Are non-fermi liquids stable to pairing?

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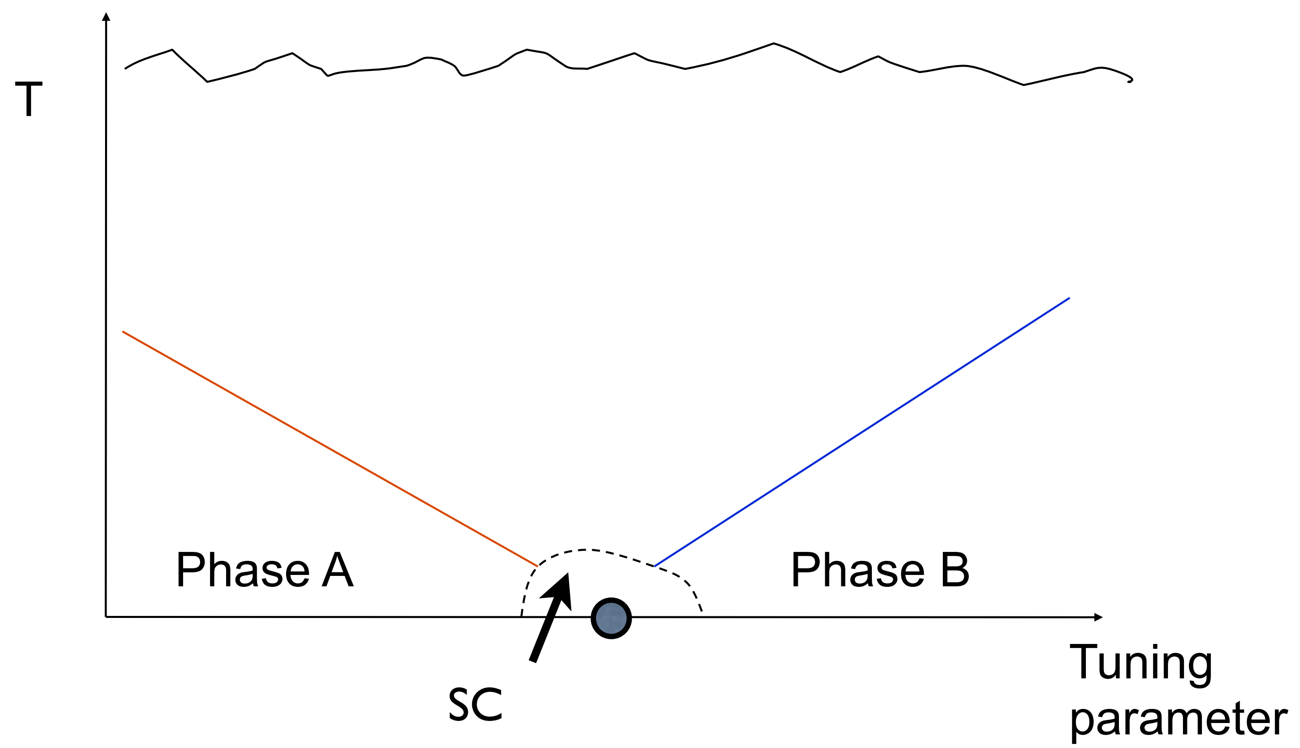
SC in correlated materials

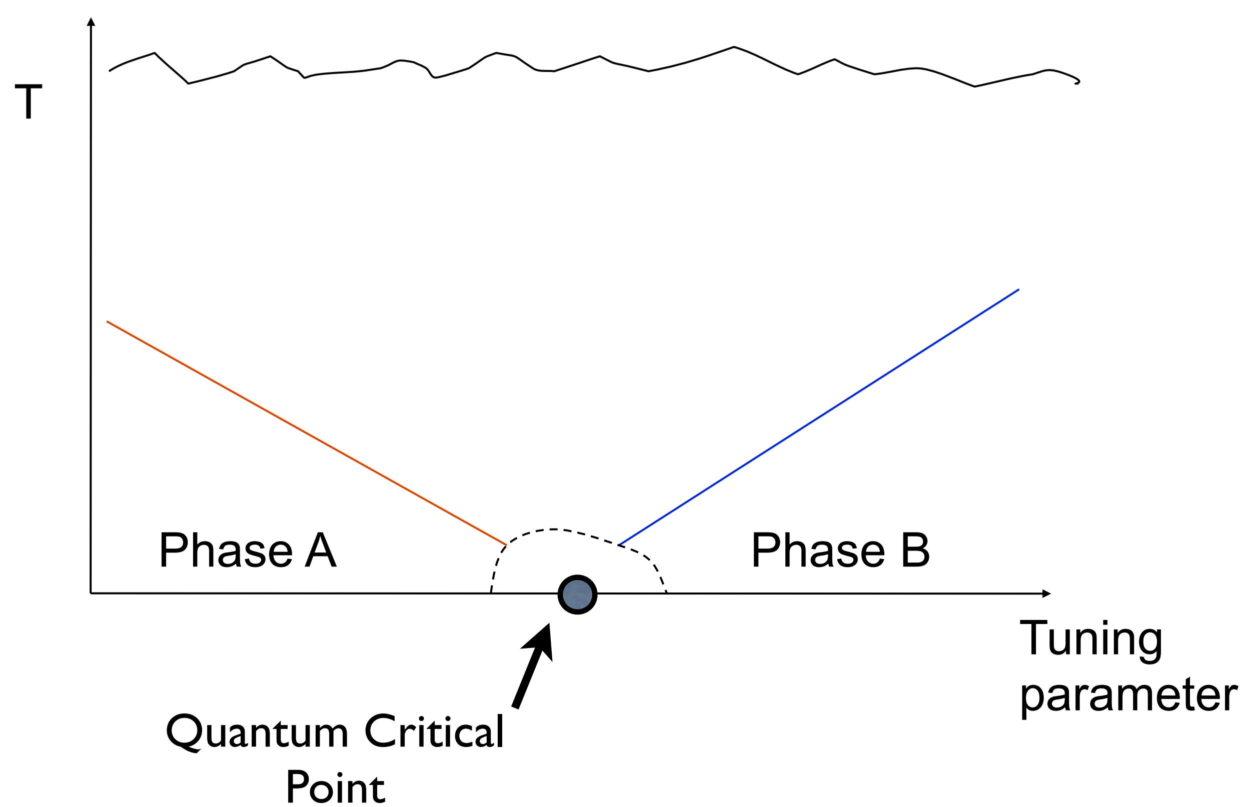
A common phase diagram



Quantum criticality in metals

Small changes of external parameters - quantum phase transitions between different competing states.

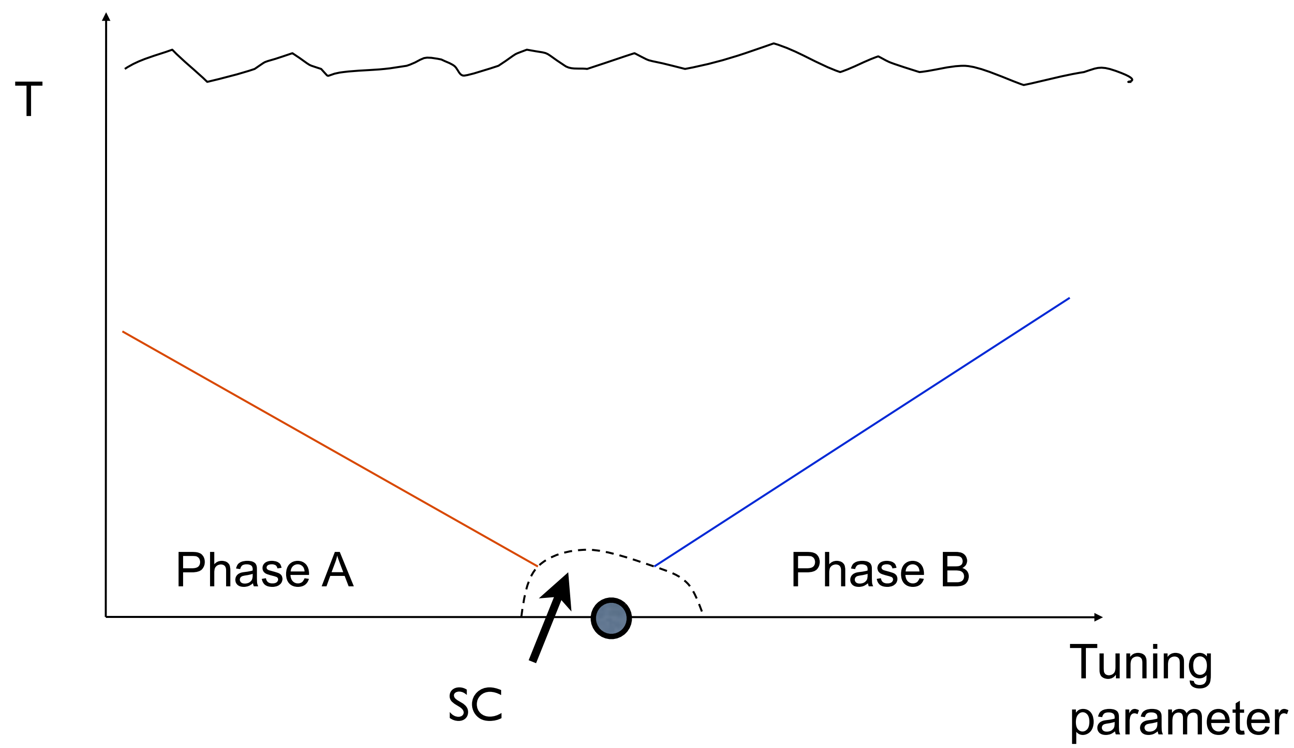
A common phase diagram



Is quantum criticality good for SC?

Many examples where superconducting Tc is optimized near QCP.

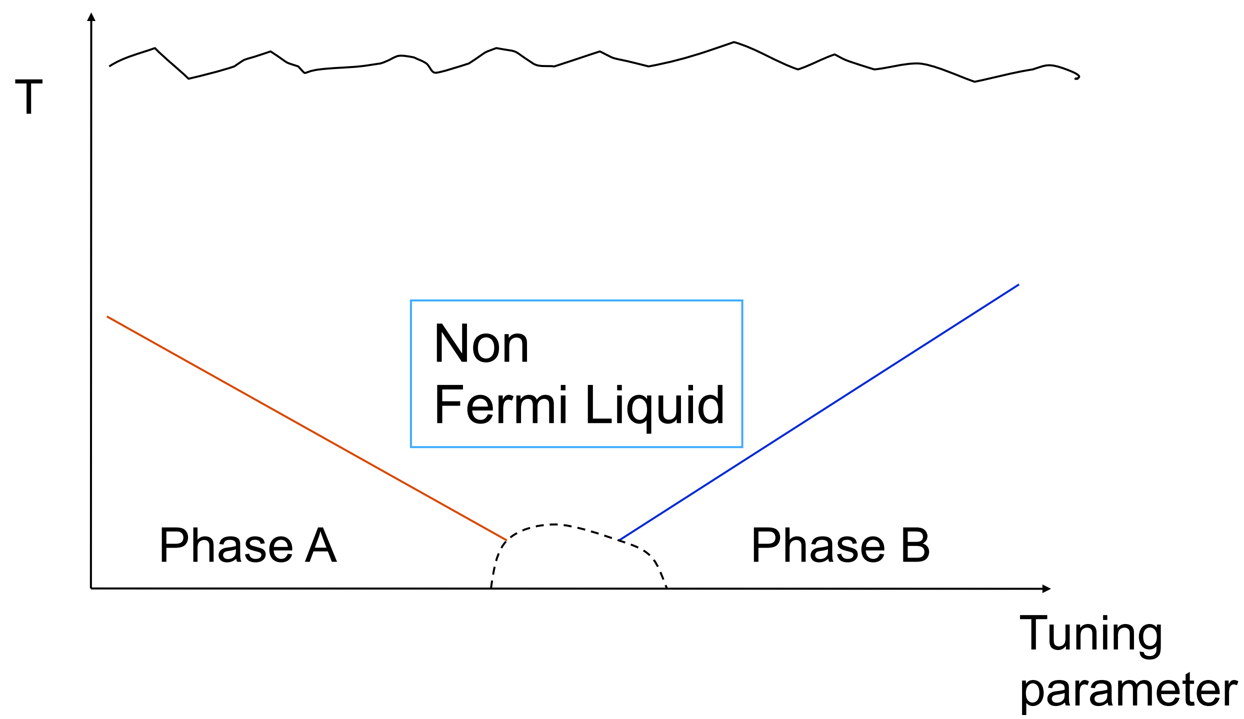
(Some heavy fermions, iron SC, cuprates?)



The elephant in the room: Non-fermi liquids

Confusion (opportunity?):

Metallic state above Tc near QCP is often not a Fermi Liquid.



Non-Fermi liquids, quantum criticality, and superconductivity

Are non-Fermi liquids more unstable to SC than Fermi liquids?

Relationship of either with quantum criticality?

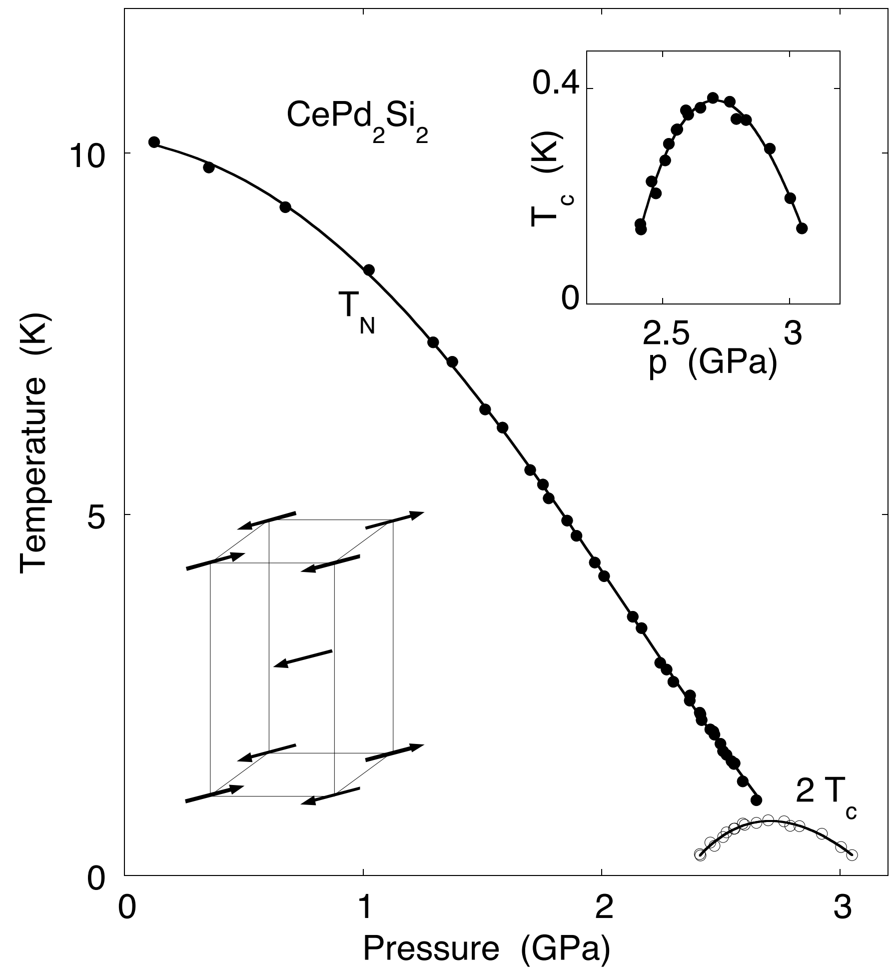
Experiment?

Some but not all quantum critical non-FLs are apparently unstable to SC.

Examples:

1. $CePd_2Si_2$, $CeIn_3$, iron arsenide, (cuprates?): ***SC enhanced near QCP.***

2. $YbRh_2Si_2$, $CeCu_{6-x}Au_x$: ***no SC near QCP***



Non-Fermi liquids, quantum criticality, and superconductivity

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Urgent need: controlled theory of some examples of non-Fermi liquids permitting access to these questions.

Two classes of quantum critical metals

Crucial: Fate of Fermi surface as a Fermi liquid metal undergoes a quantum phase transition?

Two general possibilities

Mutilate

Fermi surface evolves continuously but is distorted in some way.
(Ferromagnet, nematic, SDW, CDW,.....)

Framework: Fermi surface + critical order parameter fluctuations

Kill

Discontinuous Fermi surface evolution through a continuous transition
(2nd order Mott transition, Kondo breakdown,.....)

(Only known) framework: Slave particle methods.

Two case studies

Metlitski, Mross, Sachdev, Senthil, 14

1. Continuous Fermi surface change: Nematic quantum criticality in a metal

2. Discontinuous Fermi surface change: `Kondo breakdown' criticality in Kondo lattices.

A case study:
Onset of electronic nematic order in a metal

Electronic nematic order in a crystal

Break lattice rotation symmetry without breaking translation symmetry

Eg: Tetragonal - orthorhombic change of symmetry.

Growing number of examples in experiments.

1. Quantum Hall regime in GaAs (Eisenstein, 1998)

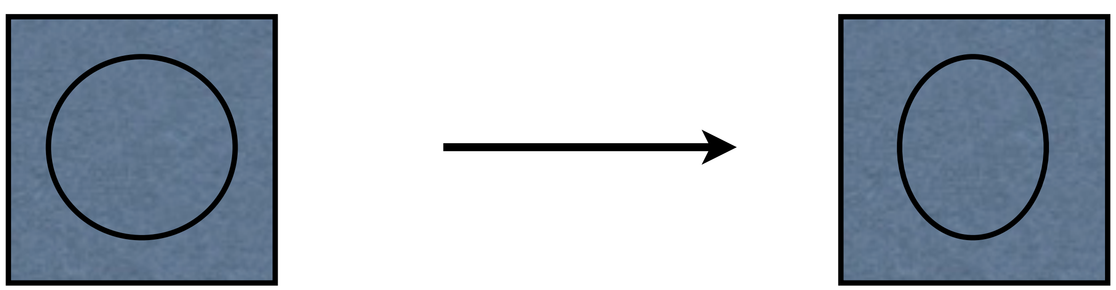
2. Ruthenate $Sr_3Ru_2O_7$ (A. MacKenzie, 2004)

3. Cuprates (Ando 2003, Keimer 2009, Taillefer 2010, Davis 2010), Iron pnictides

Sometimes nematic is a `half-way' house toward developing some other order (charge/spin stripes,.....)

Electronic nematic order and the Fermi surface

Electronic nematic in a metal leads to distortion of Fermi surface



$$\text{Order parameter } O = \sum_{\vec{K}} \left(CosK_x - CosK_y \right) c^{\dagger}_{K} c_{K}.$$

Right at the quantum phase transition, Fermi surface of electrons are coupled to critical fluctuations of nematic order parameter.

Physics at the critical point

Two competing effects

1. Landau quasiparticle destroyed by critical order parameter fluctuations
- non-fermi liquid physics

2. Order parameter fluctuations mediate an attractive interaction

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Our work: Disentangle this competition within systematic renormalization group methods.

Model

Fermi surface coupled to critical bosonic order parameter fluctuations.

First look: RPA predicts destruction of Fermi liquid.

How to really treat?

No natural small parameter;

Introduce artificial small parameter to organize a systematic approximation

(example: epsilon expansion of usual critical phenomena)

Beyond RPA

A controlled approximation (Mross, McGreevy, Liu, Senthil, 2010):

Generalize to large N with a different parameter (Nayak, Wilczek, 94) $\varepsilon = z_b - 2$ (taken small, of order 1/N)

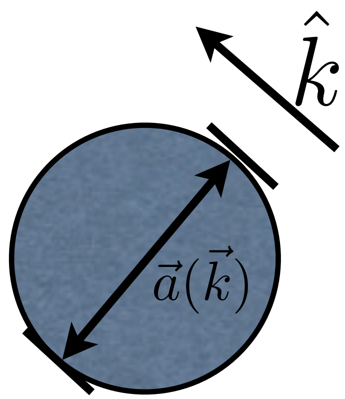
z_b = boson dynamical exponent.

Framework to address non-fermi liquid and superconductivity.

Note: Naive large-N (without small ε) fails (S.-S. Lee 2010, Metlitski, Sachdev 2010).

A useful observation

Bosons with momentum $\vec{k}$ primarily couple with patches of Fermi surface that are tangent to $\vec{k}$.



A scattering off such a boson keeps the fermion close to the Fermi surface.

Divide and conquer: The patch construction

Polchinski, '94
Altshuler et al, '94
Motrunich, Fisher, '07

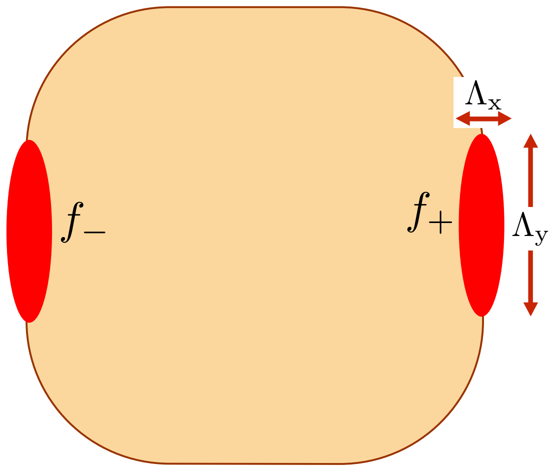
Divide Fermi surface into patches; in the low energy limit only patches with parallel normals are strongly coupled together.

Worry a posteriori about short range four fermion interactions that couple different patches.

Simple nearly circular Fermi surface: focus on two antipodal patches of Fermi surface.

Renormalization Group Analysis

Patch action has one dimensionless coupling constant



$$\alpha \equiv \frac{g^2 v_F \Lambda_y^{-\epsilon}}{(2\pi)^2}$$

Perturb in α: 1-loop RG equations

$$\frac{d\alpha}{dl} = \frac{\epsilon}{2}\alpha - \frac{\alpha^2}{N} \quad (1)$$

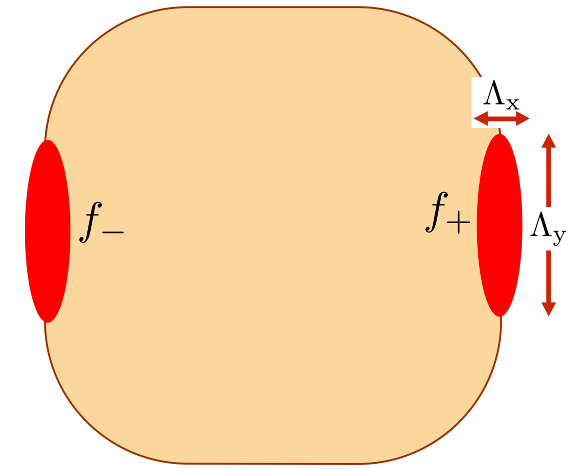
$$\frac{dv_F}{dl} = -\frac{\alpha}{N} \quad (2)$$

Fixed point at $\alpha_* = \frac{N\epsilon}{2}$.
Can analyze all NFL physics at this fixed point.

Mross, McGreevy, Liu, TS, 2010.

Pairing instability?

Beyond patch approximation



Must include inter-patch couplings that are generated under RG.

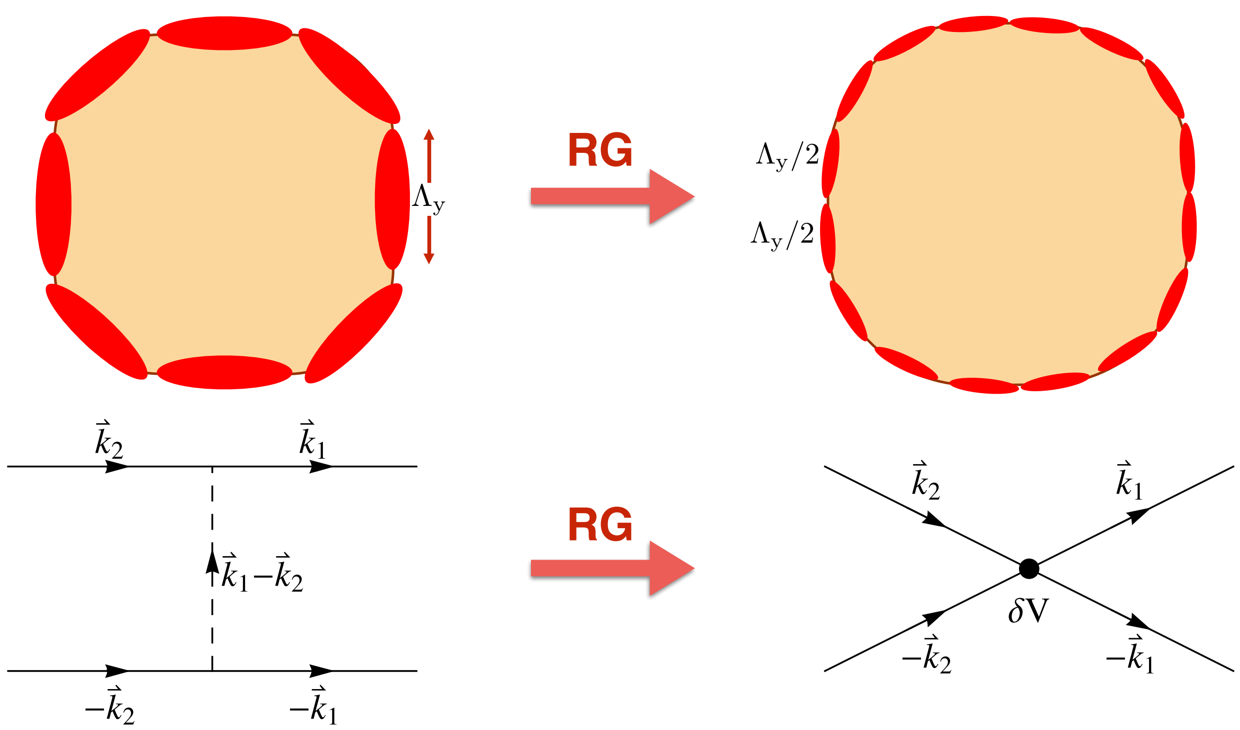
Decrease patch size => some couplings that originally were intra-patch now become inter-patch after the RG.

=> modification of usual Shankar RG flow for Cooper interaction

(physics: attractive interaction from order parameter fluctuations).

Patch + Shankar

$$\frac{d\widetilde{V}_m}{d\ell} = -\frac{\alpha}{N} - (\widetilde{V}_m)^2$$



Combine with NFL RG equations

$$\frac{d\alpha}{dl} = \frac{\epsilon}{2}\alpha - \frac{\alpha^2}{N}$$
$$\frac{dv_F}{dl} = -\frac{\alpha}{N}$$

Similar methods:
Color SC in QCD at finite density,
D. Son, 1998

Solutions: pairing instability

BCS instability independent of initial values of α or V.

Even for bare repulsive interactions, order parameter fluctuations eventually drive attraction.

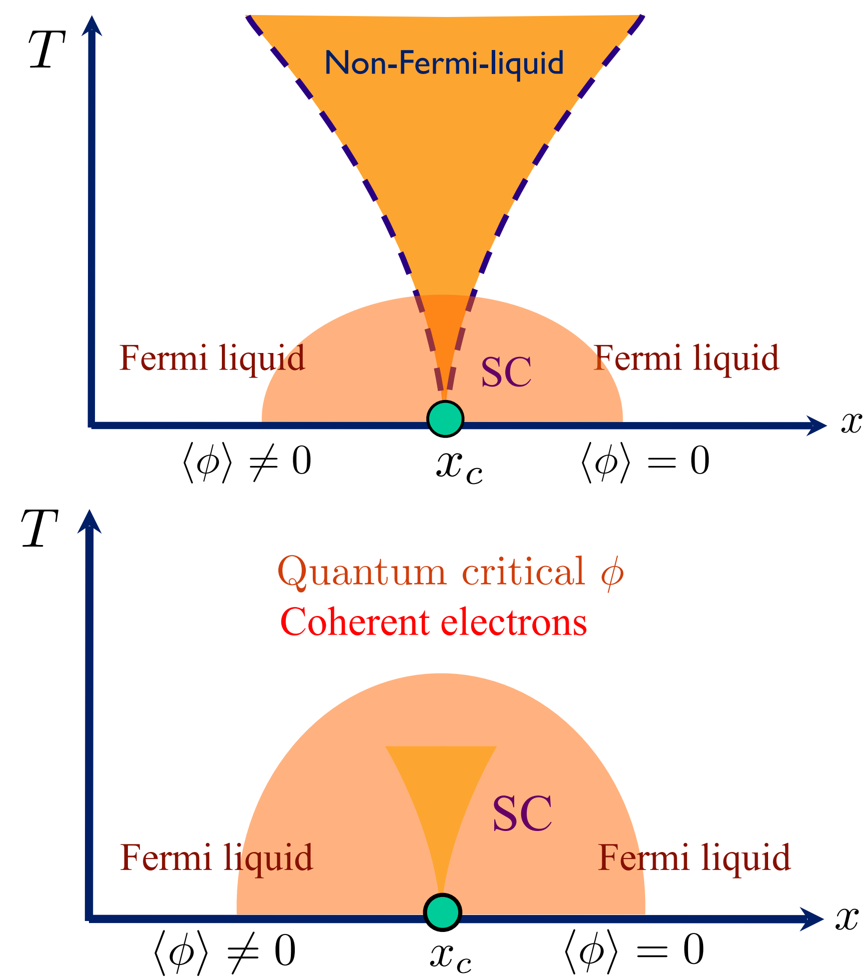
Nematic QCP always unstable to SC!

Pairing versus non-Fermi liquid

In regime where we can control our calculation (ε≪ 1), SC completely preempts NFL!

Scale of NFL ≪ SC.

For ε = 1, these scales are parametrically the same but we cannot control the theory.



Two case studies

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SC suppressed.

(Critical theory: Emergent neutral Fermi surface + gauge fields, etc

Pairing instability suppressed by gauge interactions)

Back to experiments

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Dominated by order parameter fluctuations?

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Dominated by electronic structure fluctuations?

Summary/related problems

1. Controlled theory of examples of interplay between quantum criticality, non-fermi liquid and superconductivity.

2. Methodological progress:

Solution of other related problems:
(i) Phase transition between composite Fermi liquid and Moore-Read states of 2d FQHE
(ii) Phase transitions between some distinct quantum spin liquid states of 2d magnets.