Scaling hypotheses for non-fermi liquid quantum criticality

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TS, forthcoming;
Precursor: TS, Annals of Physics, ‘06
Killing a Fermi surface

At certain quantum phase transitions in metals, an entire Fermi surface may disappear.

Examples: ① Heavy fermi liquid—antiferromagnetic metal

② Mott transition from metal to insulator

IF 2nd order, critical point will show non-Fermi liquid physics.
Non-fermi liquid metals

Growing number of examples of metals not described by Fermi liquid theory.

Notorious example: “Strange” metal of optimally doped cuprates.

Theory: Almost completely non-existent (but many interesting scattered ideas).
Quantum criticality – a route to non-Fermi liquid metals
Best studied examples: heavy electron metals

\[ \text{CePd}_2\text{Si}_2, \text{CeCu}_{6-x}\text{Au}_x, \text{YbRh}_2\text{Si}_2, \ldots \]

Magnetic metal \hspace{2cm} \text{(Quantum) critical point with striking non-fermi liquid physics} \hspace{2cm} \text{Fermi liquid}

Pressure/B-field/etc
Best studied examples: heavy electron metals

CePd$_2$Si$_2$, CeCu$_{6-x}$Au$_x$, YbRh$_2$Si$_2$,

Magnetic metal  Fermi liquid

Pressure/B-field/etc

(Quantum) critical point with striking non-fermi liquid physics

Partially filled f-orbitals form localized magnetic moments.
Best studied examples: heavy electron metals

$\text{CePd}_2\text{Si}_2$, $\text{CeCu}_{6-x}\text{Au}_x$, $\text{YbRh}_2\text{Si}_2$, ……

(Magnetic metal) Pressure/B-field/etc

(Quantum) critical point with striking non-fermi liquid physics

Partially filled f-orbitals form localized magnetic moments. Model as lattice of localized magnetic moments coupled to conduction electrons by spin exchange (the Kondo lattice)
Representative data in YbRh$_2$Si$_2$

Trovarelli et al, PRL 2000

T-dependence of resistivity at critical point: $\rho(T) \sim T$ for three decades in temperature!
Scaling of dynamic spin correlations: Inelastic neutron scattering near ordering wavevector

A. Schröder et al., Nature ’00; PRL ’98
Fermi surface and quantum phase transitions

Participation (or lack thereof) of local moments in Fermi sea strongly affects Fermi surface shape and size

$\Rightarrow$ Fermi surface may reconstruct dramatically across the quantum phase transition.
Evidence from experiments I: Hall effect in YbRh$_2$Si$_2$

$S.\text{ Paschen et al., Nature}\ 432,\ 881\ (2004)$
Evidence from experiments II: dHvA in CeRhIn5

Simpler example – Mott transition of one band systems

\( \frac{1}{2} \)-filled Hubbard model on frustrated lattice in \( d = 2 \) or 3.

\[
H = -t \sum_{\langle ij \rangle} (c^\dagger_i \sigma_\alpha c_j + h.c.) + U \sum \frac{n_i (n_i - 1)}{2}
\]

\( U = 0 \)

Fermi Liquid \( \rightarrow \) ? ? ? \( \rightarrow \) Mott insulator

? ? 2nd order Mott critical point ??
Possible experimental realization of a second order Mott transition

$k \cdot (ET)_2 Cu_2 (CN)_3$

under pressure.

One band Hubbard model on isotropic $\Delta$ lattice

No magnetic order in insulator!
Killing a Fermi surface

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Examples:
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IF 2nd order, critical point will show non-Fermi liquid physics.
How might a Fermi surface disappear?

Can a Fermi surface disappear continuously through a 2nd order transition?

Yes - if quasi-particle weight $Z$ vanishes continuously and everywhere on Fermi surface!

Concrete example: “Kondo breakdown” model (TS, Vojta, Sachdev ’84)
Electronic structure at criticality: ``Critical Fermi surface"

Crucial question: Fate of Fermi surface right at quantum critical point when $Z=0$?

Proposal: Electron spectrum continues to be gapless at Fermi surface but there is no $S$-function quasiparticle peak. "Critical Fermi surface".
Critical fermi surface: another rationale

Mott transition

Fermi liquid

Mott insulator

What is k-space locus of minimum gap in electron spectral function?

Fermi liquid: minimum gap locus = Fermi surface.

Mott insulator: Sharp minimum gap locus defines some surface.
Evolution of minimum gap surface

Approach from Mott

2nd order transition to metal $\Rightarrow$ expect entire minimum gap surface will close its gap to match onto Fermi surface of metal.

$\Rightarrow$ Fermi surface sharp at critical point.

But as $Z=0$ no sharp quasiparticle.

$\Rightarrow$ Non-Fermi liquid with sharp "critical" Fermi surface!
Some obvious consequences/questions

1. Critical Fermi surface $\Rightarrow$ unusual criticality with phenomena different from familiar critical points

2. Structure of universal singularities/scaling phenomena?
Review: scaling at bosonic quantum critical points

Example: Superfluid - Mott transition of bosons at integer filling in $d=2$ lattice

\[ H = -t \sum_{\langle ij \rangle} (b_i^+ b_j + h.c.) + U \sum_i \frac{n_i(n_i-1)}{2} \]

Single boson spectrum: Gapped in insulator
Gappedless at $k=0$ in superfluid

At critical point gap closes at $k=0$. 
Review: scaling at bosonic quantum critical points

\[ g_c \quad \text{Mott} \quad \left( g = \frac{U}{t} \right) \]
Review: scaling at bosonic quantum critical points

Boson spectral function $A_b(k, \omega) = \sum_n |\langle n \mid b_k^\dagger \mid 0 \rangle|^2 \delta(\omega - (E_n - E_0))$
Review: scaling at bosonic quantum critical points

Generally \( A_b(\mathbf{K}, \omega) \sim \frac{1}{|\omega|^{2-\eta}} F \left( \frac{c \omega}{|\mathbf{K}|^z} \right) \)

\( \eta, z \): universal exponents

\( F \): universal scaling function.

At \( T \neq 0 \) \( A_b(\mathbf{K}, \omega, T) \sim \frac{1}{|\omega|^{2-\eta}} F \left( \frac{c \omega}{|\mathbf{K}|^z}, \frac{3}{3} \right) \)

For SF-insulator transition \( z = 1 \)
Review: scaling at bosonic quantum critical points

Thermodynamics:

Free energy density $\mathcal{F}(T, \mu) \sim T^{\frac{d+z}{2}} f(\frac{\mu}{T})$

$\Rightarrow$ Specific heat $C(T, \mu=0) \sim T^{\frac{d}{2}}$

Compressibility $\kappa(T, \mu=0) \sim T^{\frac{d}{2}-1}$

Transport: Conductivity $\sigma(\omega, T) \sim T^{\frac{d-2}{2}} \Sigma(\frac{\omega}{T})$

D.C. conductivity $\sigma(0, T) \sim T^{\frac{d-2}{2}}$
Generalization to critical fermions

Fermi liquid: Gapless Fermi surface \( \vec{k} = \vec{k}_F(\Theta) \)

(\text{in } d = 2 \text{ ; } \Theta : \text{momentum direction})

Spectral function \( A(\vec{k}, \omega) \sim Z(\Theta) \delta(\omega - v_F k_{\parallel}) \)

\[ k_{\parallel} = |\vec{k}| - |\vec{k}_F(\Theta)| \]

At critical point \( Z(\Theta) \rightarrow 0 \) for all \( \Theta \).
Critical Fermi surface: scaling for single particle physics

Right at critical point expect universal singularity in $A_c(K, \omega)$ for small $\omega$,

$$k_{\perp} = |K| - |K_\parallel(\theta)|$$

For each point $\theta$ on Fermi surface postulate

$$A_c(K, \omega, T) \sim \frac{1}{15^{2/3}} \times F\left(\frac{\omega}{|k_{\parallel}|^2}, \frac{\omega}{T}\right)$$
New possibility: angle dependent exponents

A priori must allow angle dependent exponents:

\[ z = z(\theta), \quad \alpha = \alpha(\theta) \]

consistent with lattice symmetries.

Eg: Triangular lattice

\[ z(\theta + \frac{\pi}{3}) = z(\theta) \]

\[ \alpha(\theta + \frac{\pi}{3}) = \alpha(\theta) \]

Can expand \( z(\theta) = \sum_n z_n \cos(6n\theta) \), \ldots
Critical Fermi surface: scaling for single particle physics

Ground state momentum distribution

\[ n(\vec{k}) = \langle \hat{c}^{\dagger}_{\vec{k}} \hat{c}_{\vec{k}} \rangle = \frac{N}{t \to 0} \int d\omega \ A(\vec{k}, \omega) f(\omega) \]

\[ \sim \int d\omega \ \frac{1}{i\omega^{2/4}} \ f\left(\frac{\omega}{k_{\|}}^2, 0\right) \]

\[ \sim |k_{\|}|^{2-\alpha} \]

\[ n(\vec{k}) \text{ must be bounded} \Rightarrow \ z(\theta) \geq \alpha(\theta) \]
Critical Fermi surface: scaling for single particle physics

Tunneling density of states
\[ \rho(\omega) \sim \int d\omega d\mathbf{k}_\parallel A(\mathbf{k},\omega) \]
\[ \sim \int d\omega |\omega|^{\frac{1-\alpha}{2}} \]

If \( \varepsilon, \alpha \) are angle independent
\[ \rho(\omega) \sim |\omega|^{\frac{1-\alpha}{2}} \]

(Independent of space dim \( d \); each Fermi surface ‘patch’ contributes as in \( d=1 \))
Leaving the critical point

Expect scale invariant spectrum is cut off at $k_{\perp} \sim \frac{1}{\xi}$, $\omega \sim \frac{1}{\xi^{z}}$ so that

$$A_c(R, \omega) \sim \frac{1}{15^{z/2}} F_1 \left( \frac{\omega}{k_{\perp}^{z}}, \frac{k_{\perp}}{\xi} \right)$$

Expect $\xi \sim |g-g_c|^{-\nu}$ but again a priori must let $\nu = \nu(\theta)$
Example: Mott transition

Approach from Mott side:

Single particle gap $\Delta(\theta) \sim |g - g_c| 

Gaps vanish differently at different portions of Fermi surface.

Approach from Fermi liquid:

Quasiparticle residue $Z(\theta) \sim |g - g_c| \sqrt{v(\theta)(Z(\theta) - 1)}$

Fermi velocity $v_f(\theta) \sim |g - g_c| \sqrt{v(\theta)(Z(\theta) - 1)}$
Implications of angle dependent exponents

Different portions of Fermi surface will emerge out of criticality at different energy scales. Finite $T_x$ overs much richer than usual.
Finite $T$ crossovers

- Strange metal with critical FS
- Partially critical FS
- Metal with $T$-dependent Fermi arcs

Similar to cuprates (!)
Thermodynamics and transport: Two scaling models

Model 1: Each patch of Fermi surface contributes as a $d = 1$ theory (Different patches decouple at criticality)

Model 2: Various patches of Fermi surface strongly coupled at criticality

Useful criterion:
Approach from Fermi liquid – non-diverging Landau parameters necessary for Model 1.

Model 1 more likely to have angle dependent exponents on Fermi surface.
Model 1 thermodynamics

Assume in scaling limit each patch of Fermi surface contributes as a d=1 theory.

Total free energy $F(T, H) \sim \int d\sigma \frac{F_{\sigma}(T, H)}{2\pi}$

$F_{\sigma}(T, H) \sim T^{1 + \frac{1}{2} \sigma(0)} g \left( \frac{H}{T} \right)$

$H$: Zeeman field
Thermodynamics

Zero field specific heat $C(T) \sim S \delta \sigma T^{1/2}$

Spin susceptibility $\chi(T) \sim S \delta \sigma T^{1/2-1}$

Wilson ratio $W = \frac{C}{T \chi} = \text{universal constant}$

If $z(0) = z$ independent of $\theta$

$C(T) \sim T^{1/2}$, $\chi(T) \sim T^{1/2-1}$

If $z > 1$, $\chi$ will diverge even though transition is not to a ferromagnet.
Transport?

No reason to expect $\sigma \sim T^{\frac{d-2}{2}}$ ("bosonic scaling")

Total current operator $\tilde{J} \sim \int d\Omega \tilde{J}_0$

$\tilde{J}_0 \sim$ current from "patch" at $\theta$

If each patch contribution scales as in $d = 1$

$\sigma \sim T^{-\frac{1}{2}} \Sigma (\tilde{\omega})$

Non-trivial power laws possible in transport if there is a critical Fermi surface.
Model calculations with a critical Fermi surface

Simplest: "Slave rotor" description of Mott transition in 1-band Hubbard model.

Write \( c_{id} \sim e^{i\Phi_i} f_{id} \sim b_i f_{id} \)

\( b_i \sim e^{i\Phi_i} \): charge e spinless boson

\( f_{id} \): charge 0 spin-\( \frac{1}{2} \) fermion ("spinon")

\( U(1) \) gauge redundancy - local phase rotations of \( e^{i\Phi_i} \) & \( f_{id} \)
Model calculation with a critical Fermi surface

Fermi liquid: \( \langle b_i \rangle \neq 0 \)

Mott insulator: \( b_a \) gapped

(but \( f_{\alpha \lambda} \) form \( ' \)spinon Fermi surface\( ' )\).

Critical point: \( e^{i\phi_i} \) critical.

Effective theory:

\[
\mathcal{L} = \frac{1}{2} (\partial_\mu - i a_\mu) b_i^\dagger b_i + g |b|^2 + \nu |b|^4 + \bar{f} (\partial_\tau - i a_\tau - (\nabla - i \vec{a})^2) f + (\xi_{\mu \nu} \partial_\mu a_\nu)^2
\]
Model calculation with a critical Fermi surface

Crude approximation: Ignore gauge fields.

Electron Green function $\langle c \bar{c} \rangle \sim G_b G_f$

$$G_b \sim \frac{1}{(x^2 + r^2)^{\frac{2-y}{2}}} ; \quad G_f \sim \text{free fermion Green fn}$$

$$A(\mathbf{E}, \omega) \sim | \omega - v_f k_\| |^{\gamma} \Rightarrow \text{critical Fermi Surface}$$

with $\gamma = 1, \quad \alpha = -\gamma$ everywhere.
Summary

• At some metallic quantum critical points there will be an entire surface of critical fermionic modes
  - a `critical Fermi surface’.

• Presence of critical fermi surface will change the scaling phenomena associated with universal critical singularities.

• Specific scaling ansatz for single particle and thermodynamic quantities.

• Possibility of angle dependent exponents with interesting consequences (eg: metals with T-dependent Fermi arcs at intermediate temperature)
Future

• Scaling for transport?

• Verify in concrete theoretical models/experiments

• Theoretical framework to describe critical Fermi surfaces?