Superconducting Algebraic Holon Liquids

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Cuprate materials: Some indisputable basic facts

1. Zero doping: Mott insulator

2. Mott state has Neel long range spin order

3. Superconductivity in doped materials

4. d-wave order parameter symmetry for SC

Other basic facts – what kind of superconductor?

5. Flux quantization in units of $\frac{hc}{2e}$
(cheapest vortex is $\frac{hc}{2e}$)

6. Phase stiffness $\rho_s(T=0) \sim T_c \sim x$ (at not too low doping)

7. Tendency to break lattice symmetries when underdoped

Other basic facts – gapless superconductivity

8. Thermal conductivity $\lim_{T\to 0} \kappa/T \to$ constant.

9. $\rho_s(x,T) = \rho_s(x,0) - AT$

A independent of x in a wide doping range (interesting implications for T_c etc)
(Lee, Wen '98)

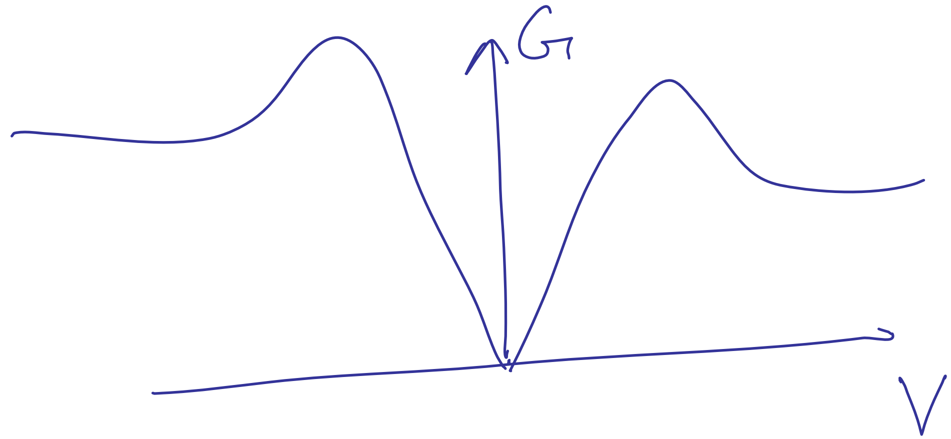
10. Specific heat $C \sim T^2$ in clean samples.

Other basic facts – nodal quasiparticles?

11. Low energy STM spectrum with

$G(V) \sim V$

at $T \gtrsim 4$ K

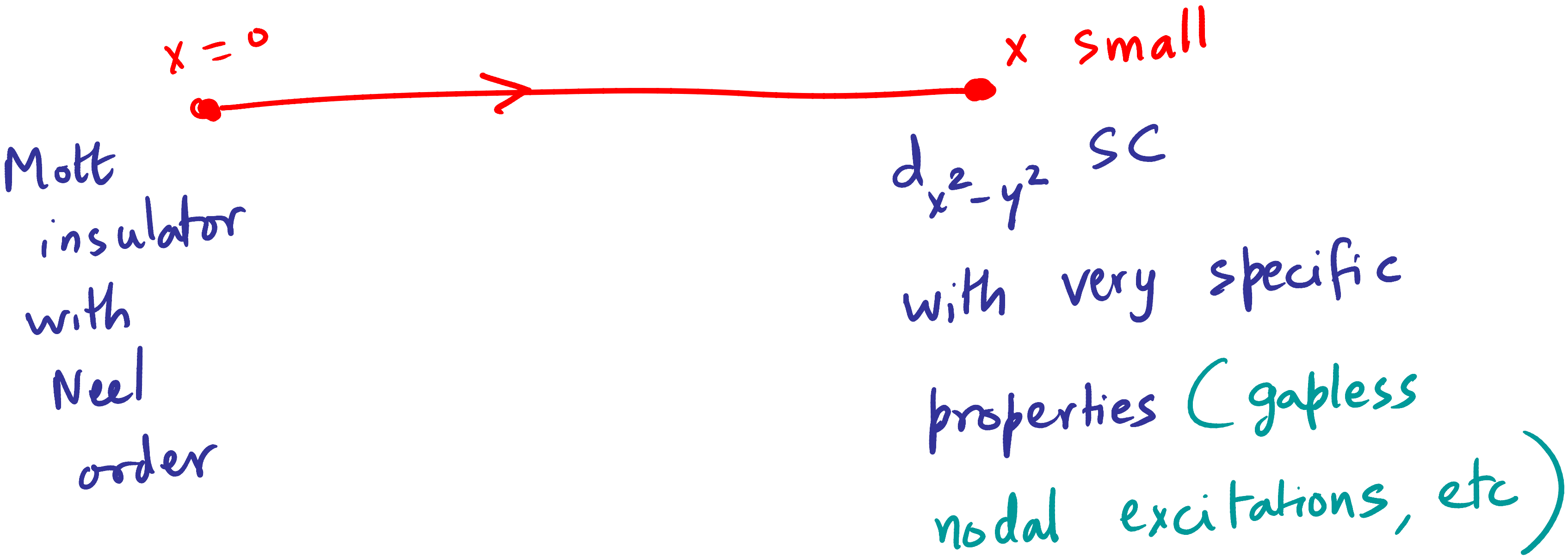


12. Nodal quasiparticles in ARPES ?

(Caution: Linewidth too broad ($\sim E$);

resolution not too high ~ few meV)

Summary of some basic experiments



A theoretical project

Can we find a few different theoretical routes that naturally incorporate these few basic facts about the undoped & underdoped cuprates ??

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Excercise in theoretical "model-building"

- much less ambitious than a full theory of cuprates

(have ignored large number of basic facts about normal state)

A theoretical project (cont'd)

Why bother ?

1. One such theoretical route must also necessarily incorporate the observed strange "normal states".

2. Even this very low ambition project is very hard !

(Existing literature does not seem to have good answers.)

This talk - explore one theoretical route which seems to do very well

Doping an antiferromagnetic Mott insulator: Low doping

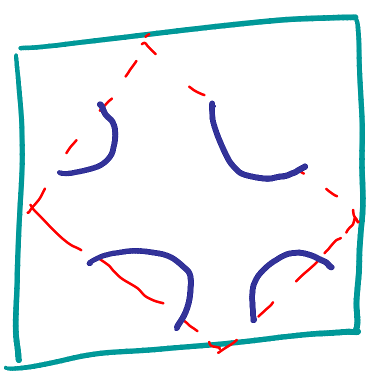
One hole - dispersion minima at $(\pm \pi/2, \pm \pi/2)$

(Kane, Lee, Read '89)

Small density of holes:

Possible state - small hole pockets centered at $(\pm \pi/2, \pm \pi/2)$ + Neel order

(Many papers '88-'91)



A metallic antiferromagnet with hole pockets

Losing the Neel order

Hole motion frustrates Neel order.

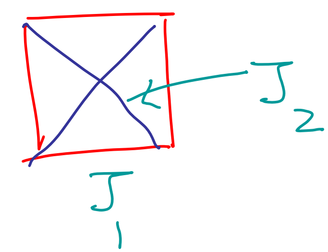
Question: What happens to the antiferromagnetic metal when the Neel order is lost at $T=0$?

Lessons from insulating quantum magnets

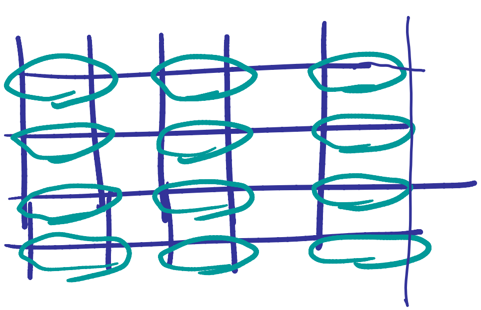
Analagous question well understood in the Mott insulator

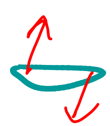
Kill Neel by increasing frustration

(Eg: $J_1 - J_2$ model)



"Natural" result: Valence bond solid paramagnets



 = singlet bond

(Read, Sachdev '89)

Deconfined quantum criticality-I

Neel - VBS transition can be 2nd order despite different broken symmetries !

(Senthil, Vishwanath, Balents, Sachdev, Fisher '04)

Critical theory : "deconfined" bosonic spinons + gauge fields

$$S = \int d^2x\, d\tau\, |(\partial_\mu - i a_\mu) z|^2 + r|z|^2 + u(|z|^2)^2 + K f_{\mu\nu}^2$$

$(z_\uparrow, z_\downarrow)$: spinons ; $a_\mu \sim U(1)$ gauge field ..

Deconfined quantum criticality-II

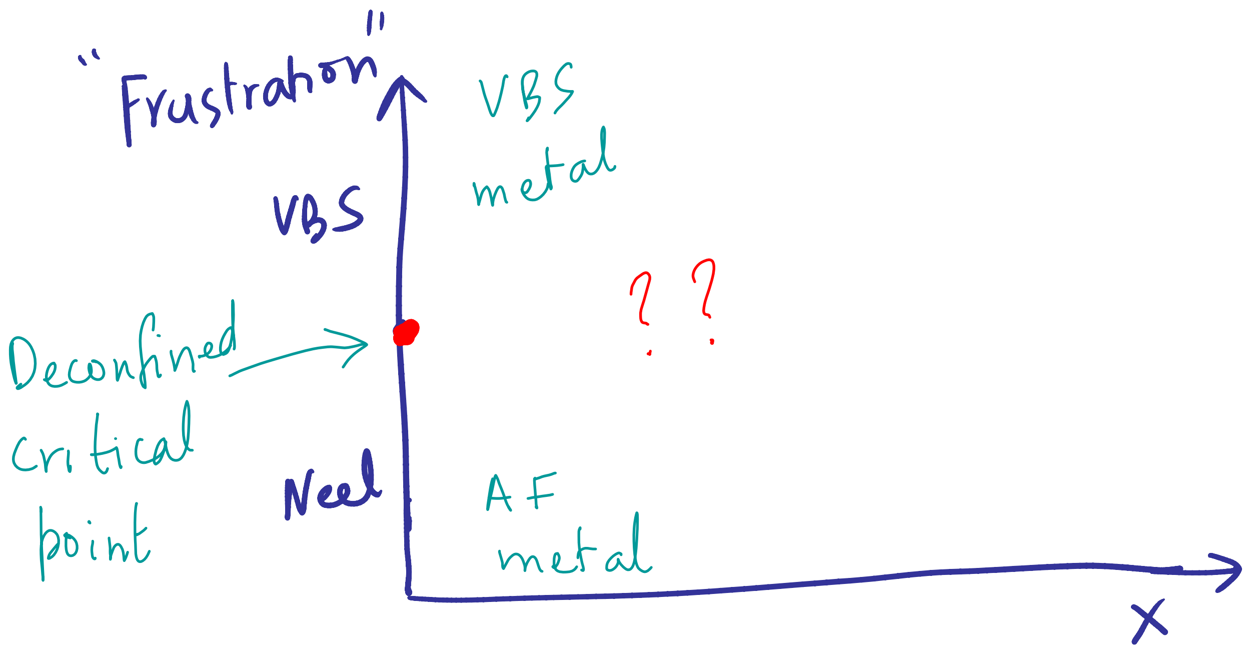
Spinons & gauge fields: Not finite energy objects in either Neel or VBS phases but emerge near the critical point.

Neel: z_α condenses ($\langle z_\alpha \rangle \neq 0$)

VBS: Subtle mechanism, really an "afterthought"
Gapped $z_\alpha \Rightarrow$ quantum spin liquid eventually unstable to VBS order due to "instanton" effects

Doping the deconfined critical point-I

Question: Fate of doping an antiferromagnet that is close to this deconfined quantum critical point ??



Doping the deconfined critical point-II

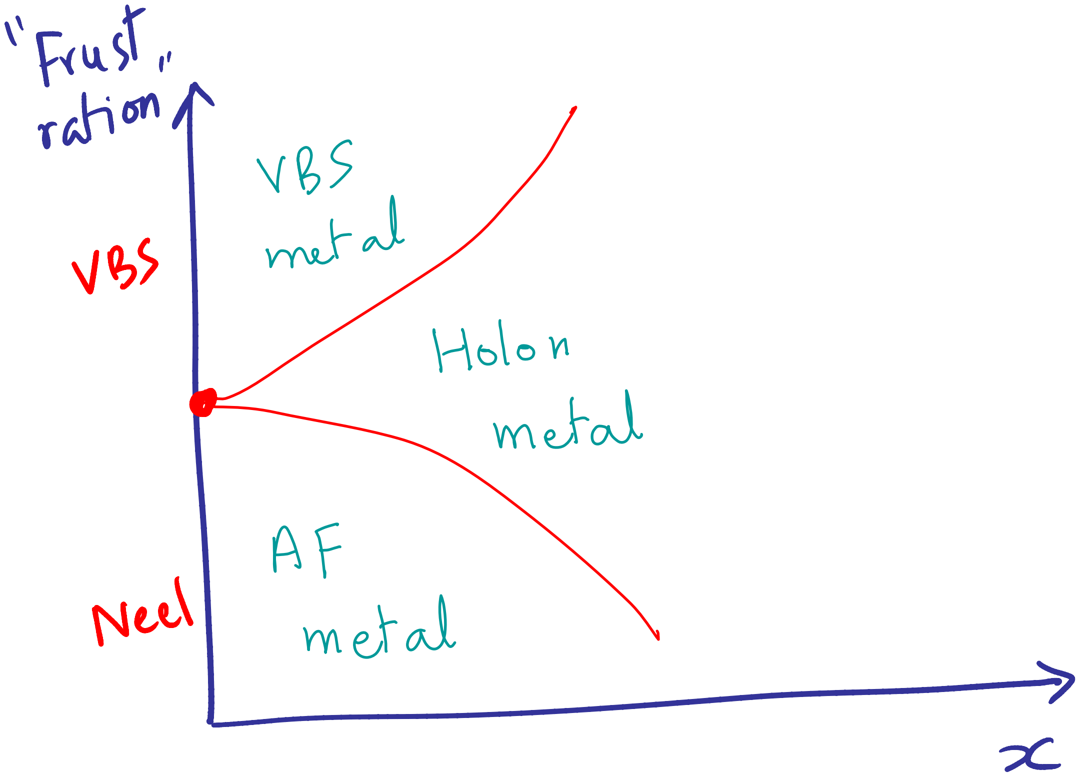
Doped hole: $c_\alpha \rightarrow z_\alpha$, $c_\alpha \rightarrow ?$

$? \sim c_\alpha z_\alpha^* \sim f$ = spinless charge-e "holon".

Guess: At doping x; f forms Fermi surface with area $\propto x$.

Confirm: Mean field calculation within "t-J" model.

The holon metal



Key points:

① Fermi surface of holons stabilizes doped metal from "instanton" effects which lead to VBS order

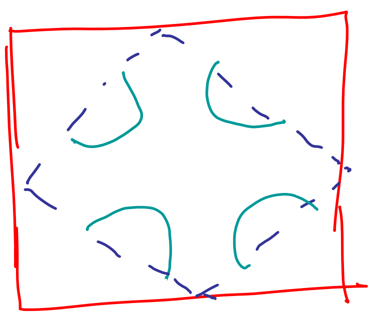
② z_α condenses ⇒ Neel order AND holons → holes of doped AF insulator.

The holon metal: Physical properties-I

1. Metallic state with no symmetry breaking

2. Two species of holons $f_{\pm}$ that live on opposite sublattices (& carry opposite gauge charge).

3. 2 small Fermi pockets per species with area $\propto x$



The holon metal: Physical properties-II

4. Holon Fermi surface ⇒ quantum oscillations in Shubnikov-de Haas, dHvA etc with frequency

$$F/\Phi_0 = \frac{2 A_k}{(2\pi)^2} = \frac{x}{2a^2}$$

5. Spin gap ⇒ No Fermi surface in ARPES!

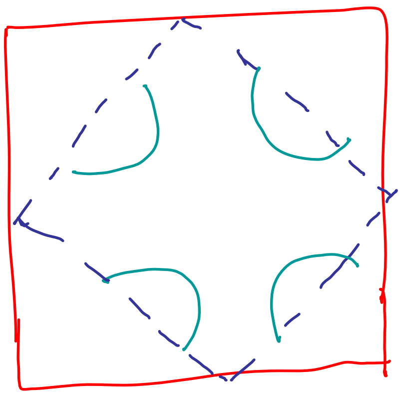
⇒ Real discrepancy between Fermi surfaces seen in quantum oscillations & ARPES

Superconductivity from the holon metal

Wen '88

Shankar '88

Lee '88



f_+ and f_- have opposite gauge charge

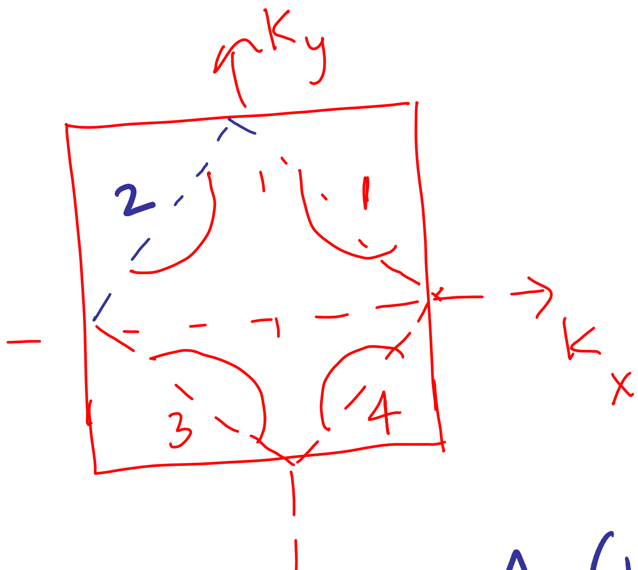
$\Rightarrow$ "gauge-mediated" attractive interaction

$\Rightarrow$ Holon metal unstable to pairing $\langle f_+ f_- \rangle \neq 0$ at low-T.

$f_+ f_- \sim$ Cooper pair $\Rightarrow$ this is a true superconductor!

Mean field theory for the superconductor

Look for $d_{x^2-y^2}$ pairing symmetry for electrons



$$\Rightarrow \Delta_1(k) = \langle f^\dagger_{1+}(k)\, f^\dagger_{3-}(-k) \rangle \sim \frac{k_x - k_y}{\sqrt{2}}$$

$$\Delta_2(k) \sim \langle f^\dagger_{2+}(k)\, f^\dagger_{4-}(-k) \rangle \sim -\frac{(k_x + k_y)}{\sqrt{2}},$$

etc

$\Rightarrow$ Gapless pairing with "nodal" holons.

Gauge fluctuations: QED_3 theory

$f_+ f_-$ order parameter is "gauge-neutral"

$\Rightarrow$ Nodal holons still coupled to gapless U(1) gauge field a_μ

$\Rightarrow$ Low energy theory of superconductor is massless QED_3

$$S = \int d^2x\, d\tau\, \bar{\psi}(\not{\partial} - i\not{a})\psi + \frac{1}{2e^2} f_{\mu\nu}^2$$

ψ : Nodal Dirac holons.

Superconducting Algebraic Holon Liquid (SAHL)

Massless QED_3 : Strongly interacting scale invariant theory.

Universal power law correlations for various physical quantities

$\Rightarrow$ Superconducting state is exotic (not smoothly connected to d-wave BCS)

"<u>Superconducting Algebraic Holon Liquid</u>"

Stability of the SAHL

Is the SAHL a stable phase of matter ?

Yes (at least within a systematic $\frac{1}{N}$ expansion)!!

1. Fermion mass term prohibited by symmetry

2. Four fermion interactions irrelevant

3. "Instantons" in gauge field also irrelevant

(Similar to stability of "algebraic spin liquids" of quantum magnets)

(Hermele, Senthil, Fisher, Lee, Nagaosa, Wen '04)

Properties of the SAHL-I
Superfluid density at T = 0

True superconductor $\Rightarrow$ non-zero ρ_s.

Pairing of holons of a holon Fermi surface of area $\sim x$

$\Rightarrow \rho_s(T=0) \sim x$

Properties of the SAHL-II
Finite T superfluid density

$T \neq 0$: thermal excitation of gapless holons

$\rho_s(T) - \rho_s(0) \sim \langle J J \rangle$ with J = electric current of unpaired holons

J : conserved density of massless QED_3

$\Rightarrow$ From scaling $\rho_s(T) = \rho_s(0) - AT$

with A = universal amplitude of massless QED_3

"A" universal $\Rightarrow$ x-independent as well!

$\Rightarrow$ SAHL has

$$\rho_s(x,T) \sim x - AT$$

for powerful & general reasons!

"A": estimate in $1/N$ expansion.

Specific heat

Low energy theory : "relativistic"

scale invariant theory

$\Rightarrow$ Low-T specific heat $C \sim T^2$

(Also expect Simon-Lee scaling with

B-field)

Thermal conductivity

Within mean field theory (i.e no gauge fluctuations), same as BCS.

Metallic thermal conductivity

$$\lim_{T \to 0} \kappa / T \to \text{constant}.$$

Tendency to break lattice symmetries

Massless QED_3 theory has slow power law fluctuations for a # of fermion bilinears

(Rantner, Wen; Franz et al.; Hermele, TS, Fisher)

$\bar{\psi} \underset{\sim}{M} \psi$

4x4 matrix

(ψ = unpaired Dirac holon)

$\Rightarrow$ Incipient order in various singlet correlations at incommensurate wave vector

$\Rightarrow$ Tendency to break lattice symmetry.

Comparison with d-wave BCS

SAHL NOT smoothly connected to d-wave BCS.

Despite this it resembles d-wave BCS in a number of probes.

(SC with d-wave symmetry, gapless nodal excitations)

Problems with SAHL?

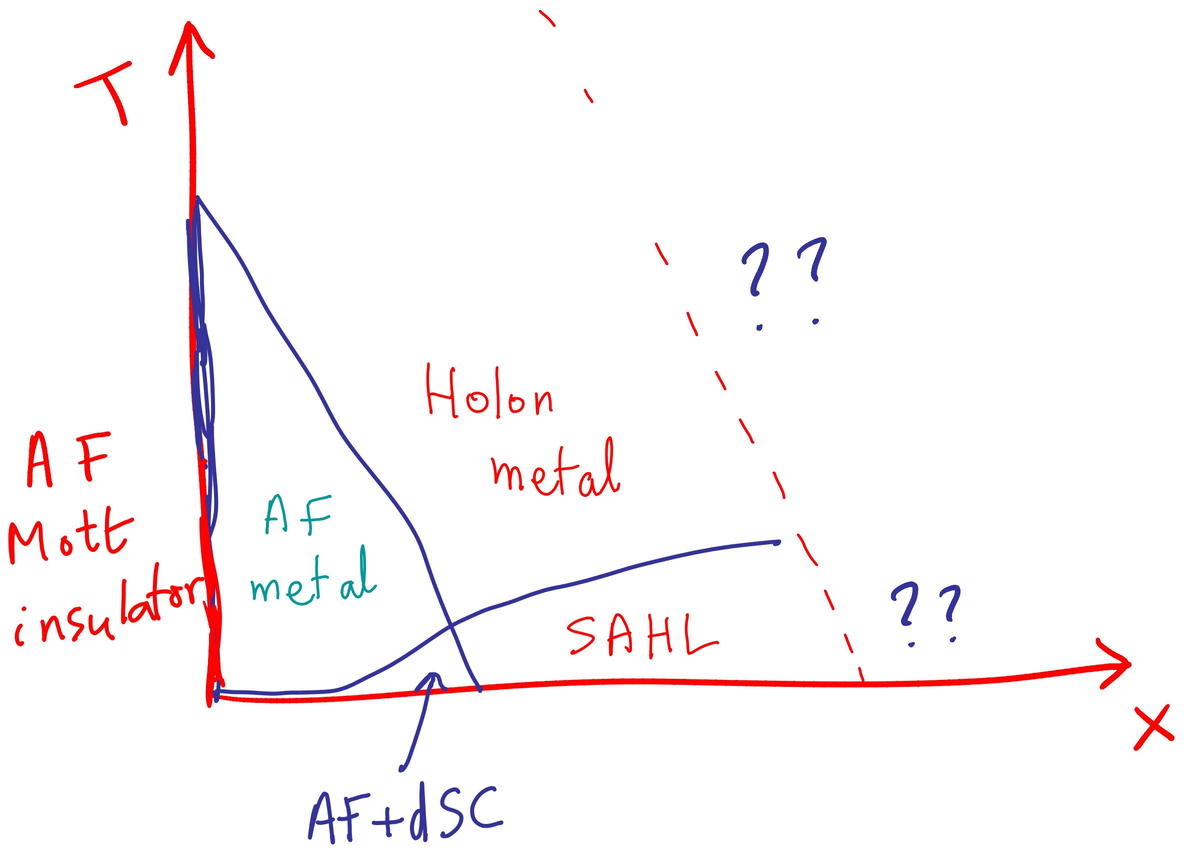
Gapless nodal excitations are spinless holons.

Spin carrying excitations are gapped

$\Rightarrow$ Gap in single electron spectrum

as measured in tunneling / ARPES.

Schematic phase diagram



How did we do?

Phenomenon	Holon Liquid	"Vanilla RVB" theories	"Stripe" based theories

How did we do?

	Phenomenon	Holon Liquid	RVB/Spin liquid theories	"Stripe" based theories
1	Mott insulator at $x = 0$	✓	✓	✓
2	Neel order at $x = 0$	✓	?	✓
3	Superconductivity with doping	✓	✓	✓
4	$d_{x^2-y^2}$ symmetry	✓	✓	✓

How did we do?

	Phenomenon	Holon Liquid	"RVB"/Spin Liquid	"Stripes"
5	$\frac{hc}{2e}$ flux quantum	✓	✓	✓
6	$\rho_s(T=0) \sim T_c \sim x$	✓	✓	✓
7	Tendency to break lattice symmetry	✓	?	✓
8	Thermal transport $\kappa/T \to$ const.	✓	✓	?
9	$d\rho_s/dT = -A$ independent of x	✓	?	?

How did we do?

	Phenomenon	Holon Liquid	"RVB"/Spin Liquid	"Stripes"
10	Specific heat $C \sim T^2$	✓	✓	?
11	STM tunneling $G(V) \sim V$	?	✓	?
12	Nodal quasiparticles in ARPES in SC	?	✓	?

General suggestions/questions-I

1. Quantum oscillations do not imply a Fermi liquid of electron quasiparticles!

Eg: (i) Holon metal : Fermi surface of spinless charge-e fermions

(ii) Composite Fermi Liquid in ½-filled Landau level of Halperin-Lee-Read

Shubnikov-de Haas in experiments but Fermi surface is "composite fermions" not electrons.

General suggestions/questions-II

2. Discrepancy between quantum oscillations & ARPES may be real!

(Not necessarily a technical issue with ARPES resolution)

Eg: Holon metal has "Fermi arcs" at $T \neq 0$ in ARPES, and quantum oscillations in transport.

General suggestions/questions-II

3. Are nodal excitations in the superconductor truly "electron-like quasiparticles" ?

(Any ARPES measurement showing $\text{Im}\,\Sigma \sim E^3$??)

4. IS GROUND STATE OF UNDERDOPED SUPERCONDUCTOR SMOOTHLY CONNECTED TO D-WAVE BCS ??