Ideas on non-fermi liquid metals and quantum criticality

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Plan

Lecture 1:
General discussion of heavy fermi liquids and their magnetism
Review of some experiments
Concrete `Kondo breakdown’ model with a non-fermi liquid critical point

Lecture 2:
Deconfined quantum criticality

Lecture 3:
Some new ideas and scaling hypotheses
Luttinger’s theorem for Fermi liquids

In a Fermi liquid, volume $V_F$ of Fermi surface is set by electron density $n$ independent of interaction strength.

$$V_F = (2\pi)^d n/2 \pmod{\text{Brillouin zone volume}}.$$ 

**Perturbative proof:** Luttinger  
**Non-perturbative topological arguments:**  
Yamanaka, Oshikawa, Affleck ($d = 1$), Oshikawa ($d > 1$).

Oshikawa: Regard as “topological quantization”.”
Heavy electron metals

Typically rare earth intermetallic alloys

Eg: CeAl$_3$, CeCu$_2$Si$_2$, UPt$_3$, etc

Huge effective mass at low-$T$, eg in

specific heat

$$\frac{m^*}{m} \sim 10^2 - 10^3 \quad (\text{hence 'heavy electron')}$$

Strongly correlated partially filled f-band

+ weakly correlated conduction bands
Simplified Anderson lattice model for heavy fermi liquids

\[
H = \sum_k \epsilon_k c_k^+ c_k + V \sum_i \left( \epsilon_{i\alpha} f_{i\alpha}^+ f_{i\alpha} + h.c. \right) \\
+ \sum_k \epsilon_{kf} f_k^+ f_k + U \sum_i n_{fi}^2 - \frac{1}{2} \sum_i \Sigma n_{fi}^2 
\]

Weak-U limit: Hybridized c and f bands

Fermi volume includes both c and f electrons but effective mass not large, and \( Z \sim o(1) \).
Strong correlations: Kondo lattice

\frac{1}{2} - \text{filling for } f\text{-band}

Large -U limit \Rightarrow each localized f-orbital is singly occupied.

Lattice of localized f-moments coupled to c-electrons

\[ H = \sum_k \epsilon_k c_k^\dagger c_k + J \sum_i \vec{S}_i \cdot \frac{\vec{c}_i \vec{\sigma} \vec{c}_i}{2} \]

\[ \vec{S}_i = f\text{-moment}; \quad J \sim o(V^2/U) \]
Luttinger’s theorem in Kondo lattices

• Kondo lattice model admits a Fermi liquid phase. (Denote ``Kondo liquid’’).

• To satisfy Luttinger’s theorem must include local moments in the count of conduction electron density

  ```Large” Fermi surface

• Fermi liquid adiabatically connected to small U Anderson model.

• Understand through (i) slave particle mean field calculation (Read et al, Millis et al)
  (ii) Oshikawa topological argument.
Physical picture: strong coupling limit

- **Large** $J_K$: Treat conduction electron hopping $t$ as perturbation.

- $t = 0$ and **half-filling** for *conduction electrons*:
  Each local moment traps a conduction electron into a singlet.
  Insulator with a gap: \textit{``Kondo insulator''}
  (Stable to small $t$)

To fit into a band picture count both **local moments** and **conduction electrons** as part of band.
Doping the Kondo insulator

Move away from half-filling for the conduction electrons:

Some unscreened free moments created by conduction holes ≈ spinful fermions.

Hopping: Fermi liquid with hole Fermi surface determined by hole density ≡ large electron Fermi surface.
``High temperature''

Low-T heavy fermi liquid

conduction electron `bare' Fermi surface

local moments

`true' Fermi surface
Slave particle formulation of Kondo lattice

Write \( \hat{S}_i = f_i^+ \sigma^z f_i \) with \( f^+_i f_i = 1 + 4i \)

Important: \( f_\alpha \) not electrons

- neutral spin-1/2 fermions ("spinons")

\[
H = \sum_k \varepsilon_k \, c_k^+ c_k - J \sum_i \left( c_i^+ f_i \right)^2 + h.c.
\]

Constraint: \( f^+_i f_i = 1 \) \( \forall i \).
Hybridization mean field theory

Treat Kondo exchange, constraint in mean field

\[ H_{MF} = \sum_k \epsilon_k c_k^{+} c_k + b \sum_k (\epsilon_k f_k^{+} f_k + \text{h.c}) + \mu \sum_k f_k^{+} f_k \]

\[ b = J \langle f_k^{+} c_k \rangle \]

\[ \langle f_i^{+} f_i \rangle = 1 \]
Theory gives (i) large Fermi surface (ii) large quasiparticle mass, etc!
Momentum distribution

\[ \langle n(k) \rangle \]

mostly c-character

mostly f-character
Comments

1. $b \sim \langle c^f \rangle \sim$ amplitude of Kondo singlet

2. Heavy quasiparticles at Fermi surface

$\sim f + bc$

$\Rightarrow$ Quasiparticle weight $Z \sim b^2 \sim \frac{m}{m^*} \ll 1$
Other known metallic states of Kondo lattice

- **Magnetically ordered** (typically antiferromagnetic) metal due to RKKY
  Favored at small $J_K$ (Doniach).

There are actually two possibly **distinct** kinds of antiferromagnetic metals.

- Other **broken symmetry** states (SC, .....)

Two kinds of antiferromagnetic metals in Kondo lattices

<table>
<thead>
<tr>
<th>(A): ``Local moment magnetic metal”</th>
<th>(B): ``Spin density wave metal”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordering of local moments (due to intermoment exchange).</td>
<td>SDW instability of heavy fermi liquid with large fermi surface.</td>
</tr>
<tr>
<td>c-electrons ≈ decoupled from local moments.</td>
<td>c- and f-electrons strongly coupled.</td>
</tr>
<tr>
<td>Typically large (staggered) moment.</td>
<td>Typically weak moment.</td>
</tr>
<tr>
<td>``f-electrons do not participate in Fermi surface”.</td>
<td>``f-electrons participate in Fermi surface”.</td>
</tr>
</tbody>
</table>
- Two kinds of magnetic metals possibly sharply distinct!!
- Distinction in topology of Fermi surface

Evolution between the 2 magnetic metals thru' a quantum phase transition!

(Is this true more generally?)
Magnetic quantum critical points in heavy fermion liquids

H. v. Löhneysen et al, PRL 1994

CeCu$_{6-x}$Au$_x$


CePd$_2$Si$_2$
Non-fermi liquid: Diverging $\tau$ coefficient, nontrivial power law resistivity, scaling in spin fluctuation spectrum etc.
Representative data: resistivity of $\text{YbRh}_2\text{Si}_2$


Trovarelli et al, PRL 2000

Magnetic metal

Fermi liquid

Non fermi liquid

T-dependence of resistivity at critical point: $\rho(T) \sim T$ for three decades in temperature!
Scaling of dynamic spin correlations:
Inelastic neutron scattering near ordering wavevector

A. Schröder et al., Nature ’00; PRL ’98
Singular specific heat
``Classical” assumption

1. NFL: Universal physics associated with quantum critical point between heavy fermi liquid and magnetic metal

2. Landau: Universal critical singularities \( \sim \) fluctuations of natural magnetic order parameter for transition

Try to play Landau versus Landau.
Which magnetic metal?

1. Heavy fermi liquid – SDW metal:

   Fluctuations of magnetic order parameter with damping due to fermionic quasiparticles (Moriya-Hertz-Millis)

   - Fail to reproduce observed NFL physics.

2. Explore alternate possibility:

   Transition between heavy fermi liquid and local moment magnetic metal.
Schematic phase structure

SDW metal

Local moment magnetic metal

Fermi liquid

Moriya-Hertz criticality

Lifshitz transition

??

Expect dramatic change of Fermi surface
Evidence from experiments I: Hall effect in YbRh2Si2

Evidence from experiments II: dHvA in CeRhIn$_5$

Questions

1. Is such a second order transition generically possible? Loss of magnetic order happens at same point as f-moments dissolving in Fermi sea: why should these 2 different things happen at the same time?

2. Theoretical description?

3. Will it reproduce observed non-fermi liquid behaviour?

Answers not known!!
General observations

• f-moments drop out of Fermi surface (↔ change of electronic structure)
  Associated time scale $t_e$.

• Onset of magnetic order
  Associated time scale $t_m$.

Both time scales diverge if there is a critical point.

$$[\text{Expect } t_e \sim |Sg|^{-\phi_e}, \quad t_m \sim |Sg|^{-\phi_m}]$$

($Sg =$ tuning parameter)
General observations

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  Associated time scale $t_m$.

Both time scales diverge if there is a critical point.

Suggestion: One time scale may diverge faster than the other so that the two competing orders are dynamically separated.

Separation between two competing orders as a function of scale (rather than tuning parameter) might make second order transition possible.
Options

1. $t_m$ diverges faster than $t_e$.  
   (electronic structure change first, magnetic order comes later)

2. $t_e$ diverges faster than $t_m$
   (magnetic order destroyed first, Fermi surface reconstruction comes later)

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Focus on option 1 as a more likely route to non-fermi liquid physics
Some implications

• **``Underlying''** transition: loss of participation of the f-electrons in forming the heavy fermi liquid.
  (View as a Mott ``metal-insulator'' transition of f-band).

• **Magnetic order:** ``secondary'' effect – a low energy complication once Kondo effect is suppressed.

• Non-fermi liquid due to fluctuations associated with change of electronic structure rather than those of magnetic order parameter.

⇒ PHYSICS BEYOND LANDAU-GINZBURG-WILSON PARADIGM FOR PHASE TRANSITIONS.
(Natural magnetic order parameter is a distraction).
Intermediate time scale physics

- f-moments drop put of Fermi surface but continue to form singlet bonds with each other

Resulting state: spin liquid of f-moments coexisting with small Fermi surface of conduction electrons (a "fractionalized Fermi liquid")

Magnetism: low energy instability of such a small Fermi surface state.
Suggested phase diagram and crossovers

T

J_H

Quantum critical

FL*

Fermi liquid

LM magnetic metal
? HOW TO PUT MEAT INTO THIS PICTURE ?
Study tractable simpler questions

1. Effects of loss of "Kondo" order?
   Study second order quantum transitions associated with loss of Kondo screening.  
   [TS, Vojta, Sachdev, '04]
   Worry about magnetism later

2. Do similar theoretical phenomena (e.g. breakdown of Landau paradigm) happen in other contexts?
   Yes! (study quantum phase transitions in insulating magnets)
   (TS, Vishwanath, Balents, Sachdev, M. Fisher)
Kondo Heisenberg models

- $J_K = 0$: Conduction electrons are decoupled from local moments and have small Fermi surface.

- Non-magnetic ground states of spin system
  (i) Spin –Peierls: break translational symmetry
  (ii) Fractionalized: can preserve translational symmetry

Focus on (ii) to discuss small Fermi surface state.
Fractionalization in $d > 1$

- Anderson: RVB spin liquid state for quantum spin models.
Couple spin liquids to conduction electrons

- **Small non-zero \( J_K \):** Perturb in \( J_K \)
- **Emergent gauge structure** of local moment system survives; conduction electrons stay sharp on a small Fermi surface*. advertised small fermi surface state.
- **A fermi liquid in peaceful coexistence with fractionalization**  `
``Fractionalized fermi liquid” (denote FL*)`
- **Large \( J_K \):** Recover large Fermi surface Kondo liquid.

(* Possible pairing instability at low T).
Physics of fractionalized fermi liquid (FL*) state

- Each local moment forms singlet with another local moment.

- Weak Kondo coupling can’t break singlets:

  Local moments and conduction electrons essentially stay decoupled.
Mean field theory

Kondo-Heisenberg model

$$H = -t \sum_{\langle rr' \rangle} c^+_r c_{r'} - \mu \sum_r c^+_r c_r + \frac{J_K}{2} \sum_r c^+_r \sigma^z c_r \cdot \vec{S}_r$$

$$\vec{S}_r = f_r^+ \sigma^z f_r$$ with $$f_r^+ f_r = 1$$

Decouple $$J_K$$ with $$b_r \sim c^+_r f_r$$ and $$J_H$$ with $$\chi_{rr'}$$.

$$\langle b \rangle \neq 0 \quad (\Rightarrow \langle \chi \rangle \neq 0)$$: Fermi liquid (FL)

$$\langle b \rangle = 0$$, $$\langle \chi \rangle \neq 0$$: Fractionalized Fermi liquid (FL*) with spinon Fermi surface.
Direct fermi volume changing transition

- Condensation of hybridization amplitude $b$ drives direct Fermi volume changing transition.

- Transition can be second order despite jump in fermi surface volume! (Z goes to zero).

- Critical point is clearly a non fermi liquid.
Mean field fermi surface evolution

Fermi liquid

``Hot``
``Cold``
Fermi surface

Fractionalized fermi liquid

Spinon
Fermi surface

``Hot``
Fermi surface

``Cold``
Fermi surface

Electron
Fermi surface

At transition $Z \sim b^2 \to 0$ on hot Fermi surface.
Fluctuations

Spinon representation of local moments has gauge redundancy

Eg: $f_r \rightarrow e^{i\theta_r} f_r$ leaves $\tilde{\sigma}_r = f_r^+ \tilde{\sigma}_r f_r$ unchanged.

$\Rightarrow b_r \sim c_r^+ f_r \rightarrow e^{i\theta_r} b_r$; $\chi_{rr'} \sim f_r^+ f_{r'} \rightarrow e^{i(\theta_r - \theta_{r'})} \chi_{rr'}$

Theory of fluctuations is a U(1) gauge theory.

FL phase: $\langle b \rangle \neq 0 \Rightarrow$ FL is Higgs phase

$FL^*: c$-fermi surface + spinons coupled to U(1) gauge field
Fluctuations (cont’d)

FL*: spinons coupled to gapless U(1) gauge field

- Near critical point: (slave) bosons and spinons coupled to a gapless U(1) gauge field
  Transition driven by condensation of slave boson.

Non-fermi liquid critical point:
Eg: Specific heat $C \approx T \ln T$ (d = 3), $T^{2/3}$ (d = 2),
singular ‘$2k_f$’ spin susceptibility along lines in k-space (d = 2),
conductivity $\approx \ln(1/T)$, ................

(Similar to gauge theories of optimally doped cuprates but bosons are at fixed chemical
potential rather than fixed density).

+other possibilities such as a $Z_2$ gauge field also exist.
How to get magnetism?

- Gauge field can confine spinons leading to magnetic long range order (particularly in $d = 2$) – low energy instability of FL* to local moment magnetic metal.

- Interesting possibility: confinement effective only in FL* phase but not at critical point

  $\Rightarrow$ Direct second order transition between heavy Fermi liquid and local moment magnetic metal but with interesting `deconfined’ critical point described in terms of spinons and gauge fields.
How to get magnetism?

• Gauge field can confine spinons leading to magnetic long range order (particularly in $d = 2$)

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⇒ Direct second order transition between heavy Fermi liquid and local moment magnetic metal but with interesting `deconfined' critical point described in terms of spinons and gauge fields.

Admittedly speculative but..................................
Evidence from a simpler context – insulating quantum magnets

• Highlights: Clear demonstration of such theoretical phenomena at (certain) quantum transitions

• Emergence of `fractional’ charge and gauge fields near quantum critical points between two CONVENTIONAL phases.
  - ``Deconfined quantum criticality” (made more precise later).

• Many lessons for competing order physics in correlated electron systems.