(Effective) Field Theory and Emergence in Condensed Matter

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Effective field theory in condensed matter physics

Microscopic models (e.g., Hubbard/t-J, lattice spin Hamiltonians, etc)

`Low energy' effective field theory

`Low energy' experiments/phenomenology
Effective field theory: *minimal* requirements/challenges

1. *Tractable*: Must be simpler to understand than original microscopic models and relate to experiments

   - continuum field theory often useful but not necessarily of the kind familiar from high energy physics.
Effective field theory: *minimal* requirements/challenges

1. **`Tractable’**: Must be simpler to understand than original microscopic models and to relate to experiments

   - continuum field theory often useful but not necessarily of the kind familiar from high energy physics.

2. **`Emergeable’**: A proposed low energy field theory must (at the very least) be **capable of emerging** from microscopic lattice models in the *`right’ physical Hilbert space* with the *right symmetries*.

   - demonstrate by calculations on `designer’ lattice Hamiltonians.

Designer Hamiltonians do not need to be realistic to serve their purpose.
Conventional condensed matter physics

Hartee-Fock + fluctuations

Structure of effective field theory:
Landau quasiparticles + broken symmetry order parameters (if any).
`Exotic’ quantum matter

Quantum spin liquids, Landau-forbidden quantum critical points, non-fermi liquid metals........

What are the useful degrees of freedom for formulating an effective field theory?

Field theory not necessarily in terms of electrons + Landau order parameters.
Emergeability

A crucial constraint on effective field theories of condensed matter systems

A proposed low energy field theory must (at the very least) be capable of emerging from microscopic lattice models in the `right’ physical Hilbert space with the right symmetries.
Emergeability

Microscopic model (UV theory): We often have a very good idea of the physical Hilbert space and global symmetries of the UV theory if not the detailed Hamiltonian.

Effective field theory (IR theory): To be emergable all its local operators must live in physical UV Hilbert space.

Global symmetries must be "non-anomalous".
A trivial example

UV theory:  Lattice model of charge-e electrons
Symmetries: Charge conservation,....

IR theory:
Non-emergable: Field theory of charge-e bosons

In physical Hilbert space all bosons must have even charge.

Emergable: Field theory of charge-2e bosons

(eg: Ginzburg-Landau theory of superconductors).
An almost trivial example

1. UV theory: Lattice model of spins/bosons

2. IR theory:

   Non-emergable: Free fermions with, eg, Dirac dispersion

   IR theory has a local fermion not legal in UV Hilbert space.
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Emergable: Free Dirac fermions coupled to $\mathbb{Z}_2$ gauge field.

Now fermions are not local....all local operators are bosonic.

Theory of a quantum spin liquid: Can demonstrate emergability of this particular theory through solvable spin models.
Remarks

Fractional quantum numbers/fractional statistics excitations can never be local objects even if they are good IR quasiparticles.

Coupling them to gauge fields in IR is a way to `hide' them from UV.

Gauge fields deconfined => effective theory of a non-trivial phase/phase transition (eg: quantum spin liquids/non-fermi liquids/Landau-forbidden criticality).
A non-trivial example

UV theory: Lattice model of spins with $U(1) \times$ time reversal in $d = 2$ space dimensions

IR field theory: massless $QED_3$ with $N_f$ fermions.

$$\mathcal{L} = \bar{\psi} \left( \gamma^\mu \left( i \partial_\mu - a_\mu \right) \right) \psi + \frac{1}{2e^2} f^2_{\mu\nu}$$

Whether this is emergable or not depends on how symmetry is implemented. Naive global symmetries:

1. $SU(N_f)$:

$$\psi \rightarrow U \psi$$

2. $U(1)$: If $a_\mu$ is non-compact, magnetic flux is conserved and generates a ‘dual’ $U(1)$. 
(Non)-emergability of massless QED3 in XY spin systems

Emergable: Physical $U(1)$ is subgroup of $SU(N_f)$.

Field theory must include terms that break all other symmetries (e.g.: instantons).

These may be irrelevant at IR fixed point (=>emergent IR symmetries).

Example of a gapless quantum spin liquid

Non-emergable: Physical $U(1) = \text{dual}' U(1)$ of non-compact gauge field*.

`Anomalous’ implementation of $U(1) \times T$.

Cannot emerge in any 2+1-d spin system but can only emerge at the surface of a 3+1-d (interacting) topological insulator (Wang, TS, 13)

A very non-trivial example

UV theory: Lattice model of bosons/spins with no symmetry in 3 space dimensions.

IR theory: Massive QED

Field content: (i) Gapless photon
(ii) Gapped electric charge
(iii) Gapped magnetic monopole.
Emergable photons

Such theories are emergable from lattice bosons.

Many `designer’ examples (Motrunich, TS, 02, Hermele, Balents, Fisher 04, Levin, Wen 05, ......)

Currently active experimental search (`quantum spin ice’ materials)

In designer models, gapped (emergent) electric charge may be boson or fermion.

Gapped (emergent) monopole is boson.
Non-emergable photons

Are there lattice boson models in a photon phase where both electric charge and magnetic charge are fermions?

No!!

Massive QED with fermion statistics for both e and m forbidden in strict 3+1-d.
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Key idea: Can think of such a phase as a (gauged) putative topological insulator of fermionic e particles.

Show such a putative topological insulator does not have a consistent surface in the right Hilbert space.

Open question: Can such a theory arise as boundary of 4+1-d theory?