# Interacting electronic topological insulators in 3 dimensions

T. Senthil (MIT)

Chong Wang, Andrew C. Potter, and T. Senthil, Science (2014)

Chong Wang, T. Senthil, Phys Rev B (2014)

Review:
T. Senthil, arxiv: 1405.4015 (to appear in Annual Reviews of Condensed Matter Physics)

# Collaborators


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Other related collaborations: A. Vishwanath (Berkeley), Cenke Xu (UCSB), Michael Levin (Chicago), N. Regnualt (Princeton), Adam Nahum (MIT post-doc)

## *Strongly correlated topological insulators*

**Interaction dominated** phases as topological insulators?

Move away from the crutch of free fermion Hamiltonians and band topology.

# Some questions about interacting topological insulators

1. Are there ***new*** phases that have no non-interacting counterpart?

2. Physical properties?

3. Experimental realization?

# Some questions about interacting topological insulators

1. <u>Are there ***new*** phases that have no non-interacting counterpart?</u>

First refine the question

2. Physical properties?

3. Experimental realization?

# How to generalize 3d topological insulators to interacting electrons?

***Minimal*** interacting generalization:

Strongly correlated spin-orbit coupled insulators

- a bulk gap and no exotic bulk excitations,
i.e, no fractional charge or unusual statistics (``short range entangled”).

Terminology: ***``Symmetry Protected Topological” (SPT)*** phases.

# Some questions about interacting topological insulators

1. Are there new phases that have no non-interacting counterpart?

First refine the question

2. Physical properties?

3. Experimental realization?

**Refined question: Phases and properties of interacting `Symmetry Protected Topological' electronic insulators***

*** Symmetry: Charge conservation, time reversal.**

# Plan of talk

1. Lightning summaries: (i) free fermion topological insulators in 3d

   (ii) bosonic topological insulators in 3d

2. 3d interacting electron TIs

- new $Z_2^3$ classification

- description of the new interacting TIs

4. 3d fermionic interacting TI/TSc with other symmetries: Beyond the 10-fold way.

# Review: free fermion 3d topological insulators

Surface states:  Odd number of Dirac cones


*Trivial* gapped/localized insulator not possible at surface so long as symmetry is preserved (even with disorder)

# Review: Free fermion topological insulators
# Axion Electrodynamics

Qi, Hughes, Zhang, 09
Essin, Moore, Vanderbilt, 09

EM response of *any* 3d insulator

$$\begin{aligned} \mathcal{L}_{eff} &= \mathcal{L}_{Max} + \mathcal{L}_{\theta} \\ \mathcal{L}_{\theta} &= \frac{\theta}{4\pi^2}\vec{E}.\vec{B} \end{aligned} \quad (1$$

Under $\mathcal{T}$-reversal, $\theta \to -\theta$.
Periodicity $\theta \to \theta + 2\pi$: only $\theta = n\pi$ consistent with $\mathcal{T}$-reversal.

Ordinary band insulator: $\theta = 0$.
Topological band insulator: $\theta = \pi$.

# Consequences of axion response: Witten effect

External magnetic monopole in EM field:

θ term => monopole has electric charge θ/2π.

(``Witten dyons”).

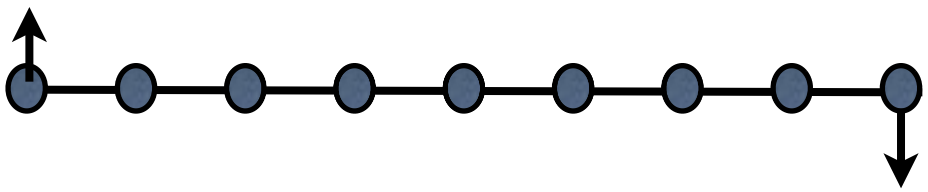
$$
\begin{aligned}
\mathcal{L}_\theta &= \frac{\theta}{4\pi^2}\vec{E}.\vec{B} \\
&= -\frac{\theta}{4\pi^2}\vec{\nabla}A_0.\vec{B} + \ldots . \\
&= \frac{\theta}{4\pi^2}A_0\vec{\nabla}.\vec{B}
\end{aligned}
$$

# A very useful detour: Bosonic topological insulators

Useful stepping stone to interacting fermionic TIs.

Many new and useful non-perturbative ideas to discuss topological insulator/SPT states.

Old example: Haldane spin-1 chain
(Symmetry protected dangling spin-1/2 edge states).



# Bosonic SPT phases

Reviews:
Turner, Vishwanath, 12,
TS, 14

d = 1: Classification/physics
Pollman, Turner, Berg, Oshikawa, 10; Fidkowski, Kitaev, 11; Chen, Gu, Wen 11; Schuh, Perez-Garcia, Cirac, 11.

d > 1: Progress in classification
1. Group Cohomology (Chen, Gu, Liu, Wen, 2012)
2. Chern-Simons approach in d =2 (Lu, Vishwanath, 2012).

Physics:
d = 2: Levin, Stern, 2012; Lu, Vishwanath, 2012; Levin, Gu, 2012; TS, Levin, 2013

*d = 3: Vishwanath, TS, 2013; Wang, TS, 2013; Xu, TS, 2013; Metlitski et al, 2013; Burnell, Chen, Fidkowski, Vishwanath, 2013; .....*

Here we will need some physical ideas on d = 3 boson TI/SPTs.

# Physics of 3d boson topological insulators

Vishwanath, TS, 2012

1. Quantized magneto-electric effect (eg: axion angle $\theta = 2\pi, 0$)

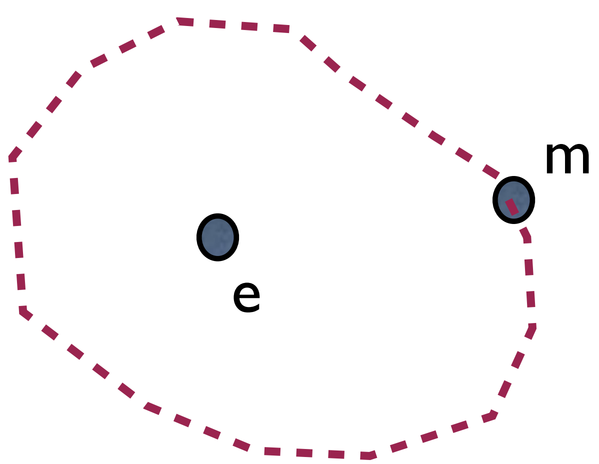
2. Emergent exotic (eg: fermionic, Kramers or both) vortices at surface

3. Related exotic bulk monopole of external EM field (fermion, Kramers, or both) (Wang, TS, 2013; Metlitski, Kane, Fisher, 2013).

Explicit construction in systems of coupled layers: Wang, TS (2013).

# Surface topological order of 3d SPTs

3d SPT surface can have intrinsic topological order though bulk does not (Vishwanath, TS, 2012).




Phase of $\pi$

Resulting symmetry preserving gapped surface state realizes symmetry `anomalously' (cannot be realized in strict 2d; requires 3d bulk).

# Electronic topological insulators

The problem: Phases and properties of correlated spin-orbit coupled electronic SPT insulators*?

* Symmetry group $U(1) \rtimes Z_2^T$

# The answer

3d electronic insulators with charge conservation/T-reversal classified by $Z_2^3$ (corresponding to total of 8 distinct phases).

3 `root' phases:
Familiar topological band insulator, two new phases obtained as electron Mott insulators where spins form a spin-SPT (``topological paramagnets'').

Obtain all 8 phases by taking combinations of root phases.

Proof

# Bulk EM response

Start with EM response of *any* 3d insulator

$$\mathcal{L}_{eff} = \mathcal{L}_{Max} + \mathcal{L}_\theta \quad (1)$$
$$\mathcal{L}_\theta = \frac{\theta}{4\pi^2}\vec{E}.\vec{B}$$

Under $\mathcal{T}$-reversal, $\theta \to -\theta$.
Periodicity $\theta \to \theta + 2\pi$: only $\theta = 0, \pi$ consistent with $\mathcal{T}$-reversal.

If there are 2 distinct insulators with $\theta = \pi$, can combine to make $\theta = 0$ insulator.
=> To look for new insulators, sufficient to restrict to $\theta = 0$.

# Bulk magnetic monopole

Witten effect => monopole charge $\frac{\theta}{2\pi}$.
At $\theta = 0$, monopole has charge 0.
Time reversal:

$$\begin{aligned} \mathcal{T}^{-1} m \mathcal{T} &= m^\dagger \\ \mathcal{T}^{-1} m^\dagger \mathcal{T} &= m \end{aligned}$$

=> Monopole transforms trivially under T-reversal.
Symmetries of monopole fixed.

Remaining possibilities:
Bosonic versus fermionic statistics of the monopole.

# Claim

Bosonic monopole:

Only allow for topological paramagnets* as new root states (simple proof next few slides)

Fermionic monopole:

Impossible in strictly 3d interacting topological insulators (somewhat difficult proof, see Wang, Potter, TS Science paper Supplement).

*Topological paramagnet = electronic Mott insulator where spins form a bosonic SPT phase.

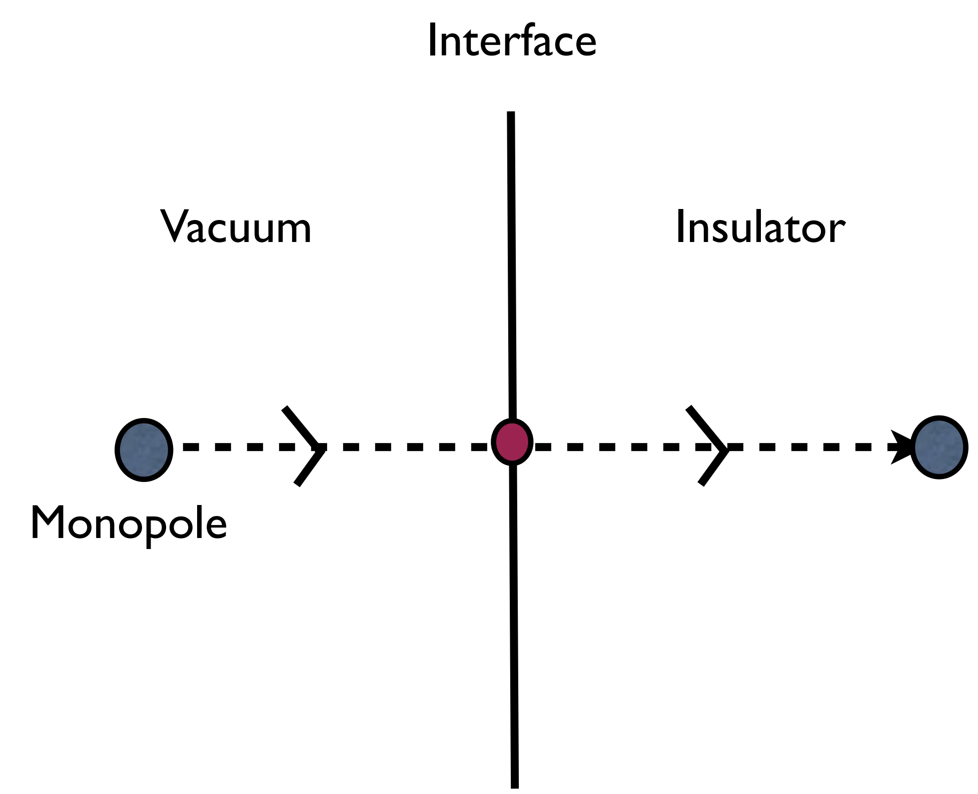
# Bosonic magnetic monopole: implications for surface effective theory

Tunnel monopole from vacuum into bulk

Tunneling event leaves behind surface excitation which has charge-0 and is a boson.

A convenient surface termination- a surface superconductor

Monopole tunneling leaves behind hc/e vortex which is a boson.




More details : Appendix of Wang, Potter, TS, Science 2014.

# Symmetry preserving surface

Disorder the superconducting surface:

Condense the bosonic hc/e vortex.

Result: symmetry preserving insulating surface with distinct topological sectors.

hc/e vortex condensate => Charge quantized in units of e.

=> Surface theory: every topological sector can be made neutral
(integer charge => bind physical electrons to make neutral).

Surface theory = $(1, \varepsilon, \ldots\ldots) \times (1,c)$ = (Neutral boson theory) $\times (1,c)$

=> Bulk SPT order is same as for neutral boson SPT (supplemented by physical electron).

# Neutral boson SPTs in electronic systems

From electrons form neutral composites which are bosons, and let these bosons form a boson SPT.

Describe as electron Mott insulators where spins form an SPT.

Strong spin-orbit => spin system only has T-reversal symmetry.

``Topological paramagnets" protected by time reversal symmetry.

Classification of 3d electron TI:

(band insulator classification) x (such topological paramagnets classification)

= $Z_2$ x (such topological paramagnets classification)

# Time reversal invariant Topological paramagnets

Classified by $Z_2^2$ with two root states conveniently characterized by surface topological order (Vishwanath, TS, 2012, Wang, TS, 2013) of deconfined $Z_2$ gauge theory.

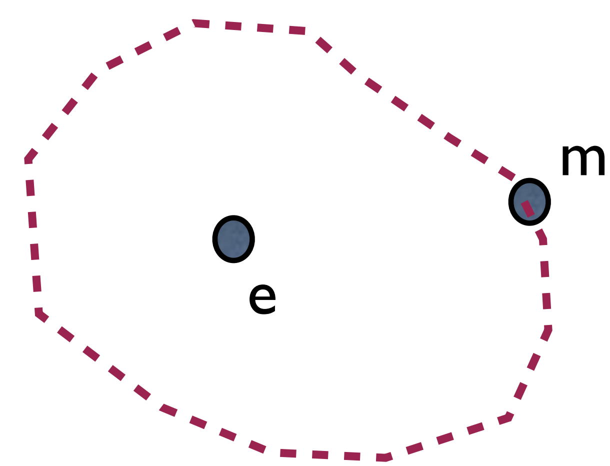
1. Topological paramagnet-I (Label: **eTmT**)

$Z_2$ topological order where both e and m are Kramers

2. Topological paramagnet-II (Label: $\mathbf{e_f\, m_f}$)

Beyond `cohomology'

``All fermion'' $Z_2$ topological order where all topological particles are fermions




Phase of $\pi$

# Classification of 3d electron TI

(Band insulator classification) x (T-reversal symmetric topological paramagnets classification)

= $Z_2$ x (such topological paramagnets classification)

= $Z_2 \times Z_2^2$ = $Z_2^3$

# Explicit construction of 3d time reversal invariant Topological Paramagnets

C. Wang, TS, 2013

Strategy: stack layers of $Z_2$ topological ordered (``toric code") phases with symmetry allowed in strict 2d.

Transition to confine all bulk topological quasiparticles but leave deconfined surface topological order.

Engineer surface topological order specific to SPT surface.

# Topological Paramagnet I (eTmT)

AV, TS 12
C. Wang, TS, 13

Surface topological order:

$Z_2$ gauge theory (``toric code”) where both e and m are Kramers doublets (not possible in 2d with T-reversal).

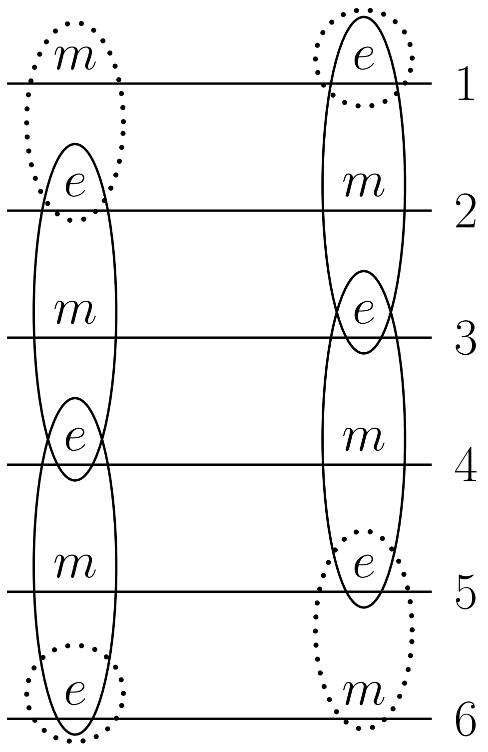
Coupled layer construction:
1. Each layer: conventional 2d $Z_2$ spin liquid where $e_i$ is Kramers doublet, $m_i$ is singlet.

2. Condense $e_i\, m_{i+1}\, e_{i+2}$ (all self and mutual bosons)

=> confine all bulk quasiparticles

3. Surface: $e_1$, $m_1e_2$ survive (at top surface)

These are mutual semions, and self-bosons
=> surface $Z_2$ topological order, both $e_1$, $m_1e_2$ are Kramers.



# Topological Paramagnet-II (efmf)

AV, TS 12
C. Wang, TS, 13
Burnell, Fidkowski, Chen, AV 13

Surface topological order:
T-reversal invariant `all fermion' Z2 gauge theory where all three topological quasiparticles are fermions

Not possible in strict 2d with T-reversal (has chiral central charge 4 mod 8).

Coupled layer construction: Start with trivial realization of T-reversal in each 2d layer, condense $\varepsilon_i m_{i+1} \varepsilon_{i+2}$ ($\varepsilon_i$ = fermion quasiparticle in layer i)

T-broken `confined' surface: quantized thermal Hall effect (in units of quantum of thermal conductance)

$$\kappa_{xy} = \pm 4.$$

# Physical characterization of the 8 interacting 3d TIs

How to tell in experiments?

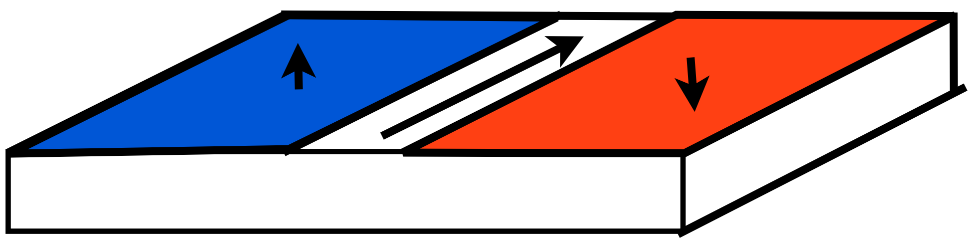
Symmetry preserving surface topological order?

Conceptually powerful characterization of surface but not very practical

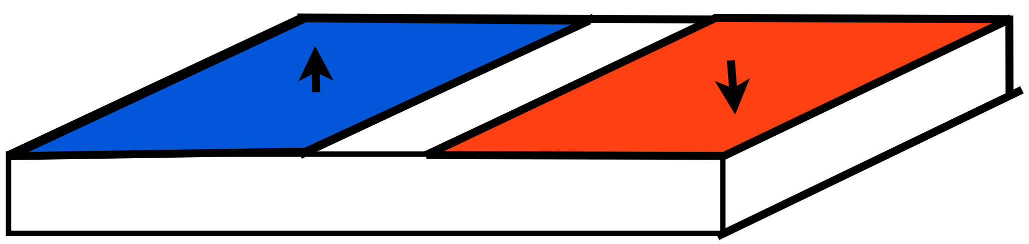
Alternate: Break symmetry at surface to produce a simple state without topological order (eg: deposit ferromagnet or superconductor)

Unique experimental fingerprint!

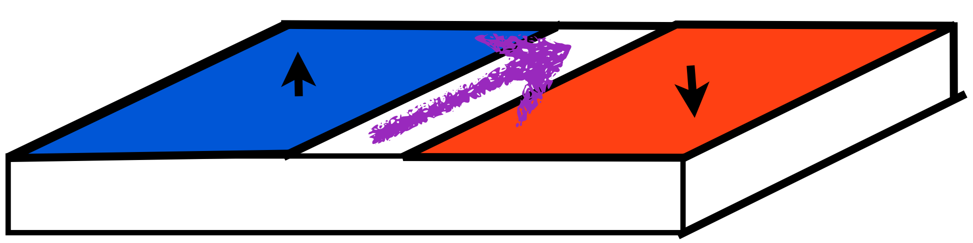
# Depositing a ferromagnet: domain wall structure



Topological band insulator: Charged one-way mode
(= edge of 2d electron IQHE)



Topological paramagnet-I (eTmT): Gapped domain wall



Topological paramagnet-II ($e_f$ $m_f$): Neutral one-way modes
(= edge of 2d $E_8$ bosonic IQHE)

# Depositing s-wave superconductor: induced quasiparticle nodes

Topological band insulator: depositing s-wave SC leads to a **gapped** surface SC with interesting Majorana zero modes on vortices (Fu, Kane 08)

Topological paramagnets (both **eTmT** and **$e_f m_f$** ):

Deposit s-wave SC => get a **gapless** nodal SC with 8 gapless Majorana cones protected by time reversal symmetry! (Wang, Potter, TS 2013)

Probe by Angle Resolved Photoemission (and tunneling, etc)

# Understanding induced quasiparticle nodes

Wang, Potter, TS, Science 2014
Wang, TS, PR B 2014

Argue in reverse:

Start with surface SC theory with 4 Dirac nodes:

$$\mathcal{L}_{free} = \sum_{i=1}^{4} \psi_i^\dagger (p_x \sigma^x + p_y \sigma^z) \psi_i, \tag{1}$$

with time-reversal acting as

$$\mathcal{T} \psi_i \mathcal{T}^{-1} = i\sigma_y \psi_i^\dagger. \tag{2}$$

Time reversal symmetric free fermion perturbations cannot gap the nodes (same as surface of certain free fermion topological SC).

But interactions can induce a gap while preserving symmetry.

# Understanding induced quasiparticle nodes

Enlarge symmetry to $U(1) \times \mathcal{T}$ with $U(1)$ acting as

$$U_\theta \psi U_\theta^{-1} = e^{i\theta}\psi \tag{1}$$

Consider pairing mass that breaks $U(1)$ and $\mathcal{T}$ but preserves a combination:

$$\mathcal{L}_{gap} = i\Delta \sum_{i=1}^{4} \psi_i \sigma_y \psi_i + h.c. \tag{2}$$

Now attempt to disorder the broken symmetry => proliferate vortices in the pairing order parameter.

However vortices have zero modes which restrict which kinds can condense.

# Understanding induced quasiparticle nodes

For N = 4 Dirac nodes, can condense strength-2 vortices.

Result: $Z_2$ topological order where $e$ and $m$ are both Kramers (eTmT)
= surface topological order of Topological Paramagnet-I.

(Can now get rid of auxiliary $U(1)$).

Closely related to a result using other methods: Classification of interacting topological SC with time reversal by Fidkowski, Chen, Vishwanath (2013)

# Other symmetries: Beyond the 10-fold way

Ideas discussed above enable us to determine stability to interactions of all the 3d free fermion topological insulators/SC represented in ``10-fold way'' and in many cases to get the full classification. (Wang, TS, PR B 2014)

SC with time reversal (class D III): Free fermion classification: $Z$

Interactions reduce this to $Z_{16}$ (`Walker-Wang' methods: Fidkowski, Chen, Vishwanath (2013)).

Our arguments (Wang, Potter, TS, 14; Wang, TS, 14) give elementary understanding of this result*, and generalize to other symmetries.

* See also Metlitski et al, arxiv:1406.3032

# Results for other symmetries (Wang, TS, 14)

Spin-orbit coupled insulators

| Symmetry class | Reduction of free fermion states | Distinct boson SPT | Complete classification |
|---|---|---|---|
| $U(1)$ only (A) | 0 | 0 | 0 |
| $U(1) \rtimes \mathbb{Z}_2^T$ with $\mathcal{T}^2 = -1$ (AII) | $\mathbb{Z}_2 \to \mathbb{Z}_2$ | $\mathbb{Z}_2^2$ | $\mathbb{Z}_2^3$ |
| $U(1) \rtimes \mathbb{Z}_2^T$ with $\mathcal{T}^2 = 1$ (AI) | 0 | $\mathbb{Z}_2^2$ | $\mathbb{Z}_2^2$ |
| $U(1) \times \mathbb{Z}_2^T$ (AIII) | $\mathbb{Z} \to \mathbb{Z}_8$ | $\mathbb{Z}_2$ | $\mathbb{Z}_8 \times \mathbb{Z}_2$ |
| $U(1) \rtimes (\mathbb{Z}_2^T \times \mathbb{Z}_2^C)$ (CII) | $\mathbb{Z}_2 \to \mathbb{Z}_2$ | $\mathbb{Z}_2^4$ | $\mathbb{Z}_2^5$ |
| $(U(1) \rtimes \mathbb{Z}_2^T) \times SU(2)$ | 0 | $\mathbb{Z}_2^4$ | $\mathbb{Z}_2^4$ |
| $\mathbb{Z}_2^T$ with $\mathcal{T}^2 = -1$ (DIII) | $\mathbb{Z} \to \mathbb{Z}_{16}$ | 0 | $Z_{16}$ (?) |
| $SU(2) \times \mathbb{Z}_2^T$ (CI) | $\mathbb{Z} \to \mathbb{Z}_4$ | $\mathbb{Z}_2$ | $Z_4 \times Z_2$ (?) |

# Results for other symmetries (Wang, TS, 14)

Spin-orbit coupled insulators

Spin-rotation invariant insulators

| **Symmetry class** | **Reduction of free fermion states** | **Distinct boson SPT** | **Complete classification** |
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# Results for other symmetries (Wang, TS, 14)

Spin-orbit coupled insulators

Spin-rotation invariant insulators

Topological SC

Topological SC with spin SU(2)

| **Symmetry class** | **Reduction of free fermion states** | **Distinct boson SPT** | **Complete classification** |
|---|---|---|---|
| $U(1)$ only (A) | $0$ | $0$ | $0$ |
| $U(1) \rtimes \mathbb{Z}_2^T$ with $\mathcal{T}^2 = -1$ (AII) | $\mathbb{Z}_2 \to \mathbb{Z}_2$ | $\mathbb{Z}_2^2$ | $\mathbb{Z}_2^3$ |
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| $SU(2) \times \mathbb{Z}_2^T$ (CI) | $\mathbb{Z} \to \mathbb{Z}_4$ | $\mathbb{Z}_2$ | $Z_4 \times Z_2$ (?) |

Many new insights:

Eg: In some cases there is no symmetry preserving surface topological order (minimal SC with spin SU(2) and T-reversal): ``symmetry-enforced gaplessness.

# Summary

1. Interacting electron TIs in 3d have a $Z_2^3$ classification

- apart from trivial and topological band insulators, 6 new TI phases with no non-interacting counterpart.

2. Unique experimental fingerprint:

Deposit ferromagnet at surface: Quantum `anomalous' ***electrical*** and ***thermal*** Hall effect of surface.
Deposit s-wave SC at surface: Presence/absence of 4 induced gapless Dirac nodes.

3. Progress in understanding interacting 3d fermion SPTs with many symmetries.

3. Hints to important open question:

what kinds of real electronic insulators may be these new 3d TIs?