Lecture 2: Deconfined quantum criticality

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General theoretical questions

- Fate of Landau-Ginzburg-Wilson ideas at quantum phase transitions?

- (More precise) Could Landau order parameters for the phases distract from the true critical behavior?

Study phase transitions in insulating quantum magnets
- Good theoretical laboratory for physics of phase transitions/competing orders.

(Senthil, Vishwanath, Balents, Sachdev, Fisher, Science 2004)
Highlights

• Failure of Landau paradigm at (certain) quantum transitions

• Rough description: Emergence of `fractional’ charge and gauge fields near quantum critical points between two CONVENTIONAL phases.
  - `Deconfined quantum criticality’

• Many lessons for competing order physics in correlated electron systems.
Phase transitions in quantum magnetism

\[ H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + \ldots \]

- Spin-1/2 quantum antiferromagnets on a square lattice.

- “……” represent frustrating interactions that can be tuned to drive phase transitions.

(Eg: Next near neighbour exchange, ring exchange,.....).
Possible quantum phases

• Neel ordered state

Order parameter – staggered magnetic moment vector

Neel ordering can be destroyed at zero temperature with suitable frustrating interactions
Example of a non-magnetic ground state - Valence Bond Solid State (VBS)

• Frozen arrangement of *singlet* bonds between pairs of local moments

• Frozen singlet pattern breaks lattice translation and rotation symmetry

\[ \Psi_{\text{bond}} = \left( \frac{1}{\sqrt{2}} \right) \]
VBS Order Parameter

• Associate a Complex Number

$\Psi_{\text{bond}}$
Neel-valence bond solid (VBS) transition

- Neel: Broken spin symmetry
- VBS: Broken lattice symmetry.

- Landau – Two independent order parameters.
  - no generic direct second order transition.
  - either first order or phase coexistence.

This talk: Direct second order transition but with description not in terms of natural order parameter fields.

Naïve Landau expectation

First order

Neel

VBS

Neel + VBS
Neel-Valence Bond Solid transition

• Naïve approaches fail
Attack from Neel ≠ Usual O(3) transition in D = 3
Attack from VBS ≠ Usual $\mathbb{Z}_4$ transition in D = 3
(= XY universality class).

Why do these fail?
Topological defects carry non-trivial quantum numbers!
Attack from VBS (Levin, TS, ‘04)
Topological defects in $\mathbb{Z}_4$ order parameter

- Domain walls – elementary wall has $\pi/2$ shift of clock angle
$Z_4$ domain walls and vortices

- Walls can be oriented; four such walls can end at point.
- End-points are $Z_4$ vortices.
$Z_4$ vortices in VBS phase

Vortex core has an unpaired spin-$1/2$ moment!!

$Z_4$ vortices are spin-$1/2$ `spinons’.

Domain wall energy $\Rightarrow$ linear confinement in VBS phase.
Z₄ disordering transition to Neel state

- As for usual (quantum) Z₄ transition, expect clock anisotropy is irrelevant. (confirm in various limits).

Critical theory: (Quantum) XY but with vortices that carry physical spin-1/2 (= spinons).
Alternate (dual) view

- Duality for usual XY model (Dasgupta-Halperin)
  Phase mode - `photon``

Vortices – gauge charges coupled to photon.

Neel-VBS transition: Vortices are spinons
=> Critical spinons minimally coupled to fluctuating U(1) gauge field*.

*non-compact
Critical theory
``Non-compact CP₁ model''

\[ S = \int d^2 x d \tau \left| (\partial_\mu - i a_\mu) z \right|^2 + r |z|^2 + u |z|^4 + (\varepsilon_{\mu \nu \lambda} \partial_\nu a_\lambda)^2 \]

\( z = \) two-component spin-1/2 spinon field
\( a_\mu = \) non-compact U(1) gauge field.

**Distinct from usual O(3) or Z₄ critical theories**.

Theory not in terms of usual order parameter fields but involve fractional spin objects and gauge fields.

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*Distinction with usual O(3) fixed point due to non-compact gauge field (Motrunich, Vishwanath, '03)
Attack from Neel
(TS, Vishwanath, Balents, Sachdev, Fisher '04)
Field theory of quantum antiferromagnets

- Deep in Neel phase (or close to it) describe by quantum O(3) non-linear sigma model field theory.

- Successful in describing experiments in cuprate Mott insulators with Neel ground states.

\[
S = \int d\tau d^2 x \frac{1}{2g} \left[ (\partial_\tau \hat{n})^2 + (\nabla \hat{n})^2 \right] + S_B
\]

\[\hat{n} = \text{Neel order parameter field}\]

- \(S_B = \text{``Berry phase'' sensitive to microscopic quantized spin at each site.}\)

- Unimportant in Neel but crucial for VBS.
Topology of quantum antiferromagnetism

• In $d=2$, there are skyrmions in the Neel field.
For the original lattice model, skyrmion number can change (shrink it down to lattice scale and let it disappear).

In $D = 2 + 1$, such an "instanton" or "monopole" event is a hedgehog configuration of the Neel field.
Berry phases and topology

For all smooth configurations of the Neel field, $S_B$ vanishes. However $S_B$ finite in the presence of monopoles.

Precise calculation (Haldane ’88): Berry phase factor associated with each monopole event that oscillates from plaquette to plaquette.

Final result: Only quadrupled monopoles survive! (Skyrmion number can change but in units of 4).
Field theory of VBS phases

• Sum over different skyrmion number changing events with Haldane phases (Read-Sachdev).

• Destruction of Neel order through monopole proliferation.

• Haldane phases of monopoles lead to VBS order.
Phase transition – tractable deformations

1. Easy plane anisotropy

2. Embed physical model in a family of models parametrized by integer $N$.
   - $N = 2$: model of interest.
   - $N = 1$ and $N = \infty$: solvable limits.
   - Same qualitative picture from all these tractable deformations
     - Quadrupled monopoles irrelevant at transition!!
Critical theory

- Universality class of O(3) model with monopoles suppressed by hand

    Early papers:
    Dasgupta & Lau ‘89, Kamal & Murthy ‘94

    Important recent progress: Motrunich & Vishwanath ‘04

    Same result as that obtained by attack from VBS!

    Quadrupled monopole fugacity = 4-fold clock anisotropy
Renormalization group flows

Clock anisotropy = quadrupled
Instanton fugacity

Deconfined critical fixed point

Clock anisotropy is `dangerously irrelevant`.
Precise meaning of deconfinement

- $Z_4$ symmetry gets enlarged to XY

$\Rightarrow$ Domain walls get very thick and very cheap near the transition.

$\Rightarrow$ Domain wall energy not effective in confining $Z_4$ vortices (= spinons)

Formal: Extra global U(1) symmetry not present in microscopic model:
Two diverging length scales in paramagnet

``Critical” \( \xi \) ``spin liquid” \( \xi_{\text{VBS}} \) VBS

\( \xi \): spin correlation length
\( \xi_{\text{VBS}} \): Domain wall thickness.

\( \xi_{\text{VBS}} \sim \xi^k \) diverges faster than \( \xi \)

Spinons confined in either phase but `confinement scale’ diverges at transition – hence `deconfined criticality’.
Analogies to heavy fermion physics

Insulating Magnet \hspace{2cm} \text{Kondo lattice}

Phases: Neel, VBS \leftrightarrow \text{Fermi liquid, magnetic metal}

Mean field: Schwinger boson \leftrightarrow \text{Slave boson}

Boson condensate: Neel \leftarrow \text{Kondo Fermi liquid}

(“Higgs”)

Confined: VBS \leftrightarrow \text{Magnetic metal}

Phase transition: Deconfined critical \leftrightarrow \text{?? Similar ??}
Other examples of deconfined critical points

1. VBS- spin liquid (Senthil, Balents, Sachdev, Vishwanath, Fisher, '04)
2. Neel–spin liquid (Ghaemi, Senthil, '06)
3. Certain VBS-VBS (Fradkin, Huse, Moessner, Oganesyan, Sondhi, '04; Vishwanath, Balents, Senthil, '04)
4. Superfluid- Mott transitions of bosons at fractional filling on various lattices (Senthil et al, '04, Balents et al, '05, ....)
5. Spin quadrupole order –VBS on rectangular lattice (Numerics: Harada et al, '07; Theory: Grover, Senthil, 07)

.........and many more!

Apparently fairly common
A `cool' new example

Grover, Senthil
(forthcoming)

s-wave superconductor - "Quantum Spin Hall" insulator on honeycomb lattice

Spin Hall insulator \[ \mathbf{J}_{\text{spin}} = \sigma \mathbf{E} \times \mathbf{E} \]

[Obtain "spin quantization axis" by spontaneously breaking spin symmetry \( \Rightarrow \) resulting phase has both conventional and topological order.]
Numerical/experimental sightings of Landau-forbidden quantum phase transitions

Weak first order/second order quantum transitions between two phases with very different broken symmetry surprisingly common….

Numerics

Antiferromagnet – superconductor
Superfluid – density wave insulator on various lattices
Neel -VBS on square lattice

Spin quadrupole order – dimer order on rectangular lattice

Experiments:
UPt$_{3-x}$Pd$_x$ SC – AF with increasing $x$. (Graf et al 2001)
Best numerical evidence: Neel-VBS on square lattice

\[ H = J \sum_{\langle ij \rangle} \hat{S}_i \cdot \hat{S}_j - Q \sum_{\langle\langle ij \rangle\rangle} (\hat{S}_i \cdot \hat{S}_j - \frac{1}{4}) (\hat{S}_k \cdot \hat{S}_l - \frac{1}{4}) \]

(\langle ij \rangle, \langle kl \rangle): parallel neighbor bonds

\( \frac{Q}{J} \) small: Neel order
\( \frac{Q}{J} \) large: VBS order
A sample scaling plot
Emergent XY symmetry for dimer order

Histogram of dimer order parameter shows full XY symmetry → irrelevance of $Z_4$ anisotropy!
Other results: critical quantum phases
(Self Organized Quantum Criticality)

- Stability of critical quantum liquid phases in two dimensions (Hermele, Senthil, Fisher, Lee, Nagaosa, Wen, ‘04)

``Deconfined critical phases” or ``Algebraic spin/charge liquids”

No quasiparticle description of low energy physics!! (Rantner, Wen, ‘02)

Huge emergent low energy symmetry/slow power law decay for many distinct orders (Hermele, Senthil, Fisher, ‘06)

Important implications for theories of underdoped cuprates
Algebraic spin liquids: Senthil, Lee, ’05, Ghaemi, Senthil, ’06
Algebraic charge liquids: Kaul, Kim, Sachdev, Senthil, ’07
Some lessons-I

- Direct 2\textsuperscript{nd} order quantum transition between two phases with different competing orders possible (eg: between different broken symmetries)

Separation between the two competing orders not as a function of tuning parameter but as a function of (length or time) scale
Some lessons-II

- Striking “non-fermi liquid” (morally) physics at critical point between two competing orders.

Eg: At Neel-VBS, spin spectrum is anomalously broad - roughly due to decay into spinons- as compared to usual critical points.

Most important lesson:
Failure of Landau paradigm – order parameter fluctuations do not capture true critical physics even if natural order parameters exist.

Strong impetus to radical approaches to non-fermi liquid physics at magnetic critical points in rare earth metals (and to optimally doped cuprates).
Outlook

• Theoretically important answer to 0\textsuperscript{th} order question posed by experiments:
  Can Landau paradigms be violated at phases and phase transitions of strongly interacting electrons?

• But there still is far to go to seriously confront non-Fermi liquid metals in existing materials……….!

Can we go beyond the 0\textsuperscript{th} order answer?