Scaling hypotheses for non-fermi liquid quantum criticality

T. Senthil (MIT)

TS, forthcoming;
Precursor: TS, Annals of Physics, ‘06

Killing a Fermi surface

At certain quantum phase transitions in metals, an entire Fermi surface may disappear.

Examples: ① Heavy fermi liquid – antiferromagnetic metal

② Mott transition from metal to insulator

IF 2nd order, critical point will show non-Fermi liquid physics.

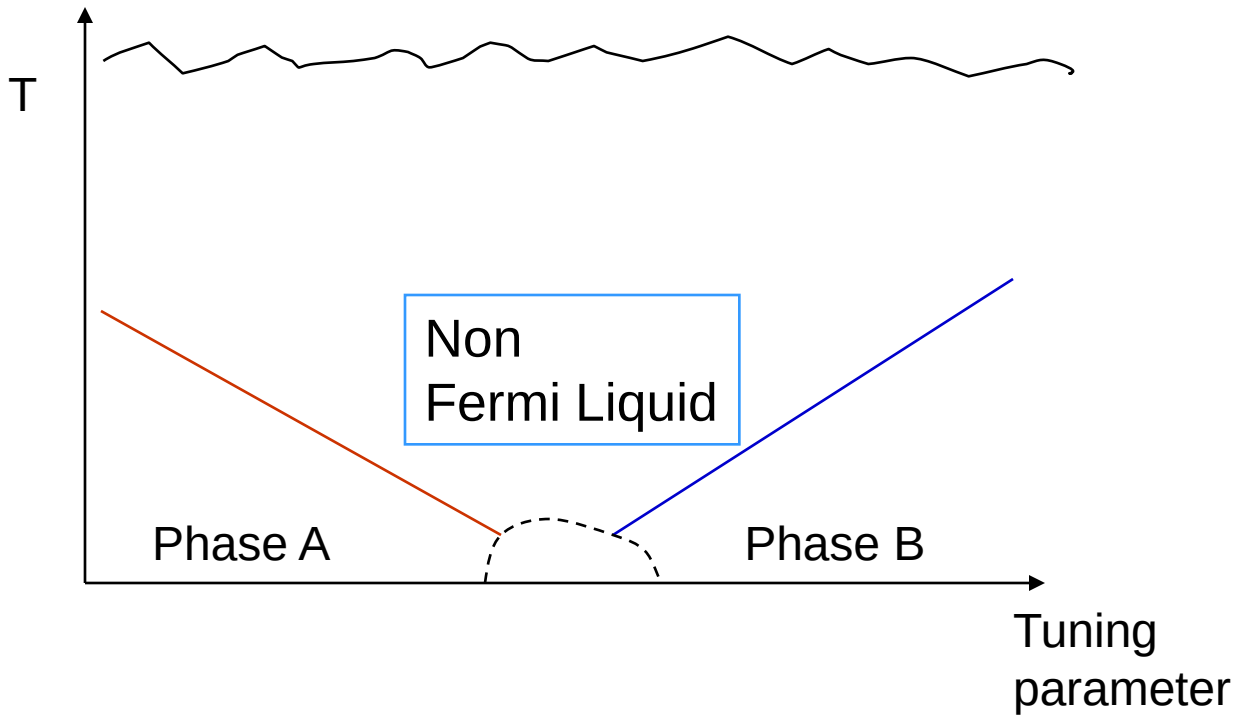
Non-fermi liquid metals

Growing number of examples of metals not described by Fermi liquid theory.

Notorious example: "Strange" metal of optimally doped cuprates.

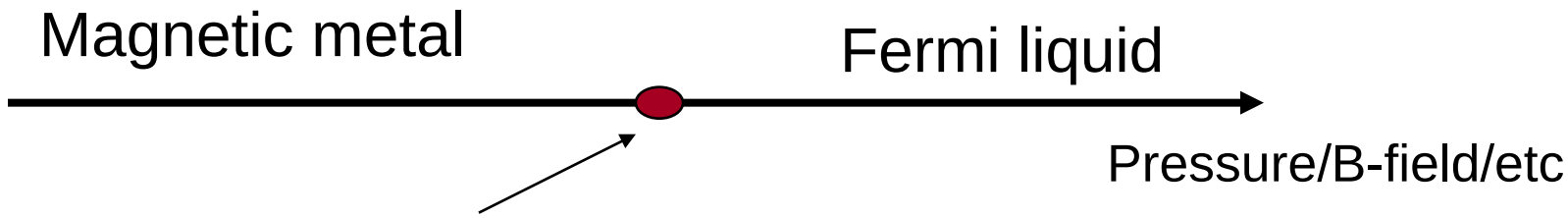
Theory: Almost completely non-existent (but many interesting scattered ideas).

Quantum criticality – a route to non-Fermi liquid metals



Best studied examples: heavy electron metals

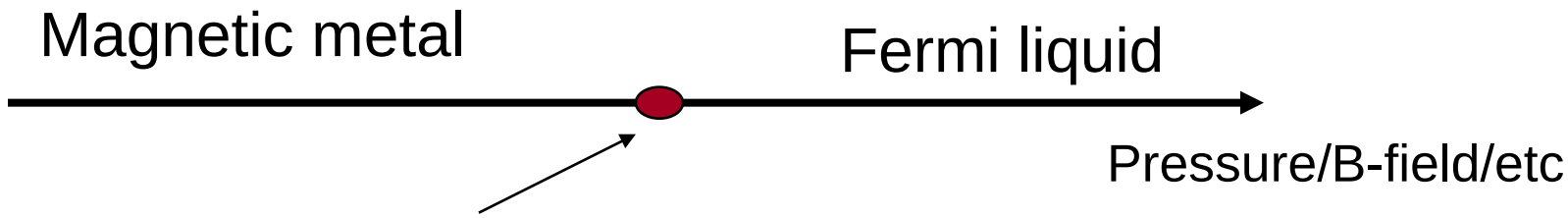
$CePd_2Si_2$, $CeCu_{6-x}Au_x$, $YbRh_2Si_2$,……



(Quantum) critical point with striking non-fermi liquid physics

Best studied examples: heavy electron metals

$CePd_2Si_2$, $CeCu_{6-x}Au_x$, $YbRh_2Si_2$,……

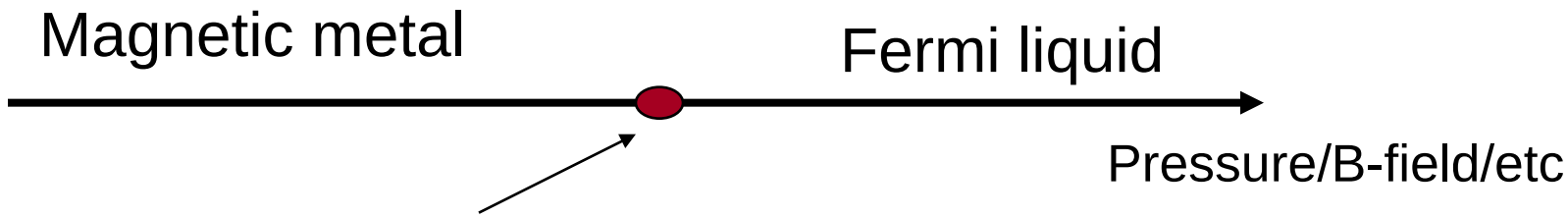


(Quantum) critical point with striking non-fermi liquid physics

Partially filled f-orbitals form localized magnetic moments.

Best studied examples: heavy electron metals

$CePd_2Si_2$, $CeCu_{6-x}Au_x$, $YbRh_2Si_2$,……

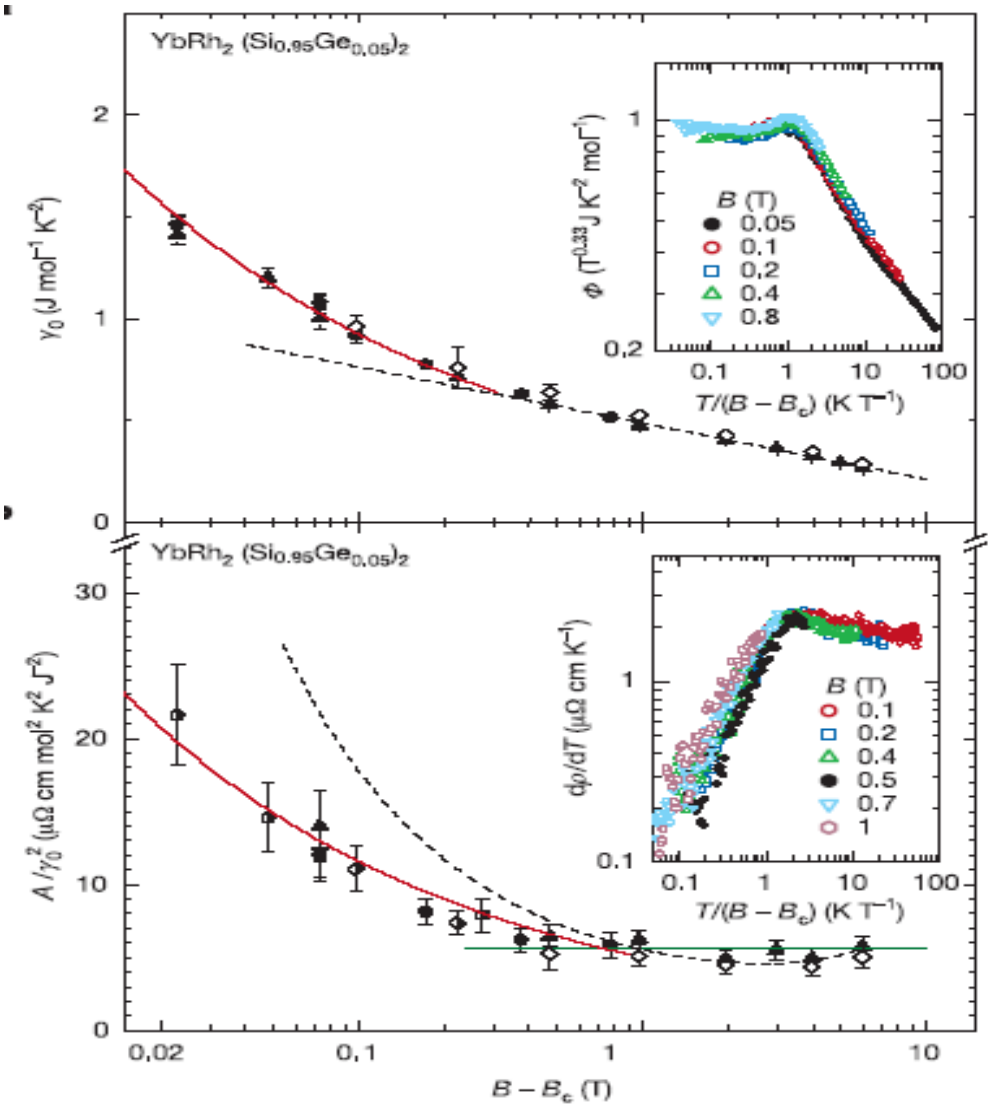


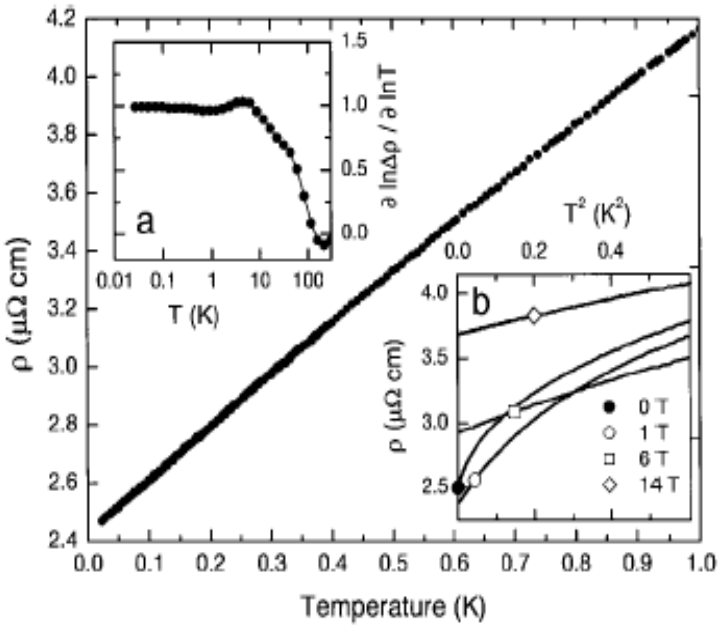
(Quantum) critical point with striking non-fermi liquid physics

Partially filled f-orbitals form localized magnetic moments.
Model as lattice of localized magnetic moments
coupled to conduction electrons by spin exchange
(the Kondo lattice)

Representative data in $YbRh_2Si_2$

Trovarelli et al, PRL 2000

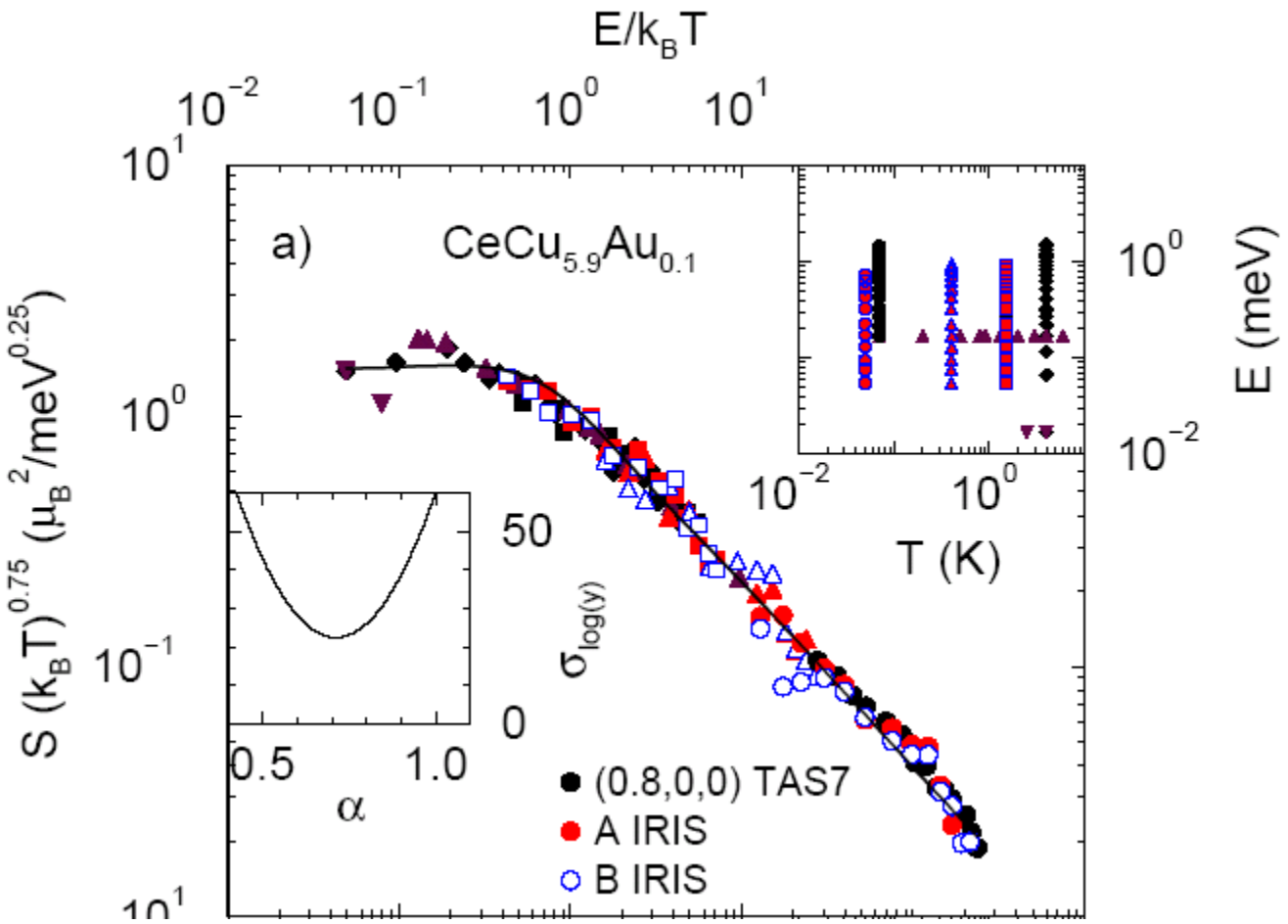




T-dependence of resistivity at critical point: ρ(T) ~ T for three decades in temperature!

Scaling of dynamic spin correlations:

Inelastic neutron scattering near ordering wavevector



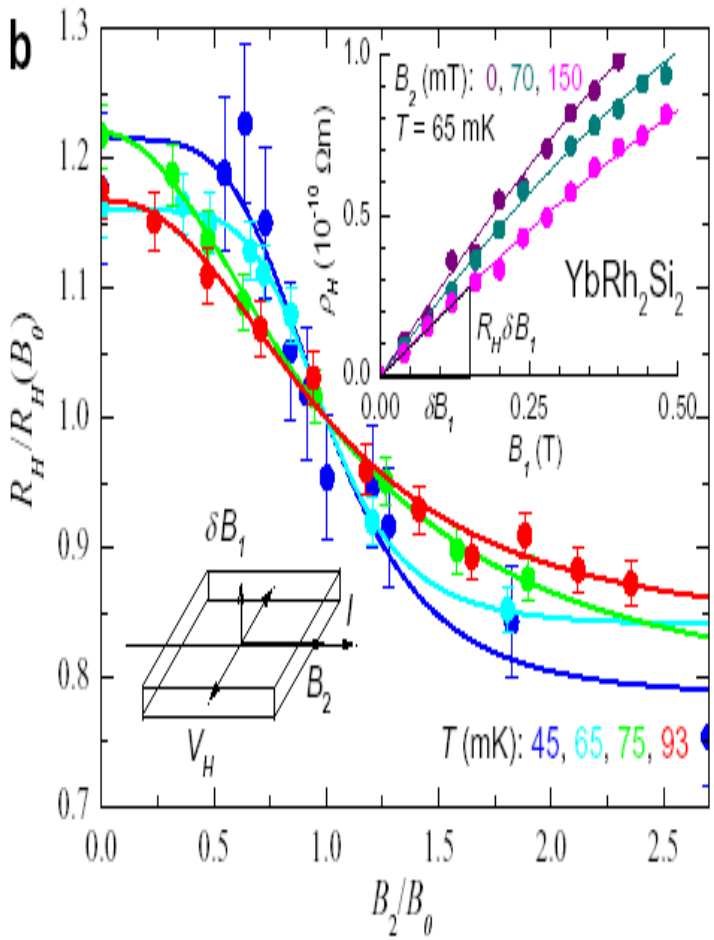
A. Schröder et al., Nature '00; PRL '98

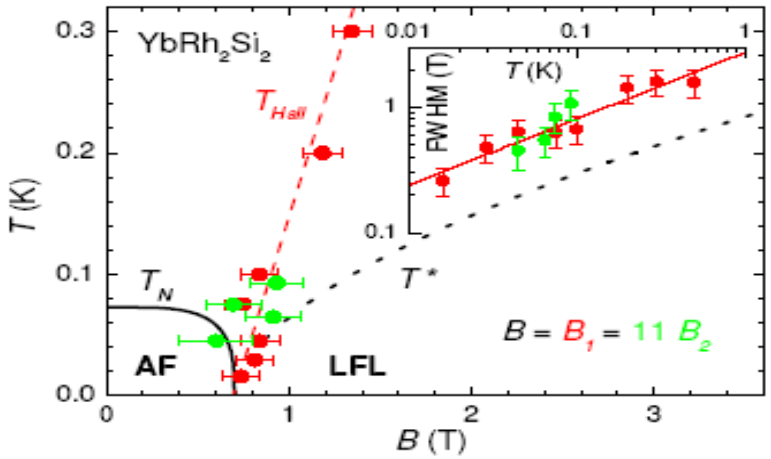
Fermi surface and quantum phase transitions

Participation (or lack there of) of local moments in Fermi sea strongly affects Fermi surface shape and size

$\Rightarrow$ Fermi surface may reconstruct dramatically across the quantum phase transition.

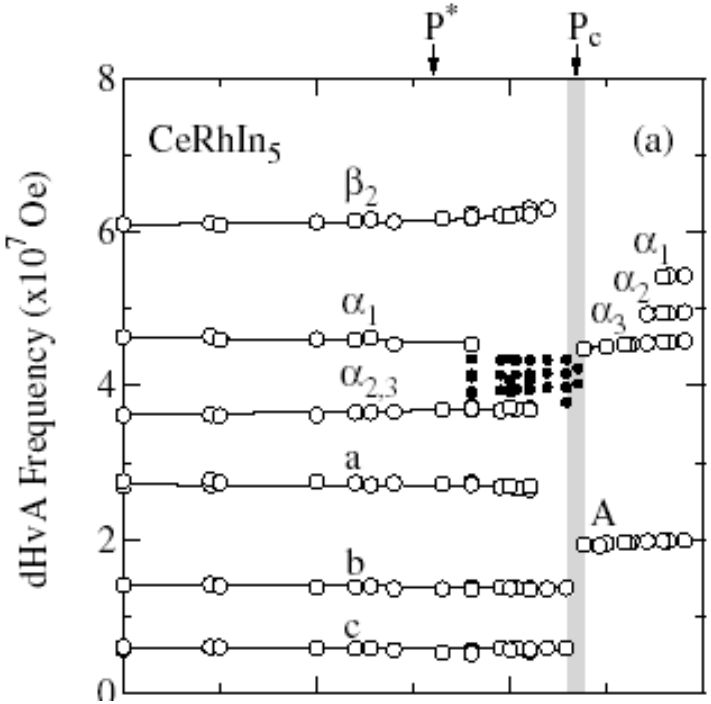
Evidence from experiments I: Hall effect in YbRh2Si2

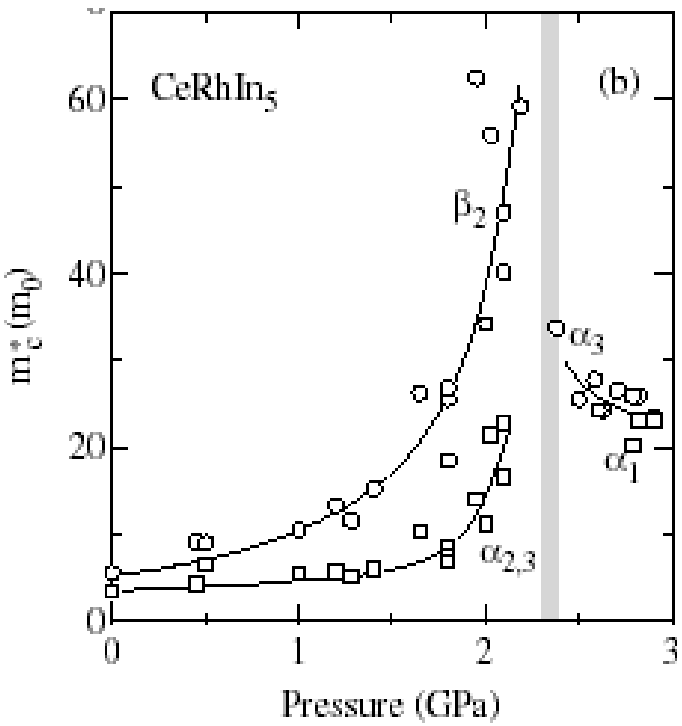




S. Paschen et al., Nature 432, 881 (2004)

Evidence from experiments II: dHvA in CeRhIn5



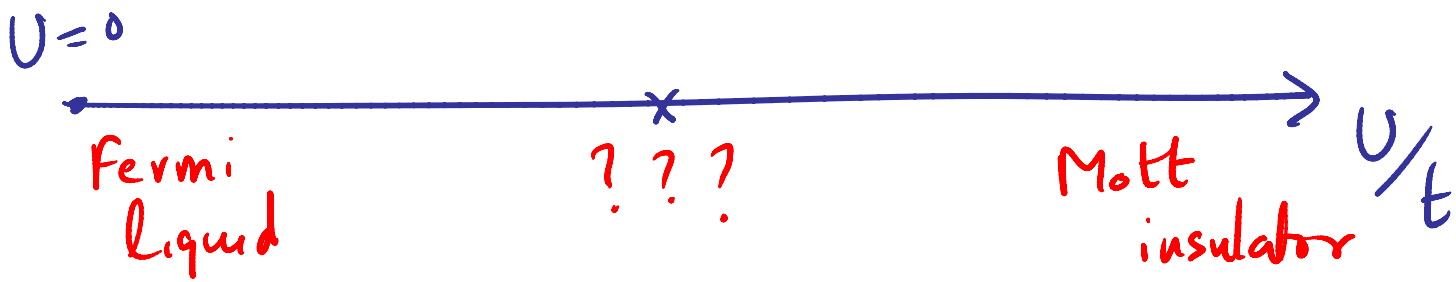


H. Shishido, R. Settai, H. Harima, & Y. Onuki, JPSJ 74, 1103 (2005)

Simpler example – Mott transition of one band systems

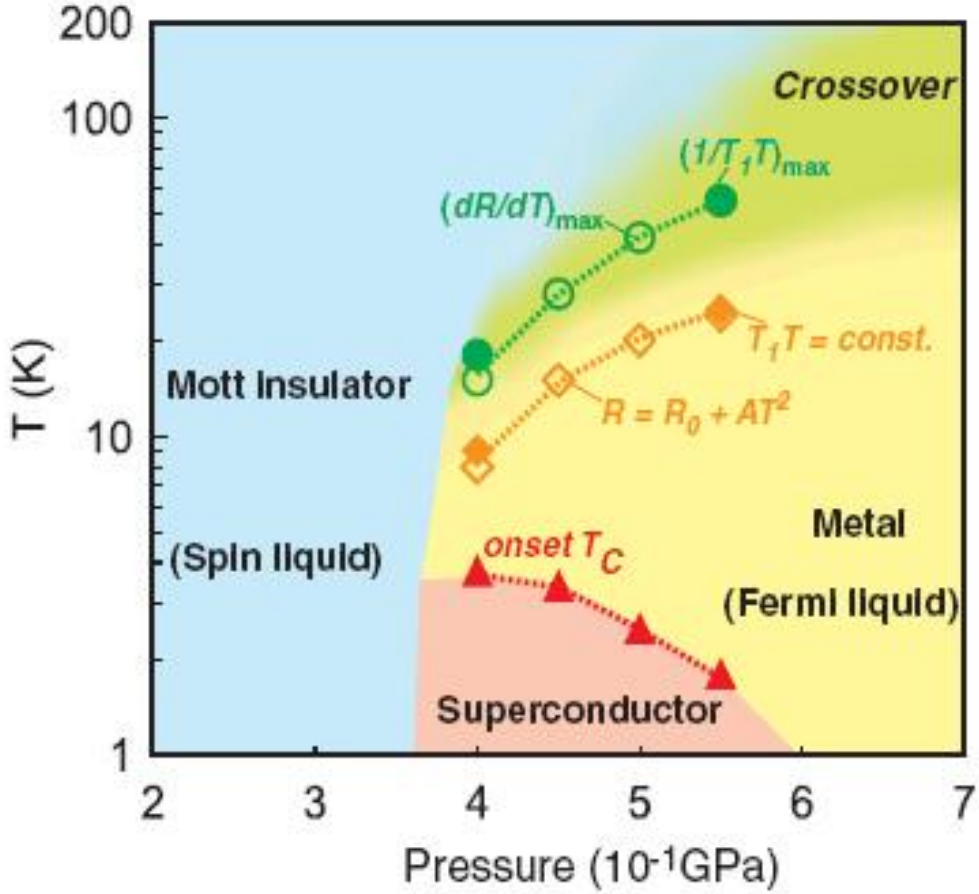
½ - filled Hubbard model on frustrated lattice in $d = 2$ or 3.

$$H = -t \sum_{\langle ij \rangle} \left(c^\dagger_{i\alpha} c_{j\alpha} + h.c \right) + U \sum_i \frac{n_i (n_i - 1)}{2}$$



?? 2nd order Mott critical point ??

Possible experimental realization of a second order Mott transition



κ-$(ET)_2Cu_2(CN)_3$ under pressure.

One band Hubbard model on isotropic Δ lattice

No magnetic order in insulator!

Killing a Fermi surface

At certain quantum phase transitions in metals, an entire Fermi surface may disappear.

Examples: ① Heavy fermi liquid – antiferromagnetic metal

② Mott transition from metal to insulator

IF 2nd order, critical point will show non-Fermi liquid physics.

How might a Fermi surface disappear?

Can a Fermi surface disappear continuously through a 2nd order transition?

Yes - if quasiparticle weight Z vanishes continuously and everywhere on Fermi surface!

Concrete example: "Kondo breakdown" model (TS, Vojta, Sachdev '04)

Electronic structure at criticality: ``Critical Fermi surface"

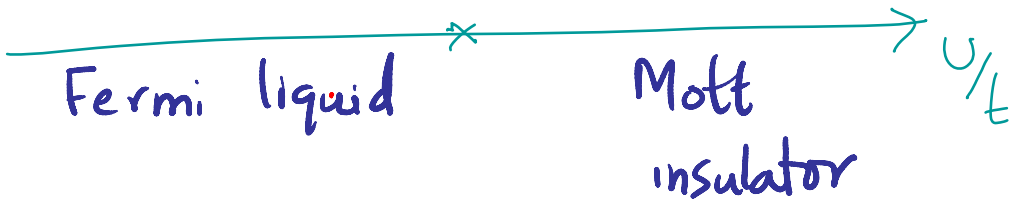
Crucial question : Fate of Fermi surface right at quantum critical point when $Z=0$?

Proposal : Electron spectrum continues to be gapless at Fermi surface but there is no δ-function quasiparticle peak.

"Critical Fermi surface".

Critical fermi surface: another rationale

Mott transition :



What is k-space locus of minimum gap in electron spectral function ?

Fermi liquid : minimum gap locus = Fermi surface.

Mott insulator : Sharp minimum gap locus defines some surface.

Evolution of minimum gap surface

Approach from Mott

2nd order transition to metal $\Rightarrow$ expect entire minimum gap surface will close its gap to match onto Fermi surface of metal.

$\Rightarrow$ Fermi surface sharp at critical point.

But as $Z = 0$ no sharp quasiparticle.

$\Rightarrow$ Non-fermi liquid with sharp "critical" Fermi surface!

Some obvious consequences/questions

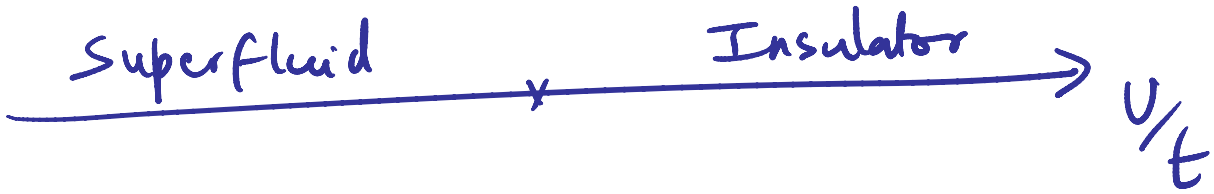
Critical Fermi surface $\Rightarrow$ unusual criticality with phenomena different from familiar critical points

1. Structure of universal singularities/scaling phenomena ?

2. Calculational framework ?

Review: scaling at bosonic quantum critical points

Example: Superfluid - Mott transition of bosons at integer filling in $d=2$ lattice



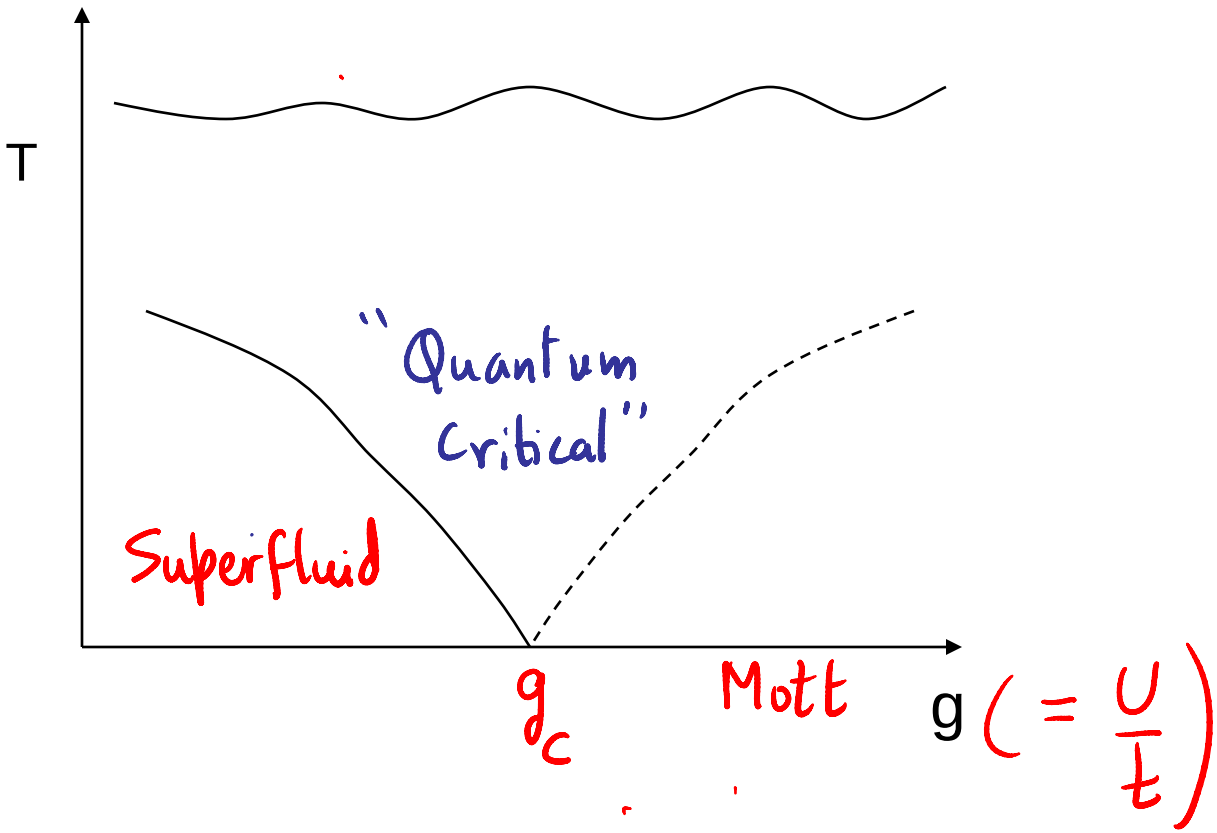
$$H = -t \sum_{\langle ij \rangle} \left(b_i^\dagger b_j + h.c \right) + U \sum_i \frac{n_i(n_i-1)}{2}$$

Single boson spectrum : Gapped in insulator

Gapless at $k=0$ in superfluid

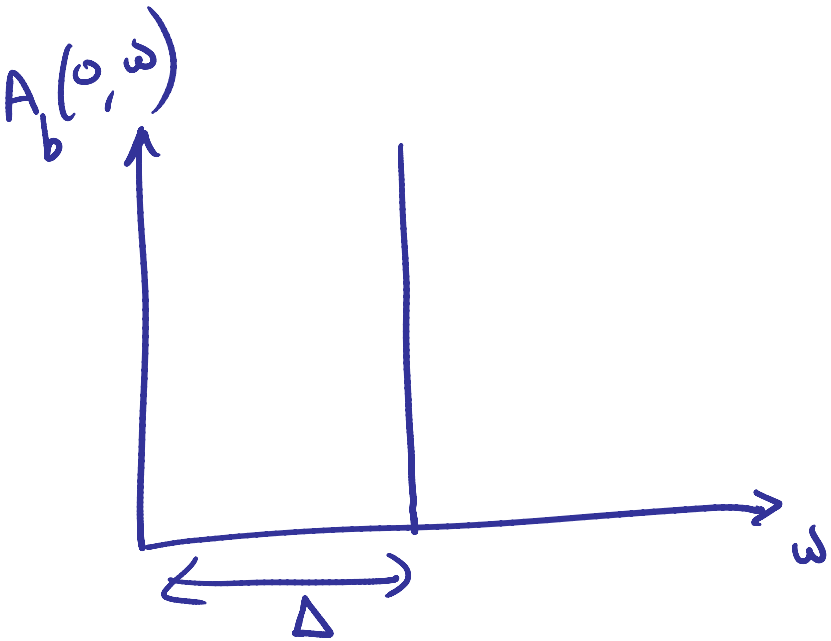
At critical point gap closes at $k=0$.

Review: scaling at bosonic quantum critical points

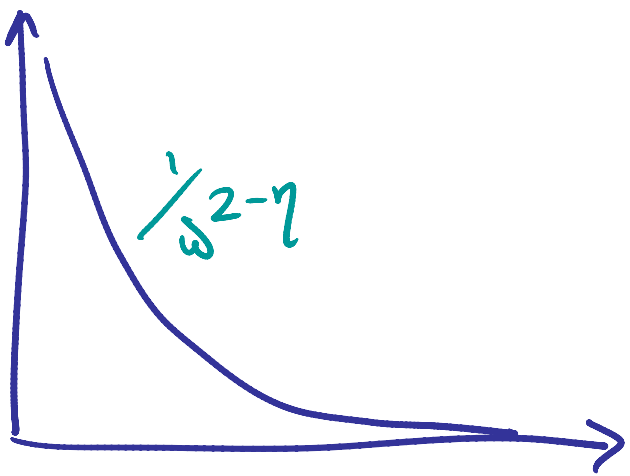


Review: scaling at bosonic quantum critical points

Boson spectral function $A_b(\vec{k},\omega) = \sum_n |\langle n| b^\dagger_{\vec{k}} |0\rangle|^2 \, \delta(\omega - (E_n - E_0))$



Mott



Critical

Review: scaling at bosonic quantum critical points

Generally $A_b(\vec{k}, \omega) \sim \frac{1}{|\omega|^{\frac{2-\eta}{z}}} F\left(\frac{c\omega}{|\vec{k}|^z}\right)$

η, z : universal exponents

F : universal scaling function.

At $T \neq 0$ $A_b(\vec{k}, \omega, T) \sim \frac{1}{|\omega|^{\frac{2-\eta}{z}}} \tilde{F}\left(\frac{c\omega}{|\vec{k}|^z}, \frac{\omega}{T}\right)$

For SF-insulator transition $z = 1$

Review: scaling at bosonic quantum critical points

Thermodynamics :

Free energy density $\mathcal{F}(T, \mu) \sim T^{\frac{d+z}{z}} f\left(\mu/T\right)$

$\Rightarrow$ Specific heat $C(T, \mu=0) \sim T^{d/z}$

Compressibility $\kappa(T, \mu=0) \sim T^{d/z - 1}$

Transport : Conductivity $\sigma(\omega, T) \sim T^{\frac{d-2}{z}} \Sigma\left(\frac{\omega}{T}\right)$

D.C conductivity $\sigma(0, T) \sim T^{\frac{d-2}{z}}$

Generalization to critical fermions

Fermi liquid : Gapless Fermi surface $\vec{k} = \vec{k}_F(\theta)$

(in, $d=2$; θ : momentum direction)

Spectral function $A(\vec{k}, \omega) \sim Z(\theta)\, \delta(\omega - v_F k_{\parallel})$

$$k_{\parallel} = |\vec{k}| - |\vec{k}_F(\theta)|$$

At critical point $Z(\theta) \to 0$ for all θ.

Critical Fermi surface: scaling for single particle physics

Right at critical point expect universal singularity in $A_c(\vec{k}, \omega)$ for small ω,

$$k_{\parallel} = |\vec{k}| - |\vec{k}_F(\theta)|$$

For each point θ on Fermi surface postulate

$$A_c(\vec{k}, \omega, T) \sim \frac{1}{|\omega|^{2/z}} F\left(\frac{\omega}{|k_{\parallel}|^z}, \frac{\omega}{T}\right)$$

New possibility: angle dependent exponents

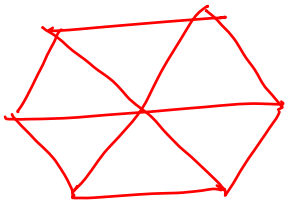
A priori must allow angle dependent exponents:

$$z = z(\theta), \quad \alpha = \alpha(\theta)$$

consistent with lattice symmetries.

Eg: Triangular lattice $z(\theta + \pi/3) = z(\theta)$

$$\alpha(\theta + \pi/3) = \alpha(\theta)$$



Can expand $z(\theta) = \sum_n z_n \cos(6n\theta)$, ...

Critical Fermi surface: scaling for single particle physics

Ground state momentum distribution

$$n(\vec{k}) = \langle c^{\dagger}_{\vec{k}} c_{\vec{k}} \rangle = \lim_{T \to 0} \int d\omega \, A(\vec{k}, \omega) f(\omega)$$

Fermi function

$$\sim \int d\omega \, \frac{1}{|\omega|^{\alpha/z}} F\left(\frac{\omega}{k_{\parallel}^{z}}, 0\right)$$

$$\sim |k_{\parallel}|^{z-\alpha}$$

$n(\vec{k})$ <u>must</u> be bounded $\Rightarrow$ $\underline{z(\theta) \geq \alpha(\theta)}$

Critical Fermi surface: scaling for single particle physics

Tunneling density of states

$$\rho(\omega) \sim \int d\theta \, dk_{\parallel} \; A(\vec{k}, \omega)$$

$$\sim \int d\theta \; |\omega|^{\frac{1-\alpha}{z}}$$

IF z, α are angle independent

$$\rho(\omega) \sim |\omega|^{\frac{1-\alpha}{z}}$$

(Independent of space dim d; each Fermi surface 'patch' contributes as in $d=1$)

Leaving the critical point

Expect scale invariant spectrum is cut off at $k_{\parallel} \sim \frac{1}{\xi}$, $\omega \sim \frac{1}{\xi^z}$ so that

$$A_c(\vec{k}, \omega) \sim \frac{1}{|\omega|^{\alpha/z}} F_1\left(\frac{\omega}{k_{\parallel}^z}, k_{\parallel}\xi\right)$$

Expect $\xi \sim |g - g_c|^{-\nu}$ but again a priori must let $\nu = \nu(\theta)$

Example: Mott transition

Approach from Mott side :

Single particle gap $\Delta(\theta) \sim |g - g_c|^{\nu(\theta) z(\theta)}$

Gaps vanish differently at different portions of Fermi surface.

Approach from Fermi liquid :

Quasiparticle residue $Z(\theta) \sim |g - g_c|^{\nu(\theta)(z(\theta) - \alpha(\theta))}$

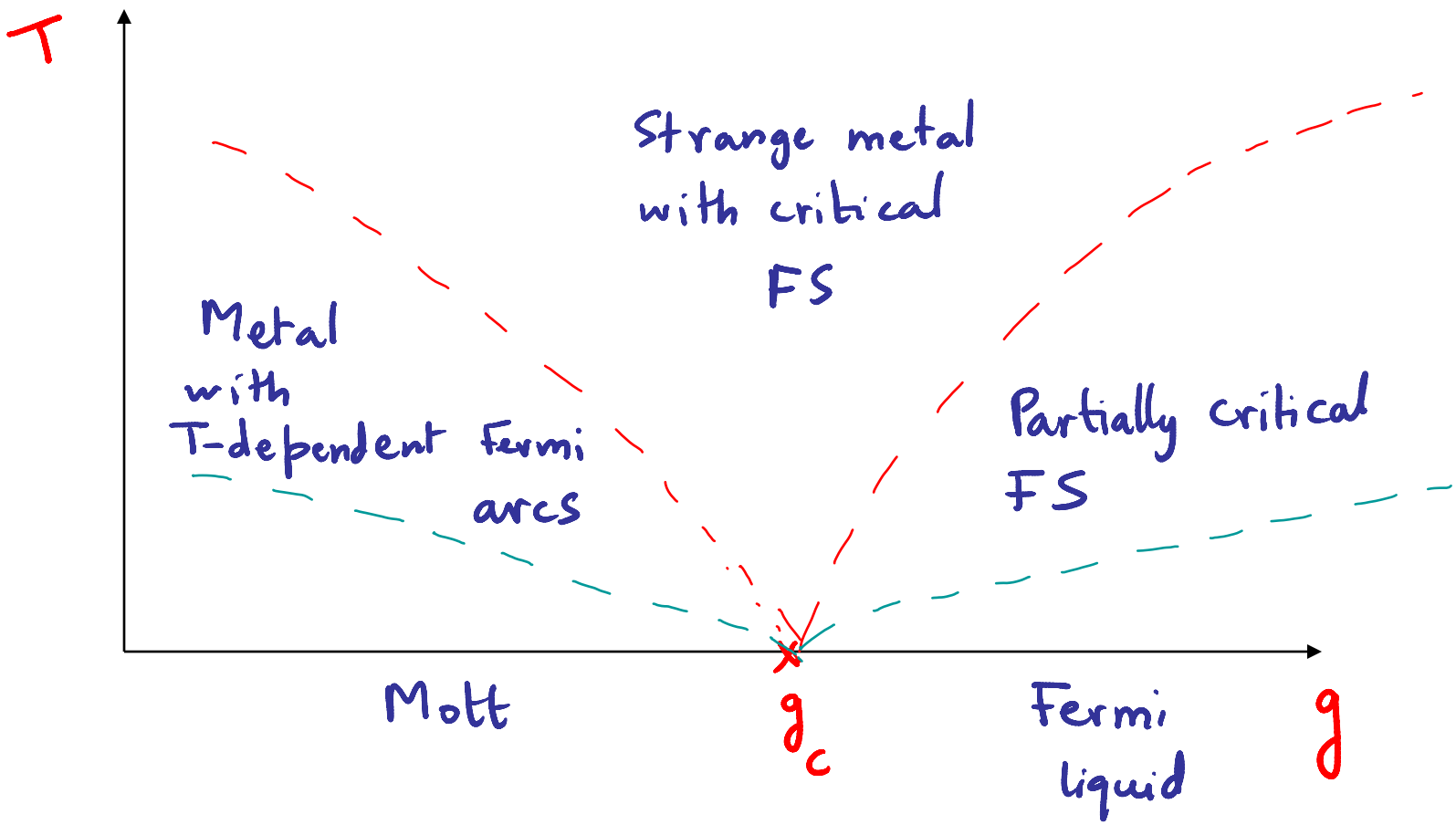
Fermi velocity $v_f(\theta) \sim |g - g_c|^{\nu(\theta)(z(\theta) - 1)}$

Implications of angle dependent exponents

Different portions of Fermi surface will emerge out of criticality at different energy scales

Finite T x overs much richer than usual.

Finite T crossovers



Similar to cuprates (!)

Thermodynamics and transport: Two scaling models

Model 1: Each patch of Fermi surface contributes as a d = 1 theory (Different patches decouple at criticality)

Model 2: Various patches of Fermi surface strongly coupled at criticality

Useful criterion:

Approach from Fermi liquid – non-diverging Landau parameters necessary for Model 1.

Model 1 more likely to have angle dependent exponents on Fermi surface.

Model 1 thermodynamics

Assume in scaling limit each patch of Fermi surface contributes as a $d=1$ theory

Total free energy $F(T, H) \sim \int d\theta \, F_\theta(T, H)$

$$F_\theta(T, H) \sim T^{1+1/z(\theta)} \, g(H/T)$$

H : Zeeman field

Thermodynamics

Zero field specific heat $C(T) \sim \int d\theta \, T^{1/z}$

Spin susceptibility $\chi(T) \sim \int d\theta \, T^{1/z - 1}$

Wilson ratio $W = \dfrac{C}{T\chi}$ = universal constant.

If $z(\theta) = z$ independent of θ

$C(T) \sim T^{1/z}$, $\chi(T) \sim T^{1/z - 1}$

If $z > 1$, χ will diverge even though transition is not to a ferromagnet.

Transport ?

No reason to expect $\sigma \sim T^{\frac{d-2}{z}}$ ("bosonic scaling")

Total current operator $\vec{J} \sim \int d\theta \, \vec{J}_\theta$

$\vec{J}_\theta \sim$ current from "patch" at θ

If each patch contribution scales as in $d=1$

$$\sigma \sim T^{-1/z} \Sigma\left(\omega/T\right)$$

Non-trivial power laws possible in transport if there is a critical Fermi surface.

Model calculations with a critical Fermi surface

Simplest : "Slave rotor" description of Mott transition in 1-band Hubbard model.

Write $c_{i\alpha} \sim e^{i\phi_i} f_{i\alpha} \sim b_i f_{i\alpha}$

$b_i \sim e^{i\phi_i}$: charge e spinless boson

$f_{i\alpha}$: charge 0 spin-$\frac{1}{2}$ fermion ("spinon")

U(1) gauge redundancy - local phase rotations of $e^{i\phi_i}$ & $f_{i\alpha}$

Model calculation with a critical Fermi surface

Fermi liquid : $\langle b_i \rangle \neq 0$

Mott insulator : b_i gapped

(but $f_{i\alpha}$ form 'spinon Fermi surface').

Critical point : $e^{i\phi_i}$ critical.

Effective theory :

$$\mathcal{L} = |(\partial_\mu - i a_\mu) b|^2 + r|b|^2 + u|b|^4 + \bar{f}(\partial_\tau - i a_0 - (\vec{\nabla} - i\vec{a})^2) f + (\epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda)^2$$

Model calculation with a critical Fermi surface

Crude approximation : Ignore gauge fields .

Electron Green function $\langle c\bar{c} \rangle \sim G_b G_f$

$$G_b \sim \frac{1}{(x^2+r^2)^{\frac{2-\eta}{2}}} \; ; \quad G_f \sim \text{free fermion Green fn}$$

$$A(\vec{k}, \omega) \sim |\omega - v_f k_{\parallel}|^{\eta} \Rightarrow \text{critical Fermi surface}$$

with $z = 1$, $\alpha = -\eta$ everywhere .

Summary

- At some metallic quantum critical points there

will be an entire surface of critical fermionic modes

- a `critical Fermi surface'.

- Presence of critical fermi surface will change the scaling phenomena associated with universal critical singularities.

- Specific scaling ansatz for single particle and thermodynamic quantities.

- Possibility of angle dependent exponents with interesting consequences (eg: metals with T-dependent Fermi arcs at intermediate temperature)

Future

- Scaling for transport?
- Verify in concrete theoretical models/experiments
- Theoretical framework to describe critical Fermi surfaces?