

# Are the cuprates doped spin liquid Mott insulators?

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Meaning of the question?  
How might we find answers?  
Don't we already know?

T.Senthil, Patrick Lee, cond-mat/0406066

# Are the cuprates doped spin liquid Mott insulators?

- ``Obvious'' answer: No!

Undoped material has antiferromagnetic order – not a spin liquid.

However ``obvious'' answer may be too quick.....

# Aspects of underdoped phenomenology (at not too low doping or temperature)

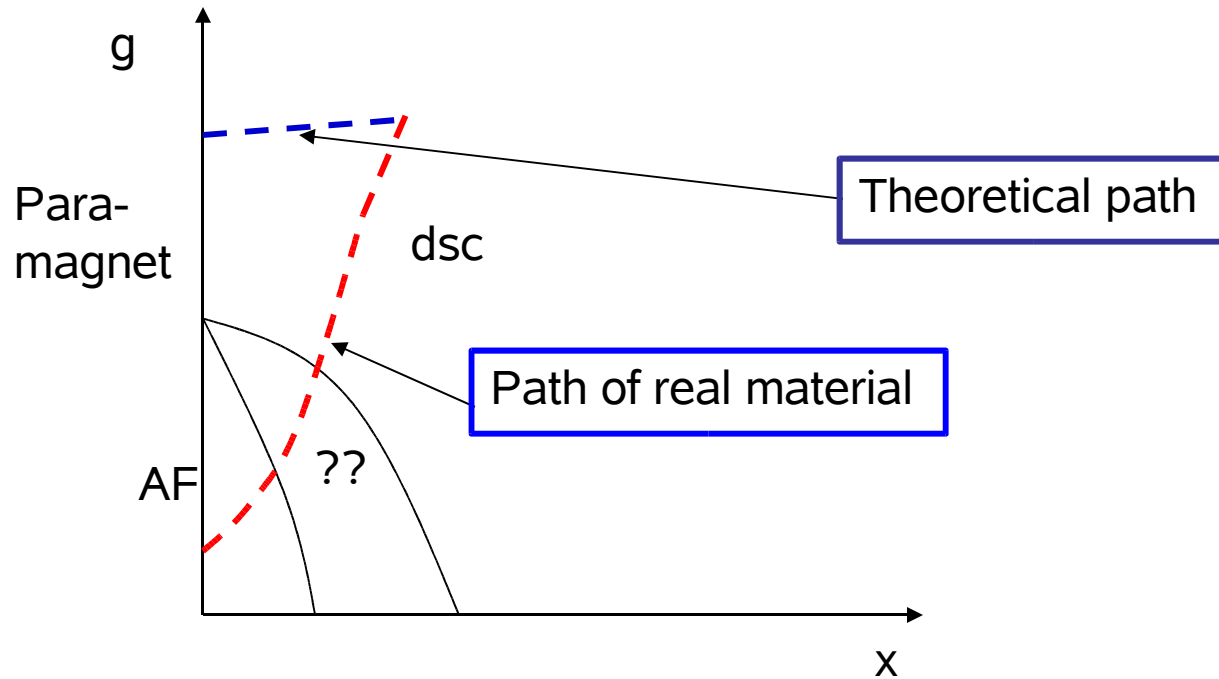
- Charge transport is by holes
- No magnetic long range order (AF LRO quickly destroyed by hole motion)
- Existence of spin gap

Perhaps useful to view as doped paramagnetic Mott insulator.

Further theoretical bonus: Superconductivity a natural outcome of doping paramagnetic Mott states

(old RVB notion – Anderson, Kivelson et al, Kotliar-Liu,.....)

# View as doped paramagnetic Mott insulator (a very old idea actually)



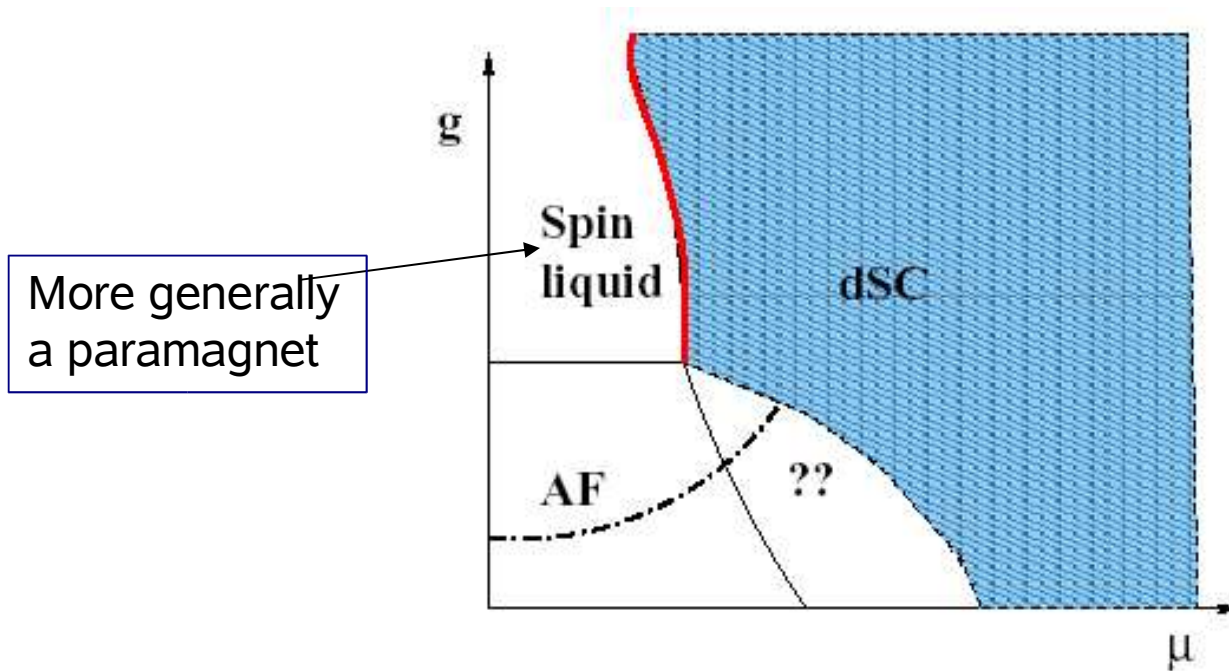
$g$  = frustration/ring exchange,....

# Questions

1. How to sharpen?
3. What paramagnet to dope?
5. How to test?

# How to sharpen?

- Useful to consider phase diagram as a function of chemical potential rather than doping



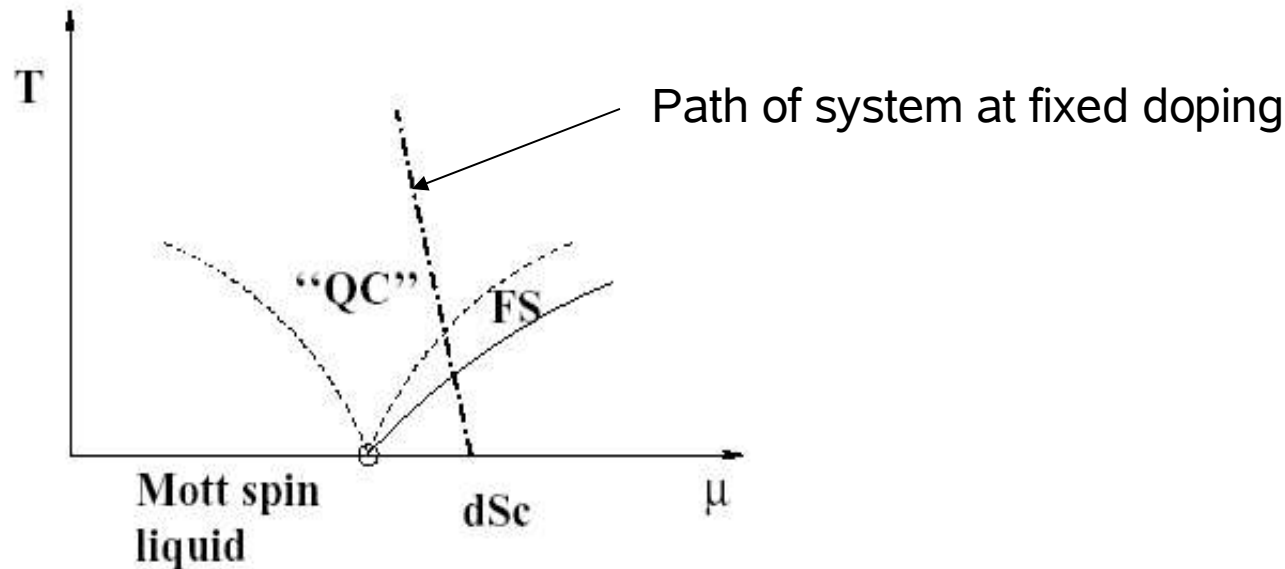
# Theoretical suggestion

(implicit in much previous work)

- Physics at moderately low doping:

Influenced by proximity to chemical potential tuned Mott transition between spin liquid Mott insulator and dSc.

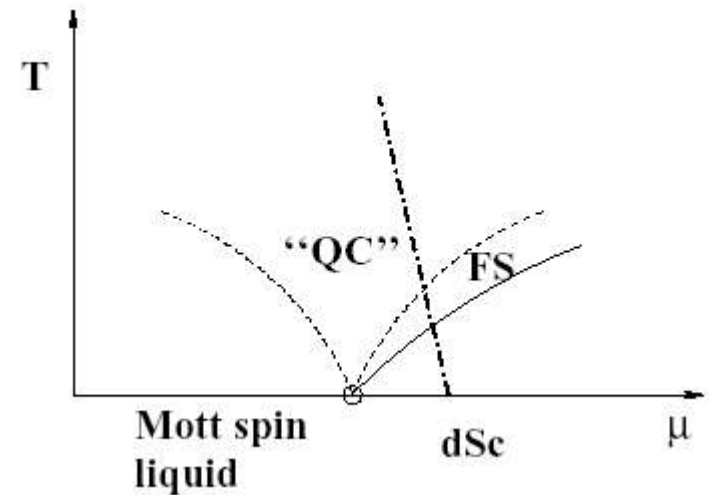
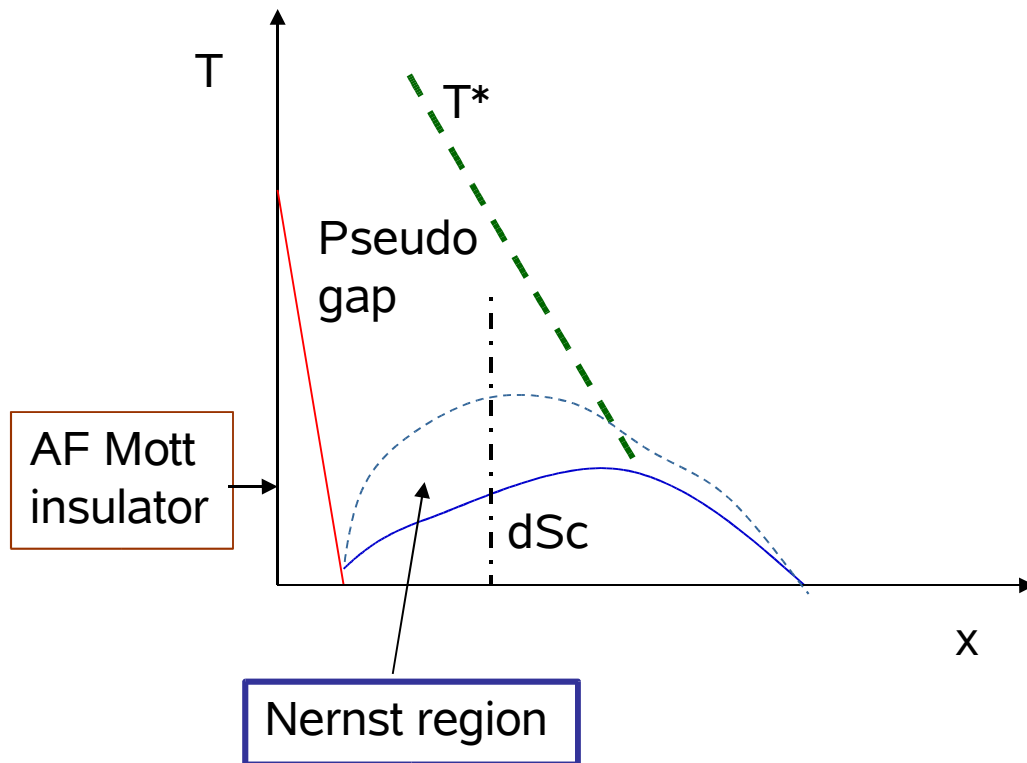
# Doping induced Mott criticality from a spin liquid



QC: "quantum critical" region of Mott transition.

FS: Fluctuating superconductor associated with  $T > 0$  superconducting transition

# Comparison to cuprate phase diagram

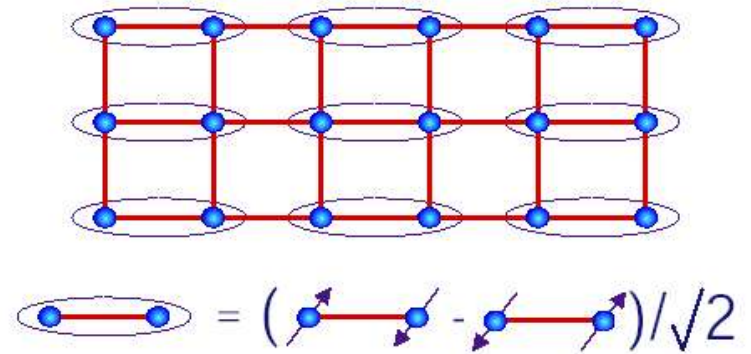


FS: Nernst region  
QC: "High- $T$ " pseudo-gap region

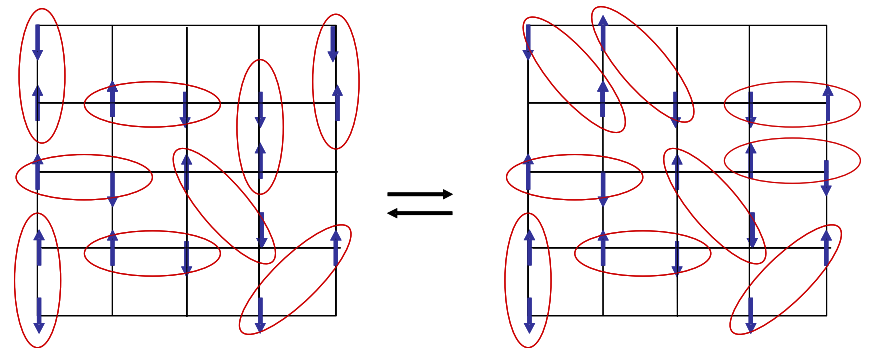
# What paramagnet to dope?

## Theoretical candidates

### 3. Valence bond solid (spin Peierls) states



### 7. Various kinds of RVB spin liquids



# What paramagnet? Some hints from experiments

- Softening of neutron resonance mode with decreasing  $x$ 
  - consider paramagnets proximate to Neel state  
i.e potentially separated by 2<sup>nd</sup> order transition.
- Gapless nodal quasiparticles in dSC
  - consider paramagnets with gapless spin excitations.

Tight constraints

=> Only few candidates: “gapless spin liquids”

# Theory of spin liquids

(enormous progress in last few years due to several people)

Spin liquid = translation invariant paramagnetic Mott state with one electron per unit cell.

Excitation spectrum of all known examples – describe in terms of spin- $1/2$  neutral spinons.

Specific examples of interest – spinons are gapless at 4 nodal points with linear dispersion

=> Very appealing starting points to dope to get dSc with nodal quasiparticles.

# Theoretical characterization of spin liquids\*

- Topological structure:

Extra 'topological' conservation law not present in microscopic spin model.

Conveniently viewed as a conserved gauge flux.

Different classes of spin liquids distinguished by nature of gauge flux.

Spinons couple minimally to corresponding gauge field.

\*abelian

# Gauge theories and spin liquids

- Conserved gauge flux – important PHYSICAL property of excitation structure of spin liquid phase

Effective theory – ‘deconfined’ gauge theory

=> Gauge description not just a calculational device

Conserved gauge flux in spin liquid can in principle be detected by experiments.

# Example of spin liquid with nodal spinons

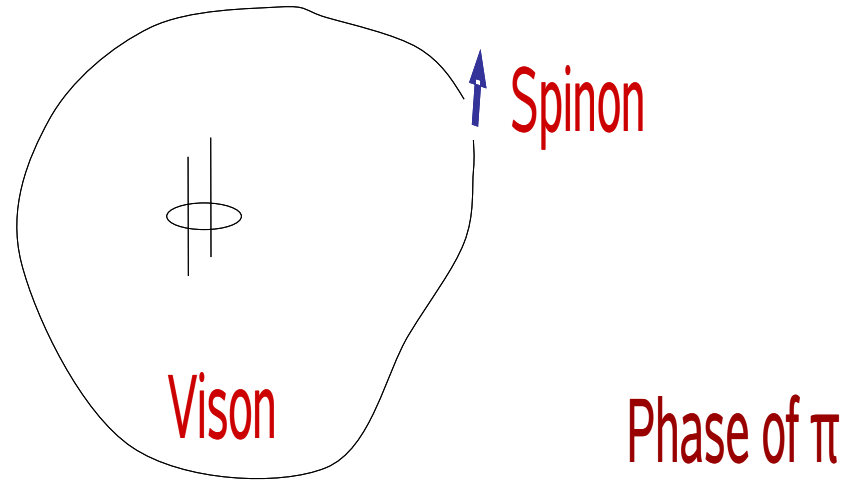
Gapless  $Z_2$  spin liquid:

Conserved  $Z_2$  gauge flux (= “vison”).

Doping a  $Z_2$  spin liquid – attractive  
theory of cuprates but apparently  
not supported by experiments

(eg: no evidence for visons or  
their

consequences – Bonn-Moler flux-  
trapping and other  
experiments).



Are there any other alternatives??

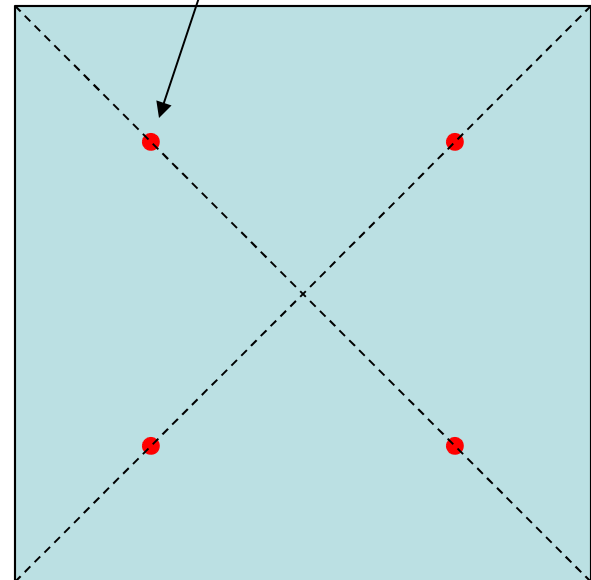
# Alternate possibility: gapless U(1) spin liquids

- Affleck-Marston '88: Flux phase of large-N spin models.

Mean field: Half-filled tight binding band of fermionic spinons ( $f$ ) with a staggered flux through each plaquette (no real breaking of lattice symmetry)

$\phi$	$-\phi$	$\phi$
$-\phi$	$\phi$	$-\phi$
$\phi$	$-\phi$	$\phi$

Band structure: four gapless Fermi points



Low energies: massless Dirac theory in  $D = 2+1$ .

# Beyond mean field

Describe by fermionic massless Dirac spinons coupled to compact U(1) gauge field.

Compactness: Allow for monopole events in space-time where the gauge flux changes by  $2\pi$ .

Ultimate fate?? Confinement??

- Doped versions: Lee, Nagaosa, Wen, .....  
(1996 - ....)

Mostly ignore possibility of confinement.

# Stability of gapless U(1) spin liquids

Hermele, TS, Fisher, Lee, Nagaosa, Wen,  
cond-mat/0404751

- Monopole events irrelevant for low energy physics – at least within a systematic  $1/N$  expansion ( $N$ : number of Dirac species)

Low energy theory is critical with no relevant perturbations (non-compact  $\text{QED}_3$ ) :

conformally invariant with power law spin correlations.

$$\mathcal{L} = \bar{\psi}_j \gamma^\mu (\partial_\mu + i a_\mu) \psi_j + \frac{1}{8\pi e^2} (\epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda)^2.$$

# Physics of the gapless $U(1)$ spin liquid

Monopoles irrelevant ~ “deconfined” spinons.

Precise meaning of “deconfinement”: extra global topological  $U(1)$  symmetry associated with gauge flux conservation.

Power-law correlations in various physical quantities (staggered magnetization, VBS order, etc....).

Gauge flux conservation is physical – can in principle be measured!

# Doping the U(1) spin liquid

A natural possibility [realized in slave boson mean field calculations (Lee, Nagaosa, Wen, ...)]

Doped holes spin-charge separate

⇒ Spin of holes carried away by spinons leaving behind spinless-charged bosons which condense to give dSc.

[Alternate:

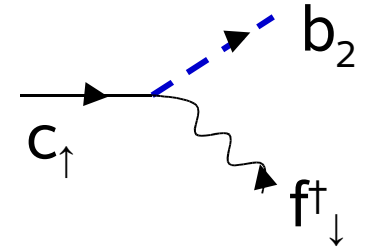
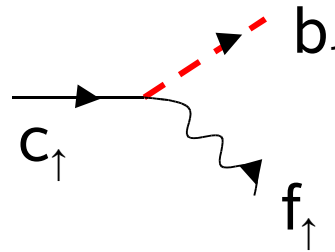
Doped holes retain spin and charge

⇒ At low doping get exotic small Fermi surface metal violating conventional Luttinger theorem]

# Decay routes of hole

- Both  $f_{\uparrow}^{\dagger}$  and  $f_{\downarrow}$  operators create an up spin

=> Expect



$$c_{\uparrow}^+ = b_1 f_{\uparrow}^+ + b_2 f_{\downarrow}$$

$b_{1,2}$  = spin - 0 bosons with physical charge  $e$  but with opposite gauge charges  $\pm 1$ .

$b_1 b_2$  = gauge neutral spin - 0 boson with physical charge  $2e$

= identify with Cooper pair. >

# Physics at finite doping

- Superconductivity: Both bosons condense ( $\Rightarrow$  physical Cooper pair is condensed).
- Nernst region: Bosons have local amplitude but phase fluctuates.
- High-T pseudogap: Bosons above their 'degeneracy' temperature ( $\Rightarrow$  'incoherent')

# Is all this really correct?

## Experimental tests

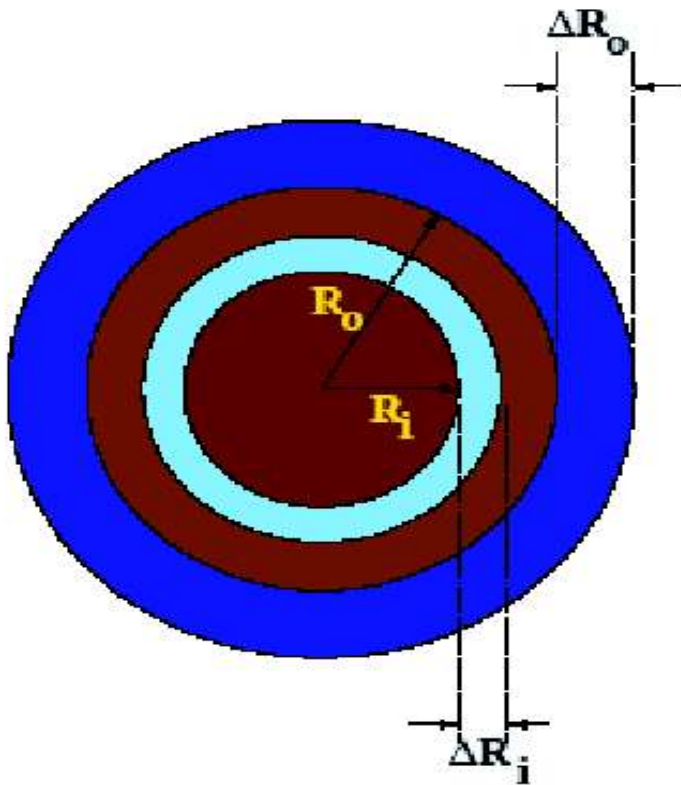
- Crucial ingredient

Conservation of gauge flux of undoped spin liquid  
approximately true at finite- $T$  in doped normal state;  
justifies use of slave particle degrees of freedom.

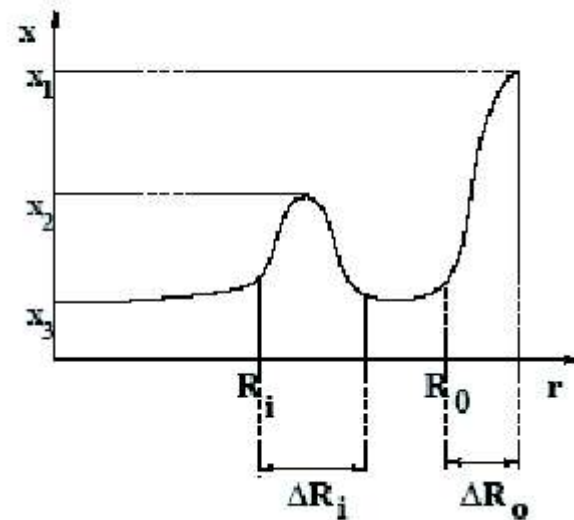
=> Crucial experiment: directly detect the gauge  
flux.

# An idea for a gauge flux detector

TS, Lee, cond-mat/0406066



Cuprate sample with spatially modulated doping as below



# Gauge flux detection

- Start with outer ring superconducting and trap an odd number of  $hc/2e$  vortices  
(choose thin enough so that there is no physical flux).
- Cool further till inner annulus goes superconducting.
- For carefully constructed device will spontaneously trap  $hc/2e$  vortex of either sign in inner annulus.

# How does it work?

- Odd  $hc/2e$  vortex inside outer ring  $\Rightarrow \pi$  flux of internal gauge field spread over the inner radius.

(Lee, Wen, 2001)

- If inner annulus sees major part of this internal flux, when it cools into SC, it prefers to form a physical vortex.
- For best chance, make both SC rings thinner than penetration depth and device smaller than roughly a micron.

## Other possible tests

- Spin physics in high-T pseudogap region expected to be only weakly modified from undoped spin liquid
- => Approximate characteristic finite-T scaling in number of measurable correlators (eg: in  $(\pi, \pi)$  spin response)

# Summary-I

- View of underdoped cuprates as doped Mott paramagnets very appealing starting point.
- Experiments tightly constrain nature of paramagnet to dope.

Current understanding of Mott paramagnet allows for only one surviving candidate – the gapless U(1) spin liquid

# Summary-II

- Gapless  $U(1)$  spin liquids potentially stable in two dimensions, have low energy “gauge fluctuations” characterized by extra conserved quantity (gauge flux) not present in microscopic model.

⇒ Gauge flux is physical.

Doped version: Proposal for experiment to detect gauge flux.

Large number of open questions ----->

# Some open questions-I

## Phenomenology

- Description of location of nominal “Fermi surface”?

(Why does leading edge more or less match band theory? How does node of dSc move away from spinon node at  $(\pi/2, \pi/2)$ ?)

- Description of Fermi arcs?

(Early crude attempt (Wen, Lee) manages to get Fermi pocket with Z small in back portion)

- Velocity anisotropy of nodal quasiparticles?

(At spin liquid fixed point, expect no velocity anisotropy for spinons – can this evolve into large anisotropy seen in dSc?)

- Slope of penetration depth versus T inside dSc?

(Simplest calculations: slope  $\sim x^2$  in disagreement with expt on moderately underdoped samples).

# Some open questions-II

## Basic theory

2. Stability of U(1) spin liquid at half-filling established for large enough  $N > N_c$ .  
Is  $N_c < 2$  so that SU(2) spin models have such phases?  
(Current numerical evidence:  $N_c$  at least  $< 4$ )

Needed: numerics to determine  $N_c$ .

What if  $N_c > 2$  – can theory be salvaged? (see TS, Lee, cond-mat/0406066 for a suggestion)

2. Better theoretical control on charge physics in doped spin liquids.
3. Better microscopic understanding for why doping might push spins into spin liquid state

# General lesson I

- Stable gapless U(1) spin liquids exist in  $D = 2+1$  (at least for SU(N) models and  $N > \text{some } N_{c1}$ ).

$N_{c1}$  possibly smaller than 2, not known at present\*.

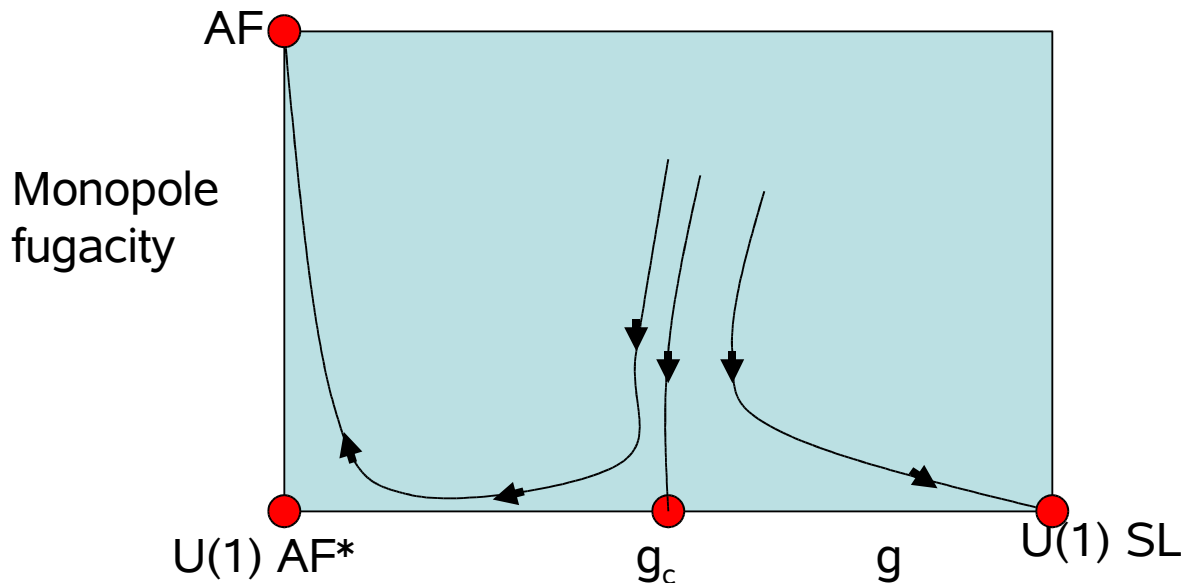
$N_{c1} < 2 \Rightarrow$  appealing description of cuprates as doped U(1) spin liquids.

\*Indications from numerics:  $N_{c1} < 4$  (Assaad, cond-mat/0406...)

# Second order transition to Neel

(induce by increasing strength of quartic spinon interaction)

- Spin density wave of spinons
- Monopoles continue to be irrelevant at critical point to Neel.
- Spinons gapped in Neel phase => monopoles no longer irrelevant, cause confinement to yield conventional Neel state.
- Deconfined critical point with dangerous irrelevant monopoles, 2 diverging length scales, etc.



# Summary, conclusions, etc - I

- Gapless spin liquids exist as stable phases in  $D = 2+1$ .

They may be accessed from conventional Neel by second order transitions.

Needed: Numerics to determine  $N_{c1}$ ,  $N_{c2}$

# Summary, conclusions -II

- U(1) SL with “gapless Dirac spinons” apparently plays an important role whether it is stable or not.

Are the cuprates doped U(1) spin liquids?

How to tell?

Detect conserved U(1) gauge flux!

