Critical fermi surfaces and non-fermi liquid metals

T. Senthil (MIT)

TS, ``Critical fermi surfaces and non-fermi liquid metals”, PRB 08,
``Theory of a continuous Mott transition in two dimensions”, PRB 08

Precursor: TS, Annals of Physics, ‘06

Killing a Fermi surface

At certain quantum phase transitions in metals, an entire Fermi surface may disappear.

Examples: ① Heavy fermi liquid – antiferromagnetic metal

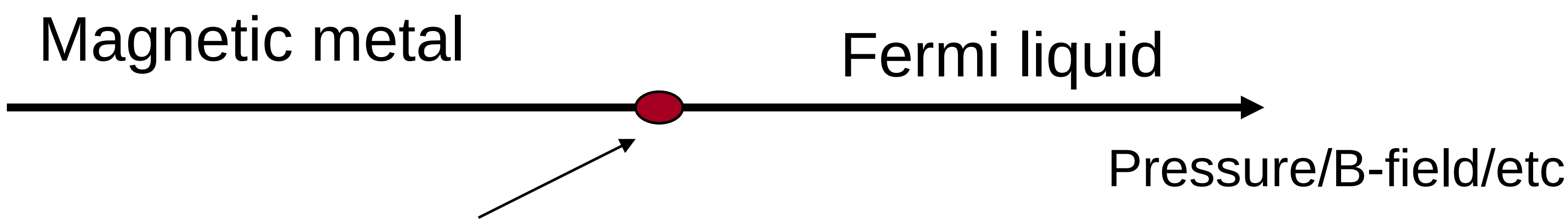
② Mott transition from metal to insulator

③ HiTc overdoped metal to underdoped metal at $T=0$?

IF 2nd order, critical point will be a non-Fermi liquid.

Example I: Heavy electron critical points

Materials: $CeCu_{6-x}Au_x$, $YbRh_2Si_2$, 115, ...



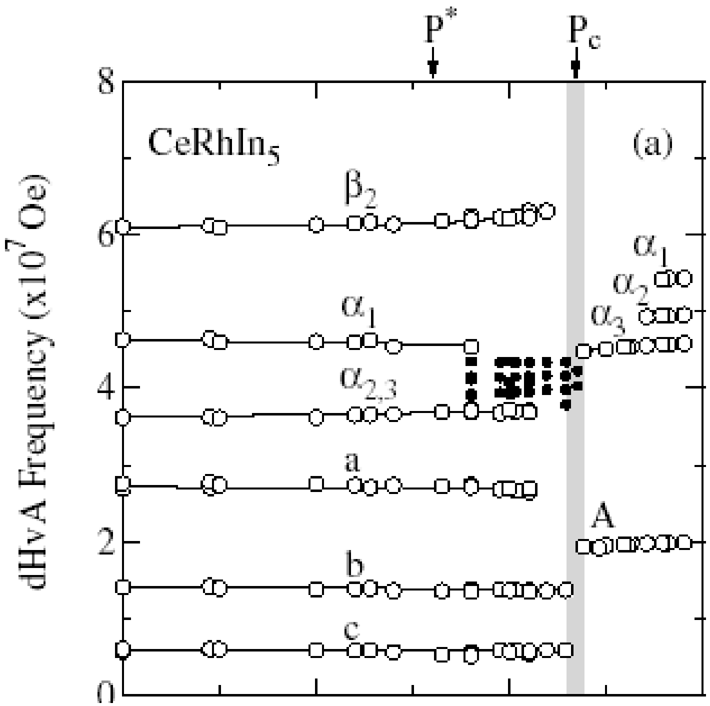
Quantum critical point with striking non-fermi liquid physics

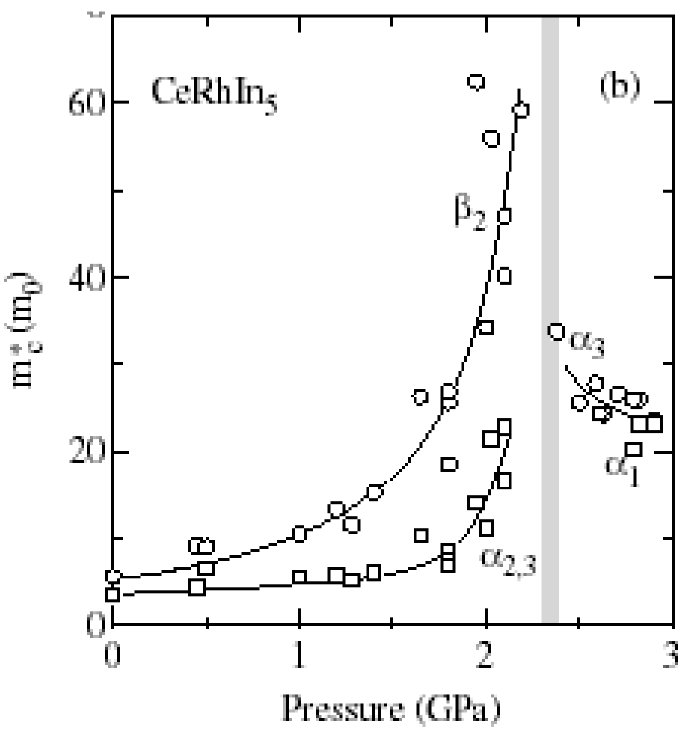
The Fermi surface may reconstruct dramatically across the quantum critical point due to loss of Kondo screening.

Evidence from (i) evolution of Hall effect in $YbRh_2Si_2$ (Paschen et al '04)

(ii) dHvA in $CeRhIn_5$ across pressure driven magnetic QPT (Shishido et al '05)

dHvA in CeRhIn5





H. Shishido, R. Settai, H. Harima, & Y. Onuki, JPSJ 74, 1103 (2005)

Entire sheets of Fermi surface disappear at the QPT

Simpler example – Mott transition of one band systems

½-filled Hubbard model on non-bipartite lattice in d=2 or 3

t/U

Mott insulator

(No Fermi surface)

????

Fermi liquid

(Sharp Fermi surface)

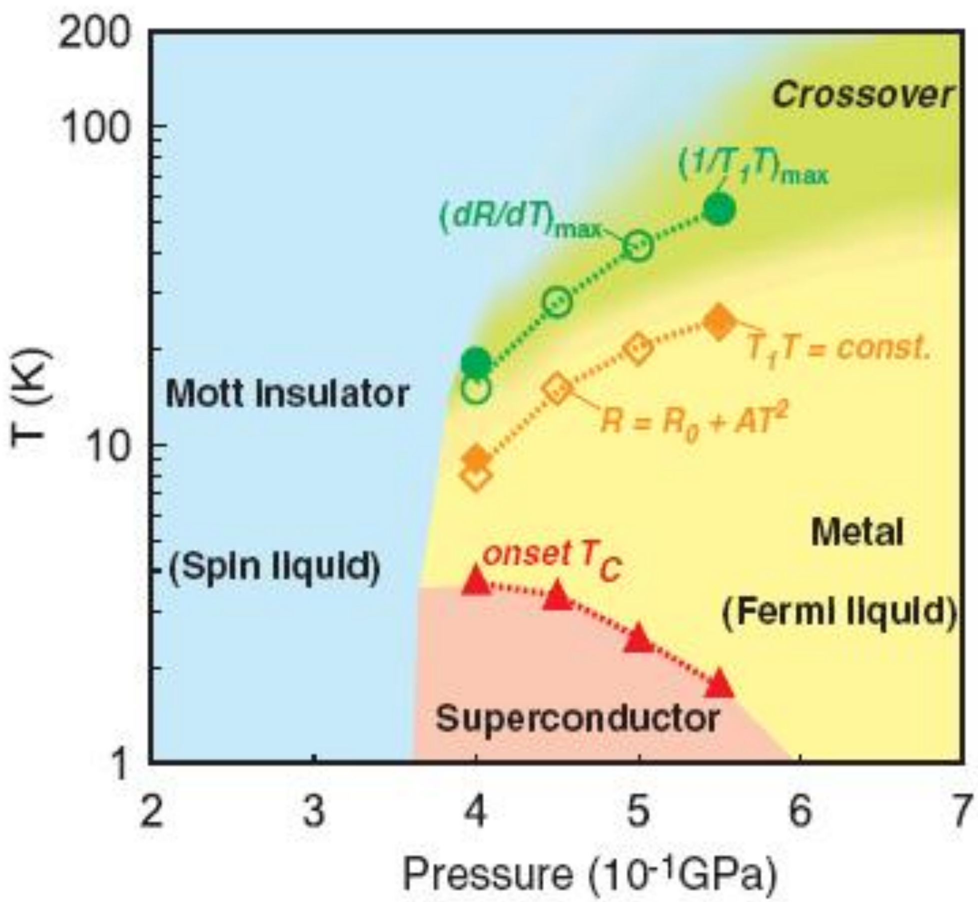
2nd order Mott critical point?

Possible realization: $\kappa-(ET)_2 Cu_2 (CN_3)_2$ under pressure

(Kanoda et al, 2003–present)

Possible experimental realization of a second order Mott transition

Kanoda et al '03 - '08



$\kappa-(ET)_2Cu_2(CN)_3$ under pressure.

One band Hubbard model on isotropic Δ lattice

No magnetic order in insulator!

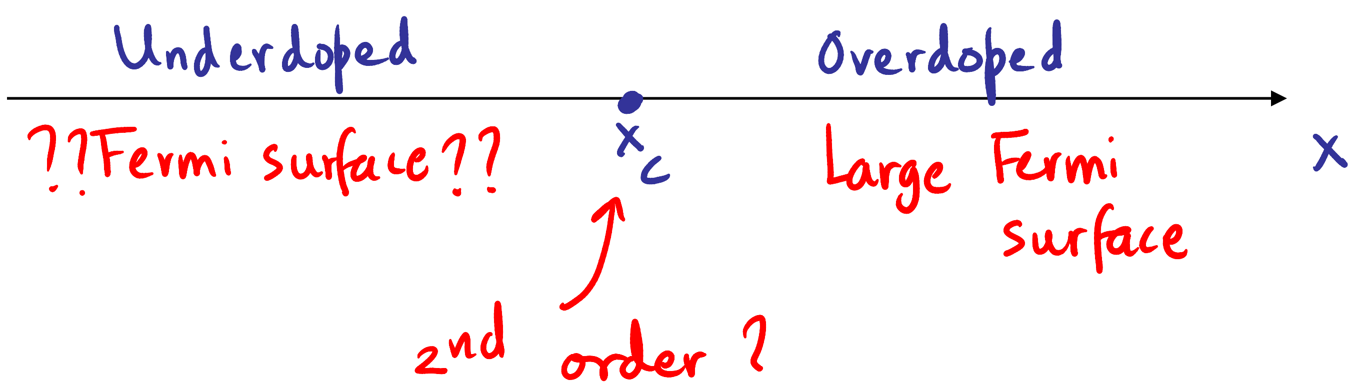
Spinon Fermi surface? (Motrunich '05, Lee & Lee '05)

Example III: HiTc ``underlying normal” metallic ground state ?

Overdoped metal : Fermi liquid with a large Fermi surface

Underdoped : Apparently metallic , unknown Fermi surface likely small pockets

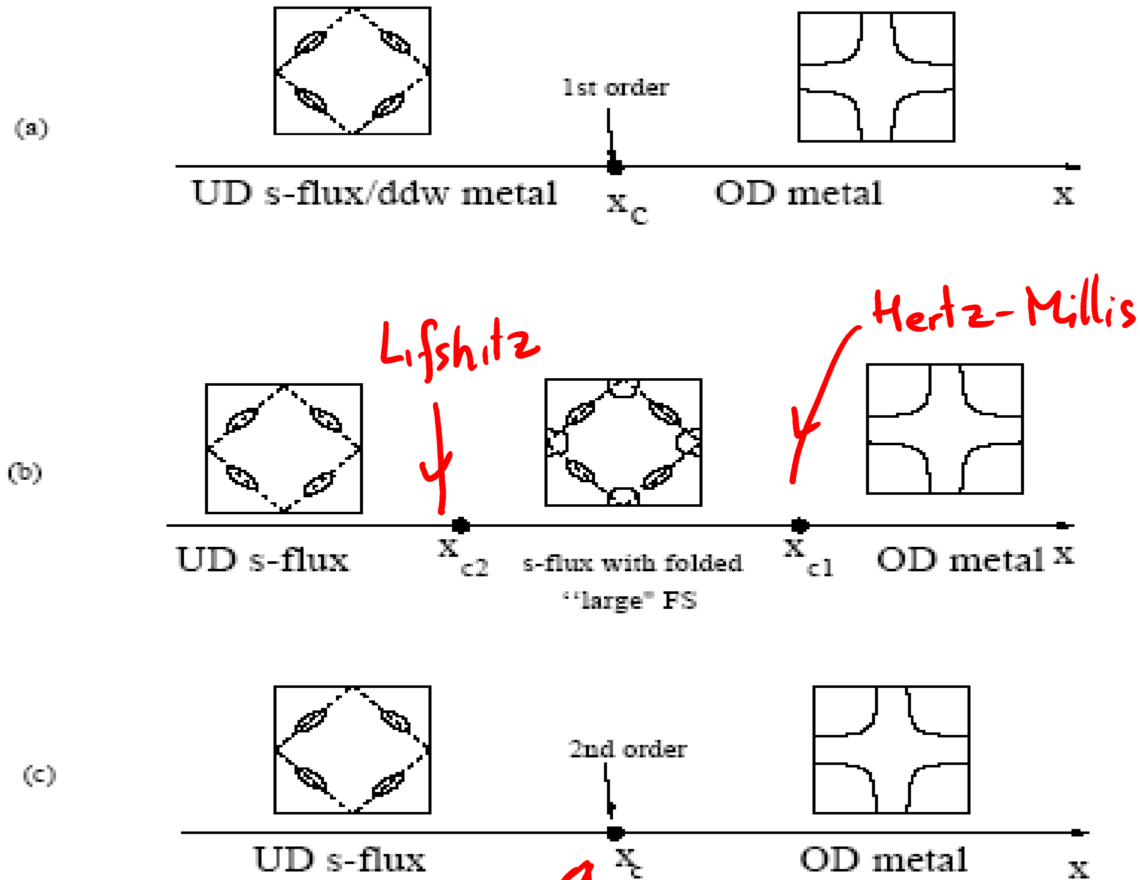
Large Fermi surface disappears for x below some x_c through a quantum phase transition.



Possible evolutions of Fermi surface with doping at T = 0

Many candidates for underdoped metal.

One example "Staggered flux/ddw" metal with small hole pockets.



Suggestion: Fig (c) is realized with a nontrivial critical point.

Non-FL critical point

How might a Fermi surface disappear?

Can a Fermi surface disappear continuously through a 2nd order transition?

One route – quasiparticle weight Z vanishes continuously and everywhere on Fermi surface!

(à la Brinckman-Rice)

Concrete example: "Kondo breakdown" model (TS, Vojta, Sachdev '04)

Electronic structure at criticality: ``Critical Fermi surface"

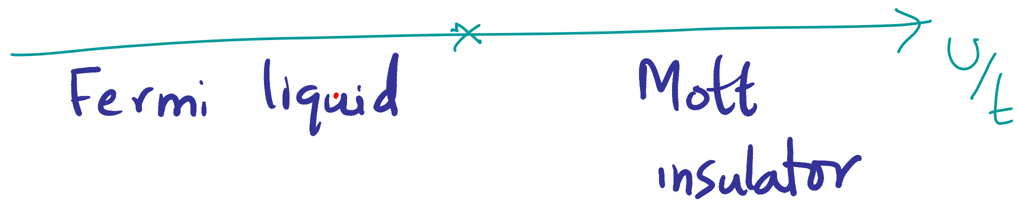
Crucial question : Nature of electronic excitations right at quantum critical point when $Z = 0$?

Claim : At critical point, Fermi surface remains sharply defined even though there is no Landau quasiparticle.

"Critical Fermi surface".

Why a critical Fermi surface?

Mott transition example:



What is gap $\Delta(\vec{k})$ in electron spectral function $A(\vec{k},\omega)$?

Fermi liquid : $\Delta(\vec{k} \in FS) = 0$

Mott insulator : Sharp gap $\Delta(\vec{k}) \neq 0$ for all $\vec{k}$

Evolution of single particle gap

Approach from Mott

2^{nd} order transition to metal $\Rightarrow$ expect Mott gap

$\Delta(\vec{K})$ will close continuously

To match to Fermi surface in metal, $\Delta(\vec{K}) \to 0$
for all $\vec{K} \in FS$.

$\Rightarrow$ Fermi surface sharp at critical point.

But as $Z = 0$ no sharp quasiparticle.

$\Rightarrow$ Non-fermi liquid with sharp "critical" Fermi surface!

Why a critical Fermi surface?

Evolution of momentum distribution

Metal with Fermi surface

(a)

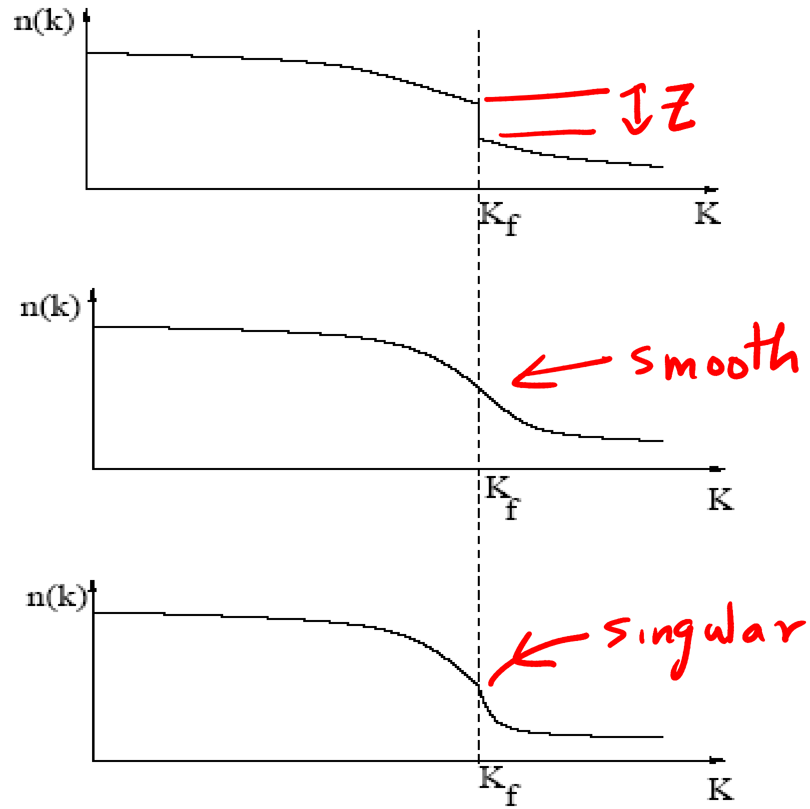
Phase where Fermi surface has disappeared

(b)

Critical point

(c)

n(K) continuous at K_F but is singular.



Killing a Fermi surface

Disappearance of Fermi surface through a continuous transition

At critical point

(a) $Z = 0$

(b) Fermi surface sharp

(Similar argument for heavy fermion critical points, hiTc, Mott critical point, etc).

Some obvious consequences/questions

Critical Fermi surface $\Rightarrow$ unusual criticality with phenomena different from familiar critical points.

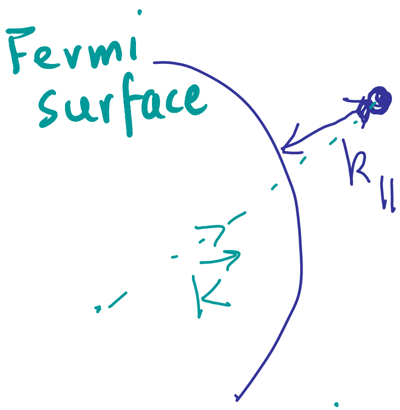
1. Structure of universal singularities/scaling phenomena?

2. Calculational framework?

Scaling phenomenology at a quantum critical point with a critical Fermi surface?

Critical Fermi surface: scaling for single particle physics

Right at critical point expect universal scale invariant singularity in $A_c(\vec{K}, \omega)$ for small ω, $k_{\parallel}$



Scaling ansatz:

For every point θ on FS

$$A_c(\vec{K}, \omega, T) \sim \frac{1}{|\omega|^{\alpha/z}} F\left(\frac{\omega}{|k_{\parallel}|^z}, \frac{\omega}{T}\right)$$

New possibility: angle dependent exponents

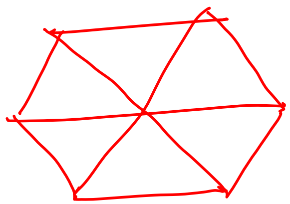
A priori must allow angle dependent exponents:

$$z = z(\theta), \quad \alpha = \alpha(\theta)$$

consistent with lattice symmetries.

Eg: Triangular lattice $\quad z(\theta + \pi/3) = z(\theta)$

$$\alpha(\theta + \pi/3) = \alpha(\theta)$$



Can expand $z(\theta) = \sum_n z_n \cos(6n\theta), \ldots$

Leaving the critical point

Expect scale invariant spectrum is cut off

at $k_{\parallel} \sim \frac{1}{\xi}$, $\omega \sim \frac{1}{\xi^z}$ so that

$$A_c(\vec{k}, \omega) \sim \frac{1}{|\omega|^{\alpha/z}} F_1\left(\frac{\omega}{k_{\parallel}^z}, k_{\parallel}\xi\right)$$

Expect $\xi \sim |g - g_c|^{-\nu}$ but again

a priori must let $\nu = \nu(\theta)$

Approach from the Fermi liquid

If Fermi liquid physics is part of scaling function

$$Z \sim |\delta g|^{\nu(z-\alpha)} \qquad (\Rightarrow z(\theta) \geq \alpha(\theta))$$

$$v_F \sim |\delta g|^{\nu(1-z)}$$

$\Rightarrow$ Specific heat $\displaystyle C_v \sim T\int_{FS} \frac{1}{v_F} \sim T\int_{FS} |\delta g|^{-\nu(1-z)}$

If ν, z are θ-dependent, not a pristine power law

Asymptopia: Dominated by portion of FS with $\max(\nu(1-z))$

Specific heat singularity

In Fermi liquid $C_v \sim \int_{FS} T \xi^{z-1}$

$\Rightarrow$ Scaling ansatz : $C_v \sim \int_{FS} T^{1/z}\, Y(T\xi^{z})$

At critical point expect $C_v \sim \int_{FS} T^{1/z}$

Again asymptopia dominated by portion of Fermi surface if z angle dependent.

Note: Spin susceptibility – different scaling models depending on fate of F^0_a as $g \searrow g_c$.

Critical 2Kf surface

2-particle response at finite q

Eg: $\chi''(q, \omega)$

Expect sharp $2K_f$ singularities associated with critical FS

$\Rightarrow$ $\chi''(q, \omega)$ has sharp critical singularities at entire surface in K-space (the $2K_f$ surface)

unlike at bosonic critical points

Separate scaling ansatz for q near $2K_f$ surface, small ω.

Implications of angle dependent exponents

(i) Different properties dominated by different portions of Fermi surface

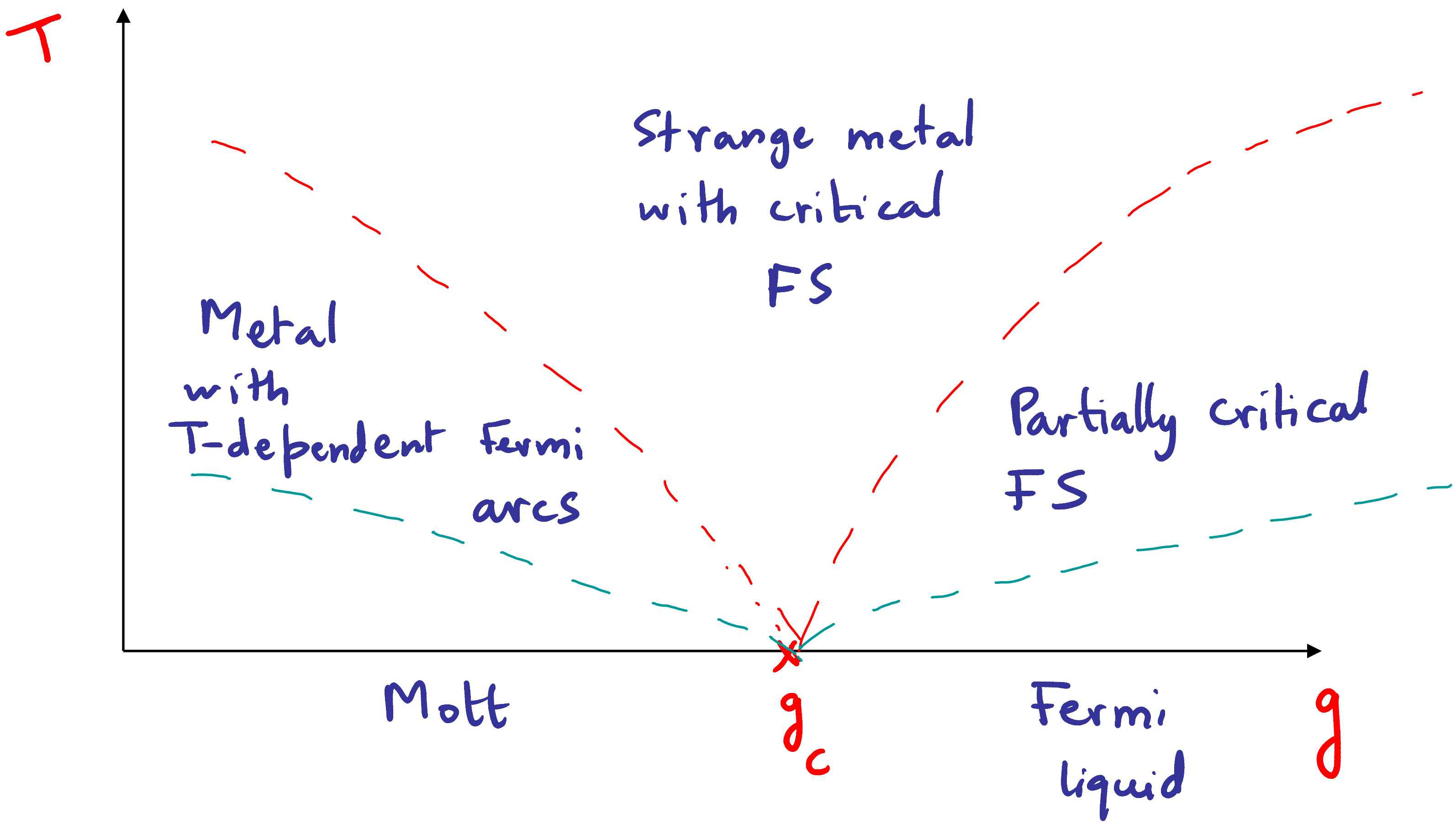
(ii) Different portions of Fermi surface will emerge out of criticality at different energy scales

Example: At Mott transition

Mott gap $\Delta(\theta) \sim |\delta g|^{z(\theta)\nu(\theta)}$.

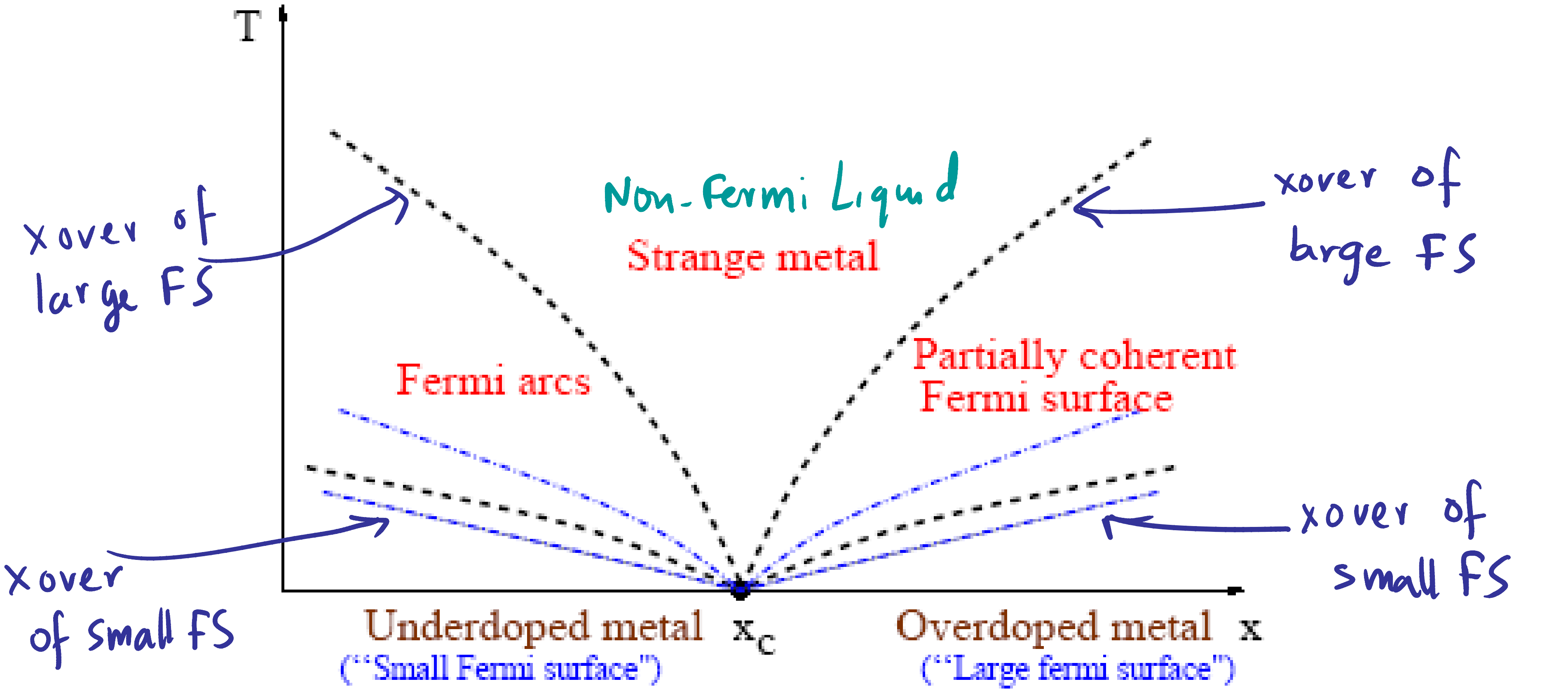
$\Rightarrow$ Finite-T x overs richer than usual

Finite T crossovers



Similar to cuprates (!)

Tentative application to proposed hiTc critical point



Calculational framework for critical Fermi surfaces

No general framework yet !

Study specific models of quantum phase transitions with disappearing Fermi surfaces.

Model calculations

Only currently existing framework for calculations seems to be a slave particle theory.

Examples:

1. Kondo breakdown model for Kondo lattices
   (TS, Vojta, Sachdev '04)
2. Theory of a continuous Mott transition in two dimensions

Possibly directly relevant to organics κ-$(ET)_2Cu_2(CN)_3$

pressure tuned thru the Mott transition.

Model calculations for the Mott transition

Slave rotor description: $c_{i\alpha} \sim e^{i\phi_i} f_{i\alpha}$

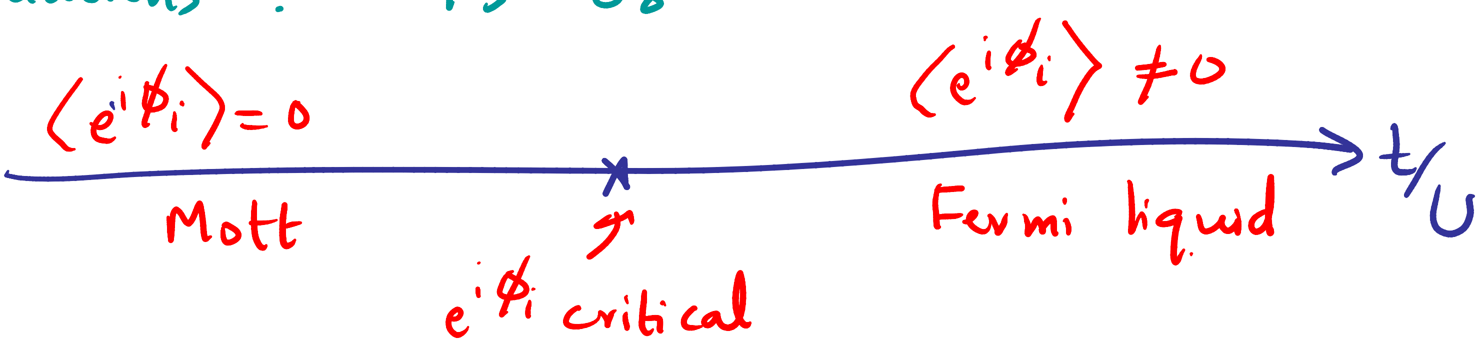
$e^{i\phi_i}$: charge-e spinless boson

$f_{i\alpha}$: neutral spin-½ fermion ("spinon")

U(1) gauge redundancy: local phase rotations of $e^{i\phi_i}$ & $f_{i\alpha}$

Mean field theory: Florens, Georges '04

Fluctuations : TS '08



Structure of critical theory

$$S_{eff} = \underbrace{S[e^{i\phi}, a]}_{\text{3D XY coupled to U(1) gauge field } a_\mu} + \underbrace{S[f, a]}_{\text{spinon Fermi surface coupled to } a_\mu}$$

RPA analysis: Effective action for transverse gauge field

$$S_{eff} = \int_{q,\omega} \left(\frac{|\omega|}{q} + \sqrt{q^2+\omega^2} \right) |a|^2$$

$$\simeq \int_{q,\omega} \left(\frac{|\omega|}{q} + |q| \right) |a|^2$$

Coupling to bosons: a_μ decouples due to Landau damping term

$\Rightarrow$ Bosons in 3D XY class

Fermions:

Same as "Coulomb" interaction case of HLR theory of $\frac{1}{2}$-filled Landau level

$$S = \underbrace{S[b]}_{\substack{\text{bosons in} \\ \text{3D XY universality} \\ \text{class}}} + \underbrace{S[f, a]}_{\substack{\text{Strongly coupled} \\ \text{spinon-gauge system} \\ \text{(same as HLR} \\ \text{with Coulomb)}}}$$

Critical Fermi surface at Mott criticality

Electron spectral function

$$A_c(\vec{K}, \omega) \sim \frac{|\omega|^\eta}{\ln\frac{1}{\omega}} F\left(\frac{\omega \ln \frac{1}{|\omega|}}{k_{\parallel}} \right)$$

Sharp critical Fermi surface but no Landau quasiparticle.

"Scaling form" with $\alpha = -\eta$, $z = 1^+$

(η = anamolous exponent in 3D XY model)

Approach from Fermi liquid

Vanishing quasiparticle residue $Z \sim \dfrac{|\delta g|^{2\beta}}{\ln \frac{1}{|\delta g|}}$

Diverging effective mass $\dfrac{m^*}{m} \sim \ln \dfrac{1}{|\delta g|}$

(but $Z \neq \dfrac{m}{m^*}$)

Diverging Landau parameters $F_0^a \sim \dfrac{m^*}{m} \to \infty$

$(\chi_0 \to \text{const.})$

$F_0^s \sim |\delta g|^{-\nu} \to \infty$

Compressibility $\kappa \to 0$ (and zero sound speed $\to \infty$)

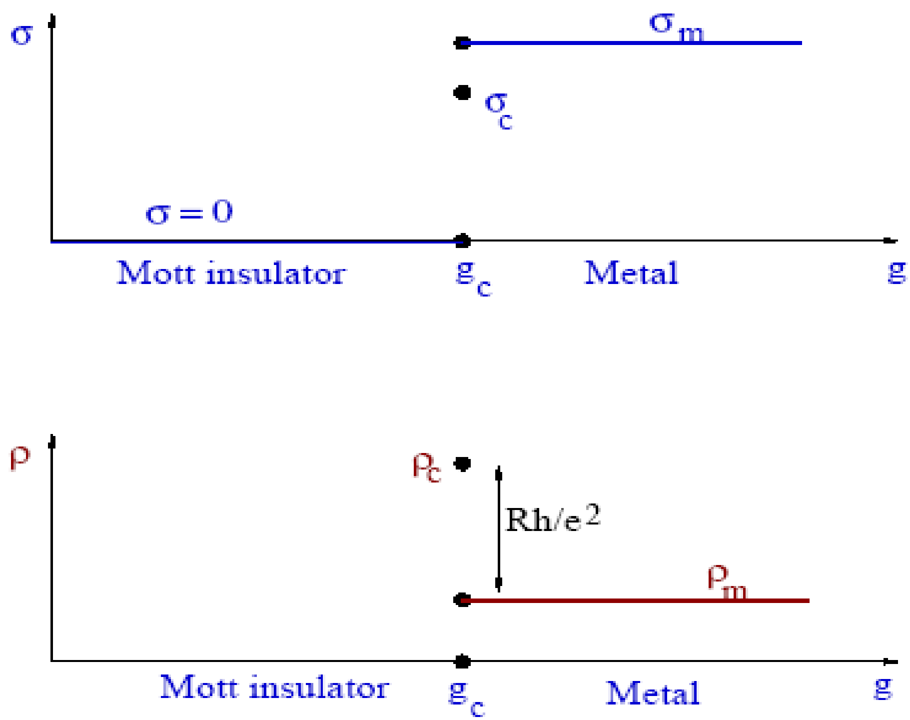
Critical thermodynamics/transport

$$C_V \sim T \ln \frac{1}{T} \quad , \quad \chi_0 \to \text{const.}$$

Resistivity $\rho = \underbrace{\rho_b}_{\text{boson}} + \underbrace{\rho_f}_{\text{fermion}}$ (Ioffe-Larkin rule)

$\rho_b = Rh/e^2 = \text{universal}$, $\rho_f \sim \rho_0 + o(T^2 \ln \frac{1}{T})$

Universal resistivity jump



$\rho = \rho_b + \rho_f$

$\Rightarrow$ in metal $\rho = \rho_f$

In insulator $\rho = \infty$

Right at Mott transition $\rho = \rho_b + \rho_f$

$\Rightarrow$ Residual resistivity jumps at Mott transition.

On approaching from metal, jump $= Rh/e^2 = \underline{\text{universal}}$.

Crossover out of criticality: Anderson is different (from Higgs)

Move from critical point to Fermi liquid:

Boson condensation scale $\sim \rho_s$ = boson stiffness

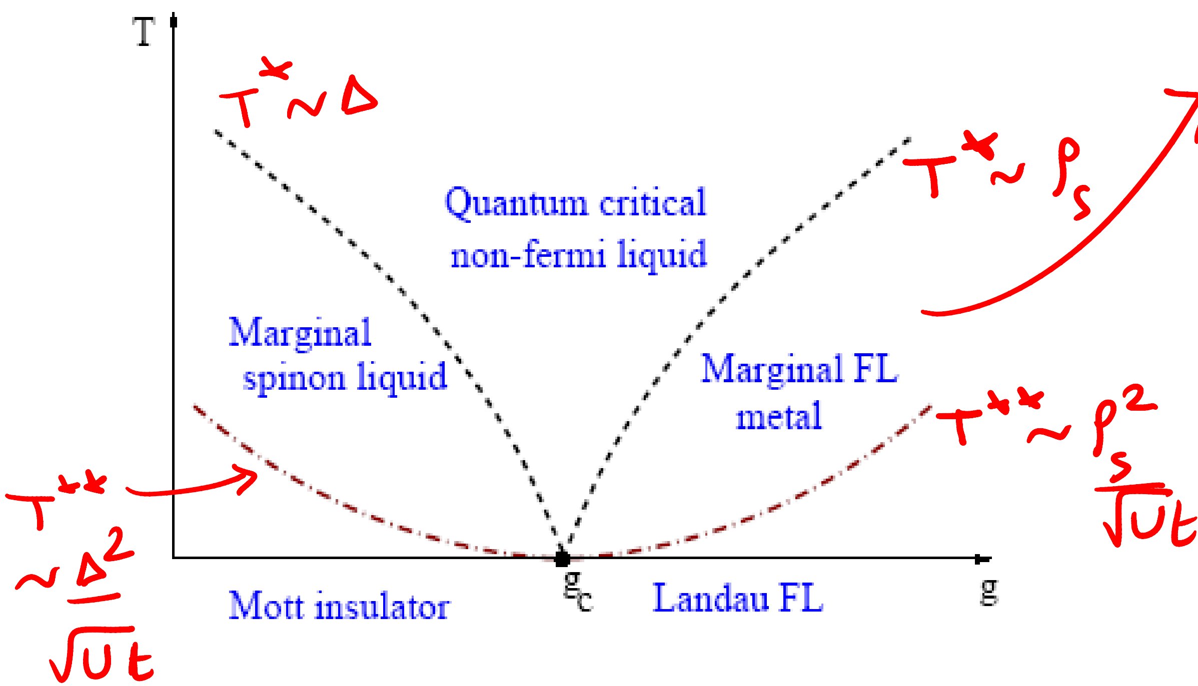
Modified gauge action $S_{eff} \sim \int_{q,\omega} \left(\frac{|\omega|}{q} + q f(\rho_s/q) \right) |a|^2$

Gauge fluctuations quenched at scale $\sim \rho_s^2 \ll \rho_s$

$\Rightarrow$ Charge & spin sectors emerge out of criticality at different scales

At intermediate scales, Bose condensation + critical spinon/gauge

Finite T crossovers: Marginal Fermi liquids



In this regime electron Greens function

$$G(\vec{k}, i\omega) \sim \frac{|\langle b \rangle|^2}{i\omega\left(1 + a \ln \frac{1}{|\omega|}\right) - \epsilon_k}$$

= Marginal FL form proposed for hiTc by Varma et al '89

Summary-I

- At some metallic quantum critical points there

will be an entire surface of critical fermionic modes

- a `critical Fermi surface'.

- Presence of critical fermi surface will change the scaling phenomena associated with universal critical singularities.

- Scaling hypotheses for single particle and thermodynamic quantities; presence of critical 2Kf surfaces,……

- Possibility of angle dependent exponents with interesting consequences (eg: metals with T-dependent Fermi arcs at intermediate temperature)

Summary-II

- Concrete theory of a continuous Mott transition in two dimensions

- demonstrate critical Fermi surface

- predict universal resistivity jump, emergence of marginal Fermi liquids

Future: Lots of challenges !