Some models and mechanisms for non-fermi liquids

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Collaborators

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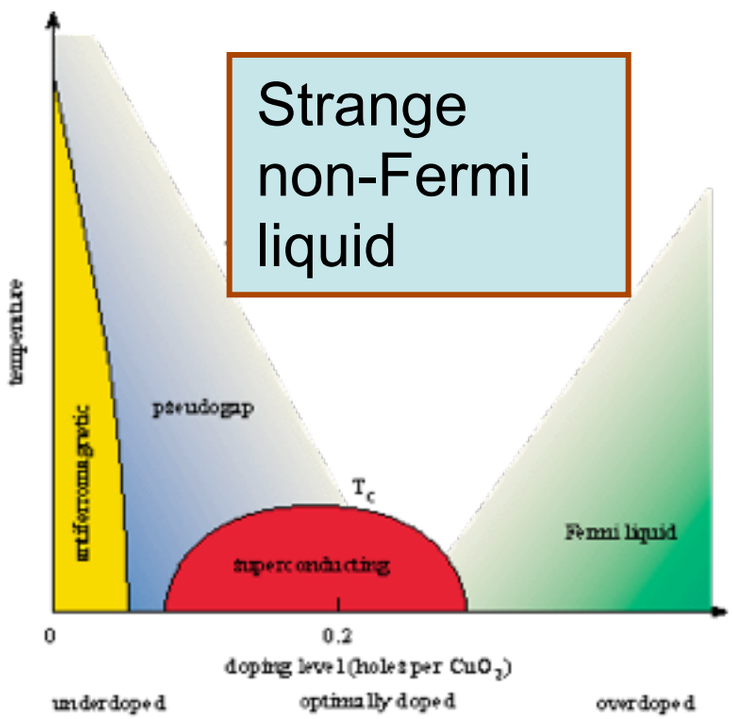
Chowdhury, Werman, Berg, TS, arXiv: 1801.06178 (appeared today!)

Cuprate strange metal regime

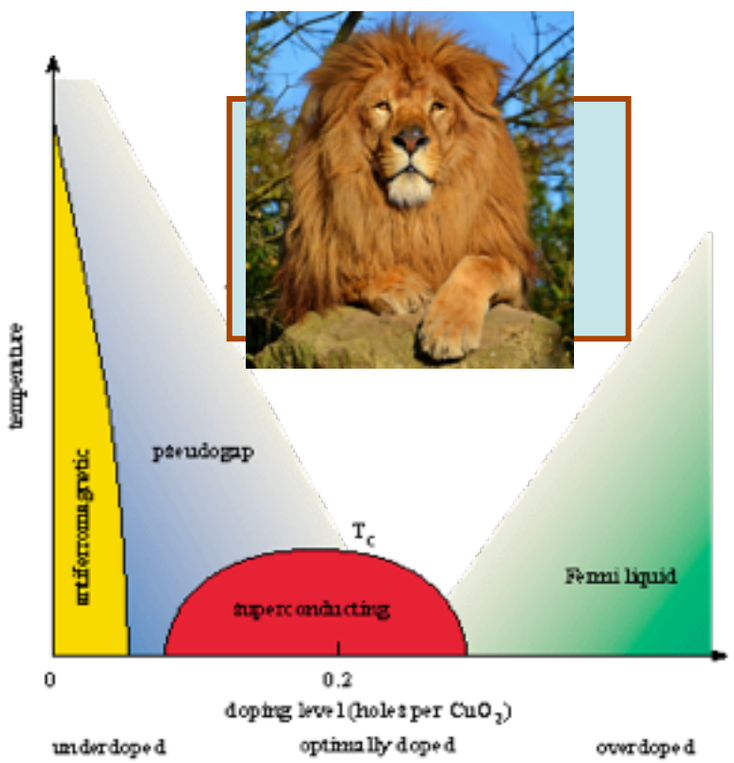


Power laws in many physical quantities distinct from that expected in a Fermi liquid.

Fermi surface but no Landau quasiparticles.

Compared to conventional textbook wisdom (= Landau fermi liquid theory) , the strange metal is a wild beast.

Cuprate strange metal regime



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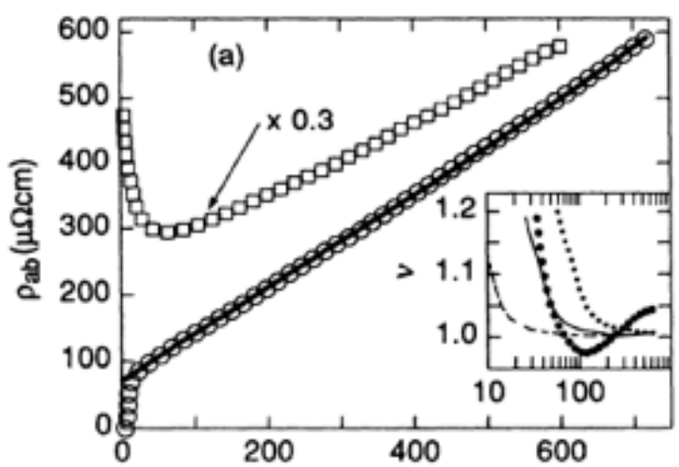
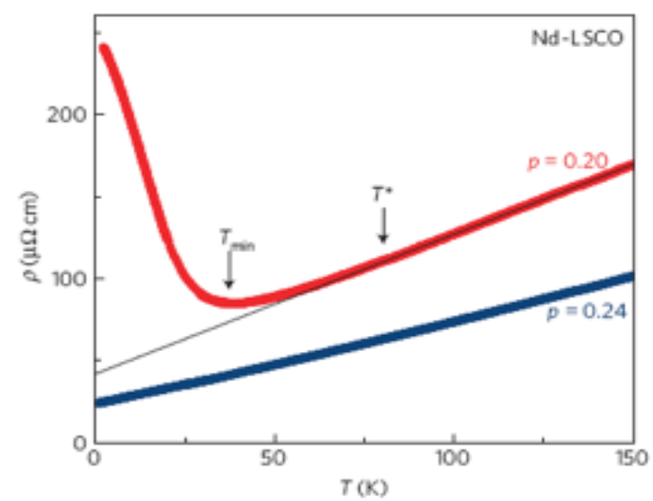
Many interesting properties

Most famous: linear resistivity





Bi-2201: Martin et al, PR B, 1990

Nd-LSCO, Daou, ..., Taillefer, Nat. Phys., 2008

Many other anomalies in other properties, eg, optical transport, ARPES, ……

Well-known phenomenological description: Marginal Fermi Liquid (MFL) (Varma et al, 1989)

?? Microscopic basis??

Q: What is the theory of the cuprate strange metal?

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A: I do not know and will not provide an answer in this talk.

Other analogous metals

Various kinds of strange/bad/non-fermi liquids show up in a wide variety of correlated metals.

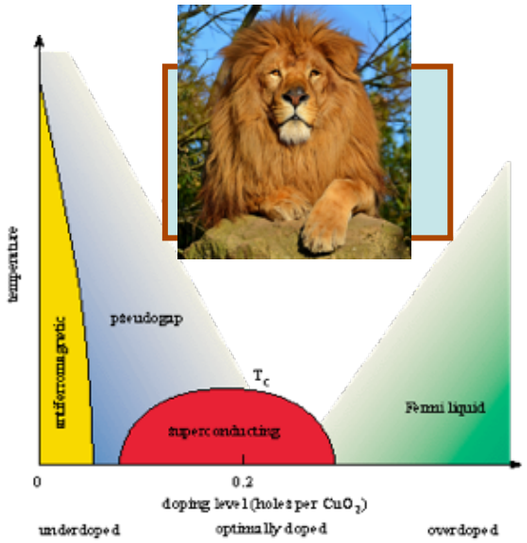
Eg: Many examples in heavy fermion systems, Ruthenates, pnictides,……..

(Various sessions at this meeting).

Others: Cobaltate $Na_{0.7}CoO_2$ (Cava, Ong, 02; Taillefer et al, 04)

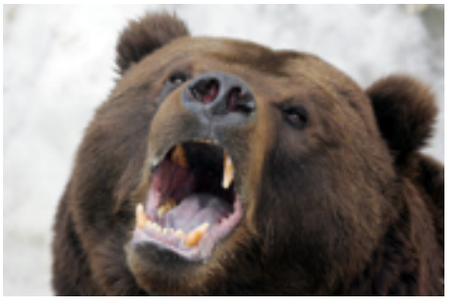
Somewhat but not exactly similar phenomenology.

These likely realize distinct ways in which Fermi liquid theory breaks down.



The wild world of strange metals

All strange metals likely not exactly the same.



Can we tame some of these beasts?

Big questions slide from James Analytis

The big questions

- How do we describe a metal with no quasiparticles?
- Even in a non-Fermi Liquid (NFL), why do we get linear-T, and should we expect universal bounds?
- How does disorder affect the NFL? Even harder to solve an already very difficult problem!

Strategies for progress in theoretical physics

Ideal world: Solve the right model rightly

If not possible then…….

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Translationally invariant non-Fermi liquid metals with critical Fermi-surfaces: Solvable models

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Today's listing: arXiv: 1801.06178

Two solvable models

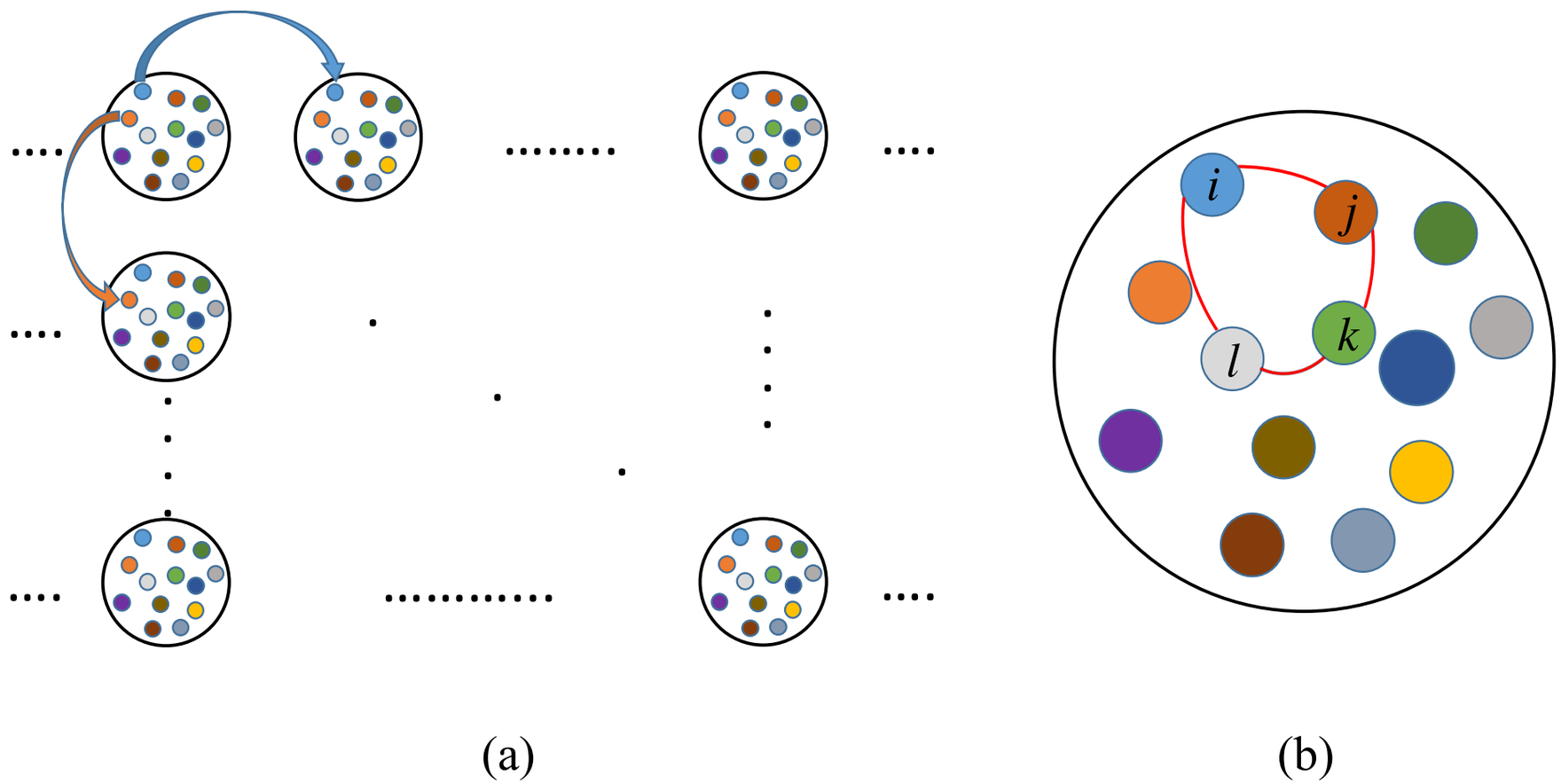
1. One band model

- ``locally critical" NFL high T regime ($T >> T_{coh}$)
- standard Landau Fermi liquid with a sharp Fermi surface at low T ($<< T_{coh}$)

2. Two band model (a light weakly correlated band coupled to a different heavy strongly correlated band)

- broad intermediate temperature Marginal Fermi Liquid with a sharp Fermi surface
- standard Landau Fermi liquid at low T

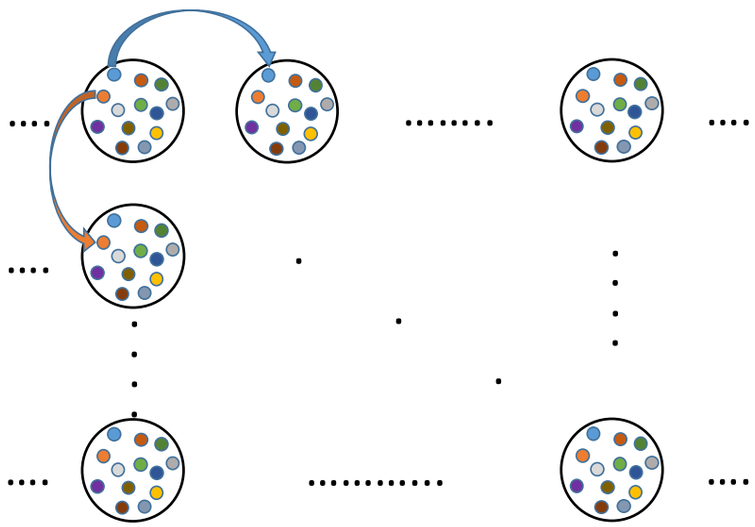
The one-band model



Lattice of strongly correlated sites coupled by electron hopping.

Each site has N orbitals coupled together by frustrated interactions.

Solvable in large-N limit.



The one band model

$$H_c = -\sum_{\vec{r},\vec{r}'}\sum_{\ell} t^c_{\vec{r},\vec{r}'} c^\dagger_{\vec{r}\ell} c_{\vec{r}'\ell} + \frac{1}{N^{3/2}}\sum_{\vec{r}}\sum_{ijk\ell} U^c_{ijk\ell} c^\dagger_{\vec{r}i} c^\dagger_{\vec{r}j} c_{\vec{r}k} c_{\vec{r}\ell}$$

Translation invariance: t_c, U^c_{ijkl} are the same at each lattice site.

Assume generic on-site inter-orbital interaction U^c_{ijkl} which depends on $(ijkl)$.

The one-band model (cont'd)

$$H_c = -\sum_{\vec{r},\vec{r}'}\sum_{\ell} t^c_{\vec{r},\vec{r}'} c^\dagger_{\vec{r}\ell} c_{\vec{r}'\ell} + \frac{1}{N^{3/2}}\sum_{\vec{r}}\sum_{ijk\ell} U^c_{ijk\ell} c^\dagger_{\vec{r}i} c^\dagger_{\vec{r}j} c_{\vec{r}k} c_{\vec{r}\ell}$$

Choose interactions from a random distribution where U^c_{ijkl} are independent, uncorrelated variables with

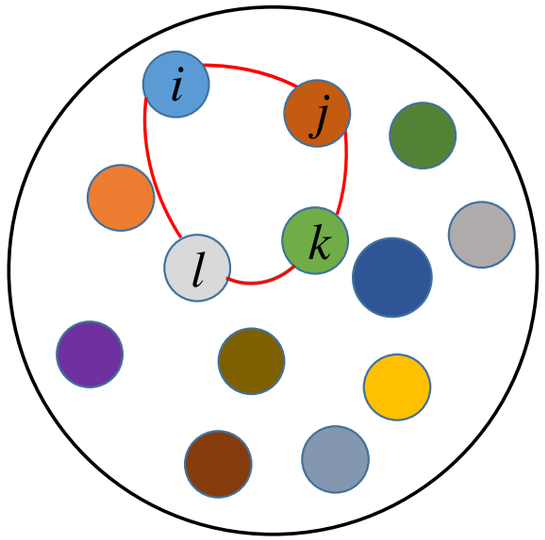
$$\overline{U^c_{ijkl}} = 0, \quad \overline{(U^c_{ijkl})^2} = U_c^2$$

Large-N limit: Properties for any generic interaction can be calculated by averaging over this random probability distribution.

Note: ***each realization*** is exactly translation invariant.

Sachdev, Ye 1992
Parcollet, Georges, 1998
Kitaev, 2015
……..

The basic ingredient: a single site
The Sachdev-Ye-Kitaev (SYK) model



$$H_{int} = \frac{1}{N^{3/2}} \sum_{\vec{r}} \sum_{ijk\ell} U^c_{ijk\ell} c^\dagger_{\vec{r}i} c^\dagger_{\vec{r}j} c_{\vec{r}k} c_{\vec{r}\ell}$$

All-to-all interactions taken from a random probability distribution.

Many novel features, for instance,

Power law Green's function $G(\omega) \sim \frac{1}{\sqrt{\omega}} F(\frac{\omega}{T})$

$T = 0$ entropy $S = N(S_0 + o(T))$

Model is self-averaging in large-N limit

Comments

1. Our models build up a translation invariant lattice of SYK ``dots'' coupled together by hopping.

2. Earlier work: Lattice of *random* SYK dots coupled by *random* hopping (Song, Jian, Balents, 17; Patel, McGreevy, Arovas, Sachdev 17).

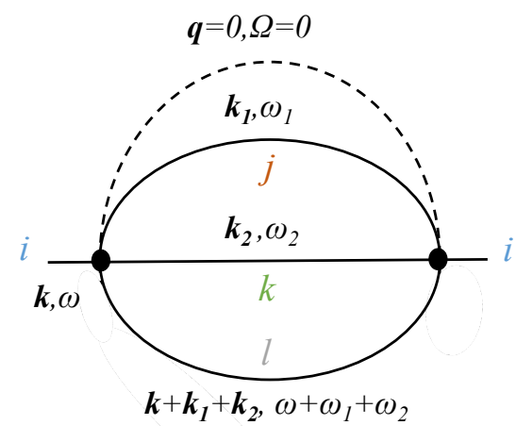
Some common features (implying translation breaking is not essential).

Allowing translation invariance further enables us to address presence/absence of ``critical Fermi surfaces'' (sharp Fermi surface without sharp quasiparticles).

Large-N solution

Exact lattice Green's function given by solving a self-consistency equation for self-energy

$\Sigma_c(\boldsymbol{k}, i\omega)$ =



$$G_c(\vec{k}, i\omega) = \frac{1}{i\omega - \epsilon_k - \Sigma_c(\vec{k}, i\omega)}$$

$$\Sigma_c(k, i\omega) = -U_c^2 \int_{\vec{k}_1, \vec{k}_2} \int_{\omega_1, \omega_2} G_c(\vec{k}_1, i\omega_1) G_c(\vec{k}_2, i\omega_2) G_c(\vec{k} + \vec{k}_1 + \vec{k}_2, i(\omega + \omega_1 + \omega_2))$$

Some similarity to DMFT equations but self-energy is a priori allowed to be momentum-dependent.

Results

Emergence of a coherence scale $T_{coh} \sim \frac{t_c^2}{U_c}$:

$T \gg T_{coh}$: $G_c(\vec{k}, i\omega)$ inherits locally critical behavior of single dot (with weak k-dependance).

$T \ll T_{coh}$: Landau Fermi liquid with sharp Fermi surface and heavy quasi-particles.

For $T \gg T_{coh}$, resistivity $\rho \propto T$.

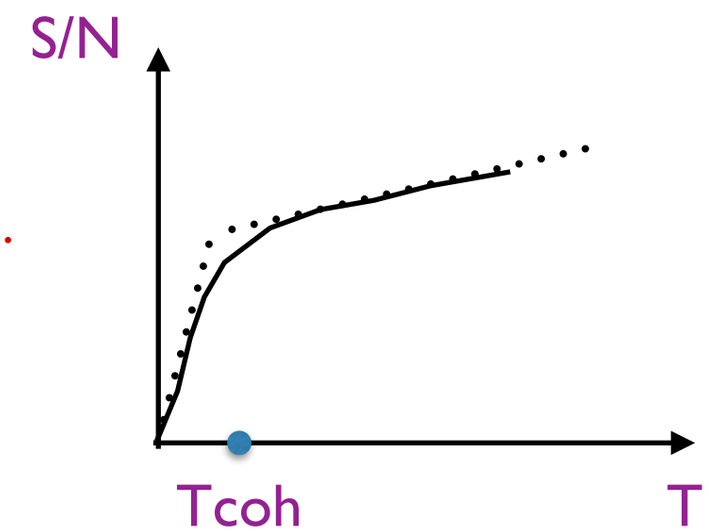
Physical picture

High T: Treat hopping t_c in perturbation theory.
Scale invariant Green's function in each site $G(\omega) \sim \frac{1}{\sqrt{\omega}}$
=> Hopping becomes scale dependent.

Effective hopping at a temperature T
$t_c^R(T) \sim t_c \left(\frac{U_c}{T}\right)^{\frac{1}{2}}$.

Perturbation theory fails at T_{coh} when effective hopping $\approx U_c$.
Natural to get FL at lower T.

Residual entropy at high-T which is relieved at low-T
=> effective mass $\sim \frac{1}{T_{coh}}$.



High-T NFL transport: Physical understanding

High-T NFL regime:
Perturbation theory in effective hopping => conductivity $\propto (t_c^R(T))^2$:

$$\sigma \sim \frac{Ne^2}{h}\left(\frac{t_c^R(T)}{U_c}\right)^2 \quad \sim \frac{Ne^2}{h}\frac{T_{coh}}{T}$$

Resistivity $\rho = \frac{h}{Ne^2}\frac{T}{T_{coh}}$
(Linear resistivity with values exceeding Mott-Ioffe-Regal limit $\frac{h}{Ne^2}$.)

Low-T Fermi liquid: $\rho(T) = AT^2$ with $A \sim \gamma^2$

Note: similar to results of Song et al (17) but in a translation invariant model.
As a matter of principle, disorder not necessary to get this kind of NFL.

Two band model

Weakly correlated c-band coupled to strongly correlated f-band.

$$H = H_c + H_f + H_{cf},$$

H_f: translation invariant lattice of SYK dots (as before) with hopping t^f, interaction U^f.

$$H_c = -\sum_{\vec{r},\vec{r}'} \sum_i t^c_{\vec{r},\vec{r}'} \left(c^\dagger_{ri} c_{r'i} + h.c \right)$$

$$H_{cf} = \frac{1}{N^{3/2}} \sum_r \sum_{ijkl} V_{ijkl} c^\dagger_{ri} c_{rk} f^\dagger_{rj} f_{rl}$$

$$\overline{V_{ijkl}} = 0, \quad \overline{(V_{ijkl})^2} = U^2_{cf}$$

Full model is translation invariant; solve as before by solving self-consistency equations for c and f self energies.

Results

(Specialize to limit $U_{cf} \ll t_c$ and $t_f \ll$ other parameters.)

Low coherence scale: $T_{coh} \sim \frac{t_f^2}{U_{ff}}$,

Low-T ($T \ll T_{coh}$): Heavy Landau Fermi liquid with separate c and f Fermi surfaces

Broad intermediate temperatures $T_{coh,f} \ll T \ll min(t_c, U_f)$:
c-electrons form a Marginal Fermi Liquid with a sharp Fermi surface.
(f are locally critical).

Focus on MFL .

Marginal Fermi Liquid solution (intermediate-T)

Physics: Scattering of c-electrons off fluctuations of critical f-electrons.

$$\Sigma_{cf} = -\frac{\nu_0 U_{cf}^2}{2\pi^2 U_f} i\omega \ln\left(\frac{U_f}{|\omega|}\right)$$

(+ weak k-dependent corrections)

Sharp c-Fermi surface satisfying Luttinger's theorem

Heat capacity $C(T) \sim T \ln(1/T)$

Finite c-electron compressibility (non-interacting value + weak corrections).

Marginal Fermi Liquid (MFL) physical properties: transport

Linear dc resistivity $$\rho_{dc}(T) \propto \frac{U_{cf}^2}{N v^2 \; U_f} T.$$

High frequency conductivity $$\sigma'_{xx}(\Omega) \propto \frac{N v^2 \; U_f}{U_{cf}^2} \frac{1}{\Omega \; \ln(1/\Omega)^2}$$

Crossover scale = ``optical scattering rate'' $$\tau_{\rm opt}^{-1} \sim T/\ln^2(1/T)$$

Note: this rate is parametrically smaller than ``Planckian limit'' despite linear resistivity.

Generalizations: Non-fermi liquids with ``critical Fermi surfaces"

Models discussed so far have 2 body interactions.

Generalization: q-body interactions with q > 2.

Two band model -
Low-T: Landau Fermi liquid.

Broad intermediate-T: Non-fermi liquid with sharp ``critical Fermi surface satisfying Luttinger theorem.

Many interesting thermodynamic and transport properties.
Example: $\rho_{dc} \sim T^{4/q}$
=> sublinear power law

Signatures of critical Fermi surface: Quantum oscillations (with calculable non-LK T-dependence), $2K_f$ singularities.

What have we learnt?

Concrete testing ground for many ideas that have been proposed about NFLs.

Example: Transport bounds

1. Need care in defining transport ``scattering rate'' in a strange metal.

Procedure in Bruin,….MacKenzie et al, 2013 leads to scattering rate parametrically smaller than Planckian bound even with linear resistivity.

Not clear which, if any, transport rate satisfies a bound that is saturated in a strange metal.

2. Can get (sub)linear resistivity in a clean metal: disorder need not play a crucial role

Where do we go?

Models studied here encouraging but not directly applicable to non-fermi liquids in generic correlated models/materials.

Inspiration to develop a coarse grained picture of generic intermediate-T non-fermi liquids.

A key feature of these models: maximal many body chaos within each dot

Conjecture: Coarse graining of generic correlated model for a strange metal
- Maximally chaotic bubbles of spatial size $l \gg a$ (microscopic length scale) but $l \ll L$ (= system size).

Universal coarse grained theory: coupled maximally chaotic bubbles.

Many interesting questions + routes to progress ahead!!