Ideas on non-fermi liquid metals and quantum criticality

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Plan

Lecture 1:

General discussion of heavy fermi liquids and their magnetism

Review of some experiments

Concrete `Kondo breakdown' model with a non-fermi liquid critical point

Lecture 2:

Deconfined quantum criticality

Lecture 3:

Some new ideas and scaling hypotheses

Luttinger's theorem for Fermi liquids

In a Fermi liquid, volume V_F of Fermi surface is set
by electron density n independent of interaction strength.

$V_F = (2\pi)^d n/2$ (mod Brillouin zone volume).

Perturbative proof: Luttinger
Non-perturbative topological arguments:
Yamanaka, Oshikawa, Affleck (d = 1), Oshikawa (d > 1).

Oshikawa: Regard as ``topological quantization''.

Heavy electron metals

Typically rare earth intermetallic alloys

Eg: $CeAl_3$, $CeCu_2Si_2$, UPt_3, etc

Huge effective mass at low-T, eg in specific heat

$\frac{m^*}{m} \sim 10^2 - 10^3$ (hence 'heavy electron').

Strongly correlated partially filled f-band
+ weakly correlated conduction bands.

Simplified Anderson lattice model for heavy fermi liquids

$$H = \sum_k \epsilon_k c_k^\dagger c_k + V \sum_i \left(c_{i\alpha}^\dagger f_{i\alpha} + h.c \right) + \sum_k \epsilon_{kf} f_k^\dagger f_k + U \sum_i \left(n_i^f \right)^2 - \mu_f \sum_i n_i^f$$

Weak-U limit : Hybridized c and f bands

Fermi volume includes both c and f electrons

but effective mass not large, and $Z \sim O(1)$.

Strong correlations: Kondo lattice

½ - filling for f-band

Large -U limit ⇒ each localized f-orbital

is singly occupied.

Lattice of localized f-moments coupled to

c-electrons

$$H = \sum_k \epsilon_k c_k^\dagger c_k + J \sum_i \vec{S}_i \cdot c_i^\dagger \frac{\vec{\sigma}}{2} c_i$$

$\vec{S}_i$ = f-moment ; $J \sim o(V^2/U)$

Luttinger's theorem in Kondo lattices

- Kondo lattice model admits a Fermi liquid phase.

(Denote ``Kondo liquid'').

- To satisfy Luttinger's theorem must include local moments in the count of conduction electron density

  ``Large'' Fermi surface

- Fermi liquid adiabatically connected to small U Anderson model.

- Understand through (i) slave particle mean field calculation

(Read et al, Millis et al)

(ii) Oshikawa topological argument.

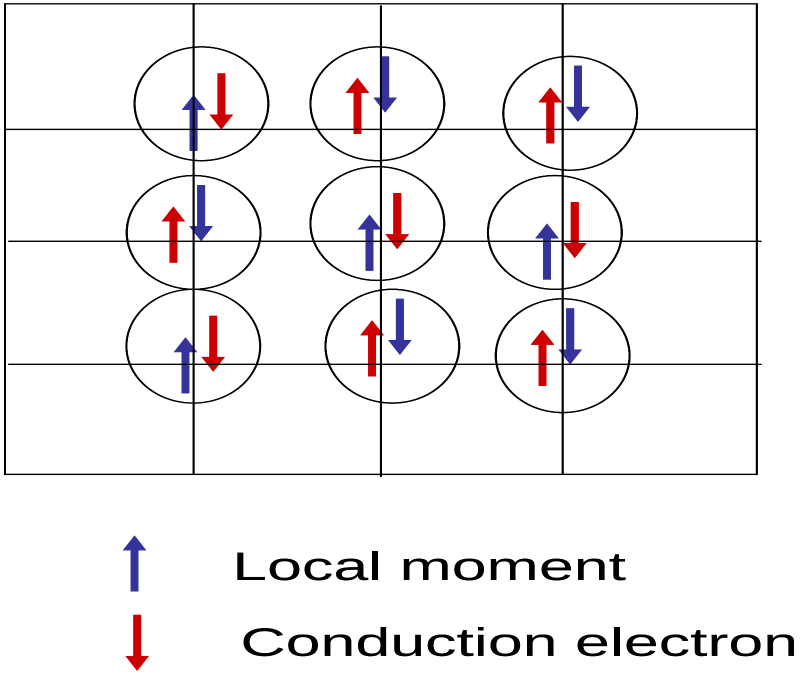
Physical picture: strong coupling limit

- Large J_K: Treat conduction electron hopping t as perturbation.

- t = 0 and half-filling for conduction electrons: Each local moment traps a conduction electron into a singlet.

Insulator with a gap: ``Kondo insulator''

(Stable to small t)



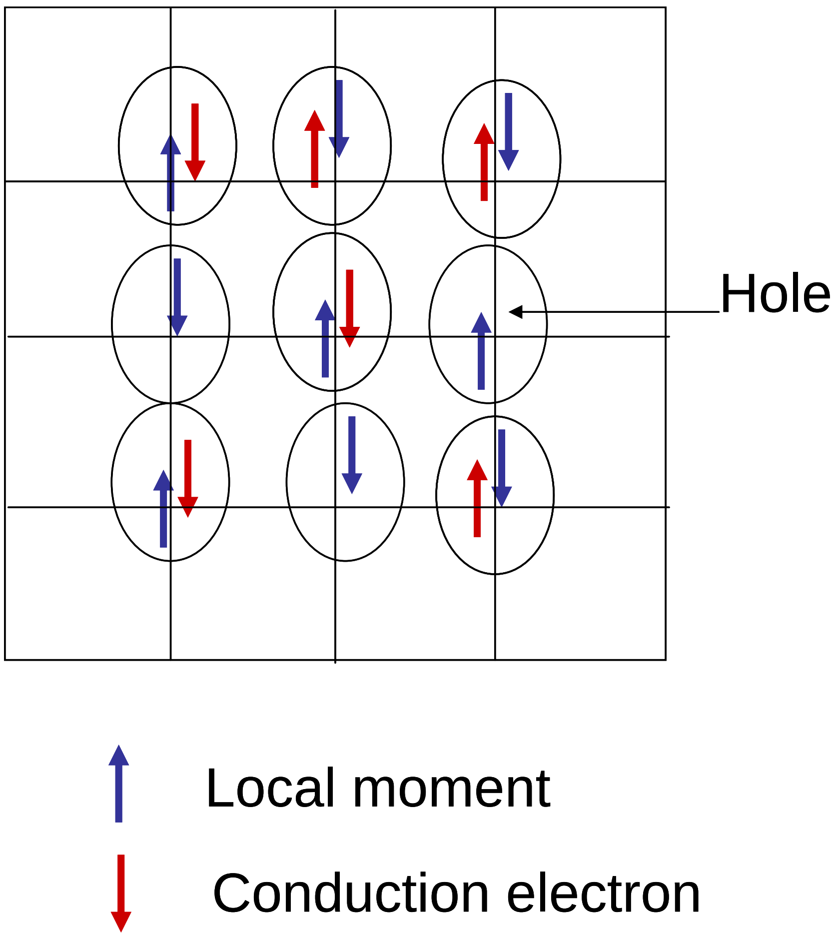
To fit into a band picture count both local moments and conduction electrons as part of band.

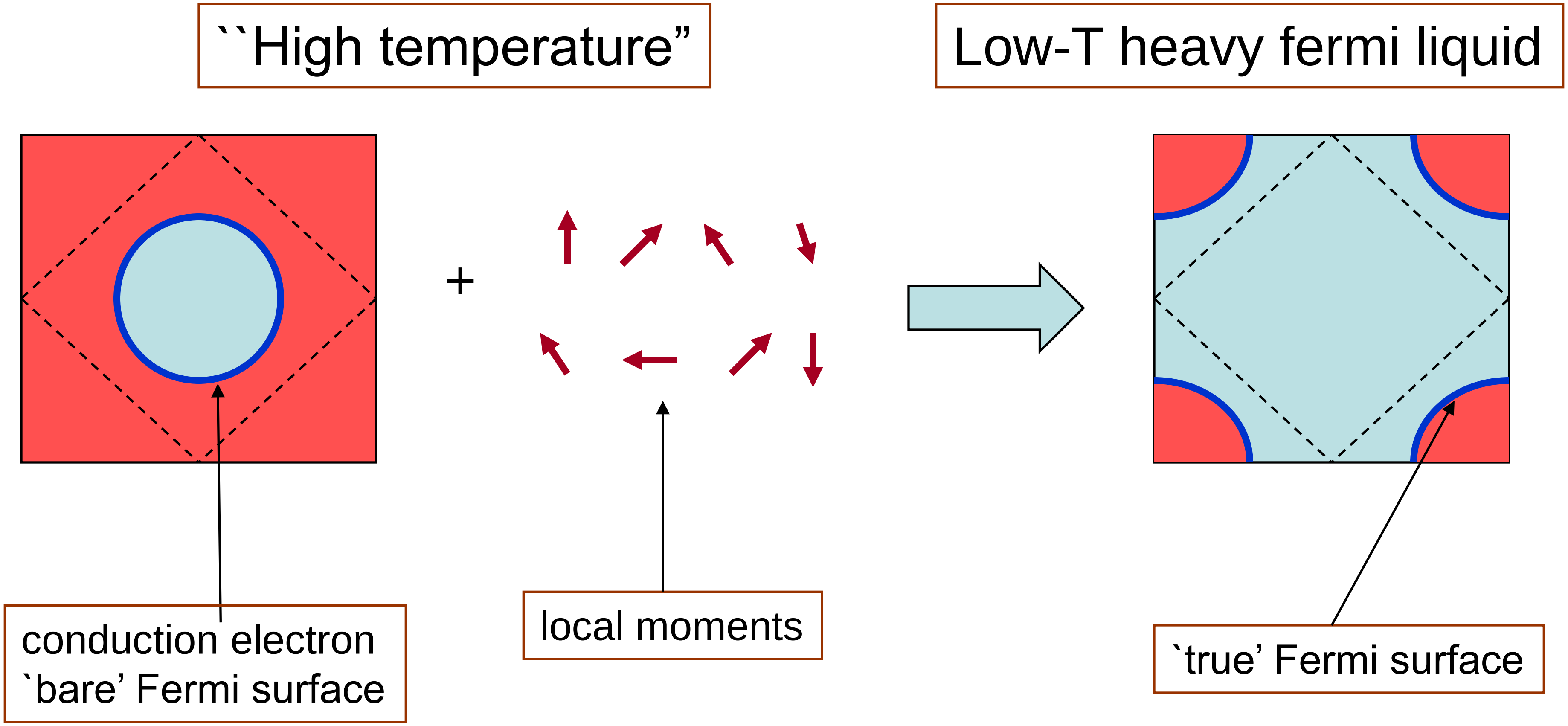
Doping the Kondo insulator

Move away from half-filling
for the conduction electrons:

Some unscreened free moments
created by conduction holes
≈ spinful fermions.

Hopping: Fermi liquid with hole
Fermi surface determined by hole
density ≡ large electron Fermi
surface.




``High temperature"
Low-T heavy fermi liquid
+
conduction electron
`bare' Fermi surface
local moments
`true' Fermi surface

(N. Read et al, '86, Millis, Lee '86)

Slave particle formulation of Kondo lattice

Write $\vec{S}_i = f_i^\dagger \frac{\vec{\sigma}}{2} f_i$ with $f_i^\dagger f_i = 1 \ \forall i$

Important: $f_{i\alpha}$ not electrons

- neutral spin-1/2 fermions ("spinons")

$$H = \sum_k \epsilon_k c_k^\dagger c_k - J \sum_i \left(|c_i^\dagger f_i|^2 + h.c \right)$$

Constraint: $f_i^\dagger f_i = 1 \ \forall i$.

Hybridization mean field theory

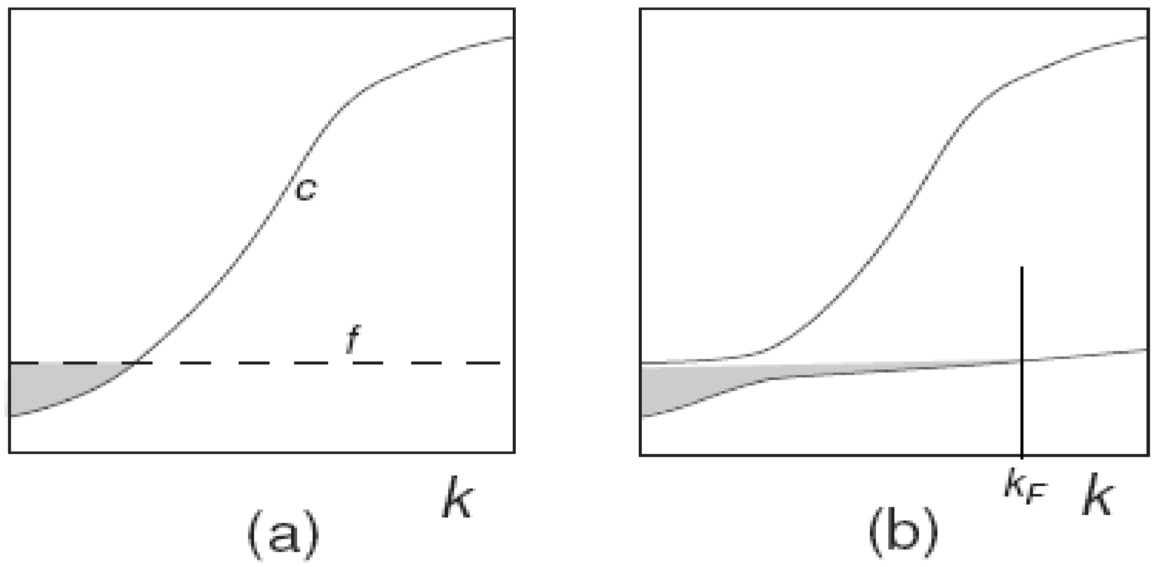
Treat Kondo exchange, constraint in mean field

$$H_{MF} = \sum_k \epsilon_k c^\dagger_k c_k + b \sum_k \left(c^\dagger_k f_k + h.c. \right) + \mu_f \sum_k f^\dagger_k f_k$$

$$b = J \langle f^\dagger_k c_k \rangle$$

$$\langle f^\dagger_i f_i \rangle = 1 .$$

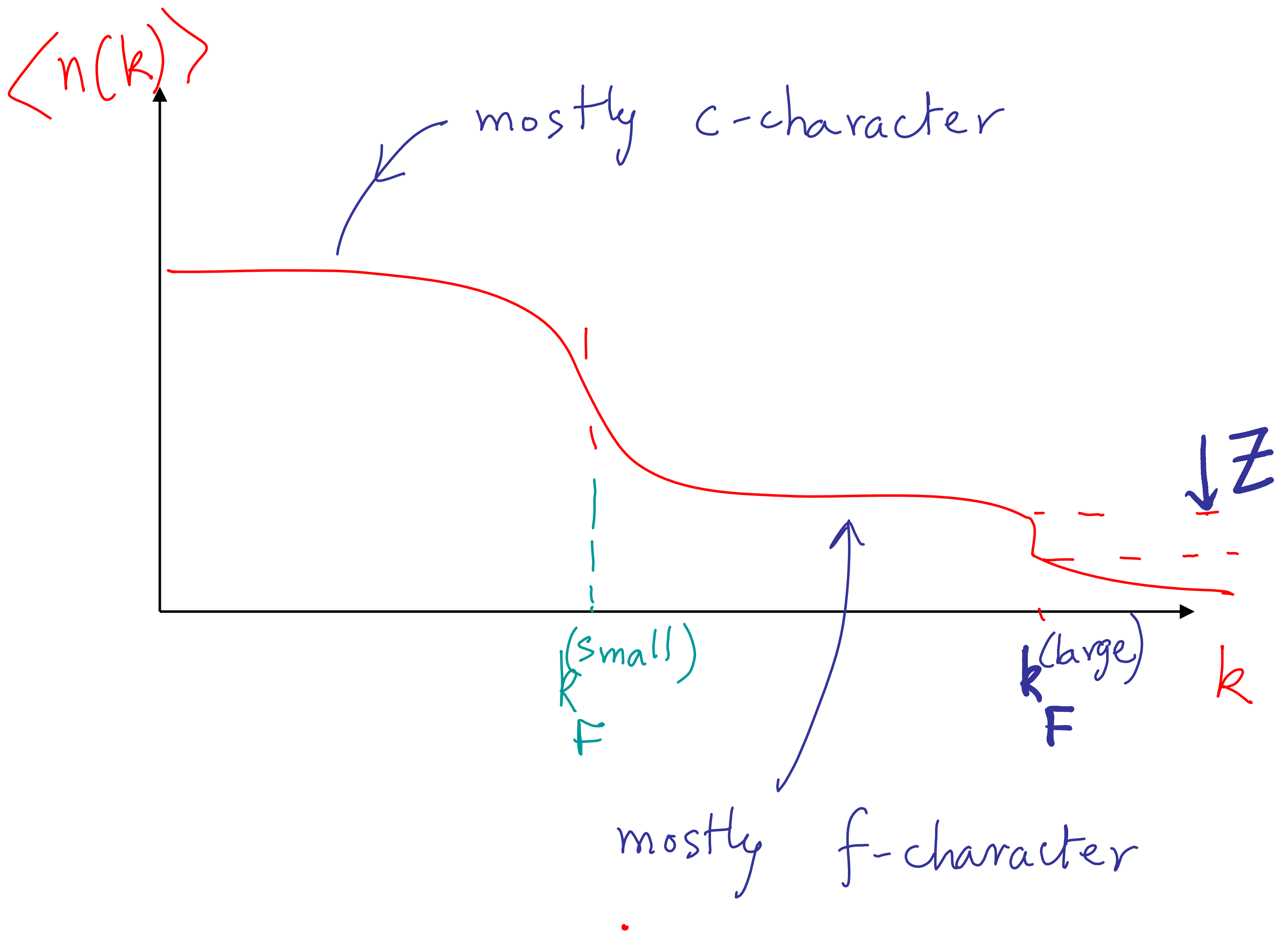
Band structure



Theory gives (i) large Fermi surface

(ii) large quasiparticle mass, etc !

Momentum distribution



Comments

1. $b \sim \langle c^{+} f \rangle \sim$ amplitude of Kondo singlet

2. Heavy quasiparticles at Fermi surface

$$\approx f + b\,c$$

$$\Rightarrow \text{Quasiparticle weight } Z \sim b^2 \sim O\left(\frac{m}{m^*}\right)$$

$$\ll 1$$

Other known metallic states of Kondo lattice

- Magnetically ordered (typically antiferromagnetic) metal due to RKKY

Favored at small J_K (Doniach).

There are actually two possibly distinct kinds of antiferromagnetic metals.

- Other broken symmetry states (SC, …..)

Two kinds of antiferromagnetic metals in Kondo lattices

(A): ``Local moment magnetic metal''

Ordering of local moments (due to intermoment exchange).

c-electrons ≈ decoupled from local moments.

Typically large (staggered) moment.

``f-electrons do not participate in Fermi surface''.

(B): ``Spin density wave metal''

SDW instability of heavy fermi liquid with large fermi surface.

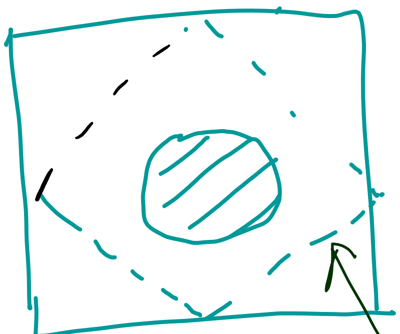
c- and f-electrons strongly coupled.

Typically weak moment.

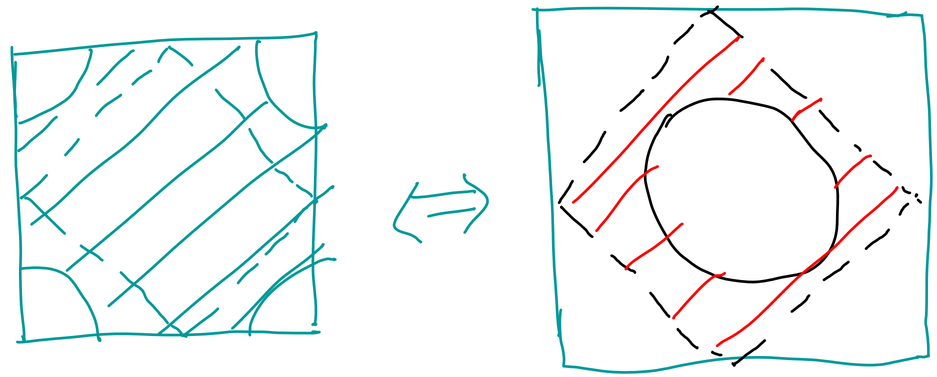
``f-electrons participate in Fermi surface''.

- Two kinds of magnetic metals possibly sharply distinct!!

- Distinction in topology of Fermi surface



versus



magnetic Brillouin zone

Fermi surface of local moment magnetic metal

+ 1 filled band

Fermi surface of SDW metal

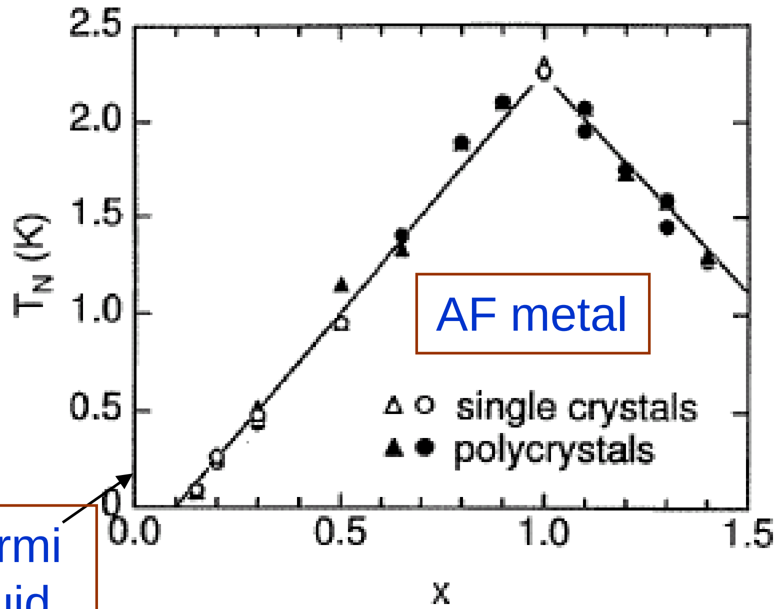
Evolution between the 2 magnetic metals thru' a quantum phase transition!

(Is this true more generally?)

Magnetic quantum critical points in heavy fermion liquids

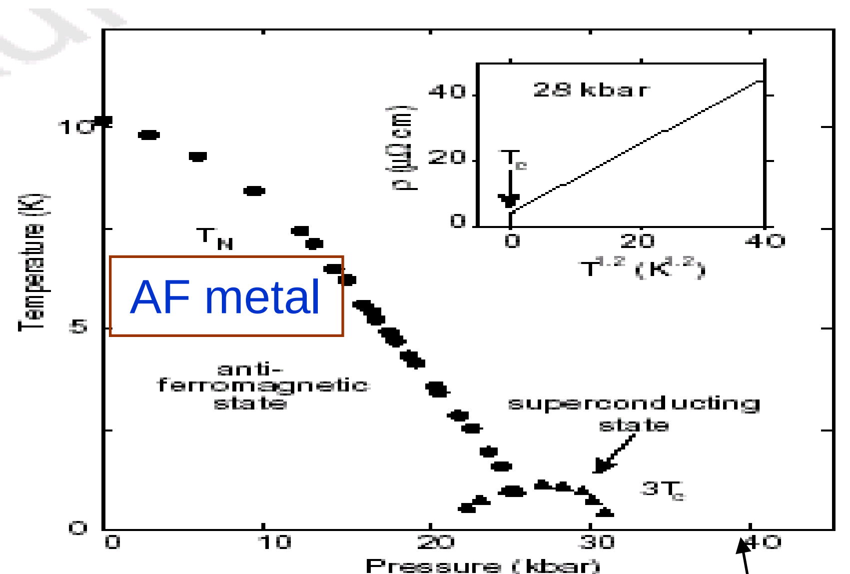
$CeCu_{6-x}Au_x$

$CePd_2Si_2$



Fermi liquid

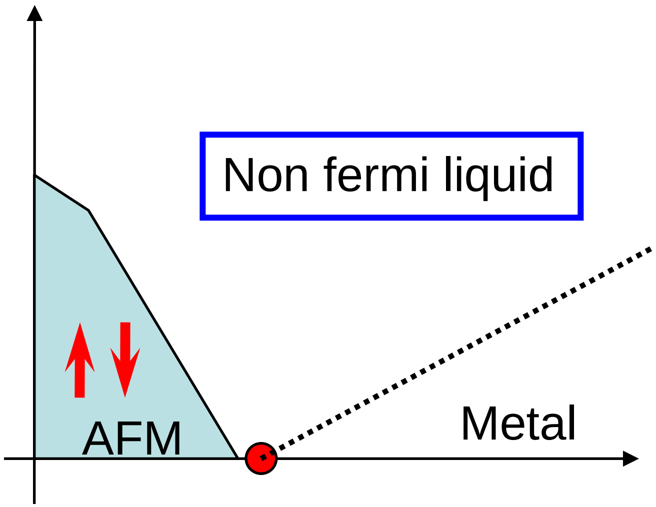
H. v. Löhneysen et al, PRL 1994



N. Mathur et al, Nature 1998

Fermi liquid

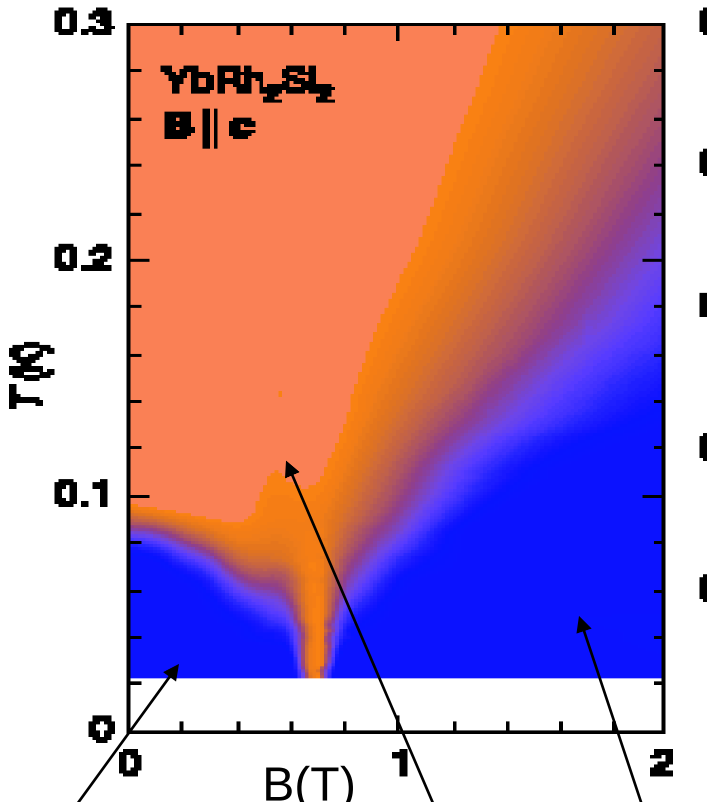
Magnetic ordering and non-fermi liquid physics



Non fermi liquid : Diverging γ coefficient, nontrivial power law resistivity, scaling in spin fluctuation spectrum etc

Representative data: resistivity of $YbRh_2Si_2$

Custers et al, Nature, 2003

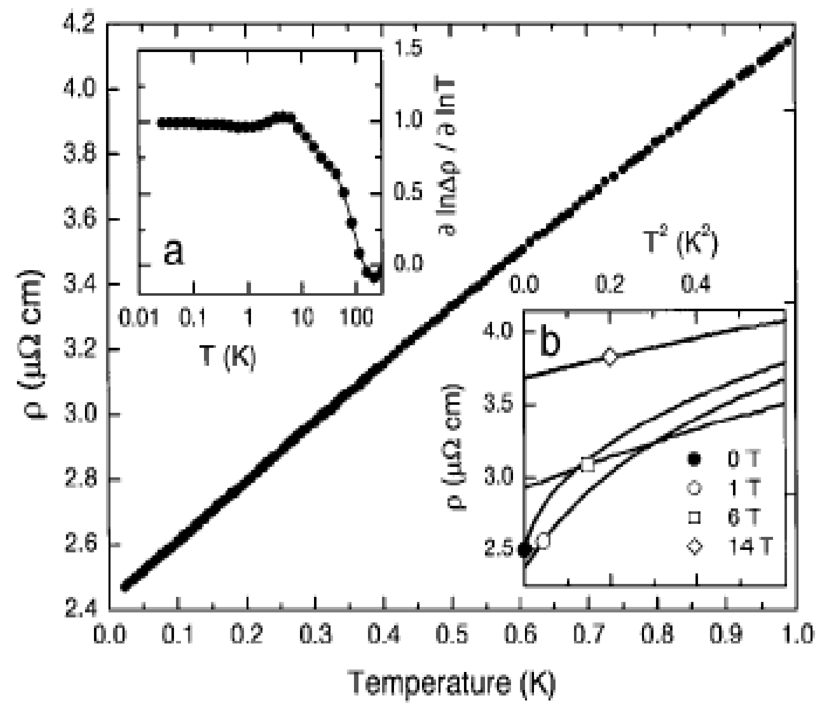


Magnetic metal

Non fermi liquid

Fermi liquid

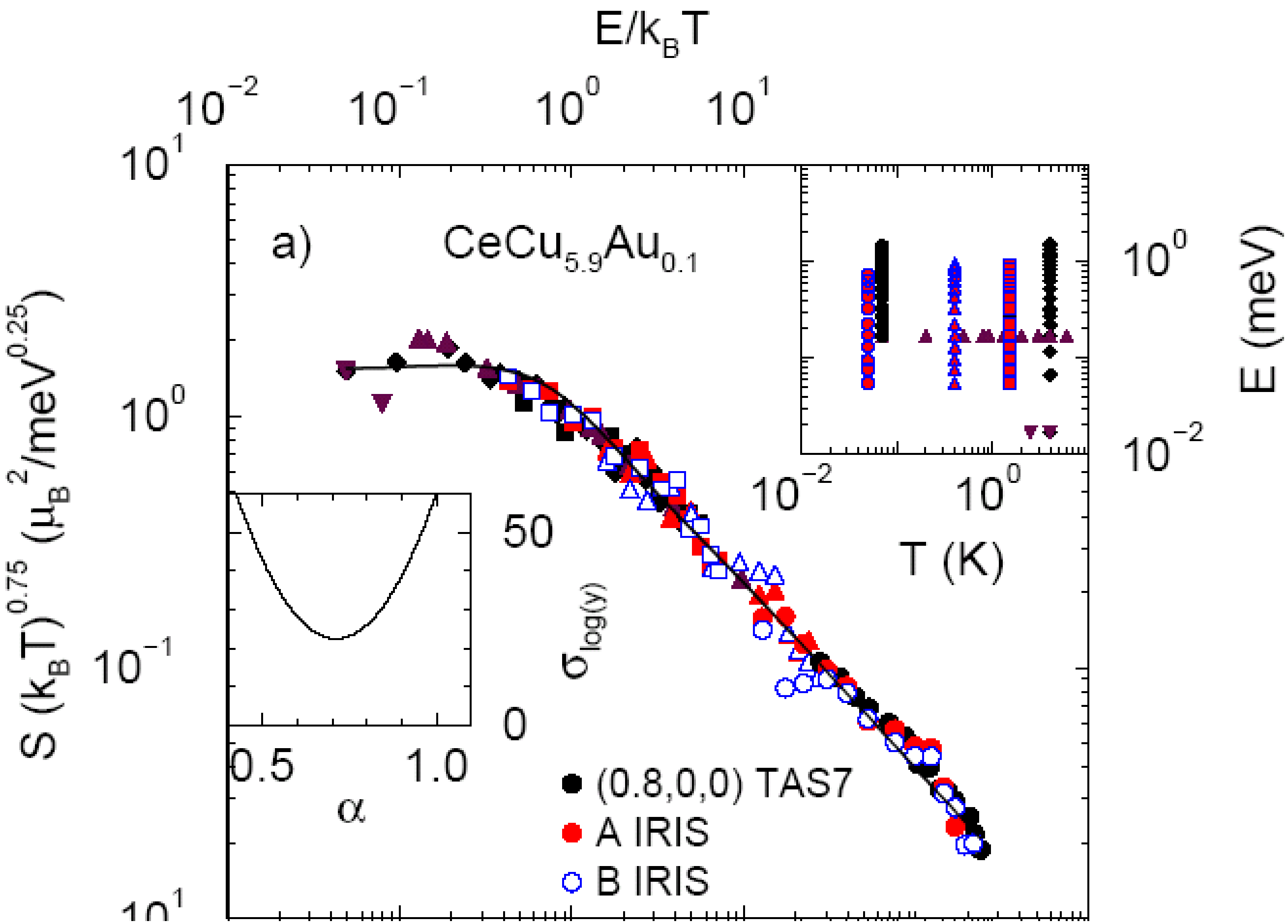
Trovarelli et al, PRL 2000



T-dependence of resistivity at critical point: $\rho(T) \sim T$ for three decades in temperature!

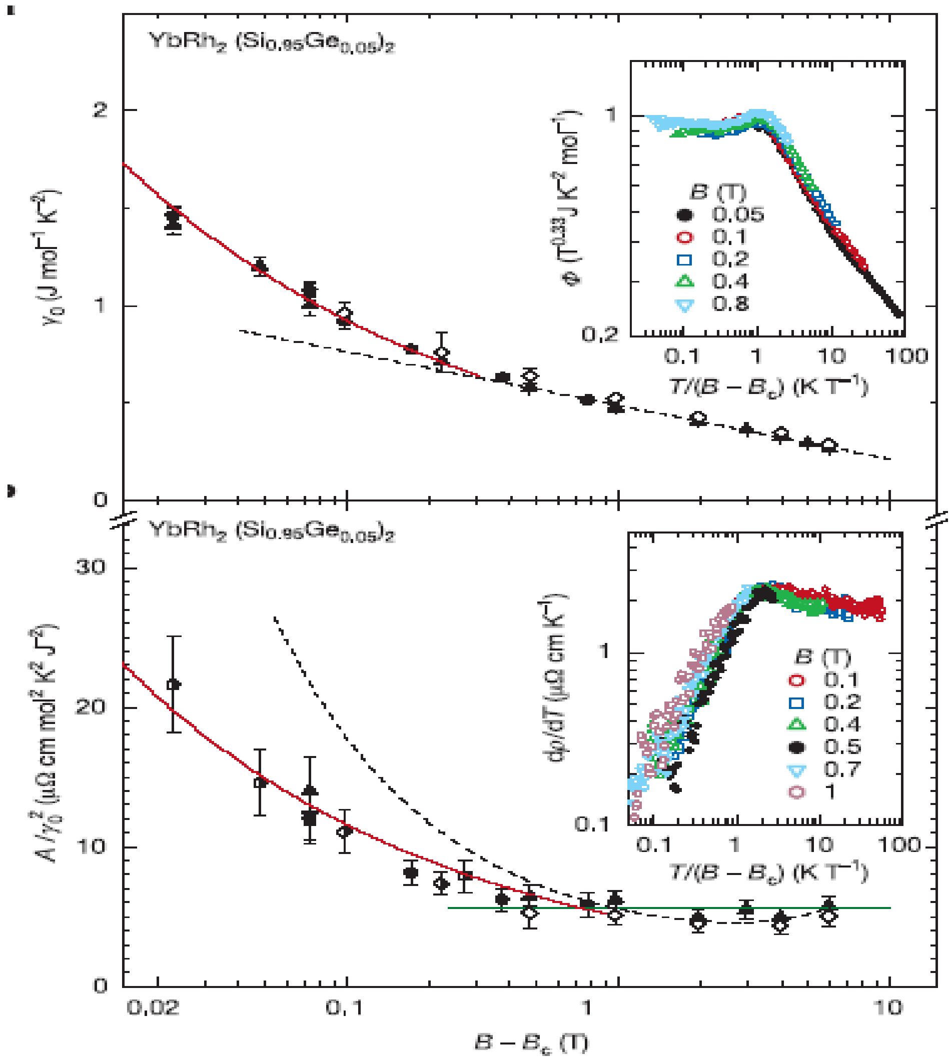
Scaling of dynamic spin correlations:

Inelastic neutron scattering near ordering wavevector



A. Schröder et al., Nature '00; PRL '98

Singular specific heat



``Classical'' assumption

1. NFL: Universal physics associated with quantum critical point between heavy fermi liquid and magnetic metal

2. Landau: Universal critical singularities ~ fluctuations of natural magnetic order parameter for transition

Try to play Landau versus Landau.

Which magnetic metal?

1. Heavy fermi liquid – SDW metal:

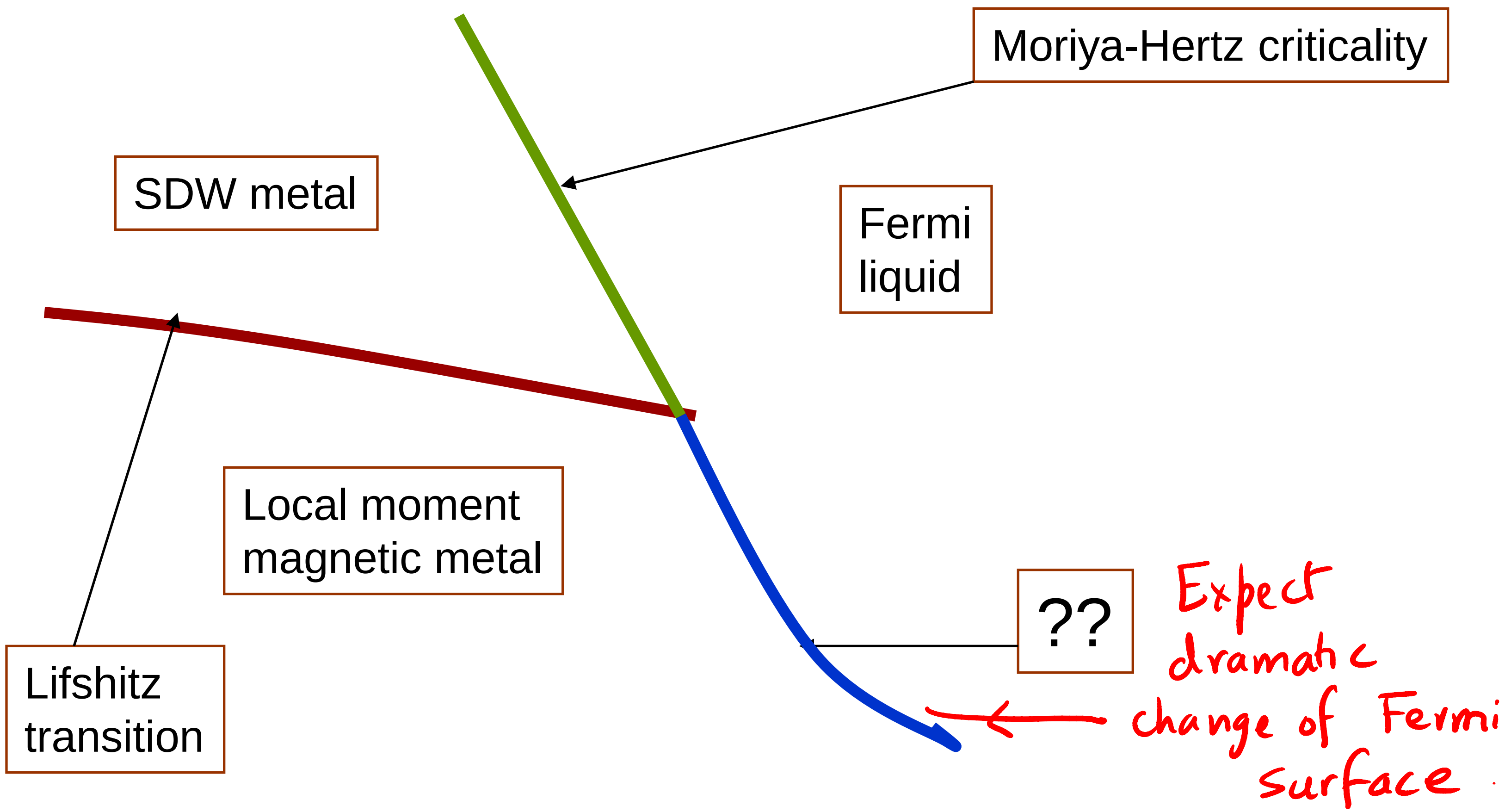
Fluctuations of magnetic order parameter with damping due to fermionic quasiparticles (Moriya-Hertz-Millis)

- Fail to reproduce observed NFL physics.

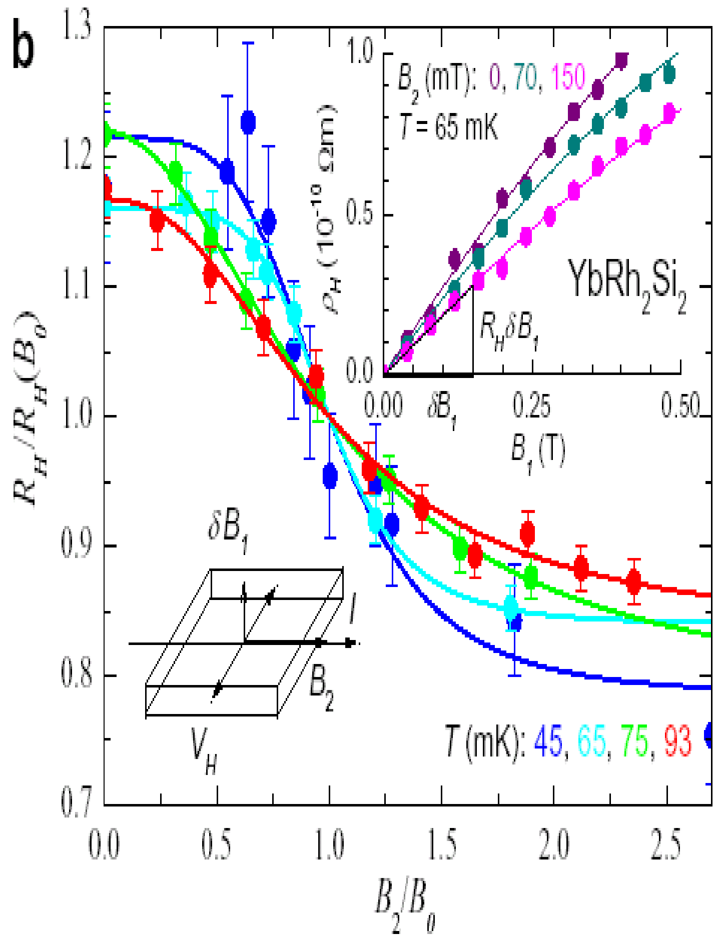
2. Explore alternate possibility:

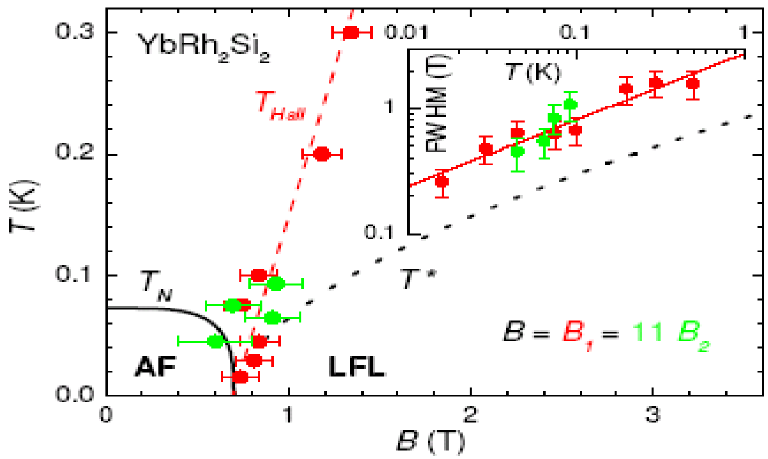
Transition between heavy fermi liquid and local moment magnetic metal.

Schematic phase structure



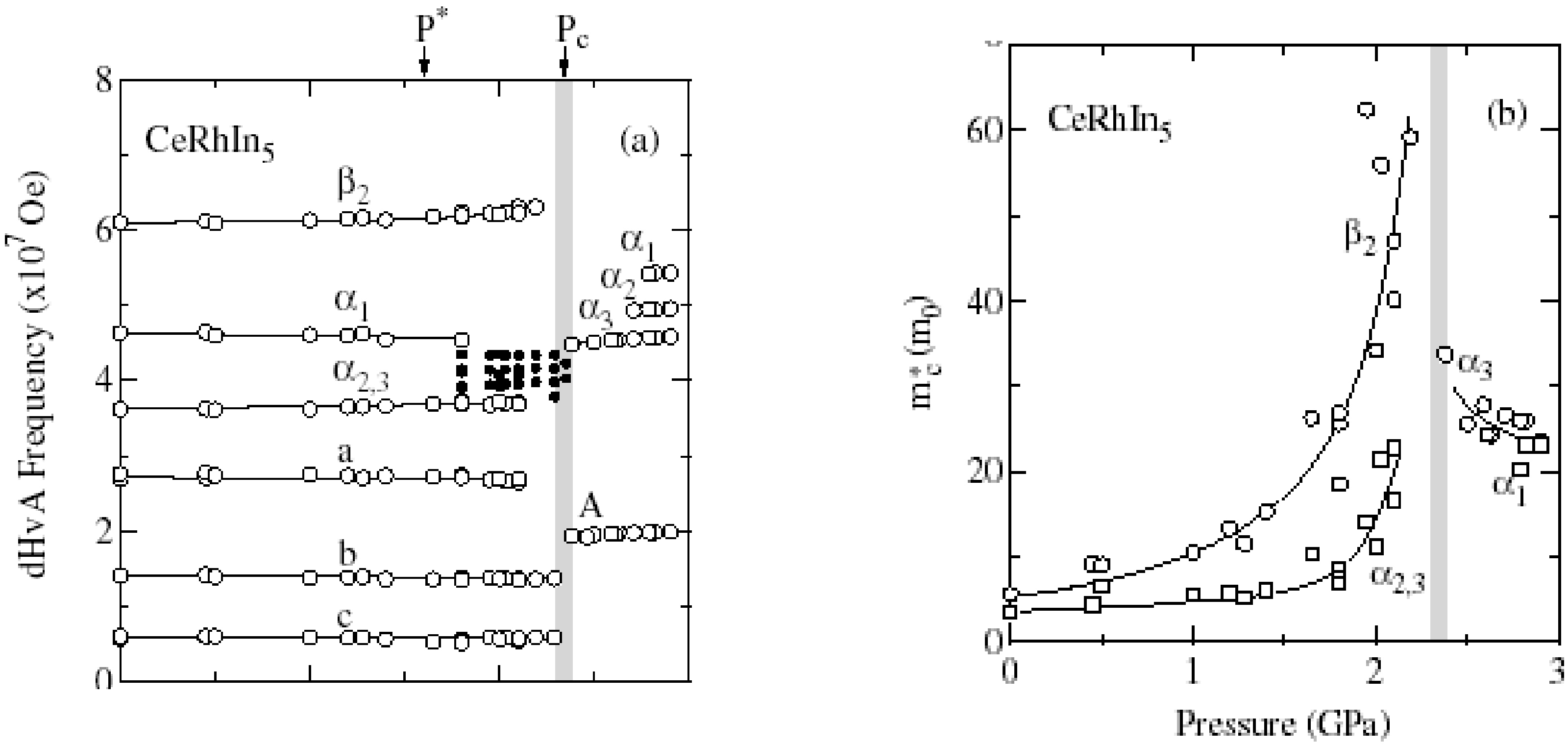
Evidence from experiments I: Hall effect in YbRh2Si2





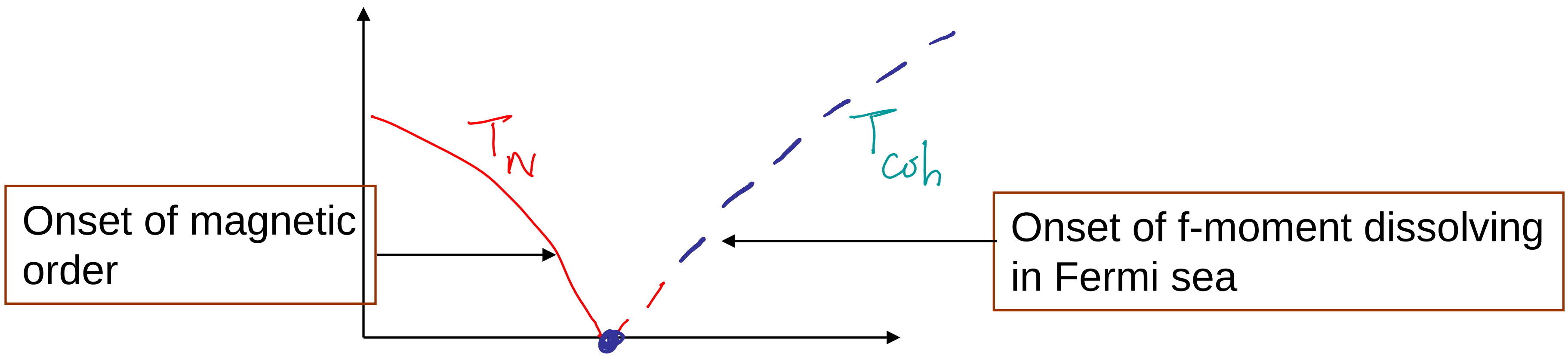
S. Paschen et al., Nature 432, 881 (2004)

Evidence from experiments II: dHvA in CeRhIn5



H. Shishido, R. Settai, H. Harima, & Y. Onuki, JPSJ 74, 1103 (2005)

Questions



1. Is such a second order transition generically possible?

Loss of magnetic order happens at same point as f-moments dissolving in Fermi sea: why should these 2 different things happen at the same time?

2. Theoretical description?

3. Will it reproduce observed non-fermi liquid behaviour?

Answers not known!!

General observations

- f-moments drop out of Fermi surface (⇔ change of electronic structure)

Associated time scale t_e.

- Onset of magnetic order

Associated time scale t_m.

Both time scales diverge if there is a critical point.

[Expect $t_e \sim |\delta g|^{-\phi_e}$, $t_m \sim |\delta g|^{-\phi_m}$

(δg = tuning parameter)]

General observations

- f-moments drop out of Fermi surface (⇔ change of electronic structure)

Associated time scale t_e.

- Onset of magnetic order

Associated time scale t_m.

Both time scales diverge if there is a critical point.

Suggestion: One time scale may diverge faster than the other so that the two competing orders are dynamically separated.

Separation between two competing orders as a function of scale (rather than tuning parameter) might make second order transition possible.

Options

1. t_m diverges faster than t_e.

(electronic structure change first, magnetic order comes later)

2. t_e diverges faster than t_m

(magnetic order destroyed first, Fermi surface reconstruction comes later)

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Separation between two competing orders as a function of scale (rather than tuning parameter) might make second order transition possible.

Focus on option 1 as a more likely route to non-fermi liquid physics

Some implications

- ``Underlying'' transition: loss of participation of the f-electrons in forming the heavy fermi liquid.

(View as a Mott ``metal-insulator'' transition of f-band).

- Magnetic order: ``secondary'' effect – a low energy complication once Kondo effect is suppressed.

- Non-fermi liquid due to fluctuations associated with change of electronic structure rather than those of magnetic order parameter.

$\Rightarrow$ PHYSICS BEYOND LANDAU-GINZBURG-WILSON PARADIGM FOR PHASE TRANSITIONS.

(Natural magnetic order parameter is a distraction).

Intermediate time scale physics

- f-moments drop put of Fermi surface but continue to form singlet bonds with each other

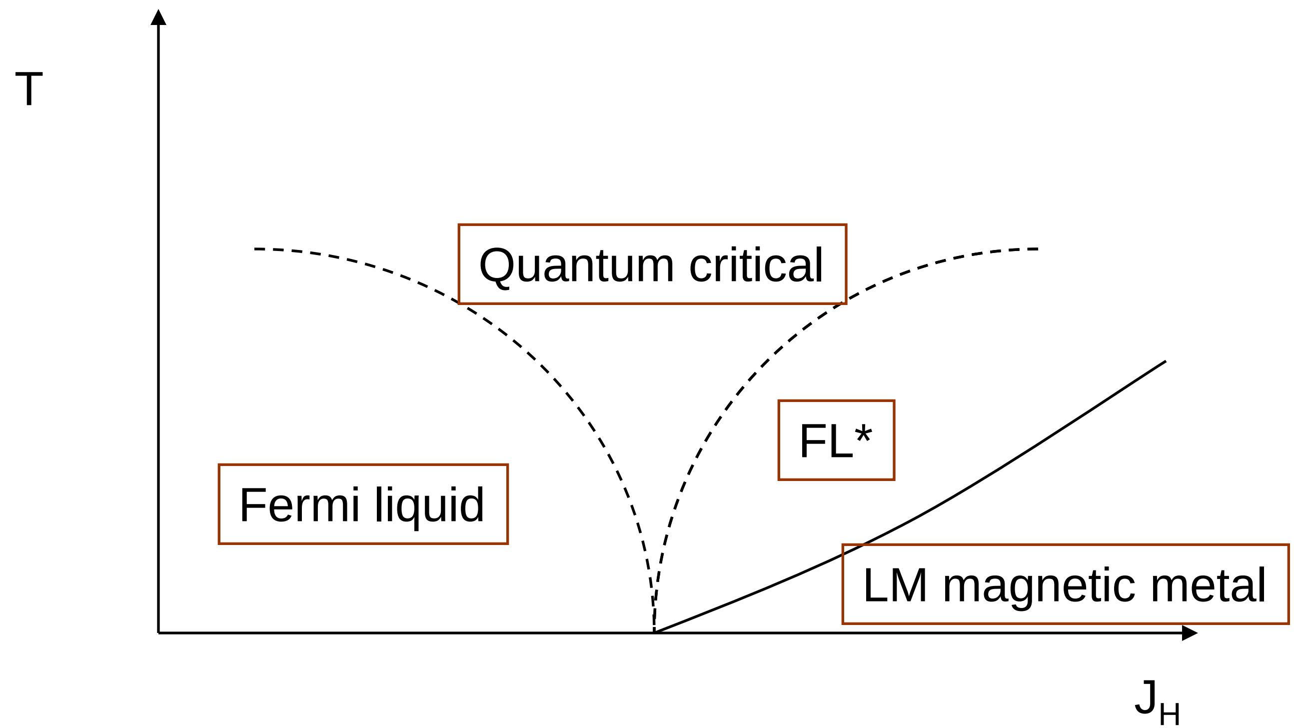
Resulting state: spin liquid of f-moments coexisting with small Fermi surface of conduction electrons

(a ``fractionalized Fermi liquid'')

Magnetism: low energy instability of

such a small Fermi surface state.

Suggested phase diagram and crossovers



? HOW TO PUT MEAT INTO THIS PICTURE ?

Study tractable simpler questions

1. Effects of loss of ``Kondo'' order?

Study second order quantum transitions associated with loss of Kondo screening. [TS,Vojta, Sachdev, '04]

Worry about magnetism later

2. Do similar theoretical phenomena (eg: breakdown of Landau paradigm) happen in other contexts?

Yes! (study quantum phase transitions in insulating magnets)

(TS, Vishwanath, Balents, Sachdev, M. Fisher)

Kondo Heisenberg models

- $J_K = 0$: Conduction electrons are decoupled from local moments and have small Fermi surface.

- Non-magnetic ground states of spin system

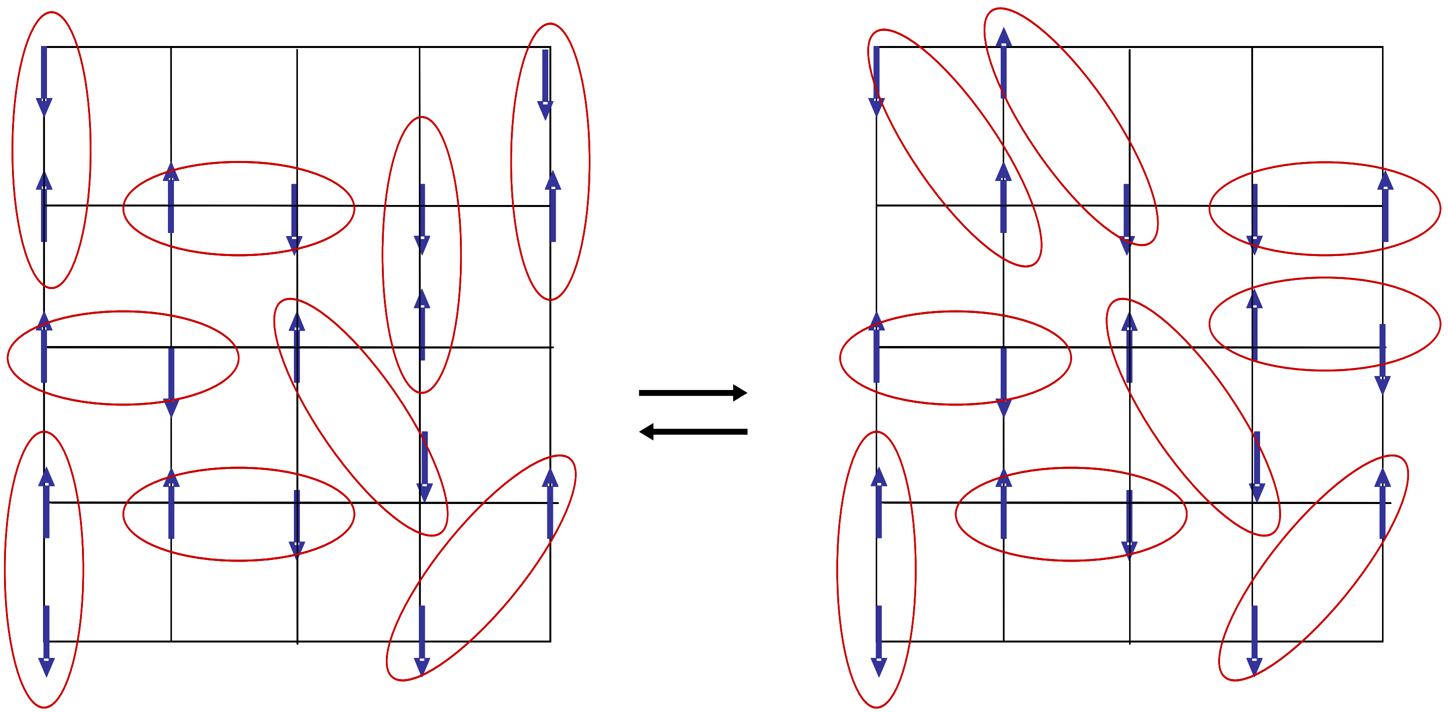
(i) Spin –Peierls: break translational symmetry

(ii) Fractionalized: can preserve translational symmetry

Focus on (ii) to discuss small Fermi surface state.

Fractionalization in d > 1

- Anderson: RVB spin liquid state for quantum spin models.



Couple spin liquids to conduction electrons

- Small non-zero J_K: Perturb in J_K

- Emergent gauge structure of local moment system survives; conduction electrons stay sharp on a small Fermi surface*.

 - advertised small fermi surface state.

- A fermi liquid in peaceful coexistence with fractionalization

  ``Fractionalized fermi liquid'' (denote FL*)

- Large J_K: Recover large Fermi surface Kondo liquid.

(* Possible pairing instability at low T).

Physics of fractionalized fermi liquid (FL*) state

- Each local moment forms singlet with another local moment.

- Weak Kondo coupling can't break singlets:

Local moments and conduction electrons essentially stay decoupled.

Mean field theory

Kondo - Heisenberg model

$$H = -t\sum_{\langle rr'\rangle} c^\dagger_r c_{r'} - \mu\sum_r c^\dagger_r c_r + \frac{J_K}{2}\sum_r c^\dagger_r \vec{\sigma} c_r \cdot \vec{S}_r + J_H \sum_{\langle rr'\rangle} \vec{S}_r \cdot \vec{S}_{r'}$$

$\vec{S}_r = f^\dagger_r \frac{\vec{\sigma}}{2} f_r$ with $f^\dagger_r f_r = 1$

Decouple J_K with $b_r \sim c^\dagger_r f_r$ and J_H with $\chi_{rr'} \sim f^\dagger_r f_{r'}$

$\langle b\rangle \neq 0$ $(\Rightarrow \langle \chi\rangle \neq 0)$: Fermi liquid (FL)

$\langle b\rangle = 0$, $\langle \chi\rangle \neq 0$: Fractionalized Fermi liquid (FL^*) with spinon Fermi surface.

Direct fermi volume changing transition

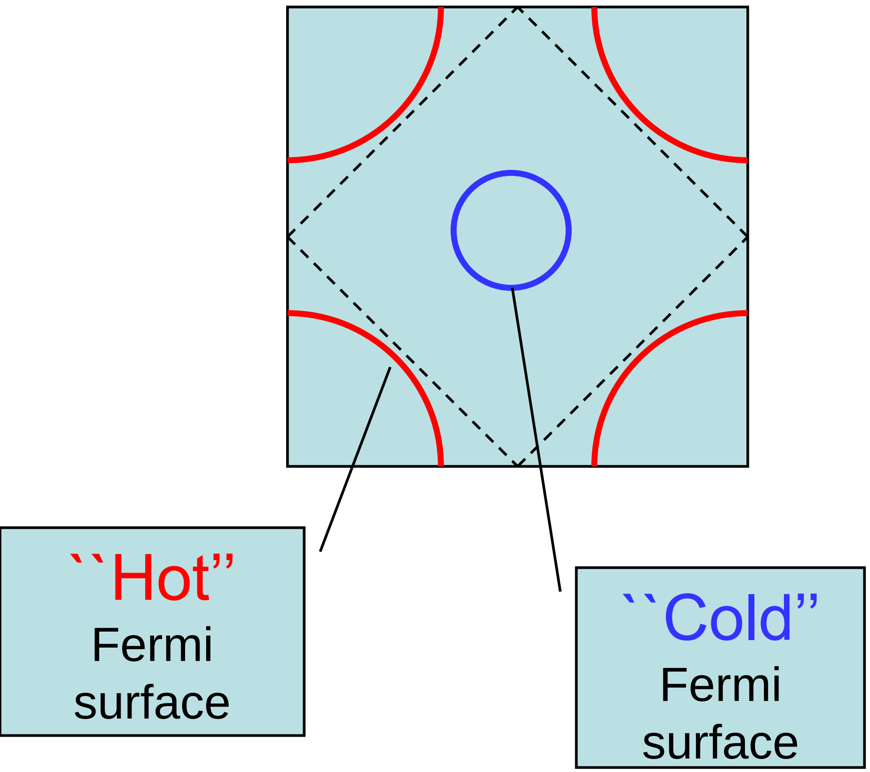
- Condensation of hybridization amplitude b drives direct Fermi volume changing transition.

- Transition can be second order despite jump in fermi surface volume!
  (Z goes to zero).

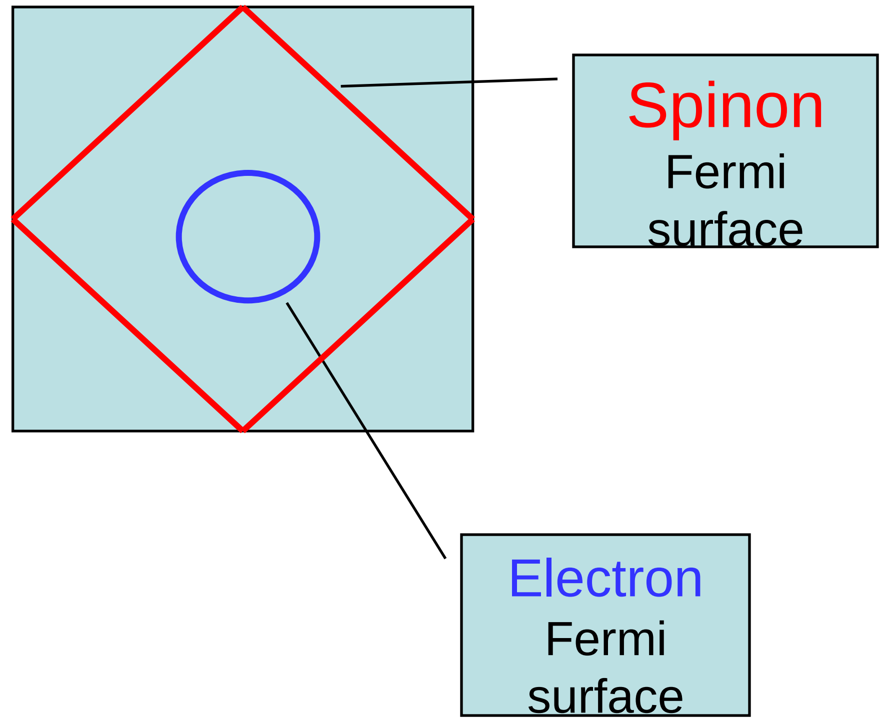
- Critical point is clearly a non fermi liquid.

Mean field fermi surface evolution

Fermi liquid



Fractionalized fermi liquid



At transition $Z \sim b^2 \rightarrow 0$ on hot Fermi surface.

Fluctuations

Spinon representation of local moments has gauge redundancy

Eg: $f_r \to e^{i\theta_r} f_r$ leaves $\vec{S}_r = f_r^\dagger \frac{\vec{\sigma}}{2} f_r$ unchanged.

$\left(\Rightarrow \ b_r \sim c_r^\dagger f_r \to e^{i\theta_r} b_r \ ; \ \chi_{rr'} \sim f_r^\dagger f_{r'} \to e^{i(\theta_{r'} - \theta_r)} \chi_{rr'} \right)$

Theory of fluctuations is a U(1) gauge theory.

FL phase: $\langle b \rangle \neq 0 \Rightarrow$ FL is Higgs phase

FL^*: c-fermi surface + spinons coupled to U(1) gauge field

Fluctuations (cont'd)

FL*: spinons coupled to gapless U(1) gauge field+

- Near critical point: (slave) bosons and spinons coupled to a gapless U(1) gauge field

Transition driven by condensation of slave boson.

Non-fermi liquid critical point:
Eg: Specific heat C ≈TlnT (d = 3), $T^{2/3}$ (d = 2),
singular `$2k_f$' spin susceptibility along lines in k-space (d = 2),
conductivity ≈ ln(1/T),……………

(Similar to gauge theories of optimally doped cuprates but bosons are at fixed chemical potential rather than fixed density).

+other possibilites such as a Z_2 gauge field also exist.

How to get magnetism?

- Gauge field can confine spinons leading to magnetic long range order (particularly in d = 2) – low energy instability of FL* to local moment magnetic metal.

- Interesting possibility: confinement effective only in FL* phase but not at critical point

$\Rightarrow$ Direct second order transition between heavy Fermi liquid and local moment magnetic metal but with interesting `deconfined' critical point described in terms of spinons and gauge fields.

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- Gauge field can confine spinons leading to magnetic long range order (particularly in d = 2)

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Admittedly speculative but……………………….

Evidence from a simpler context – insulating quantum magnets

- Highlights: Clear demonstration of

such theoretical phenomena at (certain) quantum transitions

- Emergence of `fractional' charge and gauge fields near quantum critical points between two CONVENTIONAL phases.

- ``Deconfined quantum criticality'' (made more precise later).

- Many lessons for competing order physics in correlated electron systems.