective electric field $\vec{e} = 2\pi(\hat{z} \times \vec{j})$

Fluctuating particle density/current => vortices see fluctuating effective ``electromagnetic field" (represent in terms of effective scalar/vector potentials).

Dual vortex description: Vortices + U(1) gauge field.

Charge-vortex duality of bosons

$$\mathcal{L} = |\partial_\mu \phi|^2 + r|\phi|^2 + u|\phi|^4$$

Physical boson

$$\mathcal{L}_v = |(\partial_\mu - i a_\mu)\phi_v|^2 + r_d|\phi_v|^2 + u_d|\phi_v|^4 + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2$$

Vortex field

Internal gauge field

Dual description of phases

Boson superfluid:

Charge condensed $\Longleftrightarrow$ Vortices costly (energy gap)

Boson Mott Insulator

Charge gapped $\Longleftrightarrow$ Vortices condensed

Suggests both field theories correctly describe the same second order superfluid-insulator phase transition.

Charge-vortex duality of bosons: Field theoretic version

$$\mathcal{L} = |\partial_\mu \phi|^2 + r|\phi|^2 + u|\phi|^4 \longleftrightarrow \mathcal{L}_v = |(\partial_\mu - i a_\mu)\phi_v|^2 + r_d|\phi_v|^2 + u_d|\phi_v|^4 + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2$$

Tune one parameter

Tune one parameter

IR Conformal Field theory (CFT)

Passes numerical checks (Nguyen, Sudbo, 99; Katanjie et al, 04)

Applications of duality of bosons

This basic charge-vortex duality of bosons underlies
our understanding of many novel correlated boson phenomena.

Examples:

1. Fractional quantum Hall heirarchy (D.-H. Lee, M. P. A. Fisher 1989)

2. Fractional charge (spin) in 2d boson insulators (quantum magnets)
(TS, Fisher 2000)

3. Non-Landau quantum criticality in boson/spin systems
(TS, Vishwanath, Balents, Fisher, Sachdev 2004)

Other applications: theory of SC-insulator transition in
thin films (Fisher 1992)

Later in the talk: revisit duality of bosons.

Is there a similar duality for fermions in 2d?

Yes! After > 35 years, in 2015, there is a generalization to fermions.

Wang, TS 2015; Metlitski, Vishwanath 2015
(Inspiration from Son 2015 ideas on composite fermions in quantum Hall effect)

Seiberg, TS, Wang, Witten 2016;
Karch, Tong 2016.

Charge-vortex duality for fermions

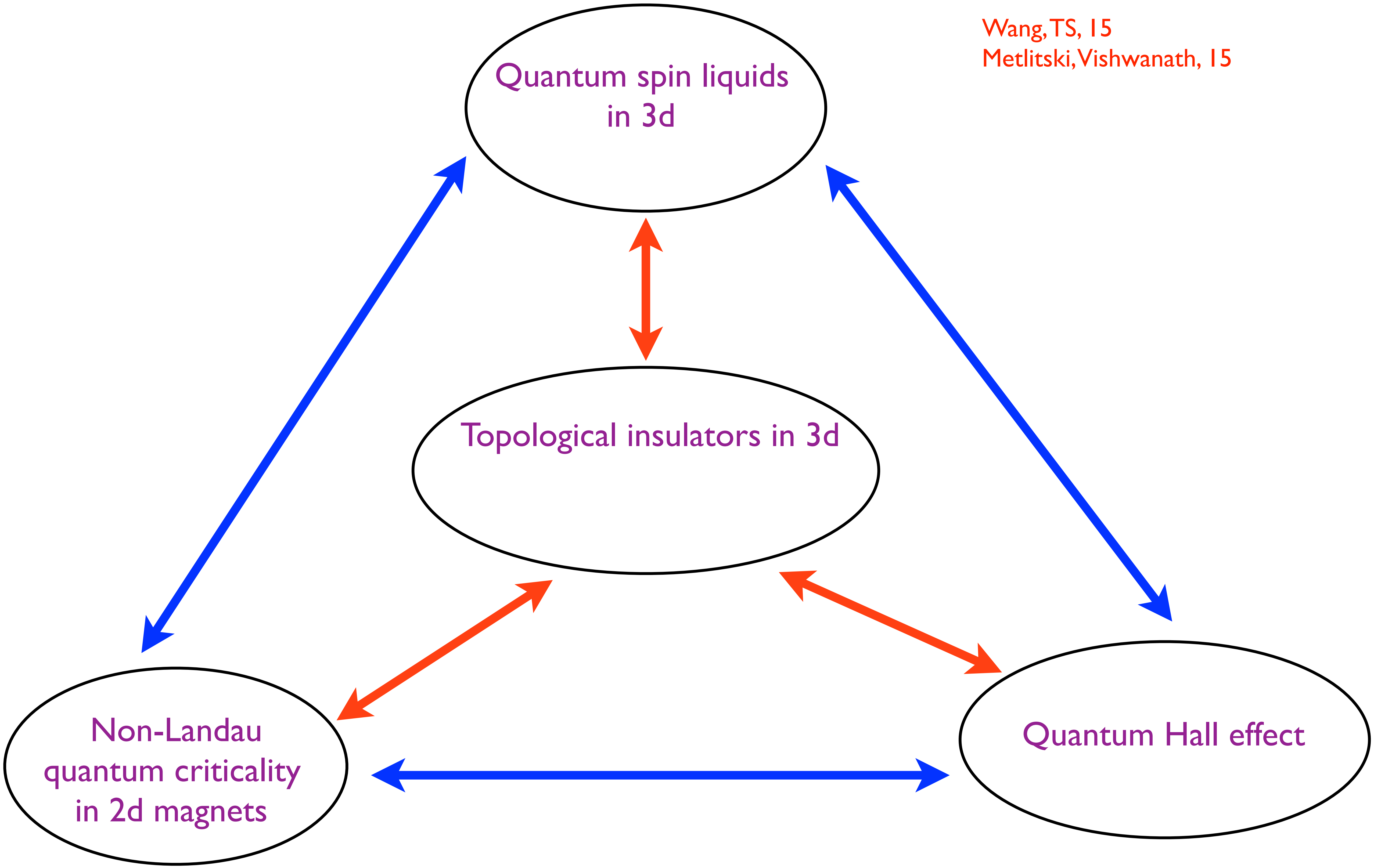
New non-perturbative window into many correlated fermion problems.

Many powerful applications expected - several already explored.

This duality is an outgrowth of deep and surprising connections between many different topics in correlated quantum matter.

Deep connections between many apparently different problems

Wang, TS, 15
Metlitski, Vishwanath, 15

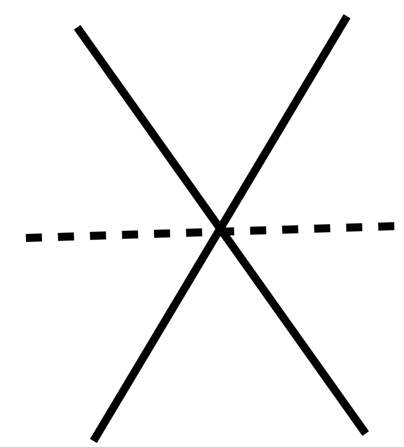


Charge-vortex duality for fermions

Wang, TS, 15
Metlitski, Vishwanath, 15

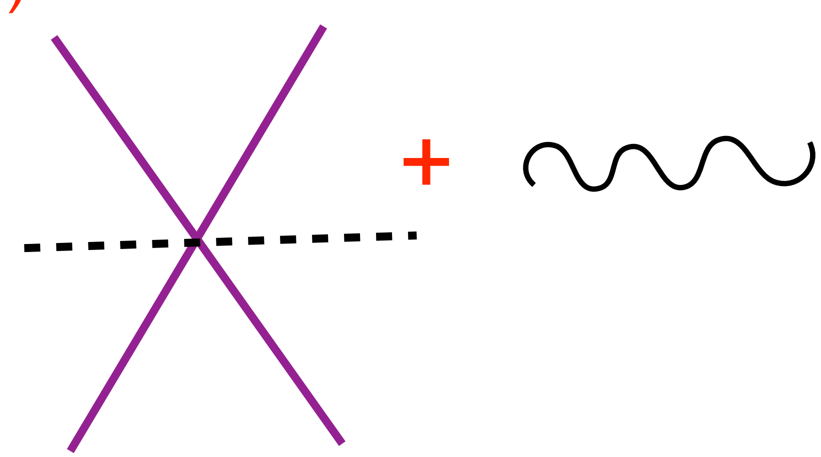
Theory A: Single massless Dirac fermion

$$\mathcal{L} = \bar{\psi}\left(-i\gamma^{\mu}\partial_{\mu}\right)\psi$$



Theory B: Dual massless Dirac fermion + U(1) gauge fields (*)

$$\mathcal{L}_v = \bar{\psi}_v\left(-i\gamma^{\mu}(\partial_{\mu} - ia_{\mu})\right)\psi_v + \frac{1}{2e^2}\left(\epsilon_{\mu\nu\lambda}\partial_{\nu}a_{\lambda}\right)^2$$



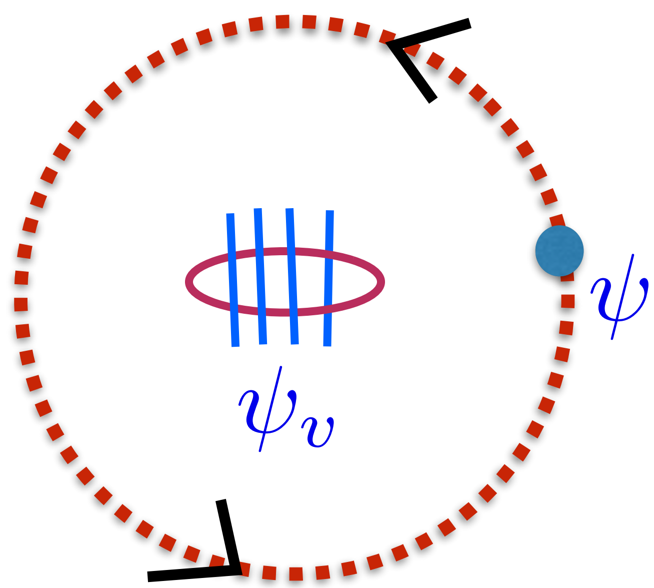
(*) For a more precise version see Seiberg, TS, Wang, Witten 16

Charge-vortex duality for fermions

$$\mathcal{L} = \bar{\psi}\,(-i\gamma^\mu \partial_\mu)\,\psi \longleftrightarrow \mathcal{L}_v = \bar{\psi}_v\,(-i\gamma^\mu(\partial_\mu - i a_\mu))\,\psi_v + \frac{1}{2e^2}\,(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2$$

Electron

Interpret: ψ_v is a 4π "vortex" in the electron.



4π phase

Charge-vortex duality for fermions

$$\mathcal{L} = \bar{\psi}\left(-i\gamma^\mu \partial_\mu\right)\psi \quad \longleftrightarrow \quad \mathcal{L}_v = \bar{\psi}_v\left(-i\gamma^\mu(\partial_\mu - i a_\mu)\right)\psi_v + \frac{1}{2e^2}\left(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda\right)^2$$

Electron

Interpret: ψ_v is a 4π "vortex" in the electron.

ψ_v sees

(i) particle density as effective magnetic field $b = 4\pi\rho$

(ii) particle current as effective electric field $\vec{e} = 4\pi\hat{z}\times\vec{j}$

Justifications

A consistency check:

Dual vortex theory has same operators and symmetries as the `charge' theory.

Derivations:

1.   Through understanding of bulk 3d topological insulator (for which this theory is a surface) (Wang, TS 15; Metlitski, Vishwanath 15).

2.   Construct both theories as systems of coupled 1d quantum wires (and use 1d dualities) (Mross, Alicea, Motrunich 16).

Comment

This fermion-fermion duality is central to many frontier issues in modern condensed matter physics.

It is crucially connected to other dualities of 2+1-D field theories.

(More in next 2 talks)

Rest of this talk: new perspectives on superfluid-insulator transition of bosons

Charge-vortex duality for bosons, revisited

Standard charge-vortex duality: Dual vortex field is bosonic

$$\mathcal{L} = |\partial_\mu \phi|^2 + r|\phi|^2 + u|\phi|^4 \longleftrightarrow \mathcal{L}_v = |(\partial_\mu - i a_\mu)\phi_v|^2 + r_d|\phi_v|^2 + u_d|\phi_v|^4 + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2$$

However in 2d, we can always transmute statistics by ``flux-attachment''.

Can we treat the vortices as fermions?

Alternate duality for the 2+1-D XY model ?

Fermionic vortex dual for bosons

$$|D_A \phi|^2 + |\phi|^4 \longleftrightarrow i\bar{\chi} D_a \chi + \frac{i}{2\pi} adA - \frac{i}{4\pi} AdA$$

Probe background gauge field

Wilson-Fisher (3D XY)

``Massless Dirac fermion + $U_{1/2}(1)$"
(1/2 level CS term for a comes from regularization with massive fermion)

Can check that local operators match.

SF-insulator transition $\longleftrightarrow$ Integer Quantum Hall transition + U(1) gauge field

Interpret: χ is a fermionized vortex (obtained through flux attachment)

Chen, Fisher, Wu, 1993; Barkeshli, McGreevy, 2012.

An old puzzle: time reversal

XY fixed point is time reversal symmetric.

$$|D_A\phi|^2 + |\phi|^4$$

Flux attachment is clearly problematic for time reversal.

$$i\bar{\chi}D_a\chi + \frac{i}{2\pi}adA - \frac{i}{4\pi}AdA$$

``Massless Dirac fermion + $U_{1/2}(1)$"

Apparently not time reversal symmetric.

Resolution (Seiberg, TS, Wang, Witten 16):

T-reversal manifest for Wilson Fisher theory.
Fermionic version: T-reversal takes fermion to its fermionic dual.

Check by an easy short calculation.

A deeper understanding: connection to EM duality in 3+1-D

Punchline:

All these 2+1-D dualities follow from standard electric-magnetic duality of U(1) gauge theory in 3+1-D.

Condensed matter context: Relevant 3+1-D systems are ``quantum spin liquids" where the gauge fields are emergent

Possible examples are in quantum spin ice materials.

Properties of U(1) quantum spin liquids

Particles:

1. Gapless photon

2. `Magnetic monopole' - the `M' particle

3. `Electric monopole' - the `E' particle

Emergence of photon ***necessarily*** accompanied by emergence of E and M particles.

Microscopic lattice models (many in last 10+ years): Motrunich, TS, 02; Hermele et al, 04, Moessner 03, Banerjee, Isakov, Damle, Kim 08

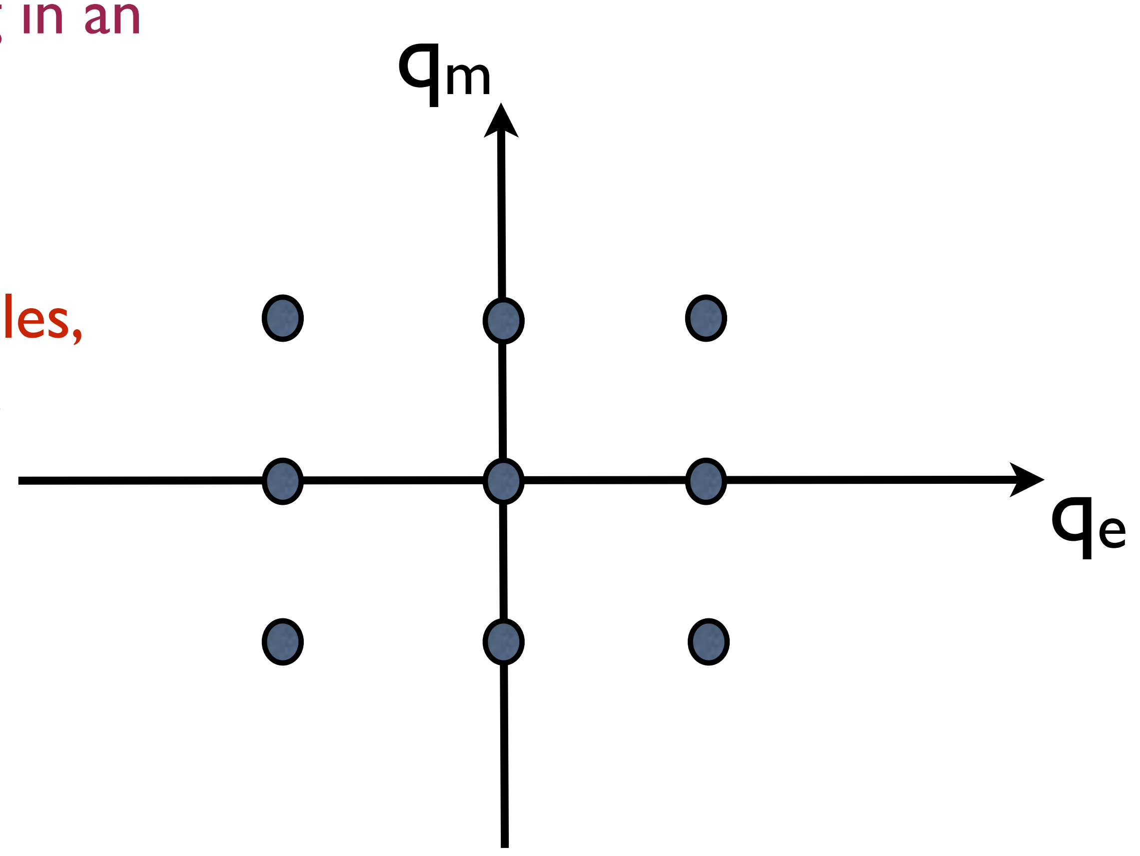
Experimental candidates: $Yb_2Ti_2O_7$, $Pr_2Zr_2O_7$, $Pr_2Sn_2O_7$, $Tb_2Ti_2O_7$..?

Time reversal symmetric U(1) quantum spin liquids

Here we are interested in U(1) spin liquids with time reversal symmetry (and with a boundary).

An example: Both elementary electric charge E, and elementary magnetic charge M are bosons, transforming in an obvious way under time reversal.

In general must consider full lattice of charges, monopoles, and their bound states as allowed by Dirac quantization.

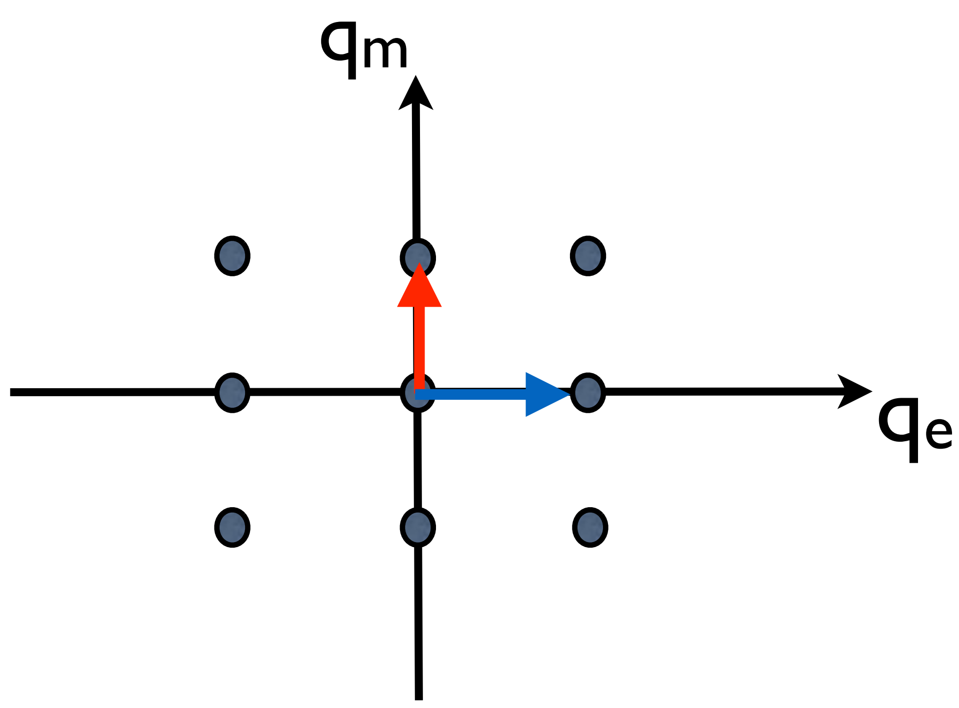


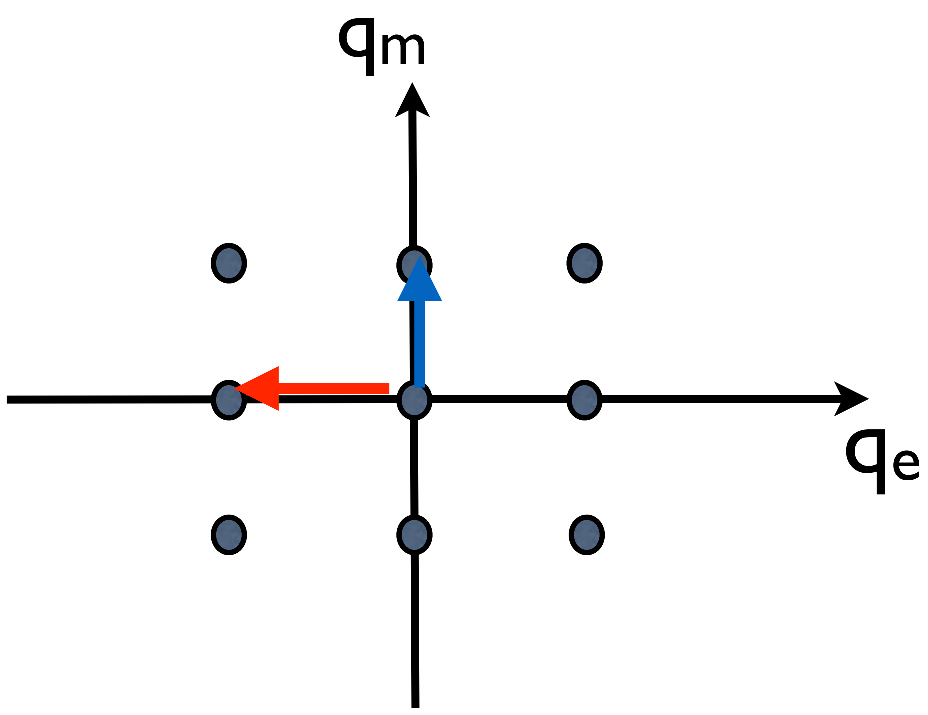
Many equivalent descriptions (Dualities)

In principle can use any basis to describe the lattice.

Pick a basis and couple the basic particles to the U(1) gauge field with appropriate coupling constant to reproduce charge-monopole lattice.

Caveat: T-reversal symmetry action may be non-trivial.





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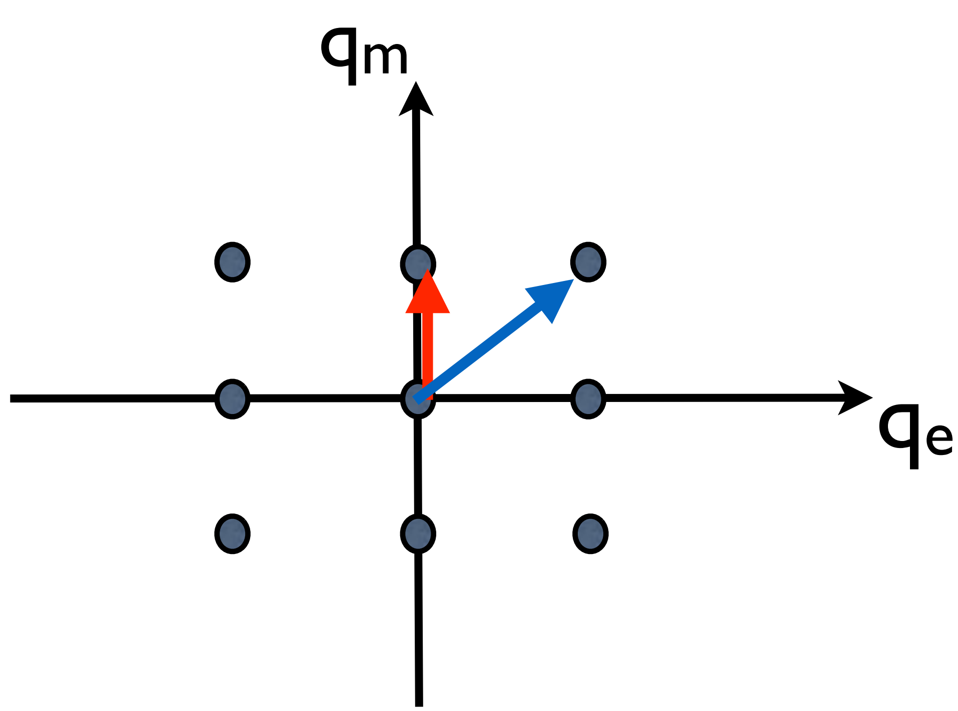
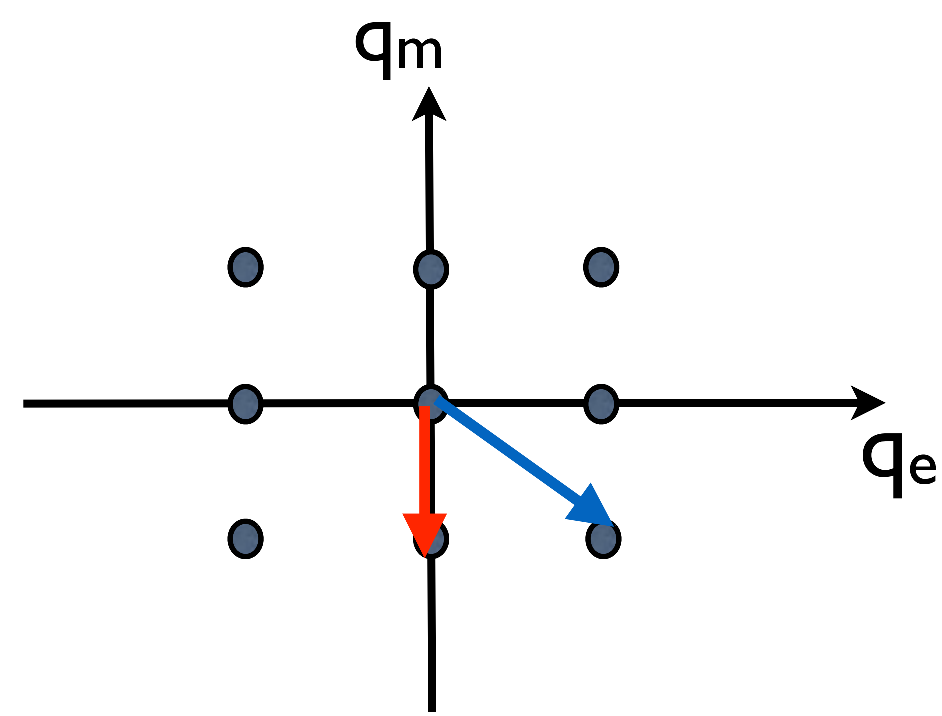
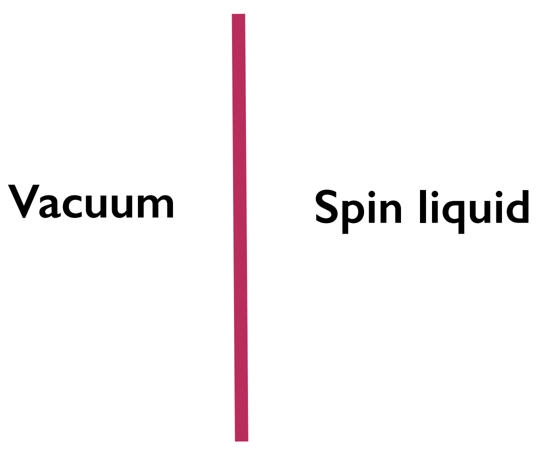




Boundary states



What happens if there is a surface to the vacuum?

Weak coupling limit of bulk => get 2+1-D surface theory with a ***background*** U(1) gauge field

Different choices of basis particles in bulk give different descriptions of the same surface

=> Theory of surface states admits many equivalent dual descriptions.

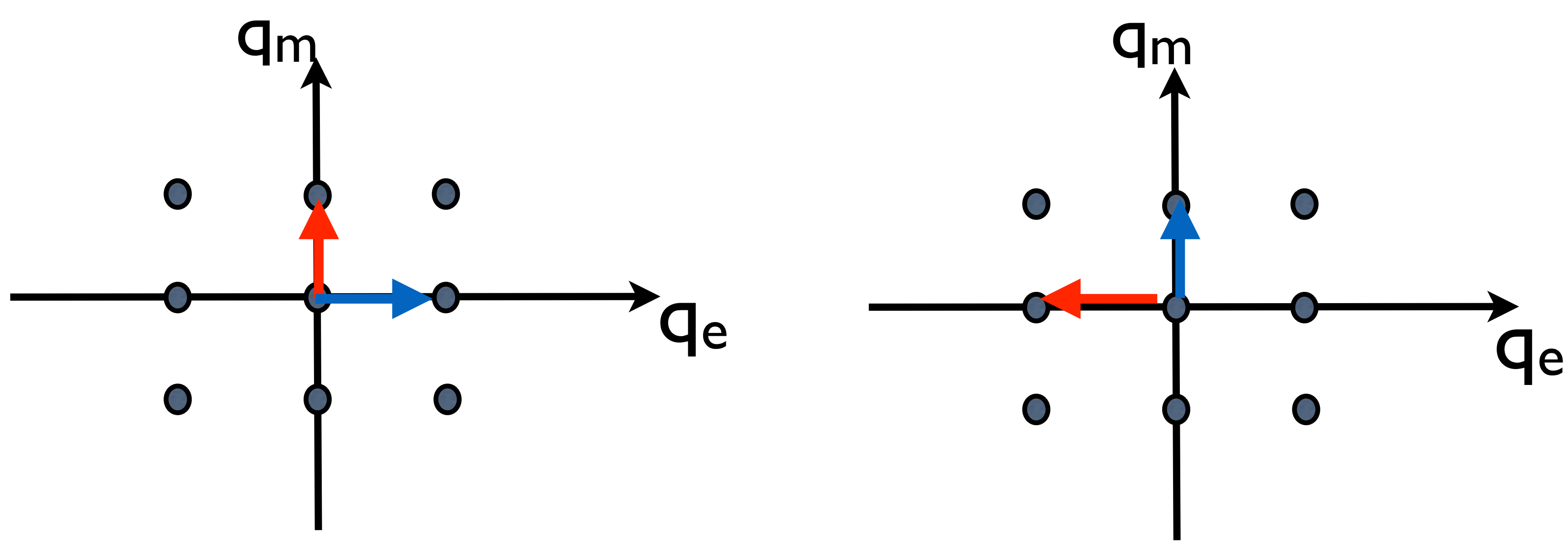
Recover many of the 2+1-D dualities of theories with a global U(1) symmetry.

Illustration: Bosonic charge-vortex duality

Take (1,0) and (0,1) particles to be bosons.

Tune boundary to Wilson-Fisher fixed point of (1,0) (``electric" description).

``Magnetic" description: Tune boundary to critical point of Abelian-Higgs model => precisely bosonic charge-vortex duality

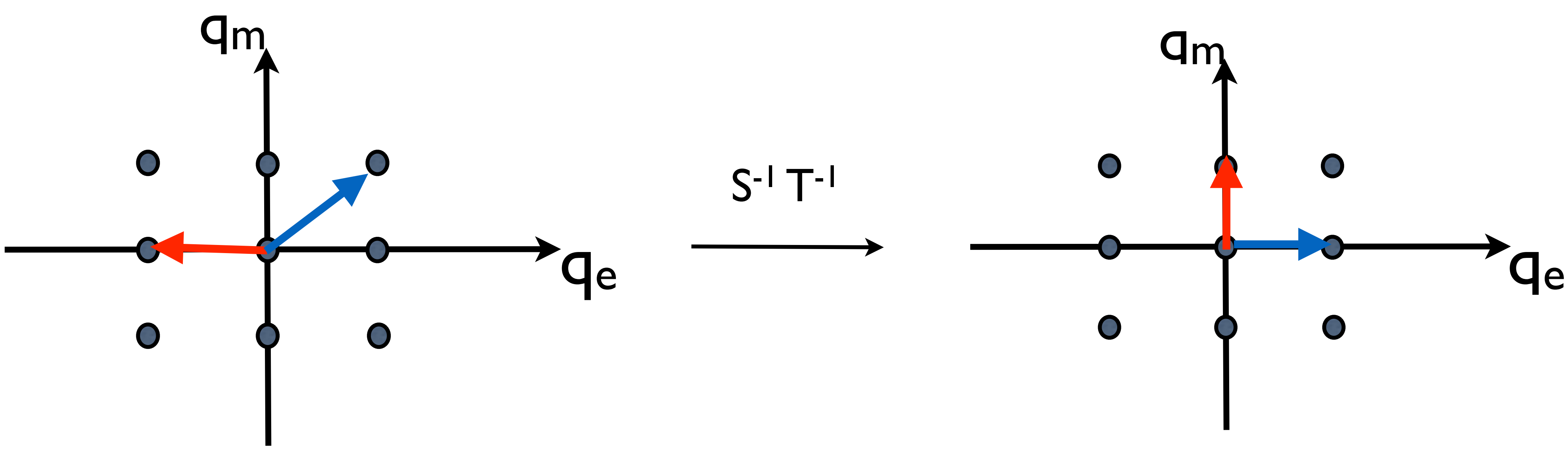


Boson-fermion duality

Bulk: With (1,0) and (0,1) particles being bosons, (1,1) and (1, -1) are fermions.

Change of basis: Use (1,1) fermion as `particle' and (-1, 0) boson as `monopole'.

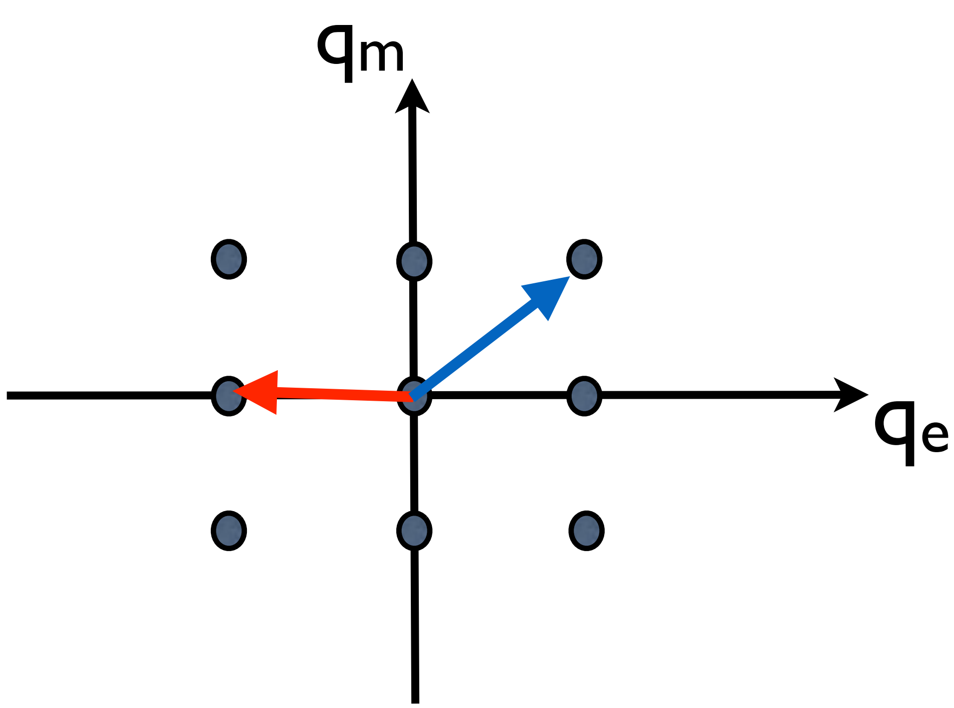
Boundary: Fermionic vortex theory dual to Wilson-Fisher

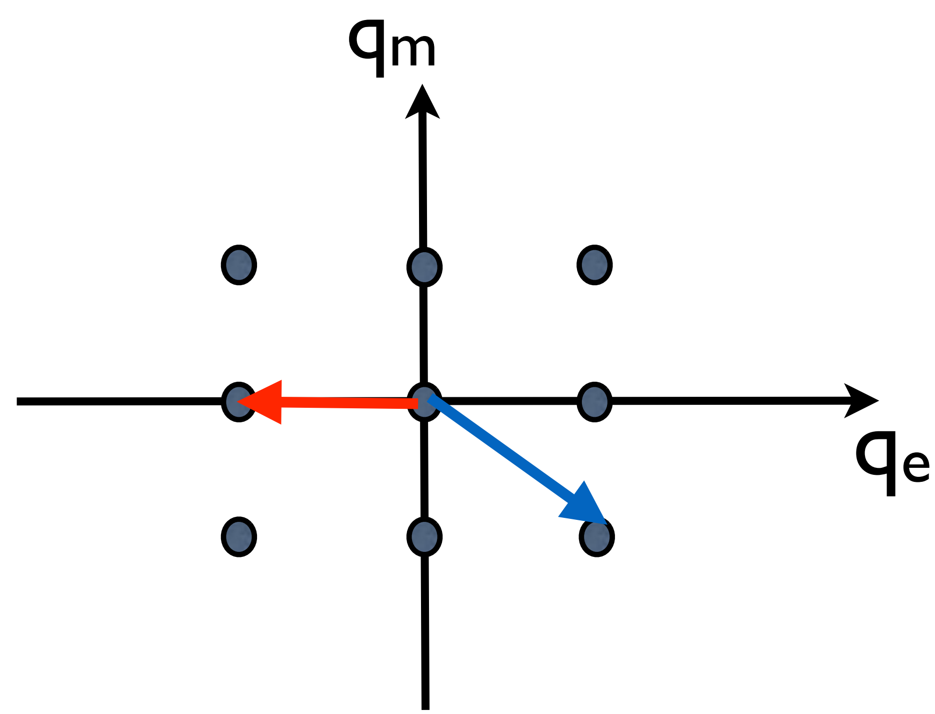


Bulk duality and time reversal

Bulk basis below clearly has non-trivial T-reversal transformation.

This corresponds precisely to non-trivial boundary T-reversal of fermionic dual of Wilson-Fisher.





Summary

Dualities provide a non-perturbative window into correlated many body systems.

Familiar and powerful for 1+1-D physics, and for 2+1-D bosons.

Dualities for 2+1-D Dirac fermions, and generalizations:
new window to view and solve some diverse problems.

Applications/future challenges/opportunities:

Quantum Hall problems (see Chong Wang talk)

Non-Landau quantum criticality (see Metlitski talk)

SC - insulator or SC- metal transition in thin films? (Kapitulnik et al 1990s - present)
(recent attempts by Raghu, Mulligan using dualities).