Topological insulators of bosons/spins

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Thanks: X.-G. Wen, M.P.A. Fisher

Collaborators, references, ...

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Refs: 1.Integer Quantum Hall Effect for bosons: A physical realization, T. Senthil and Michael Levin, arXiv:1206.1604

2. Physics of three dimensional bosonic topological insulators: Surface Deconfined Criticality and Quantized Magnetoelectric Effect, A. Vishwanath, T. Senthil, arXiv:1209.3058

Exotic insulating phases of matter

1. Gapped phases with ``topological quantum order'', fractional quantum numbers
(eg, fractional quantum Hall state, gapped quantum spin liquids)

Emergent non-local structure in ground state wavefunction
(``long range quantum entanglement'')

2. Simpler but still interesting: Topological insulators

Non-trivial surface states often protected by symmetry

No fractional quantum numbers/topological order in the bulk
(``short range entangled'')

Strongly correlated topological insulators

Interaction dominated phases as topological insulators?

Move away from the crutch of free fermion Hamiltonians and band topology.

This talk: Topological Insulators of bosons

Why study bosons?

1. Non-interacting bosons necessarily trivial - so must deal with an interacting theory right away

Necessitates thinking more generally about TI phases without the aid of a free fermion model.

2. Natural realizations in quantum spin systems

Is there a spin analog of a topological insulator, i.e a ``Topological Paramagnet''
(as distinct from a ``Quantum spin liquid'')?

(Other realizations: cold atoms)

3. Correlated bosons are stepping stone to correlated fermions (probably).

Topological Insulators of bosons?

Can bosons be in a ``topological insulator'' state with

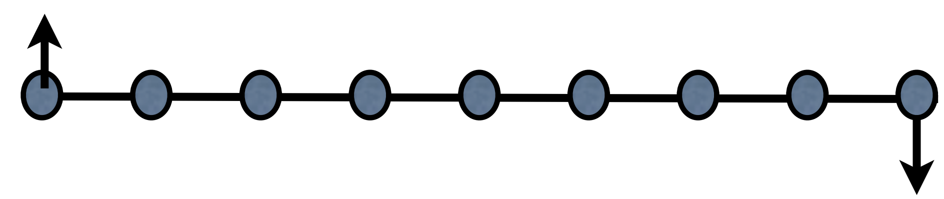
(1) no fractionalized excitations or topological order

(2) a bulk gap

(3) non-trivial surface states protected by global (internal) symmetry?

An old example: Haldane spin chain

$$\mathcal{H} = J \sum_i \vec{S}_i . \vec{S}_{i+1}$$



$\vec{S}_i$: spin-1 operators

Unique ground state on ring, no bulk fractional excitations,
but dangling spin-1/2 moments at edge.

Edge states protected by symmetry (SO(3) x time reversal)

A symmetry protected topological paramagnet.
First example of an interacting topological insulator.

Topological Insulators of bosons?

Can bosons be in a ``topological insulator'' state with

(1) no fractionalized excitations or topological order

(2) a bulk gap

(3) non-trivial surface states protected by global (internal) symmetry?

What about $d > 1$?

Yes! (according to recent progress in general abstract classification of short ranged entangled phases)

1. Cohomology classification (Chen, Liu, Gu, Wen, 2011)
2. ?? (Kitaev, unpublished)
3. Chern-Simons classification in $d = 2$ (Lu, Vishwanath, 2012)

This talk

1. Integer quantum Hall state of bosons (d = 2) (TS, Levin, 2012)

Simple, possibly experimentally relevant, example of the kind of state the formal classification shows is allowed to exist.

2. Topological insulators of bosons in d = 3 (Vishwanath, TS, 2012)

- Quantized magneto-electric effect (axion angle $\theta = 2\pi$)

- Surface `deconfined criticality', emergent fermionic vortices, ...

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Two component bosons in a strong magnetic field

Two boson species b_I each at filling factor $\nu = 1$

$$H = \sum_I H_I + H_{int} \tag{1}$$

$$H_I = \int d^2x b_I^\dagger \left(-\frac{\left(\vec{\nabla} - i\vec{A}\right)^2}{2m} - \mu \right) b_I \tag{2}$$

$$H_{int} = \int d^2x d^2x' \rho_I(x) V_{IJ}(x - x') \rho_J(x') \tag{3}$$

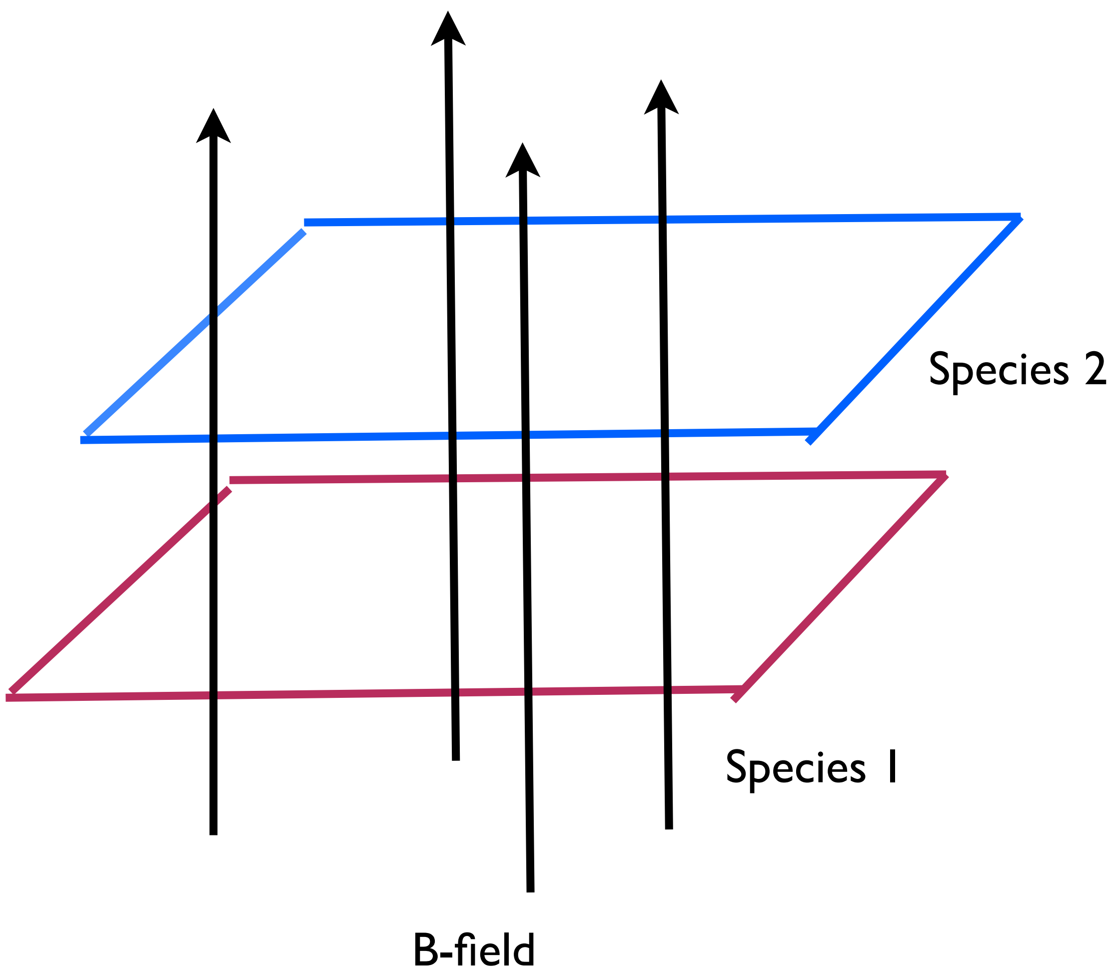
External magnetic field $\vec{B} = \vec{\nabla} \times \vec{A}$.
$\rho_I(x) = b_I^\dagger(x) b_I(x) =$ density of species I

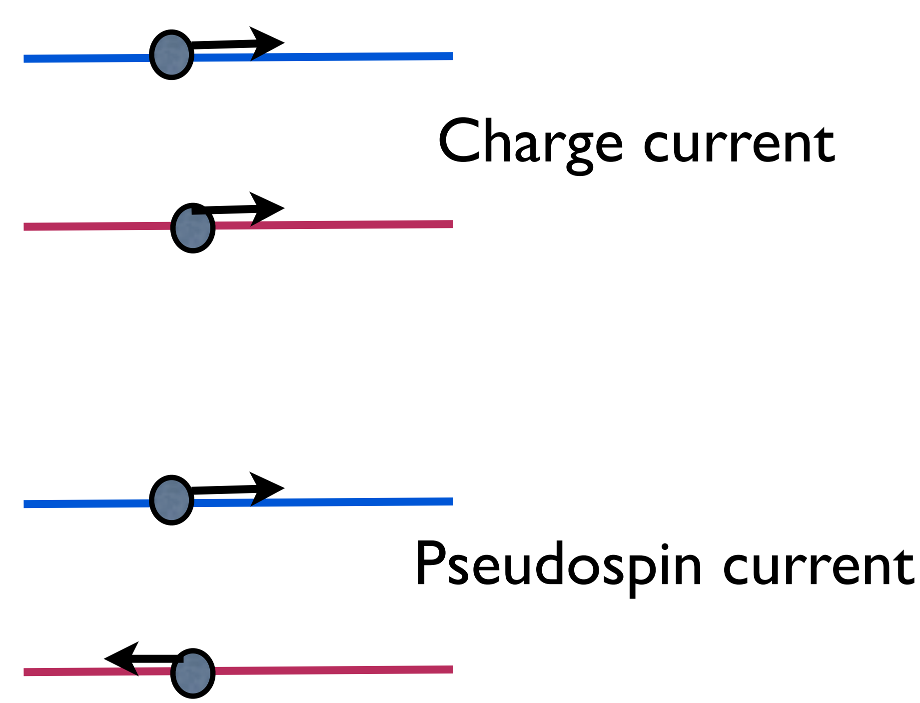
Symmetries and picture

Number of bosons N_1, N_2 of each species separately conserved: two separate global U(1) symmetries.

Total charge = N_1+N_2

Call N_1 - N_2 = total ``pseudospin''





Later relax to just conservation of total boson number

Guess for a possible ground state

If interspecies repulsion V_{12} comparable or bigger than same species repulsion V_{11}, V_{22}, particles of opposite species will try to avoid each other.

Guess that first quantized ground state wavefunction has structure

$$\psi = \text{``}\ldots\ldots \prod_{i.j}(z_i - w_j)\text{''} e^{-\sum_i \frac{|z_i|^2+|w_i|^2}{4l_B^2}} \tag{1}$$

z_i, w_i: complex coordinates of the two boson species.

Flux attachment mean field theory

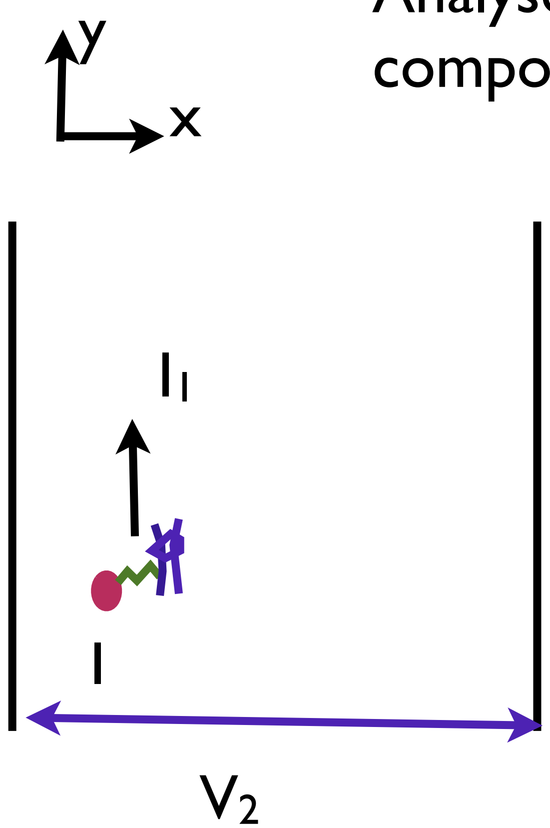
$\prod_{i.j}(z_i - w_j)$: particle of each species sees particle of the other species as a vortex.

Flux attachment theory:

Attach one flux quantum of one species to each boson of other species.
"Mutual composite bosons"

$\nu = 1$ => on average attached flux cancels external magnetic flux.
Mutual composite bosons move in zero average field.

Physical properties



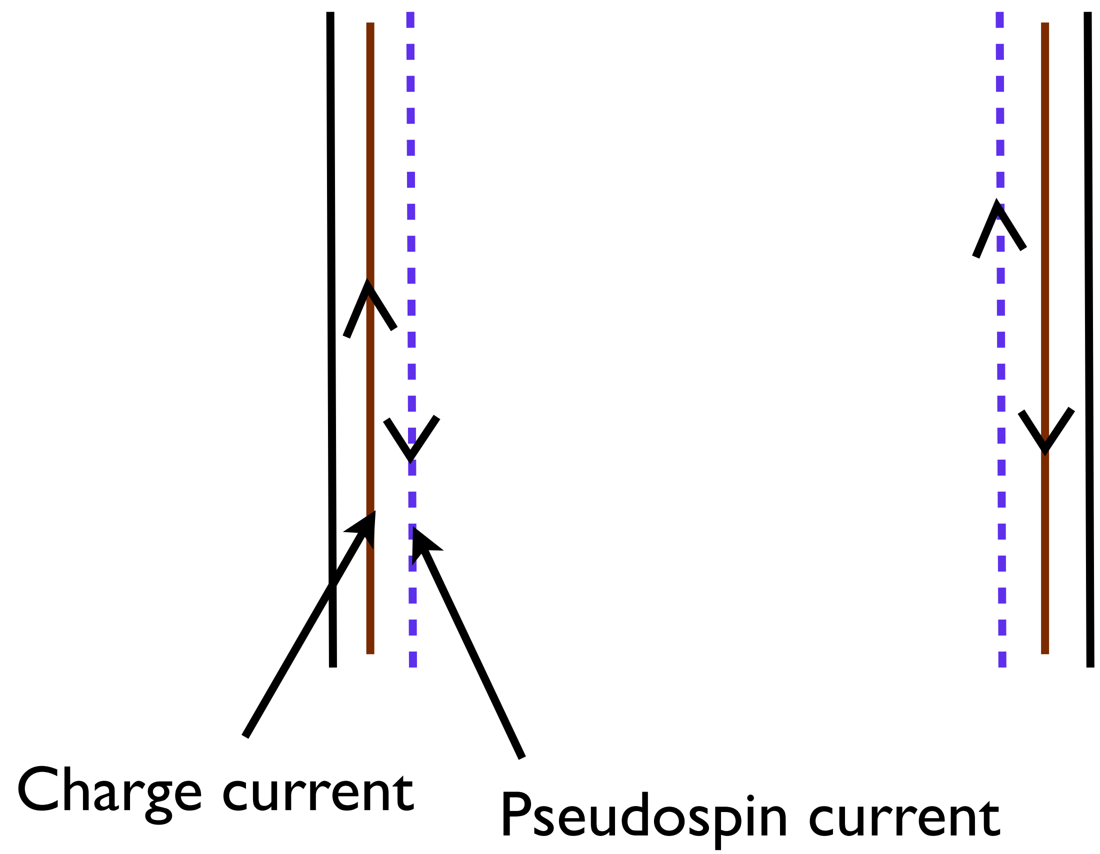
Analyse through usual Chern-Simons Landau Ginzburg theory for composite bosons

$$
\begin{aligned}
I_{1y} &= \frac{e^2}{h} V_{2x} \\
I_{2y} &= \frac{e^2}{h} V_{1x}
\end{aligned}
$$

Electrical Hall conductivity $\sigma_{xy} = 2$
Pseudospin Hall conductivity $\sigma^s_{xy} = -2$.

"Integer quantum Hall effect"

Edge states

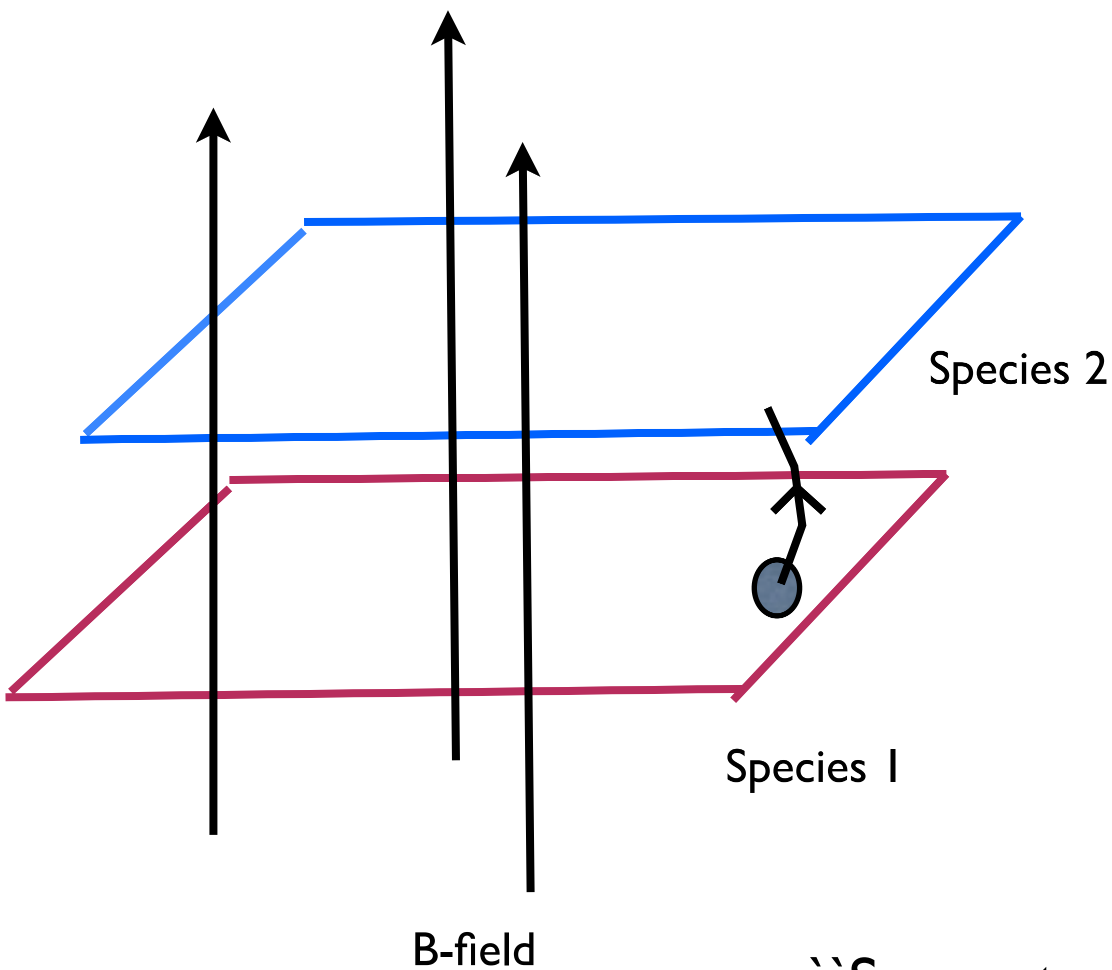


Comments:

1. Counterpropagating edge states but only one branch transports charge.

2. Thermal Hall conductivity = 0

Symmetry protection of edge states



Include interspecies tunneling:
Pseudospin not conserved,
only total particle number conserved.

Counterpropagating edge modes cannot backscatter due to charge conservation.

Edge modes are preserved so long as total charge is conserved.

``Symmetry Protected Topological Phase'' of bosons*

*Describe by two component Chern Simons theory with ``K-matrix'' with |det K| = 1: no bulk topological order/fractionalization.

Ground state wavefunction

(ignore interspecies tunneling)

Naive guess
$$\Psi\left(\{z_i, w_j\}\right) = \prod_{i,j} (z_i - w_j) \cdot e^{-\sum_i \frac{|z_i|^2+|w_i|^2}{4}} \qquad (1)$$

Problem: Unstable to phase separation (see using Laughlin plasma analogy)

Fix, for example, using ideas initiated by Jain (1993) for some fermionic quantum Hall states

$$\begin{aligned}\Psi_{flux} &= P_{LLL} \prod_{i<j} |z_i - z_j|^2 \cdot \prod_{i<j} |w_i - w_j|^2 \\ &\cdot \prod_{i,j} (z_i - w_j) \cdot e^{-\sum_i \frac{|z_i|^2+|w_i|^2}{4}} \qquad (1)\end{aligned}$$

P_{LLL}: projection to lowest Landau level

Wavefunction is pseudospin singlet; suggests stabilized by pseudospin SU(2) invariant Hamiltonian.

Microscopics: a simple Hamiltonian

$$
\begin{aligned}
H &= \sum_I H_I + H_{int} && (1)\\
H_I &= \int d^2x b_I^\dagger \left(-\frac{\left(\vec{\nabla} - i\vec{A}\right)^2}{2m} - \mu \right) b_I && (2)\\
H_{int} &= \int d^2x d^2x' \rho_I(x) V_{IJ}(x - x')\rho_J(x') && (3)
\end{aligned}
$$

Simple and **realistic** interaction:

$$
\begin{aligned}
V_{II}(\vec{x}) &= g_s \delta^{(2)}(\vec{x})\\
V_{12}(\vec{x}) &= g_d \delta^{(2)}(\vec{x})
\end{aligned}
$$

$g_s = g_d$: Pseudospin $SU(2)$ invariance

What is ground state?

SU(2) symmetric point: recent exact diagonalization work show an incompressible, spin singlet state (Grass et al 2012, Furukawa, Ueda 2012)

Candidates:
1. Boson IQHE

2. A non-abelian spin singlet state (Ardonne, Schoutens 1999)

Regnualt, TS (exact diagonalization on torus with 16 particles): unique ground as expected for boson IQHE

Prospects - experiments

Ultracold atoms in strong artificial magnetic fields?

The delta function repulsion is realistic and controllable.

Challenge: get fields high enough to be in the quantum Hall regime

Comments

- State described has $\sigma_{xy} = 2, \kappa_{xy} = 0$.

  Can obtain states with $\sigma_{xy} = 2n, \kappa_{xy} = 0$ by taking copies.

- For bosons, IQHE necessarily has σ_{xy} even.

Comments

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  Can obtain states with $\sigma_{xy} = 2n, \kappa_{xy} = 0$ by taking copies.

- For bosons, IQHE necessarily has σ_{xy} even.

Simple argument (TS, Levin 12).
Thread in 2π flux - pick up charge σ_{xy}.
Resulting particle has statistics $\pi\sigma_{xy}$.
No topological order => only boson excitations, so σ_{xy} even.

This talk

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Simple, possibly experimentally relevant, example of the kind of state the formal classification shows is allowed to exist.

2. **Topological insulators of bosons in d = 3** (Vishwanath, TS, 2012)

- Quantized magneto-electric effect (axion angle $\theta = 2\pi$)

- Surface `deconfined criticality', emergent fermionic vortices, ...

Review: free fermion 3d topological insulators

Characterize by

1. presence/absence of non-trivial surface states

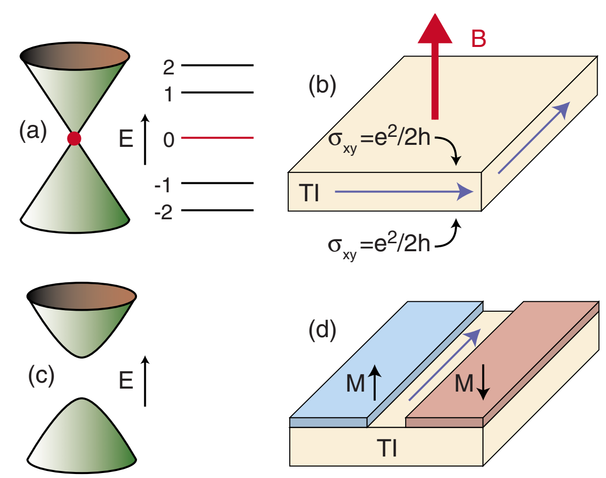
2. EM response

Surface states: Odd number of Dirac cones

Trivial gapped/localized insulator not possible at surface so long as T-reversal is preserved (even with disorder)

Review: free fermion topological insulators

EM response: Surface quantum Hall Effect



If surface gapped by B-field/proximity to magnetic insulator, surface Hall conductance

$$\sigma_{xy} = \left(n + \frac{1}{2}\right)\frac{e^2}{h}$$

Domain wall between opposite T-breaking regions: chiral edge mode of 2d fermion IQHE

Review: Free fermion topological insulators
Axion Electrodynamics

Qi, Hughes, Zhang, 09
Essin, Moore, Vanderbilt, 09

EM response of *any* 3d insulator

$$\begin{aligned} \mathcal{L}_{eff} &= \mathcal{L}_{Max} + \mathcal{L}_\theta \qquad (1) \\ \mathcal{L}_\theta &= \frac{\theta}{4\pi^2} \vec{E}.\vec{B} \end{aligned}$$

Under $\mathcal{T}$-reversal, $\theta \to -\theta$.
Periodicity $\theta \to \theta + 2\pi$: only $\theta = n\pi$ consistent with $\mathcal{T}$-reversal.
Domain wall with $\theta = 0$ insulator: Surface quantum Hall effect

$$\sigma_{xy} = \frac{\theta}{2\pi}$$

Free fermion TI: $\theta = \pi$.
Interpretation of periodicity:
$\theta \to \theta + 2\pi$: deposit a 2d fermion IQHE at surface.
Not a distinct state.

Boson topological insulators: EM response

Vishwanath, TS, 2012

For bosons, $\theta = 2\pi$ is distinct from $\theta = 0$.

Surface Hall conductivity $\sigma_{xy} = 1$, i.e, half of 2d boson IQHE state.
-cannot be obtained by depositing 2d boson IQHE state (which has σ_{xy} even).
Surface of $\theta = 2\pi$ requires the 3d bulk.

$\theta \to \theta + 4\pi$ is trivial.

``Landau-Ginzburg'' Theory of surface states

General effective theory for surface of $\theta = 2\pi$ boson TI ?

Required feature: **No trivial gapped insulator**

Surface either breaks defining global symmetry, or has topological or other exotic order.

Similar to Lieb-Schultz-Mattis restrictions (and extensions by Oshikawa, Hastings) for bosons on clean lattices at commensurate fractional filling.

Difference: Protected even in absence of translation symmetry.

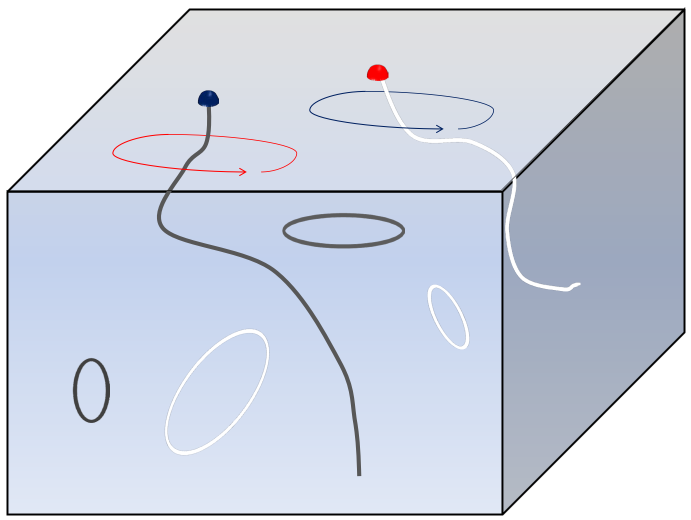
Theoretical strategy: first assume U(1) x U(1) global symmetry + T-reversal, then break to single U(1) (similar to d = 2).

Two approaches

1. Dual description of surface in terms of point vortices = points where vortex lines of bulk penetrate surface.

Demand that there is no trivial vortex that can condense to give a trivial insulator.

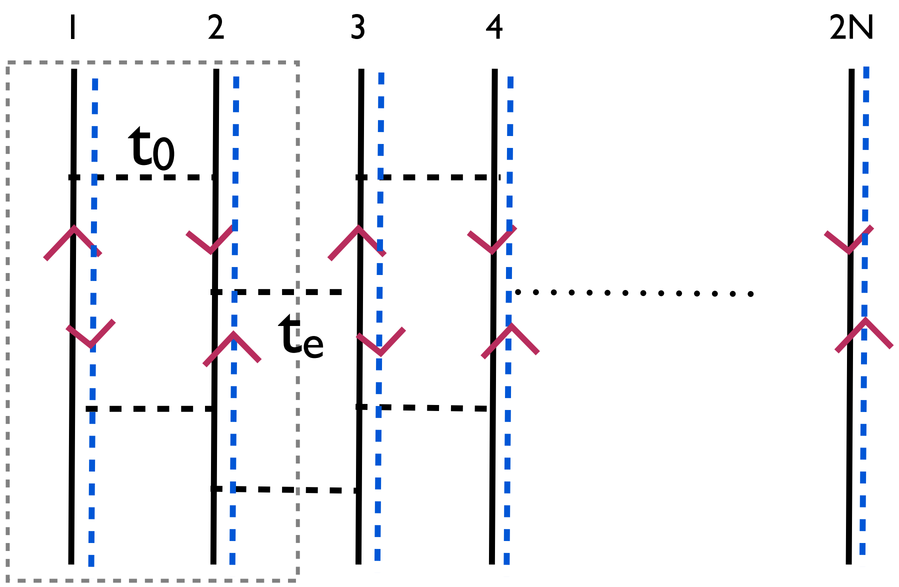
Implement: vortices transform projectively under internal global symmetry (i.e they fractionalize global symmetry)

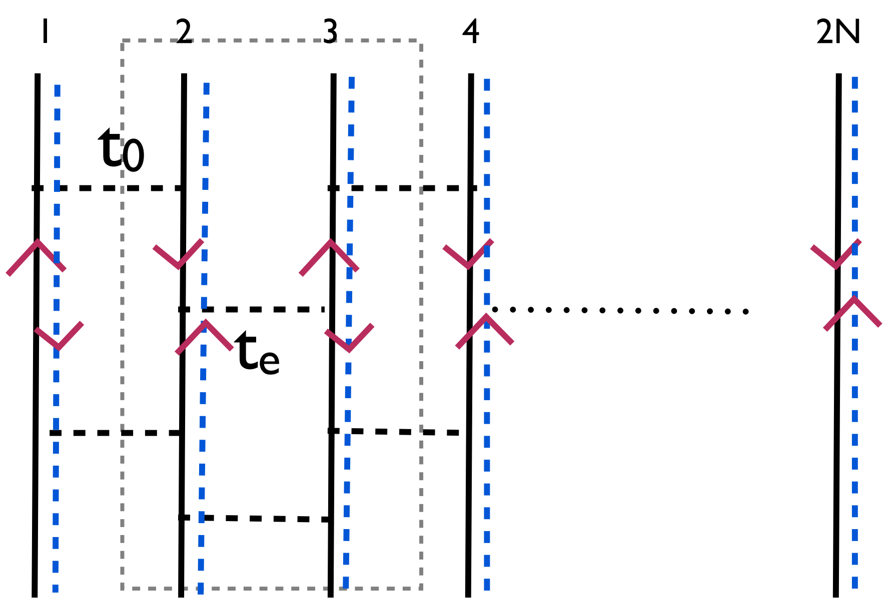


$(U(1) \times U(1)) \times Z_2^T$: Vortex of each boson species carries charge $\pm\frac{1}{2}$ of other species.

Approach 2: relate to 2d boson IQHE plateau transition

Surface state: fluctuating between two T-breaking states with $\sigma_{xy} = \pm 1$
=> theory of 2d IQHE plateau transition of bosons.
Describe through network model.





Edge channels of 2d boson IQHE coupled by boson hopping.

Transition: all hoppings equal.

Continuum limit: Surface field theory of two boson fields + topological term.

Key effect: mutual semion statistics of vortices of the two boson species

Surface Landau-Ginzburg effective field theory

$$\begin{aligned}\mathcal{L}_1 &= \sum_{s=\pm} \left|(\partial_\mu - i\alpha_{2\mu})\,\psi_{2s}\right|^2 + ... \\ &\quad + \frac{1}{2\kappa_1}\left(\epsilon_{\mu\nu\lambda}\partial_\nu \alpha_{2\lambda}\right)^2 \end{aligned} \tag{1}$$

$\psi_{2\pm}$: vortices of boson 1 but carry electric charge $\pm\frac{1}{2}$ of boson 2.
Density of boson 1: flux density of gauge field α_2.

"Non-compact CP^1 model" (NCCP^1)

Equivalent dual description in terms of different fields $\psi_{1\pm}$ = vortices of boson 2 carrying electric charge $\pm\frac{1}{2}$ of boson 1.

Perturb by all symmetry allowed terms.

Comments

1. Field theory familiar from studies of deconfined quantum criticality in 2d quantum magnetism (TS et al, 2004).

Naturally incorporates restriction of no trivial gapped insulator.

Phases of field theory either break symmetry or have surface topological order (or more exotic).

2. Key difference with ``standard'' deconfined criticality in quantum magnetism:

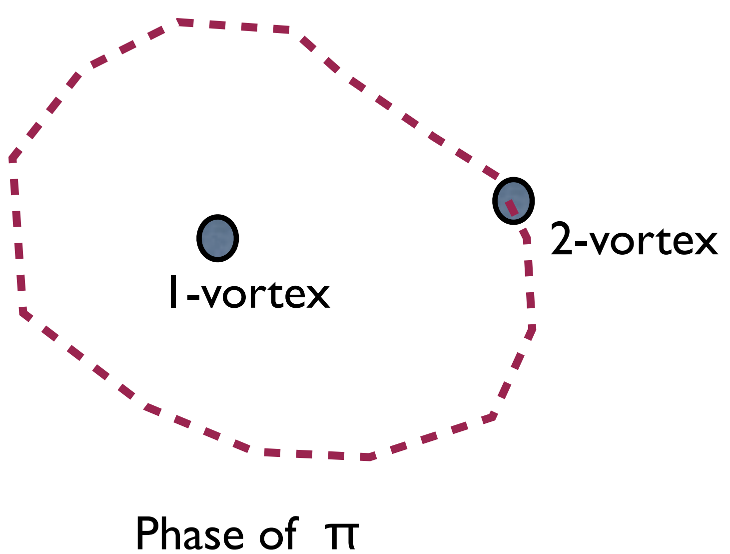
Different realization of symmetry.

Robust to disorder, new allowed perturbations, etc.

Important perturbation

Inter-species tunneling to break U(1) x U(1) to U(1)

Confines vortices in one boson to vortices in the other boson - single common vortex



These vortices are mutual semions => bound common vortex is a fermion!

Fermionic vortex does not carry fractional quantum numbers but cannot condense => no trivial gapped insulator.

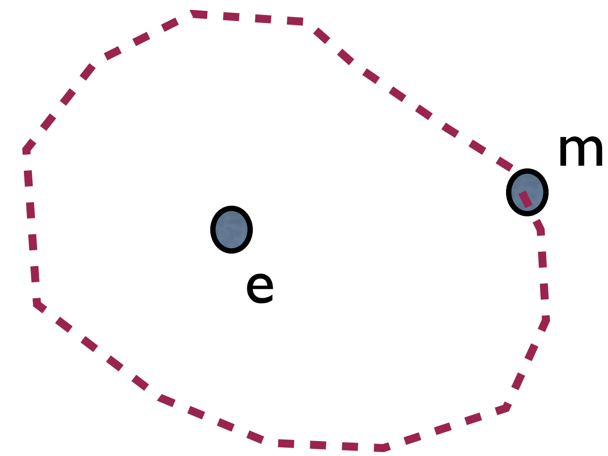
Symmetry preserving surface phases
Eg: surface topological order

Simplest: Z2 surface topological order (surface ``toric code") driven by paired vortex condensation

Usual topological quasiparticles e, m, ε(= e+m)

charge 1/2 bosons

charge-0 fermion



Phase of π

Destroy surface topological order => condense e or m necessarily breaks U(1); no trivial phase.

Strict 2d: this charge assignment not possible with T-reversal symmetry.
Surface ``toric code" implements symmetry in a way not allowed in strict 2d.

Other results: bulk effective field theories

1. Bulk topological field theory - ``BF'' + theta

$$2\pi\mathcal{L}_{3D} = \sum_I \epsilon^{\mu\nu\lambda\sigma} B^I_{\mu\nu}\partial_\lambda a^I_\sigma + \Theta \sum_{I,J} \frac{K_{IJ}}{4\pi}\epsilon^{\mu\nu\lambda\sigma}\partial_\mu a^I_\nu \partial_\lambda a^J_\sigma$$

$I = 1, 2$

Two-form gauge fields $B^I_{\mu\nu}$: dual of conserved boson current j^I_μ.
Magnetic field lines of a^I_μ: vortex lines
Θ term: fractional charge on surface vortices.

2. Bulk SO(5) non-linear sigma model with (O(2) x O(2)) x Time reversal anisotropy and theta term at $\theta = 2\pi$.

Summary

1. 2d boson TI: IQHE is prototype.
Physical realization in 2-component bosons. Even integer Hall conductance

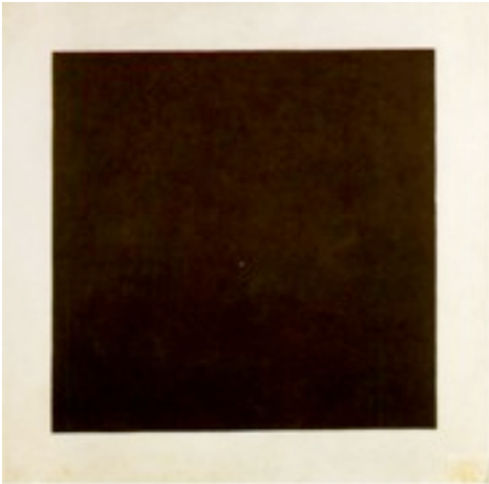
2. 3d boson TI with T-reversal invariance: Axion angle $\theta = 2\pi$.
Surface theory: deconfined critical + perturbations
Fermion vortices and associated exotic phases.

3. Spin system interpretation: Topological paramagnet in 3d.

Surface state either breaks symmetry or is in a surface quantum spin liquid which cannot exist in strict 2d.

Microscopic models? Experiments?

Other work on classification/field theory/properties of 3d boson TI:
Chen, Gu, Liu, Wen, **arXiv:1106.4772**, Cenke Xu, arXiv:1209.4399 , Swingle, arXiv:1209.0776, Metlitski, Kane, Fisher, upcoming



Black Square, 1915

Kazimir Malevich