Half-filled Landau level, topological insulator surfaces, and three dimensional quantum spin liquids

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Collaborators, etc

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Co-explorers: M. Metlitski (KITP →PI), A. Vishwanath (Berkeley), D. Son (Chicago)

Primary references

1. Son, Phys. Rev. X 5, 031027 (2015) (*arXiv:1502.03446*)

2. C. Wang, and TS, arXiv:1505.03520 and arXiv:1505.05141

3. M. Metlitski and A. Vishwanath, arXiv:1505.05142.

4. C. Wang and TS, arXiv:1507.08290

5. Scott D. Geraedts, Michael P. Zaletel, Roger S. K. Mong, Max A. Metlitski, Ashvin Vishwanath, Olexei I. Motrunich, arXiv:1508.04140

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3. M. Metlitski and A. Vishwanath, arXiv:1505.05142. (M & A talks tomorrow)

4. C. Wang and TS, arXiv:1507.08290 (Synthesis, physical description) This talk

5. Geraedts, Zaletel, Mong, Metlitski, Vishwanath, Motrunich, arXiv:1508.04140 (Mong tomorrow)

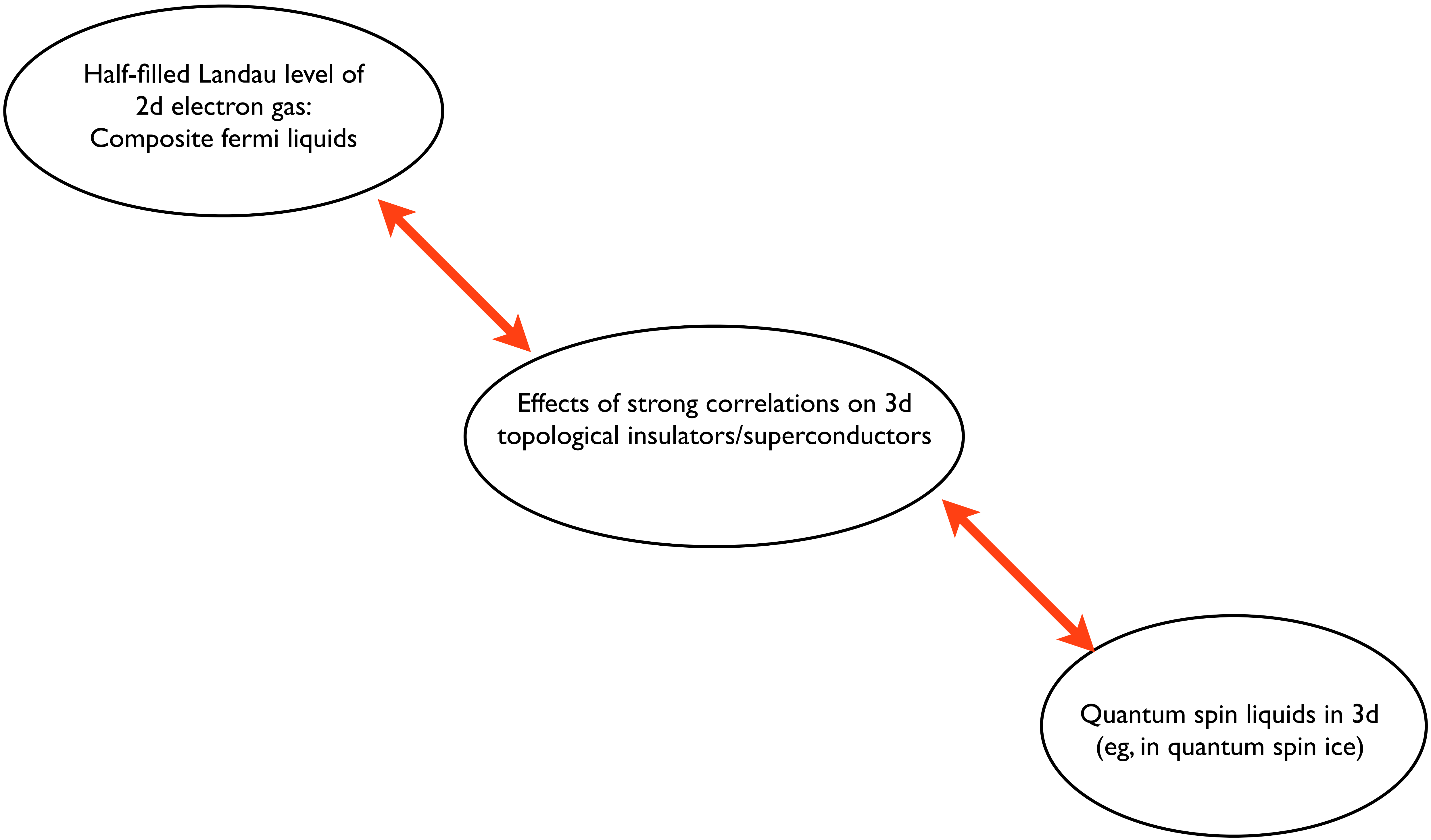
Some other recent pertinent papers (unfortunately not able to discuss in this talk)

1. Mross, Messin, Alicea, PRX 2015 (an exotic gapless surface state of 3d TI)

2. Other recent half-filled LL work: Kamburov, Shayegan et al, PRL 2014 (expt); Barkeshli, Mulligan, Fisher, arxiv, 2015 (theory), Murthy, Shankar arxiv, 2015 (theory)

3. Many papers in the last 3 years on physics aspects of 3d SPT phases
Review: TS, Ann. Rev. Cond. Matt., 2015

Deep connections between 3 apparently different problems



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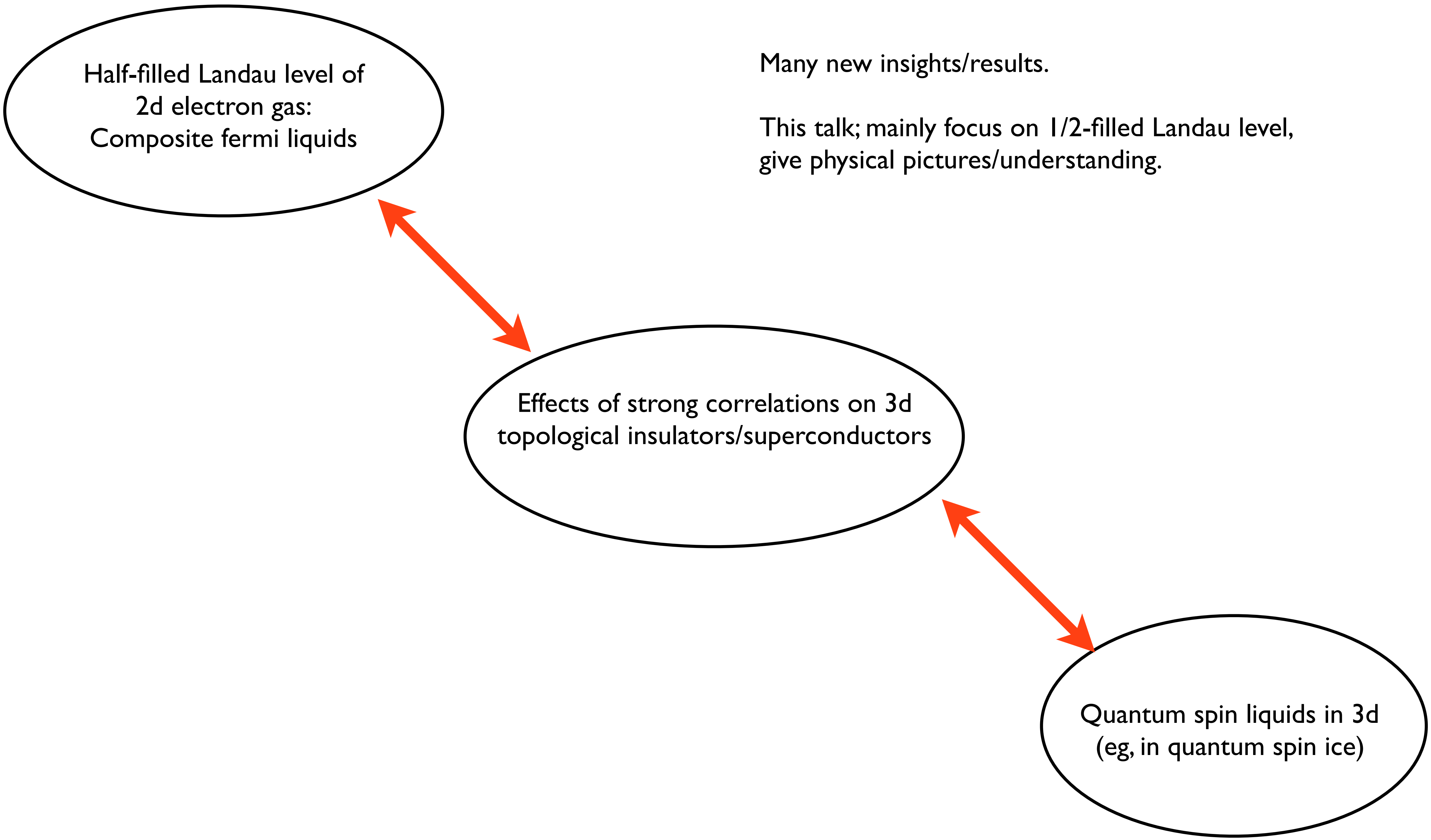


Many new insights/results.

This talk; mainly focus on 1/2-filled Landau level, give physical pictures/understanding.

1/2-filled Landau level: the problem

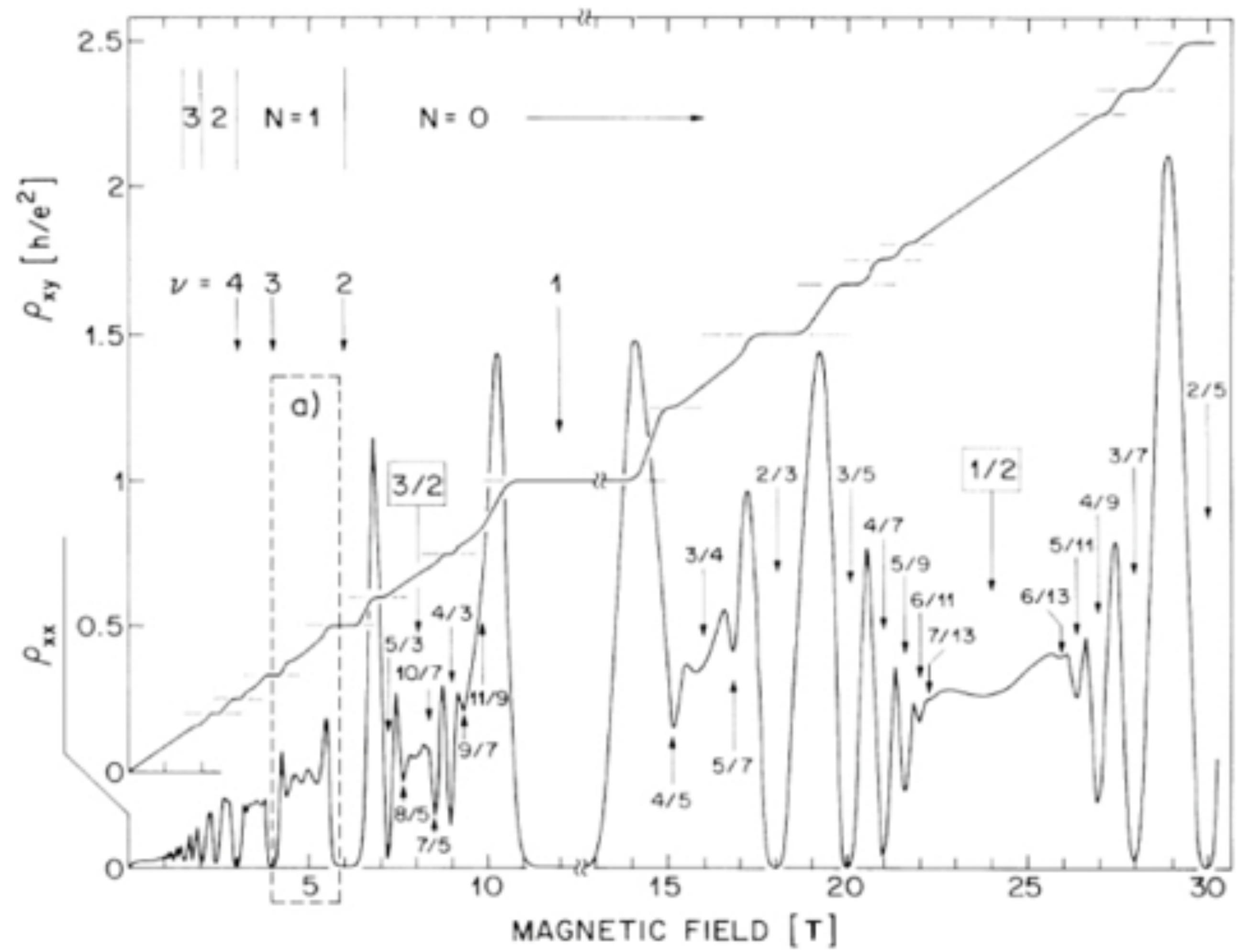
2d electron gas in a strong magnetic field in the ``quantum Hall regime''

Filling factor ν = 1,2., 3, ... (IQHE)

ν = 1/3, 1/5, ... (FQHE)

ν = 1/2 ???
(other 1/(2m))???

Experiment: Metal with $\rho_{xx} \neq 0$, $\rho_{xy} \neq 0$, but $\rho_{xx} \ll \rho_{xy}$.



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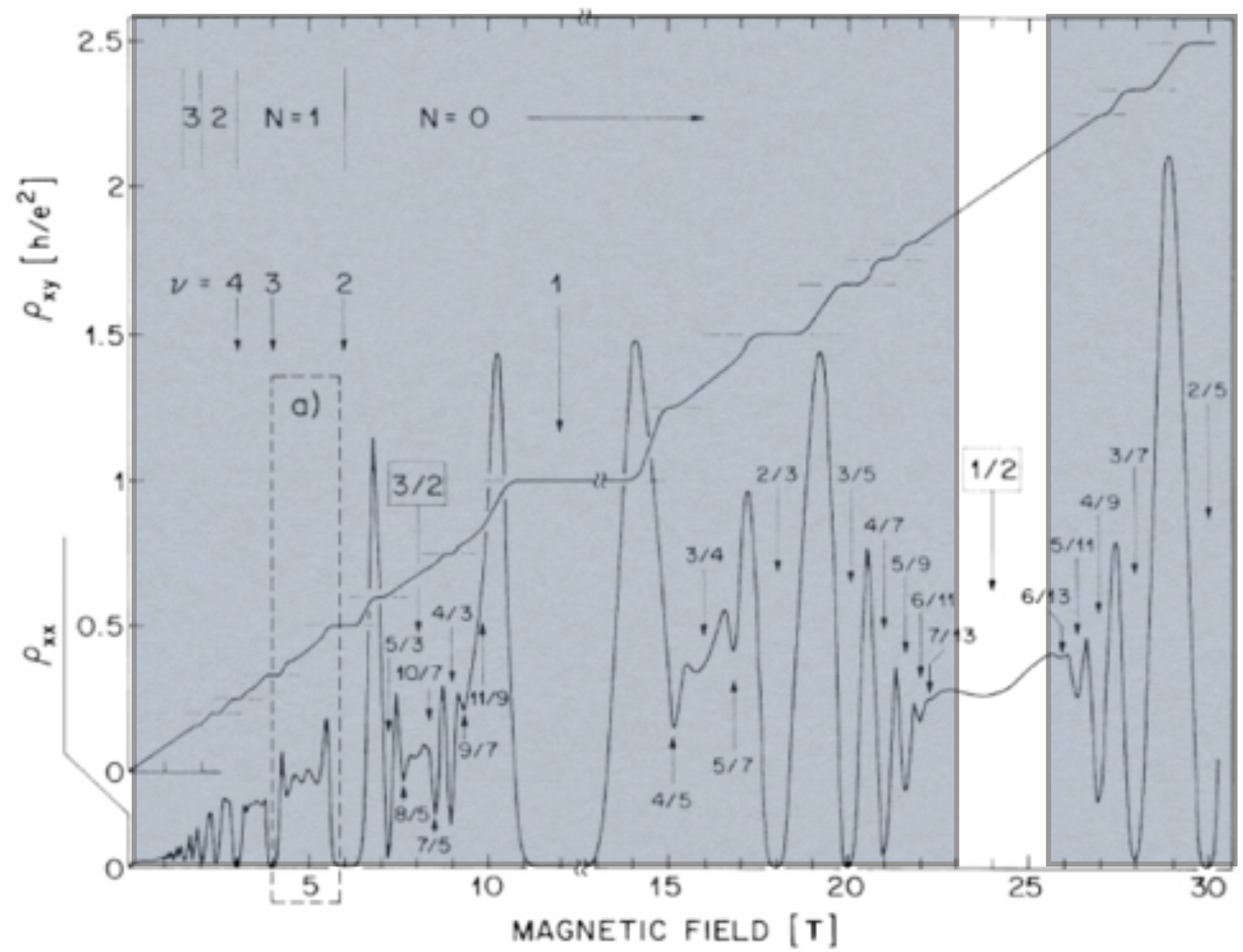
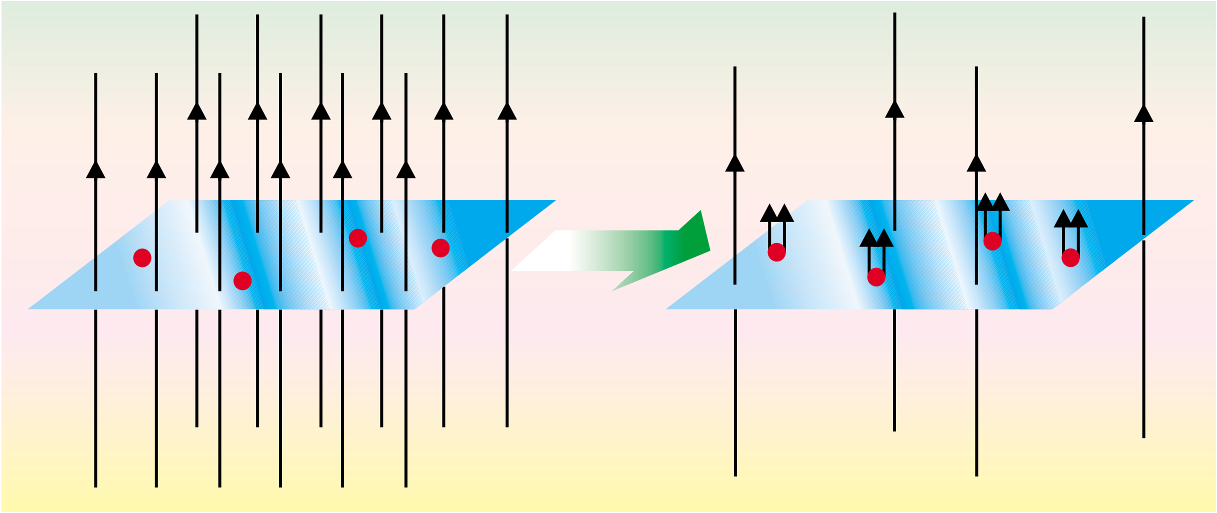


Composite fermi liquid theory
(Halperin, Lee, Read 1993)

Bind 2 flux quanta to electron:
Form composite fermions (Jain 89)
moving (at ν = 1/2) in effective field B* = 0



=> form Fermi surface of composite fermions

Effective theory:

$$\mathcal{L} = \bar{\psi}_{CF}\left(i\partial_t - a_0 - iA_0^{ext} + \frac{(\vec{\nabla} - i(\vec{a} + \vec{A}^{ext})^2}{2m}\right)\psi_{CF} + \frac{1}{4\pi}a_\mu \epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda \quad (1)$$

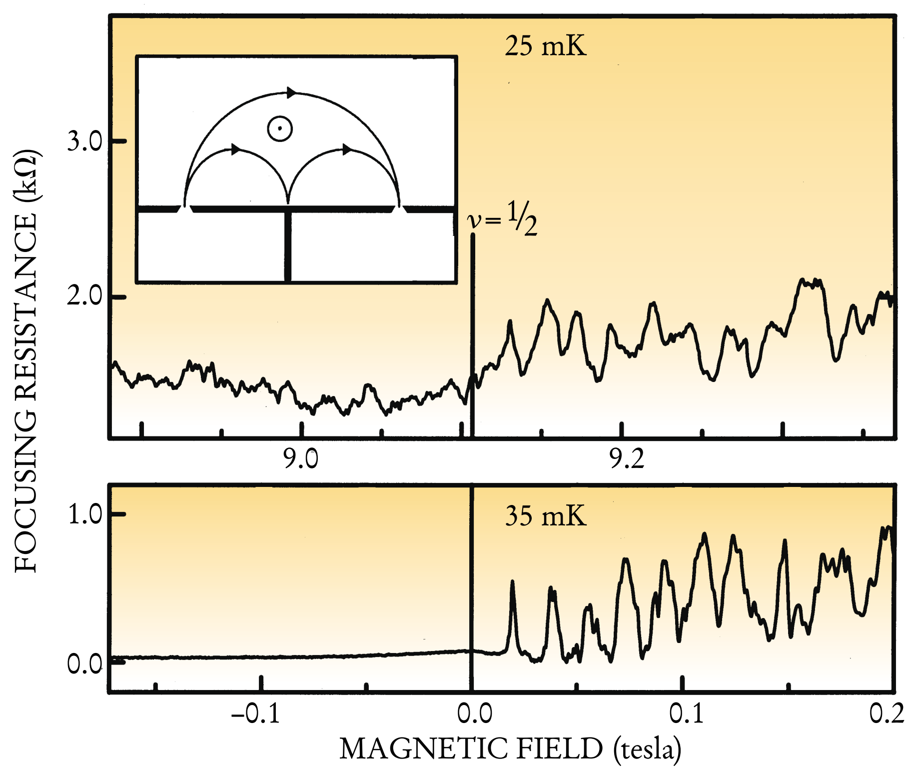
Away from ν = 1/2, composite fermions see effective B*≠0 but reduced from external field B.

Some experimental verification of composite fermions

Many groups: Willett, Stormer, Tsui, Shayegan, Goldman,.....

Examples:

1. Slightly away from $\nu = 1/2$, composite fermions move in cyclotron orbits with radius >> electron cyclotron radius.



2. Confirmation of composite fermion Fermi surface (geometric resonances, Shubnikov-deHaas oscillations)

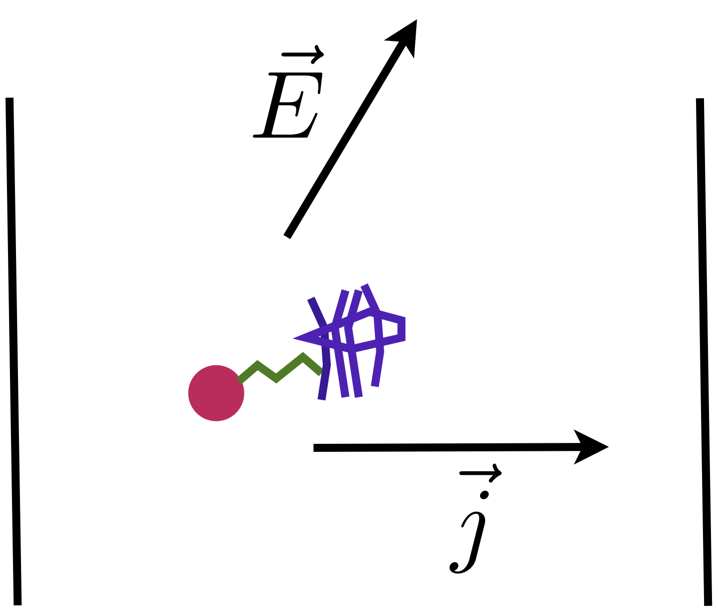
3. Successful theory of experimental finding $\sigma_{xx}(q) \sim q$ (surface acoustic wave)

4. Successful description of many FQHE states in terms of filled composite fermion Landau levels (Jain).

Transport phenomenology of composite fermi liquids

Electrical current = current of composite fermions

but the motion of attached flux induces additional transverse voltage drop.



$$\vec{E} = \overleftrightarrow{\rho*}\vec{j} + \frac{2h}{e^2}\hat{z} \times \vec{j}$$

ρ^*_{ij}: resistivity tensor of composite fermions.

Measured resistivity tensor $\rho_{ij} = \rho^*_{ij} + \frac{2h}{e^2}\epsilon_{ij}$

$(\epsilon_{ij} = -\epsilon_{ji})$

$\rho_{xx} \ll \rho_{xy} \sim \frac{2h}{e^2} \Rightarrow \sigma_{xx} \simeq \frac{\rho_{xx}}{\rho^2_{xy}}$

Heat transport and Wiedemann-Franz violation in composite fermi liquids

Wang, TS, 2015

When composite fermions move, they directly transport heat => very different heat and electrical transport

Conventional metal @ $T \to 0$ (even at $B \neq 0$, with disorder, interactions):

Heat conductivity $\kappa_{xx} = L_0 T \sigma_{xx}$ ("Wiedemann-Franz law")
Lorenz number $L_0 = \frac{\pi^2 k_B^2}{3e^2}$

Composite fermi liquids: $\kappa_{xx} \neq L_0 T \sigma_{xx}$ but $\kappa_{xx} = L_0 T \frac{\left(e^2/2h\right)^2}{\sigma_{xx}}$

Then $\frac{\kappa_{xx}}{L_0 T \sigma_{xx}} = \left(\frac{\rho_{xy}}{\rho_{xx}}\right)^2 > 10^3$

Huge violation of conventional Wiedemann-Franz !

Unfinished business in theory

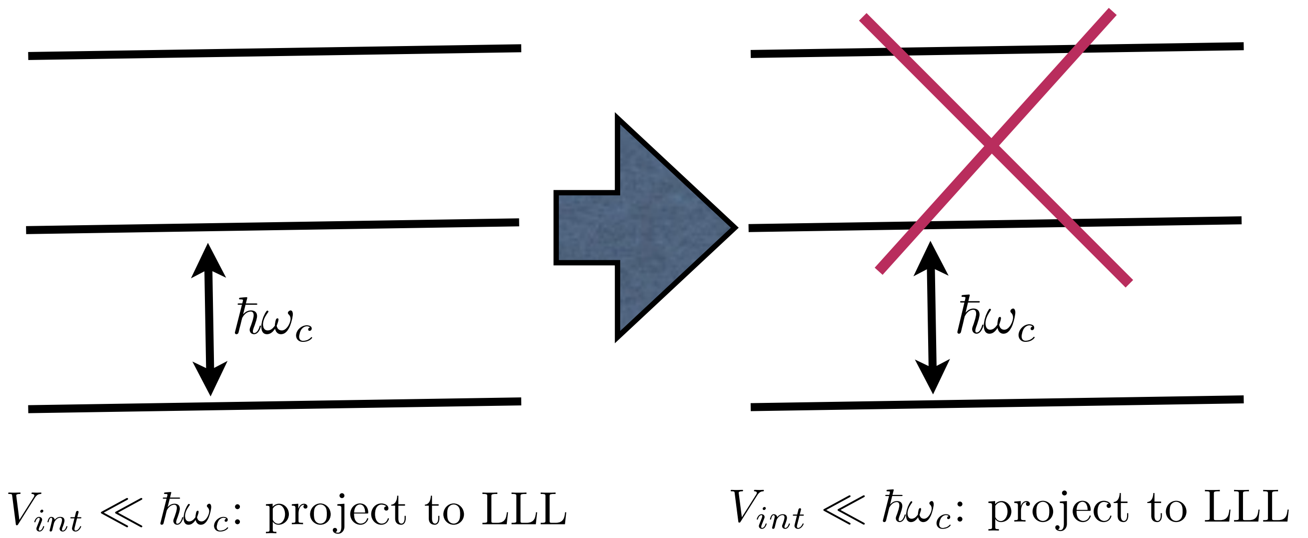
1. Theory should be defineable within the Lowest Landau Level (LLL) but HLR is not in the LLL.

Many refinements in the late 90s (Shankar, Murthy; Read; Halperin, Stern, Simon, van Oppen; D.-H. Lee, Pasquier, Haldane,.......) but dust never settled.

2. Particle-hole symmetry

A symmetry of the LLL Hamiltonian (with eg, 2-body interactions) but not manifest in HLR.

Issue identified in the 90s (Grotov, Gan, Lee, Kivelson, 96; Lee 98) but no resolution.

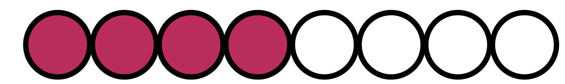


Particle-hole symmetry in LLL

At ν = 1/2, regard LLL as either ``half-empty or half-full'':

Start from empty level, fill half the LL

or start from filled LL and remove half the electrons



Formal:
Electron operator $\psi(x,y) \simeq \sum_m \phi_m(x,y) c_m$ after restriction to LLL ($\phi_m(x,y)$: various single particle wave functions in LLL).

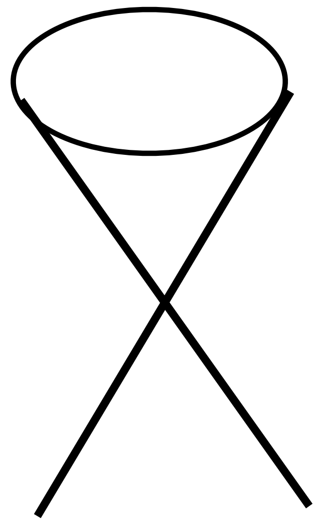
Particle-hole: **Antiunitary symmetry** C
$C\psi C^{-1} = \psi^\dagger = \sum_m \phi_m^*(x,y) c_m^\dagger$

Full symmetry of 1/2-LLL (with, eg, 2-body interaction) is U(1) X C

Particle-hole symmetric CFL: a proposal

Is the Composite Fermion a Dirac Particle? (Son , 2015)

Composite fermion ψ_v forms a single Dirac cone tuned away from neutrality:



Anti-unitary p/h (C) acts in same way as time reversal usually does on Dirac fermion:

$$C\psi_v C^{-1} = i\sigma_y \psi_v => C^2 \psi_v C^{-2} = -\psi_v$$

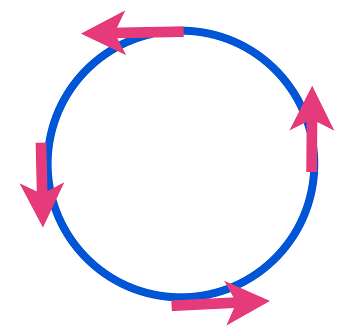
Composite fermion is Kramers doublet under C.

Same as single Dirac cone surface of spin-orbit coupled 3d topological insulators (with C replacing time-reversal) but with an extra coupling to a dynamical U(1) gauge field.

Particle-hole symmetric CFL: a proposal (cont'd) (Son, 2015)

Effective Lagrangian

$$L = \bar{\psi}_v \left(-i\gamma^\mu(\partial_\mu - a_\mu)\right)\psi_v - \mu_v \bar{\psi}_v \gamma_0 \psi_v + \frac{1}{4\pi}\epsilon_{\mu\nu\lambda} A^{ext}_\mu \partial_\nu a_\lambda + L_{bg}[A^{ext}] \quad (1)$$



Low energy theory: focus on states near Fermi surface.

Meaning of Dirac?

As CF goes around FS, pick up π Berry phase.

Particle-hole symmetric CFL: a proposal (cont'd) (Son, 2015)

Effective internal magnetic field $B^* = \vec{\nabla} \times \vec{a} = B - 4\pi\rho.$
Composite fermion density $n_v = \frac{B}{4\pi}$

B^* same as in HLR but composite fermion density n_{CF} is different.
In HLR $n_{CF} = \rho$ (at any filling)

At $\nu = 1/2$, $n_v = n_{CF}$ but they are different away from it.
This slight difference is crucial!

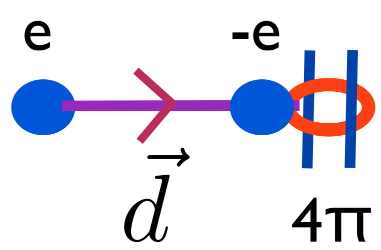
Particle-hole symmetric composite fermions: a physical picture (Wang, TS, 2015)

Old physical picture of composite fermion in LLL

N. Read, 1994; many subsequent papers in late 90s.

CF: electron bound to 4π-vortex: Vortex has depletion of charge -e
but is displaced from electron

=> neutral dipole.



LLL wave function (Rezayi-Read 94)

$$\psi(z_1, ..., z_N) = P_{LLL} det(e^{i\vec{k}_i \cdot \vec{r}_j}) \prod_{i<j} (z_i - z_j)^2$$

$$\vec{d} \perp \vec{k}$$

Plane wave factors $e^{i(k^*z + kz^*)/2}$ push vortex out of electron
in direction perpendicular to momentum.

Dipole moment perpendicular to CF momentum.

However this picture is not p/h symmetric (charge changes
sign but vorticity does not).

New picture of composite fermion in LLL

Fermion wavefunctions in LLL $\psi(z_1, z_2, ..., z_N) = \prod_{i<j}(z_i - z_j) f(z_1, ..., z_N)$

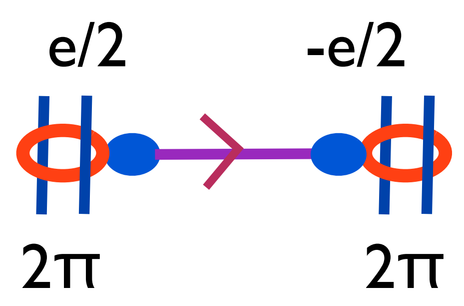
$f(z_1, ...,z_N)$: a symmetric polynomial.

=> one vortex is exactly on electron due to Pauli.

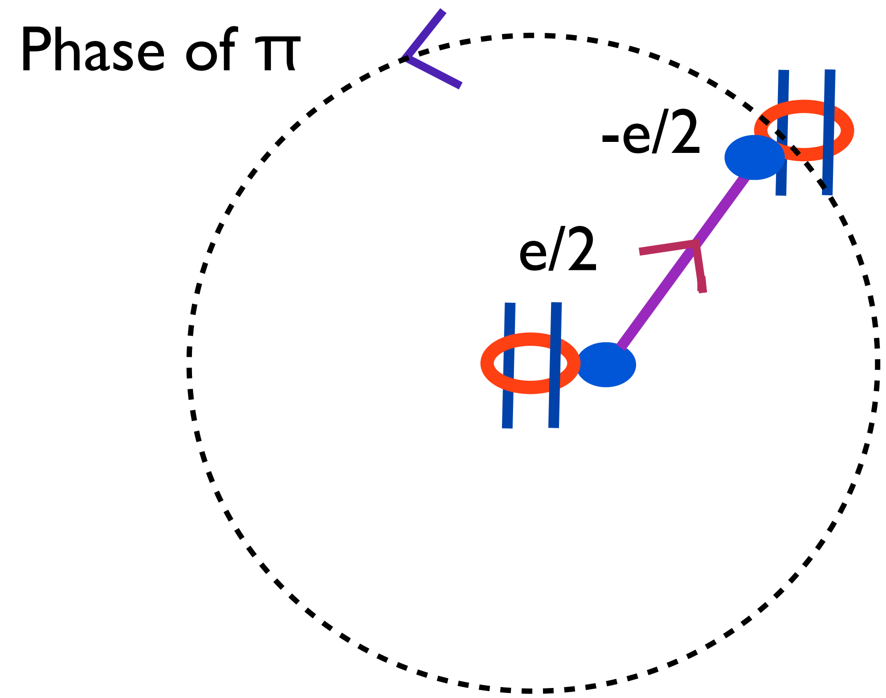
$\nu = 1/2$: can regard $f(z_1,....z_N)$ as wavefunction of bosons at $\nu = 1$ which can also form a compressible composite fermi liquid state.

=> second vortex is displaced from first in direction perpendicular to CF momentum.

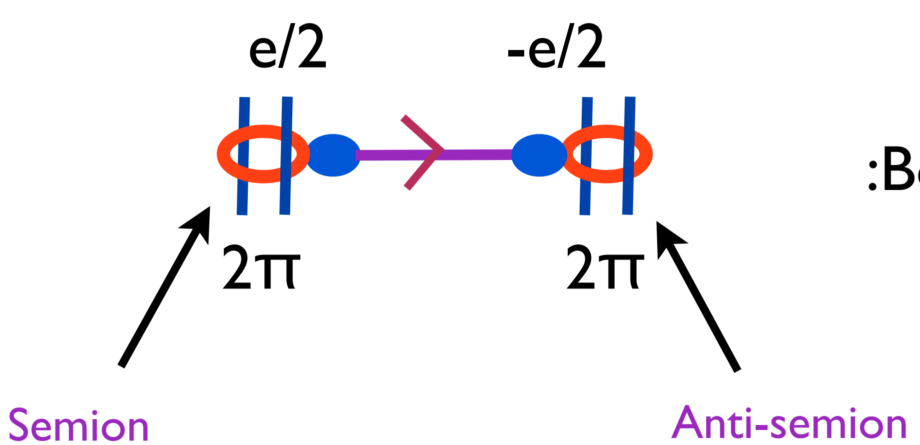
Each vortex has charge -e/2 => single vortex exactly on electron has charge +e/2 and the displaced vortex has charge -e/2.



Internal structure of composite fermion(*) in LLL



Two ends have mutual statistics of π

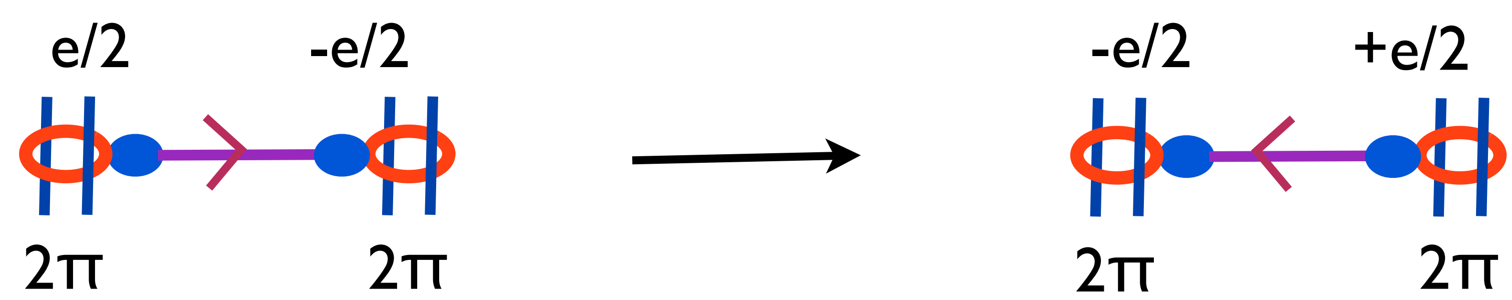


:Bound state (of course) is a fermion.

* Assume ends are well separated compared to vortex size.

New picture of composite fermion in LLL (cont'd)

C:



Anti-unitary C interchanges relative coordinates of the two ends.

As two ends have mutual statistics of π, at low energies they form a ``spin''-1/2 doublet which is Kramers under C.

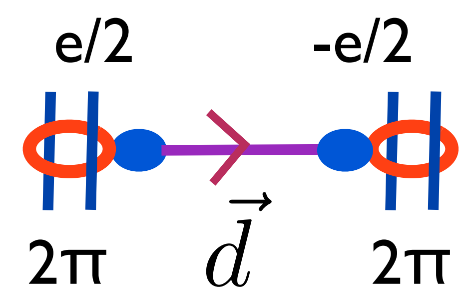
New picture of composite fermion in LLL (cont'd)

Non-zero CF momentum => non-zero dipole moment

=> ``spin" of composite fermion polarized perpendicular to it's momentum

(spin-momentum locking expected of Dirac fermion).

Composite fermion goes around FS => momentum rotates by 2π => spin rotates by 2π => Berry phase of π as expected for a Dirac fermion.



$$\vec{d} \perp \vec{k}$$

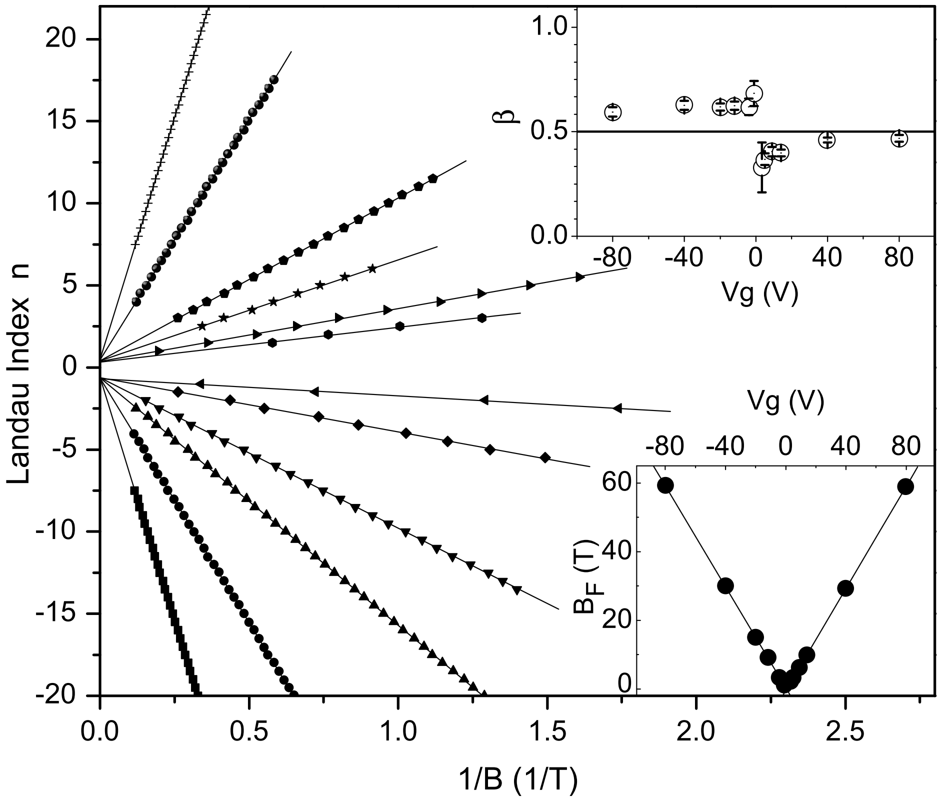
$$=> \langle \vec{\sigma} \rangle \perp \vec{k}$$

Measuring the π Berry phase

Recall: Conventional metals

(eg, graphene)

Measure in Shubnikov-DeHaas (SdH) oscillation data as function of magnetic field at fixed density:



SdH in graphene:
Zhang, Tan, Stormer, Kim, 2005

Resistivity minima at fields $\frac{1}{B_n} = \frac{n+\frac{1}{2}}{F}$

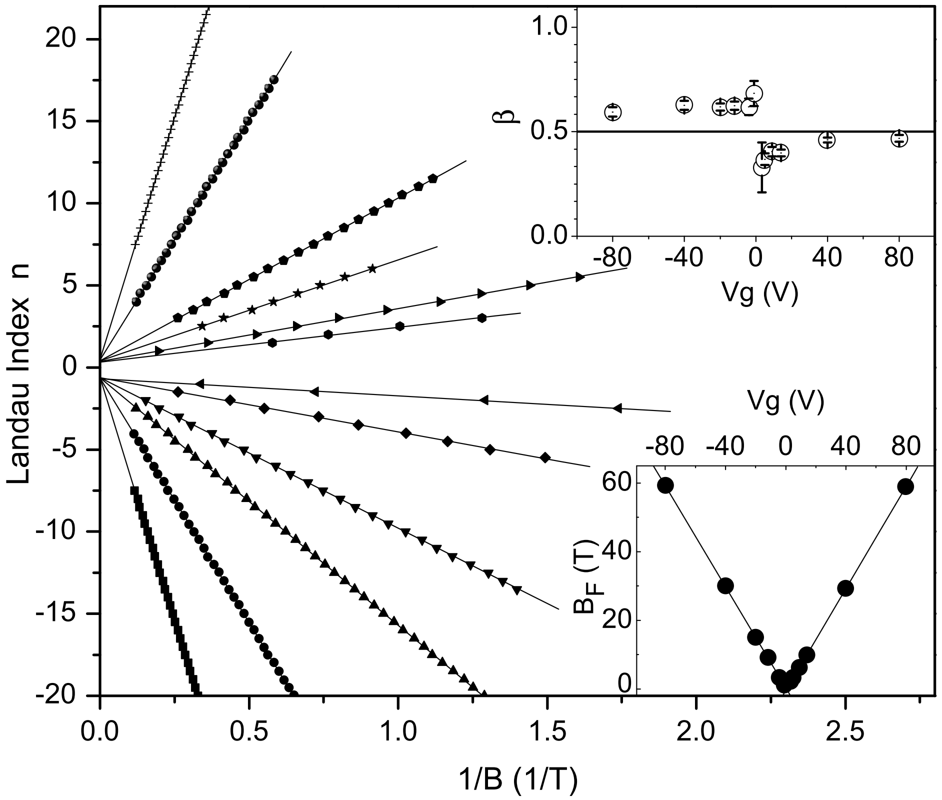
n = integer; F = frequency of oscillations.

Measuring the π Berry phase

Recall: Conventional metals

(eg, graphene)

Measure in Shubnikov-DeHaas (SdH) oscillation data as function of magnetic field at fixed electron density:



SdH in graphene:
Zhang, Tan, Stormer, Kim, 2005

Resistivity minima at fields $\frac{1}{B_n} = \frac{n + \frac{1}{2}}{F}$

Measures Berry phase.

n = integer; F = frequency of oscillations.

Measuring the π Berry phase

Conventional metals

Measure in SdH data as function of magnetic field B at fixed electron density:

Composite Fermi liquids:

Measure in SdH data as a function of ***effective magnetic field*** B* at fixed ***composite fermion density.***

Particle-hole symmetric composite fermi liquid:

$$
\begin{aligned}
n_v &= \frac{B}{4\pi} \\
B^* &= B - 4\pi\rho
\end{aligned}
$$

Plot SdH resistivity minima as a function of 1/B* by varying ρ at fixed B.

Measuring the π Berry phase

Plot SdH resistivity minima as a function of 1/B* by varying ρ at fixed B.

These occur at ``Jain fillings" ν = n/(2n+1)

$$=> \frac{1}{B_n^*} = \frac{n+\frac{1}{2}}{2B}$$ ⇒ **π Berry phase !!**

Contrast with successful interpretation of same data within old theory (no Berry phase):

Crucial difference: Composite fermion density = ρ and no Berry phase (old theory)

versus composite fermion density = B/(4 π) and π Berry phase (new theory)

1/2-filled LL and correlated TI surfaces

Derivation of and more insight into Son's proposal

LLL p/h symmetry is ``anomalous”

C is an emergent low-energy symmetry of a single Landau level (with eg, effective 2-body interactions); not a microscopic local symmetry.

Question:
Can we ``UV complete” the lowest Landau level such that C is an exact local microscopic symmetry?

Answer (see later):

No! (in a strictly 2d system)

Yes - if we regard the Landau level as living at the surface of a 3d topological insulator (or more generally a 3d ``Symmetry Protected Topological” (SPT) phase).

C is an ``anomalous” symmetry.

p/h symmetric LL as a surface of 3d fermion SPT: Preliminaries

Consider (initially free) fermions with ``weird" action of time-reversal (denote C):

$C \rho C^{-1} = - \rho$

ρ = conserved ``charge" density.

Full symmetry = U(1) x C
(called class AIII in Topological Insulator/Superconductor literature*)

Eg: Triplet time reversal-invariant superconductor where physical S_z is conserved and plays the role of such a ρ.

*distinct symmetry from usual spin-orbit coupled insulators which have $U(1) \ltimes C$ symmetry (i.e, ρ is usually even under time reversal).

p/h symmetric LL as a surface of 3d fermion SPT (cont'd)

Free fermion insulators with U(1) x C symmetry: Z classification (Schnyder, Ryu, Furusaki, Ludwig, 08; Kitaev, 08).

Surface: n Dirac cones (n = integer).

C symmetry guarantees that surface Dirac cone is exactly at neutrality.

Electron-electron interactions: $Z \rightarrow Z_8$ ($\times Z_2$) (C. Wang, TS; 14; Metlitski, Chen, Fidkowski, Vishwanath 14)

free fermion states that survive interactions

Beyond free fermions: no band theory description

Focus on n = 1 which is stable to interactions.

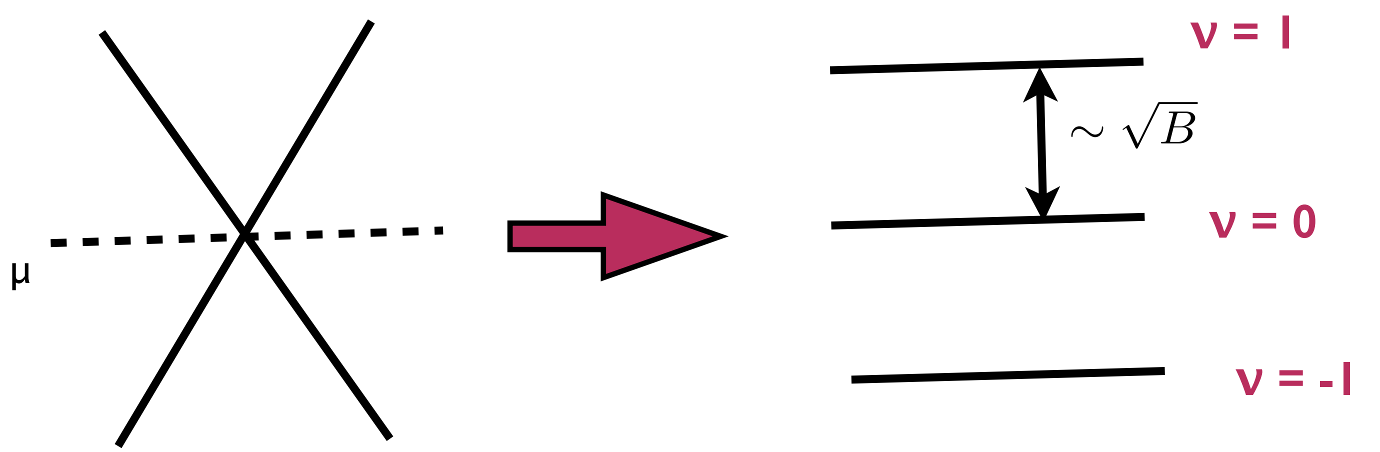
Surface: Single Dirac cone.

p/h symmetric LL as a surface of 3d fermion SPT (cont'd)

ρ is odd under C => `electric current' is even.

External E-fields are odd but external B-fields are even.

=> Can perturb surface Dirac cone with external B-field.



C-symmetry: ν = 0 LL is exactly half-filled.

Low energy physics: project to 0LL

With interactions => map to usual half-filled LL

Comments

1. Half-filled LL obtained in system with a local microscopic U(1) x C symmetry.

2. Surface of such a 3d SPT is a ***natural home*** for the half-filled LL with p/h symmetry.

3. Realization as SPT surface => impossible to UV complete the half-filled LL in a strictly 2d system while keeping p/h exact.

Implication: Study p/h symmetric half-filled LL level by studying correlated surface states of such 3d fermion SPTs.

Correlated surface states of fermion SPTs in 3d: generalities

Free fermion theory: Dirac cones

Interactions:

(1) ``Ferromagnet'' (break C symmetry)

(2) ``Superconductor'' (break U(1) symmetry)

(3) Symmetry preserving gapped state
(must have anyons with ``anomalous'' symmetry implementation.)

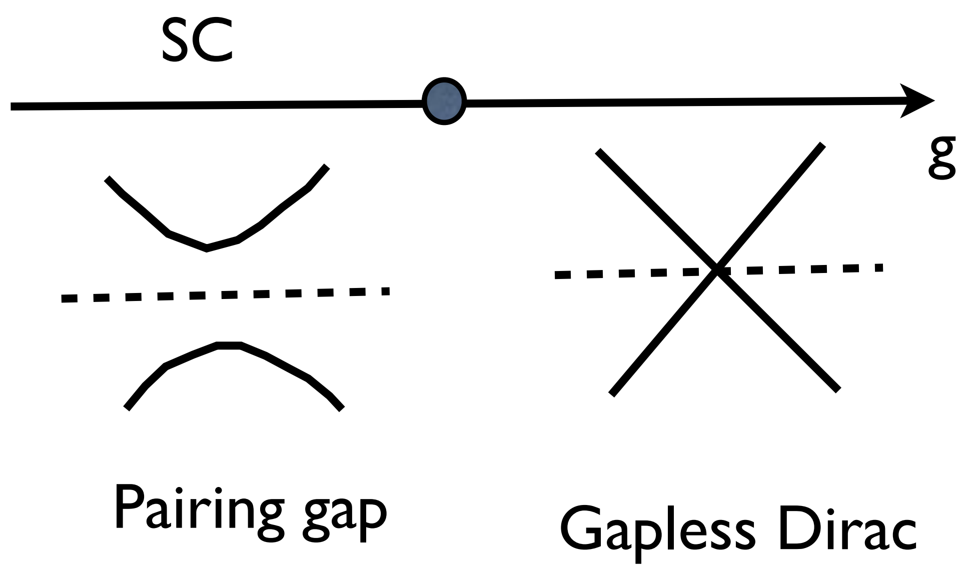
Surface superconductor as a gateway to other states

Surface superconductor is a powerful ``base'' for describing the surface phase diagram

Strategy:

(i) Understand structure of vortices in surface SC

(ii) ``Quantum disorder'' superconductor through phase fluctuations:
=> Proliferate vortices to obtain symmetry preserving state.



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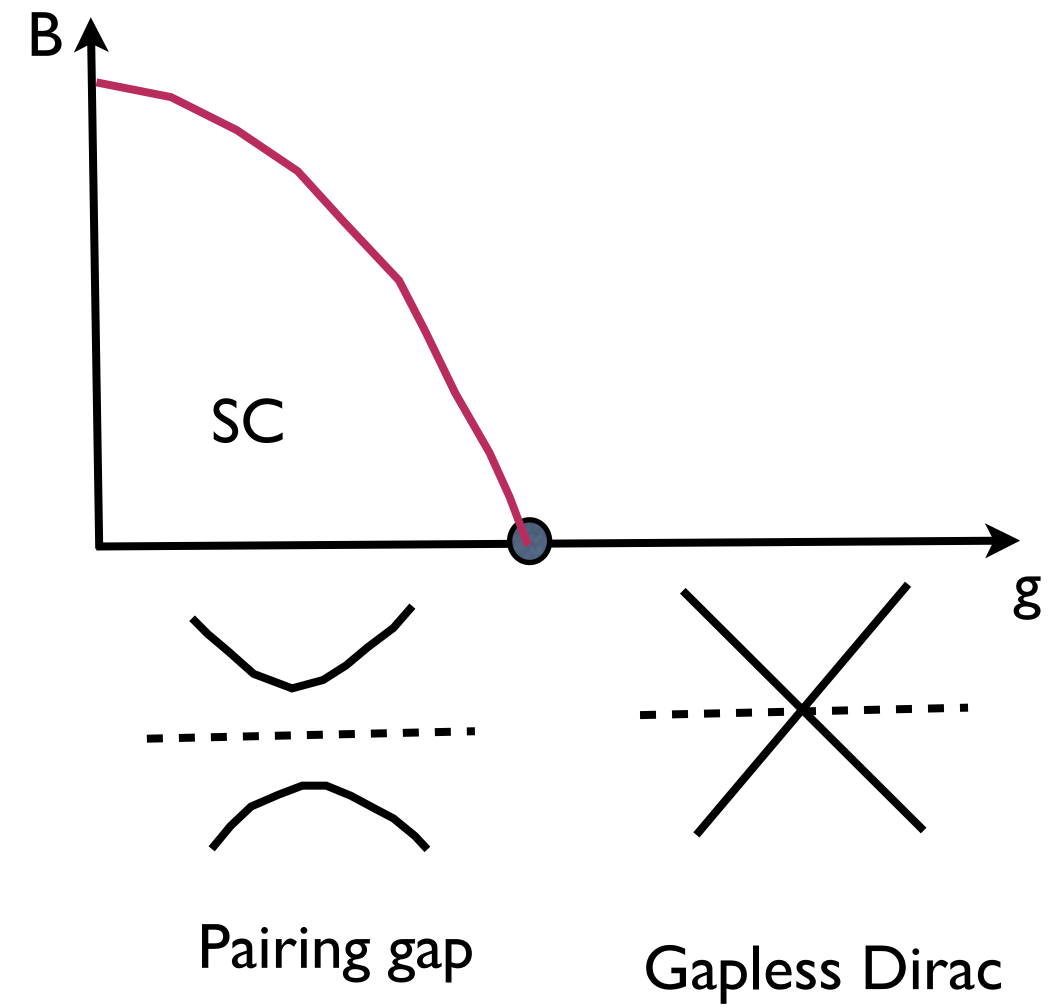

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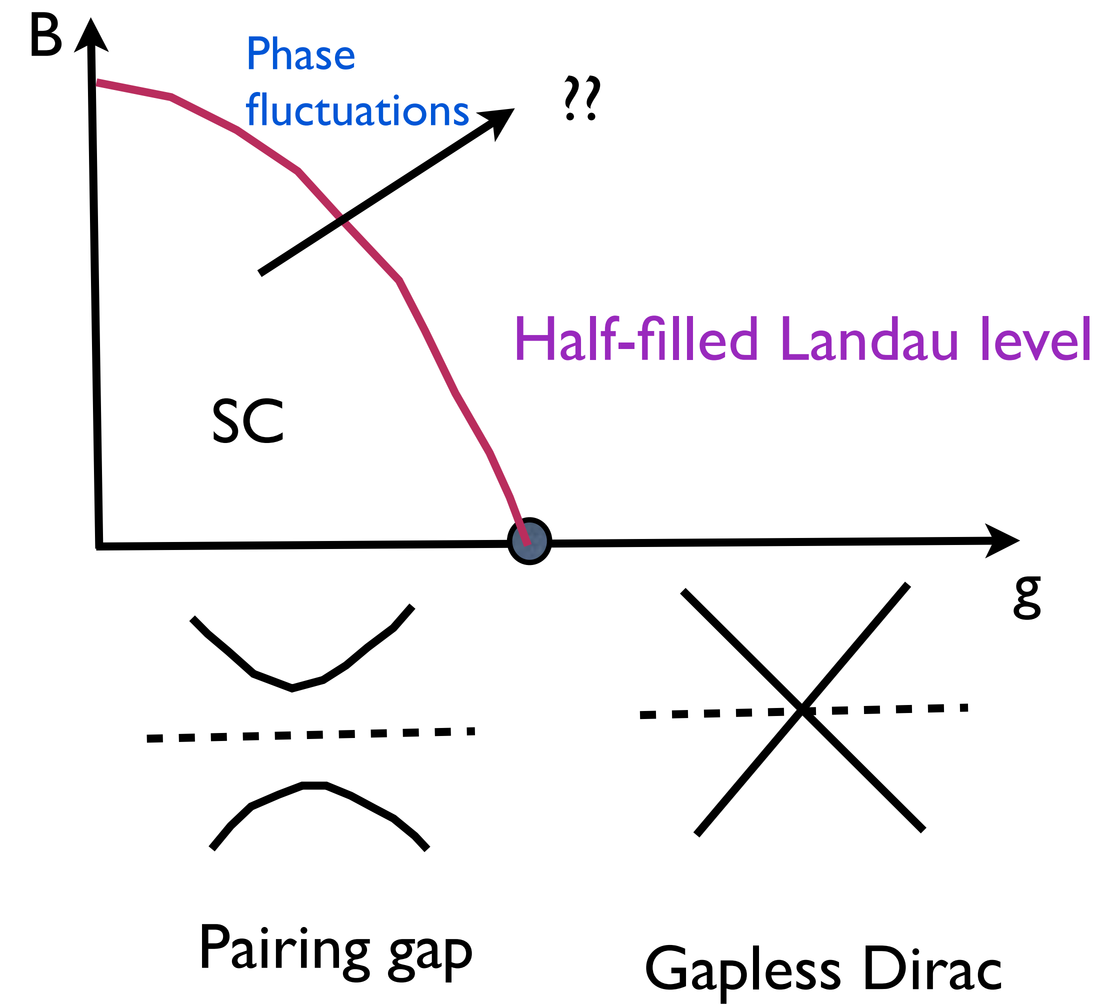


Vortex structure (*) in surface SC

1. Study zero modes of Bogoliubov quasiparticles on vortices (a la Fu, Kane following Read-Green)

- odd strength vortices have unpaired Majorana mode: non-abelian (=> cannot simply condense)

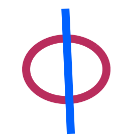
2. Powerful approach for even strength vortices: relate to bulk physics

(*) Caveat: Turn off vortex coupling to Goldstone zero sound fluctuations of the SC, and put them back as minimal coupling to a dynamical U(1) gauge field
(usual principles of 2+1-d charge-vortex duality: Dasgupta-Halperin, 1980; Fisher, Lee, 1989)

Vortex structure in surface superconductor: preview of results

mh/2e vortices with m odd: Unpaired Majorana zero mode

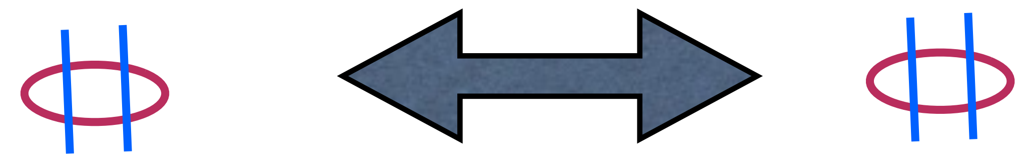
π vortex



``Non-abelian”

h/e vortices: two species interchanged by C

C:

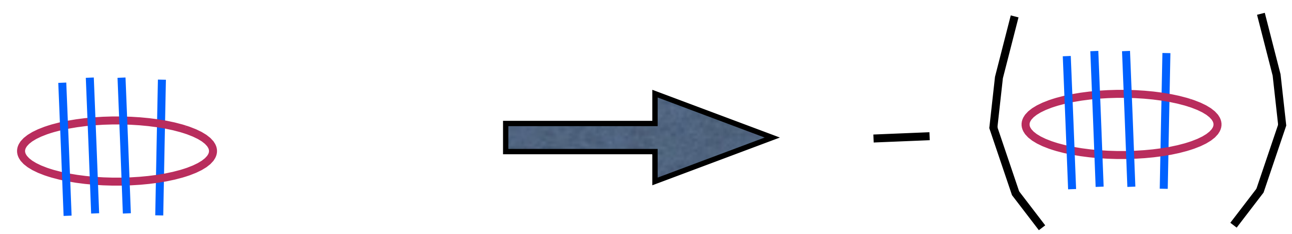


2π- vortex v_{2+} (semion)

2π- vortex v_{2-} (anti-semion)

2h/e vortex: Fermion, Kramers under C

C^2:



4π- vortex v_4 (fermion)

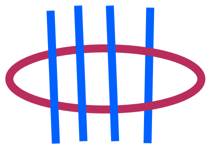
4h/e vortex: trivial boson

Vortex structure in surface superconductor: preview of results (cont'd)

Surface SC can be described as (anamolous) topological ordered state of the v_4 vortex

(same as a surface topological order of standard spin orbit coupled topological insulators)

Disorder the SC in a B-field:
simplest is to proliferate the Kramers fermion v_4 vortex to form a quantum vortex liquid.



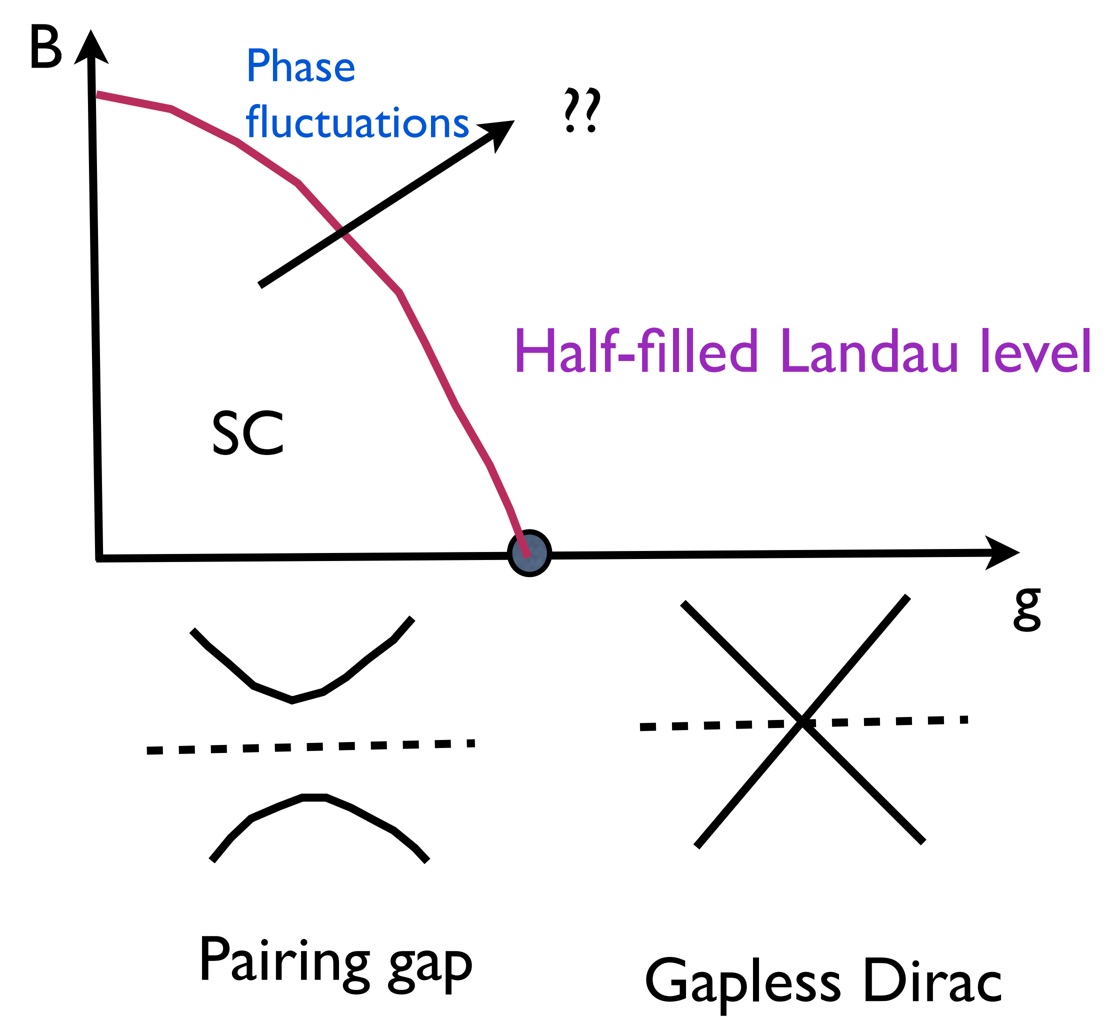
4π- vortex v_4 (fermion)

Vortex structure in surface superconductor: preview of results (cont'd)

Surface SC can be described as (anamolous) topological ordered state of the v4 vortex

(same as a surface topological order of standard spin orbit coupled topological insulators)

Proliferate the v4 vortex => get single Dirac cone coupled to non-compact U(1) gauge field (Son's proposed theory).



Characterizing the U(1) x C fermion SPT in 3+1-d

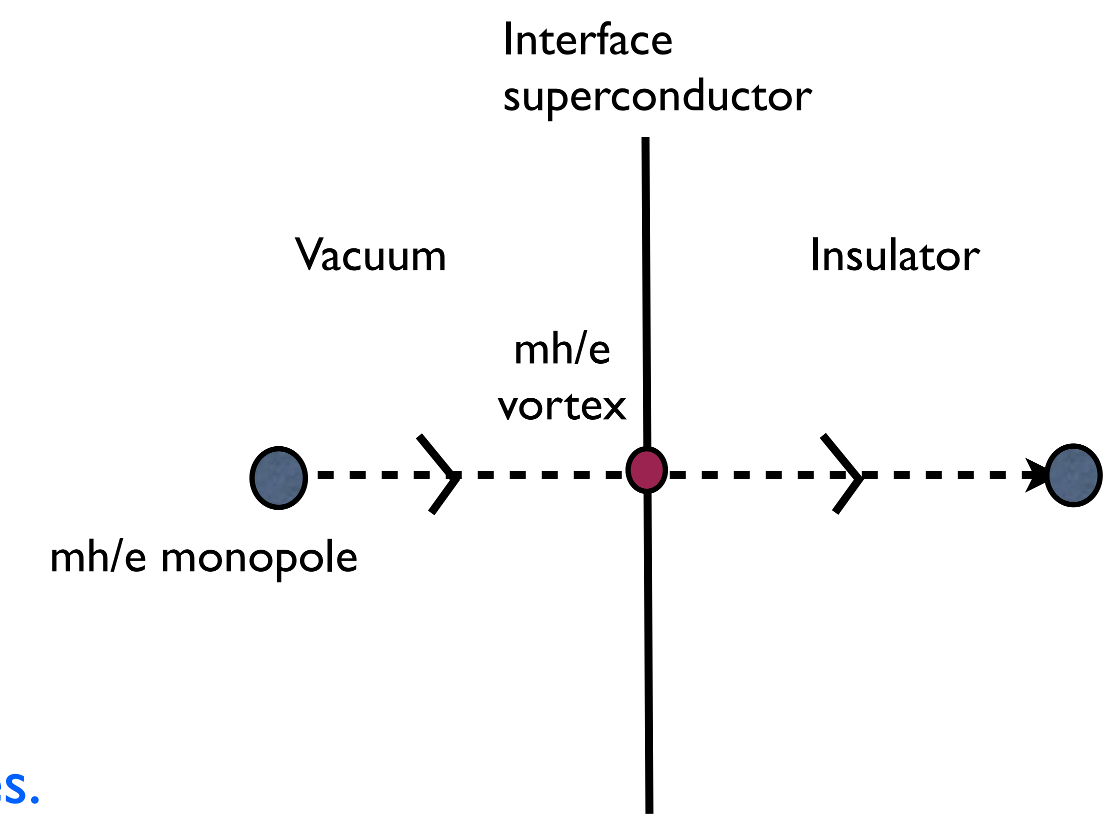
Gauge the U(1) and study monopoles.

After gauging the resulting phase is a U(1) quantum spin liquid.

Strength-q_m monopole has flux q_m h/e

If surface is in SC phase, tunneling a monopole from the vacuum leaves behind a mh/2e vortex with m = 2 q_m

=> Even strength vortices related to bulk monopoles.



Specialize to n = 1 U(1) x C fermion TI:
``Electromagnetic" response and the Witten effect

EM response of *any* 3d insulator

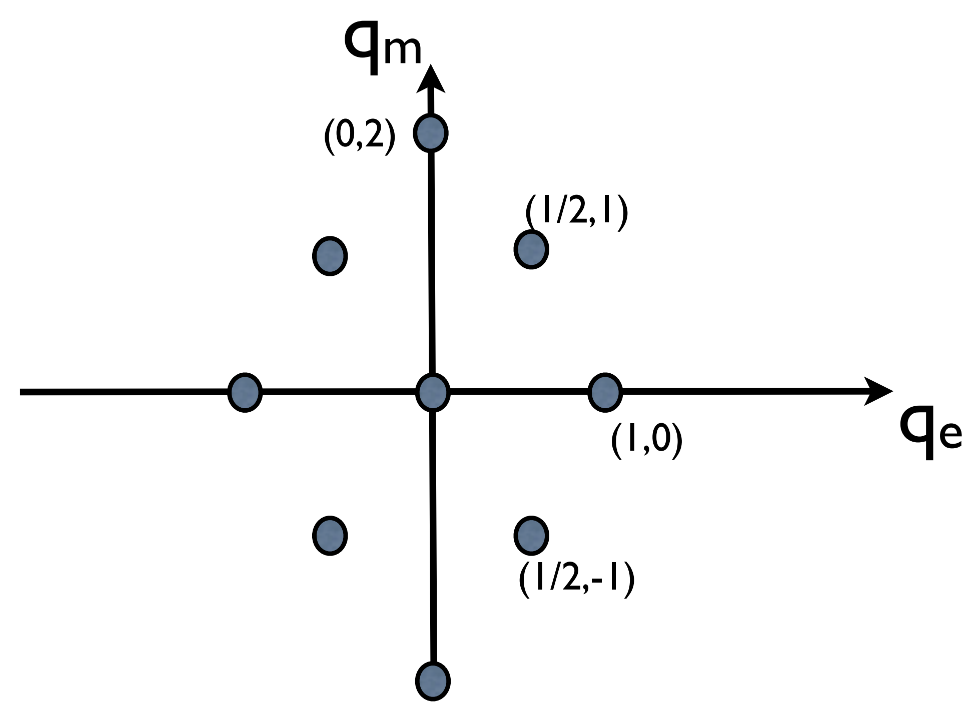
$$\begin{aligned} \mathcal{L}_{eff} &= \mathcal{L}_{Max} + \mathcal{L}_{\theta} \\ \mathcal{L}_{\theta} &= \frac{\theta}{4\pi^2} \vec{E}.\vec{B} \end{aligned} \quad (1)$$

$n = 1$ $U(1) \times C$ fermion TI: $\theta = \pi$.

Witten effect:

θ term => strength-1 monopole has electric charge θ/2π = 1/2 (+ integer)

Bulk properties: Charge-monopole spectrum



1. (1/2, 1) and (-1/2, 1) dyons are both bosons which are interchanged under C.

2. Their bound state (0,2) is a Kramers fermion (Wang, Potter, TS, 13; Metlitski, Kane, Fisher, 13)

3. Bulk monopole - boundary vortex correspondence:
Strength-2 monopole ~ 4π vortex v_4

=> Justifies previous claim that v_4 is a Kramers fermion.

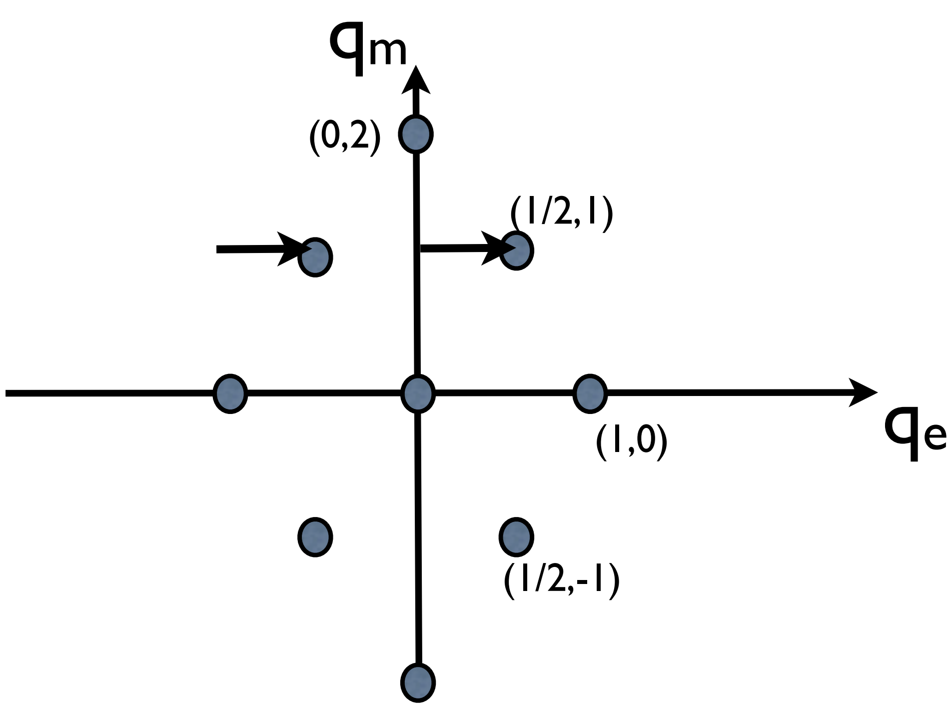
A dual description of the gauged topological insulator

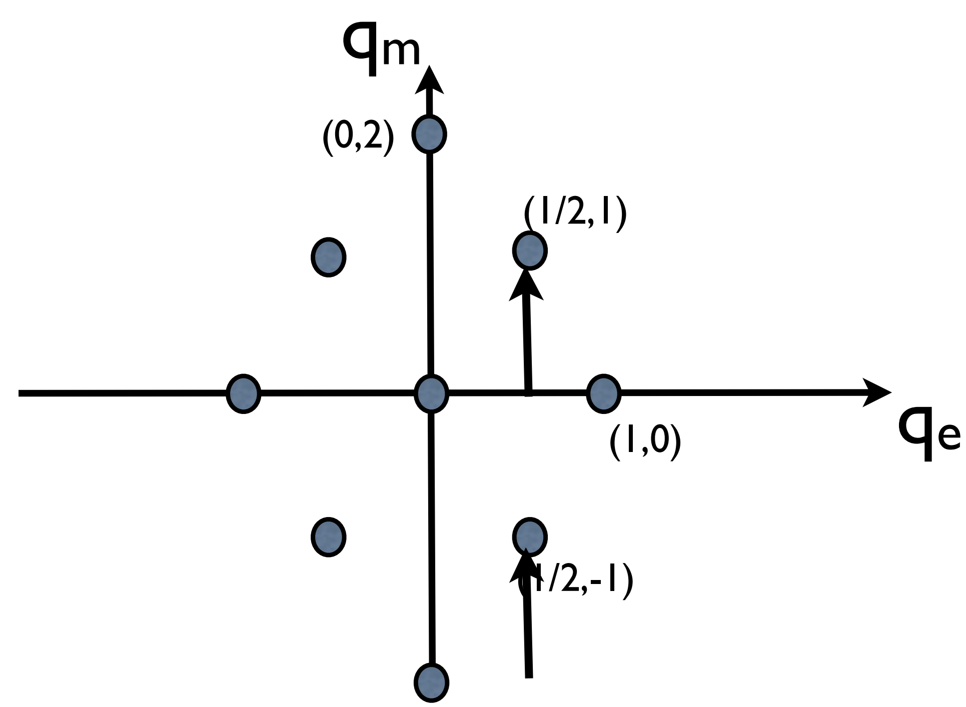
Can view charge-monopole lattice in two equivalent ``dual'' ways.

1. n = 1 TI of (1,0) fermion with $U_e(1) \times C$ symmetry

or

2. TI of (0,2) Kramers fermion with $U_m(1) \ltimes C$ symmetry





Dual description of the surface superconductor

`Electric' point of view: Surface superconductor (break Ue(1) but preserve C)

`Magnetic' point of view: Vortex insulator (preserves Um(1) ⋉ C symmetry)
=> symmetry preserving surface topological order of the monopole TI

Quantum disordering the superconductor: vortex metal phase

Proliferate 4π Kramers fermionic vortex v_4

Density of vortices = $B/(4\pi)$.

Make these vortices mobile => resulting state must still inherit the anomalous implementation of $U_m(1) \ltimes C$ of the surface

=> Form a ***single*** Dirac cone of v_4 vortices tuned away from neutrality.

A ``vortex metal''

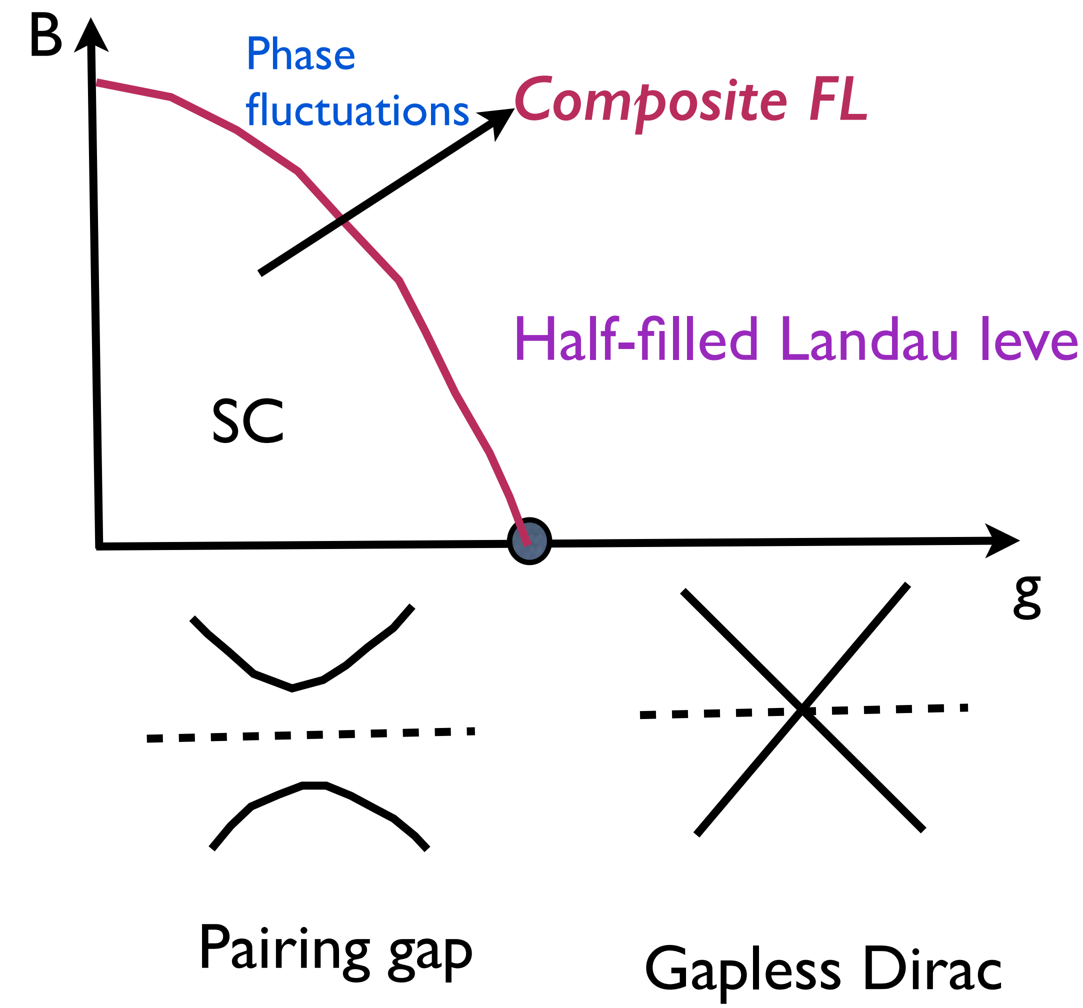
Include coupling to zero-sound mode of superconductor: Vortices couple to non-compact U(1) gauge field (as usual).

Justification of Son's proposal for half-filled LL

Surface SC can be described as (anamolous) topological ordered state of the v_4 vortex

(same as a surface topological order of standard spin orbit coupled topological insulators)

Proliferate the v_4 vortex => get single Dirac cone coupled to non-compact U(1) gauge field (Son's proposed theory).



Comments: Composite fermi liquids as vortex metals

Vortex metal (as opposed to charge metal) viewpoint of CFL:

1. Composite fermion density = density of 4π vortices

(as opposed to density of electrons in usual HLR).

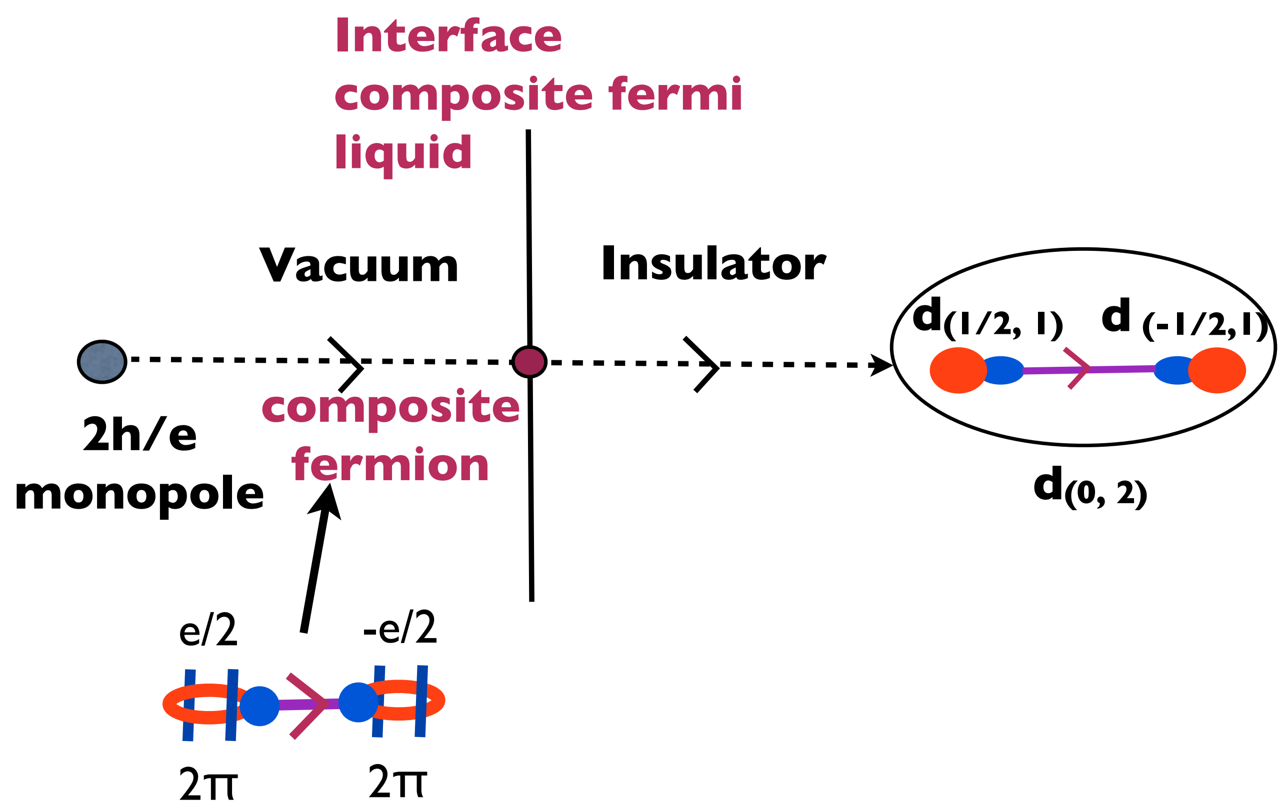
2. Simple physical understanding of transport (electrical, heat, Wiedemann-Franz violation)

Longitudinal electrical conductivity α composite fermion resistivity

Transverse electrical conductivity = e2/2h (exactly)

3. Similar perspective: nice earlier work on CFL of bosons at $\nu = 1$ (Alicea, Hermele, Motrunich, Fisher, 05) matching LLL theory of Read (98).

Physical picture of the composite fermion: further justification



CFL as a surface state of the 3d fermion SPT:

Boundary composite fermion ~ bulk (0,2) monopole = bound state of two dyons which are bulk avatars of the two ends of the dipole forming the composite fermion.

Comments/summary

1. Old issue of p/h symmetry in half-filled Landau level: simple, elegant answer

2. Surprising connection to correlated 3d TI surfaces

3. Many other results

- clarification of many aspects of correlated surfaces of 3d TIs

- particle-vortex duality for 2+1-d massless Dirac fermions?

- classification of time reversal invariant 3d spin liquids with emergent gapless photon

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