Overview of Landau and non-Landau quantum critical points: field theories and microscopic quantum models

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Outline

1. Orientation: Landau and non-Landau ordering in quantum matter

2. Landscape of quantum phase transitions

3. Remarks on Landau QCPs

4. Non-Landau QCP: a simple example

5. Detour: tractable model of metallic quantum criticality

6. Remarks on deconfined criticality at Neel-VBS transitions in 2d

7. Entanglement properties of non-Landau quantum critical points*

*B. Swingle and TS, in progress

Orientation

Conventional ordered phases of matter:

Concepts of broken symmetry/ Long Range Order (LRO)

Characterize by Landau order parameter.

Examples

$$|\uparrow\uparrow\uparrow\uparrow \ldots\ldots\ldots\rangle \qquad\qquad |\uparrow\downarrow\uparrow\downarrow \ldots\ldots\ldots\rangle$$

Ferromagnet | Antiferromagnet

Landau Order

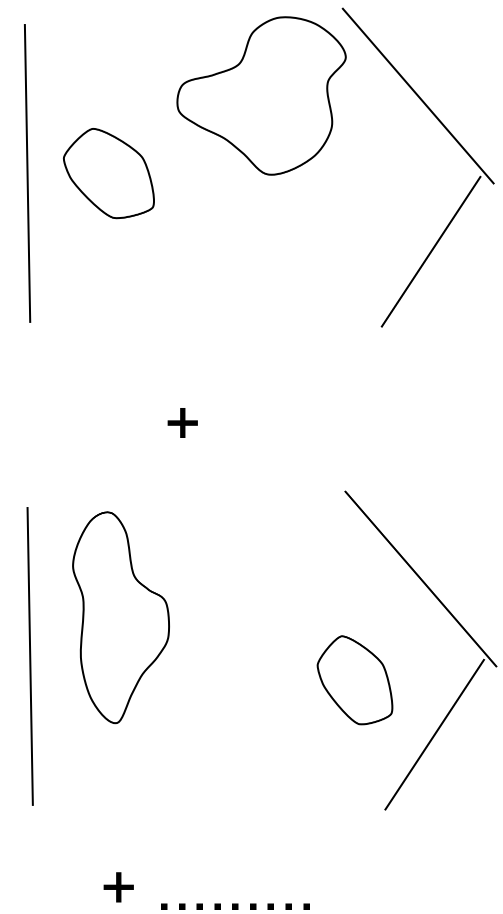
Known for several millenia

Non-Landau order I: Topological order

Examples: FQHE, gapped quantum spin liquids in d > 1, ...

Emergence of **sharp** quasiparticles with fractional quantum numbers, long range statistical interactions.

Low energy effective theory: a topological field theory



Known only since 1980s

Non-Landau order II: Beyond topological order

Some known examples: Gapless quantum spin liquids, 1/2 filled Landau level

Most familiar: Landau fermi liquid

Others: all non-fermi liquid states in d > 1, bose metals, ,,,,,,,

Protected gapless excitations: in simple cases these can be given a quasiparticle description.

But in some such phases **elementary** excitations may not exist,
i.e, no quasiparticle description of
excitation spectrum.

Probably crucial to understand current experiments on candidate spin liquid materials, non-fermi liquid `strange' metals in cuprates and other systems.

Slowly evolving understanding in last 20 years.

Landau versus non-Landau order

Landau order captured by LRO in correlation function of local order parameter field.

Non-Landau order: non-local global property of ground state wavefunction protects universal properties.

Non-locality enables emergence of quasiparticles with fractional quantum number or in some cases death of any quasiparticle structure.

Entanglement in many body ground states

Landau ordered phases: prototypical ground state wavefunction is **direct product** of **local** degrees of freedom.

$$|\uparrow\uparrow\uparrow\uparrow \,\ldots\ldots\ldots\rangle$$

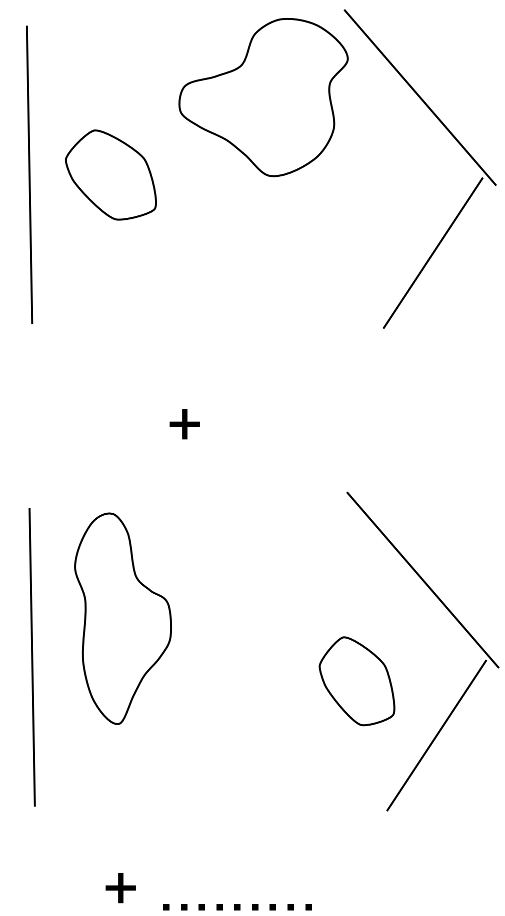
``Fixed point'' wavefunction for ordered state only has Short Range Entanglement (SRE).

$$|\uparrow\downarrow\uparrow\downarrow \,\ldots\ldots\ldots\rangle$$

Entanglement in many body ground states

Non-Landau phases: non-local structure in ground state wavefunction;
Cannot deform smoothly to direct product of local degrees of freedom

Describe as Long Range Entanglement (LRE)

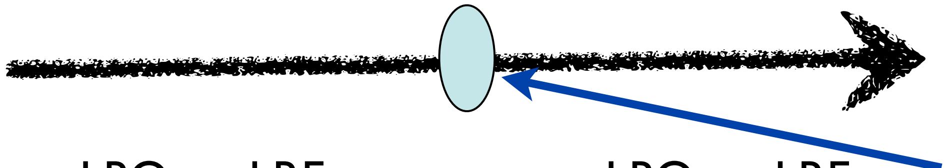


Key question: how to directly characterize LRE?

Topological order: Topological entanglement entropy (Levin Wen; Kitaev, Preskill, 06)

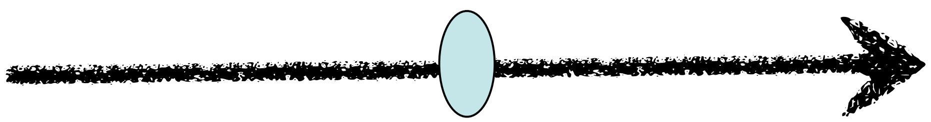
Beyond topological order: ??

Landscape of quantum phase transitions



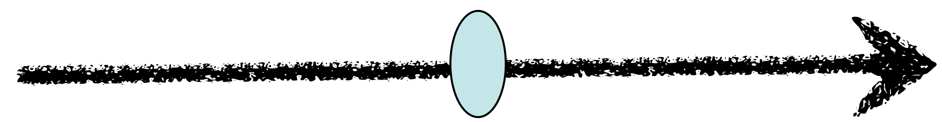
LRO, no LRE — no LRO, no LRE

Landau-Ginzburg-Wilson paradigm only works here!



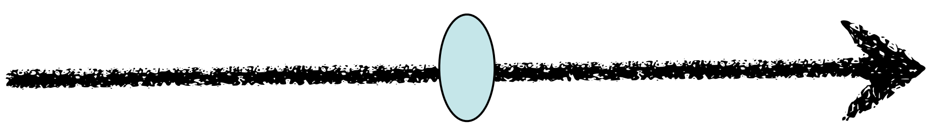
LRE, no LRO — no LRO, no LRE

Eg: Topological quantum spin liquid
- trivial paramagnet



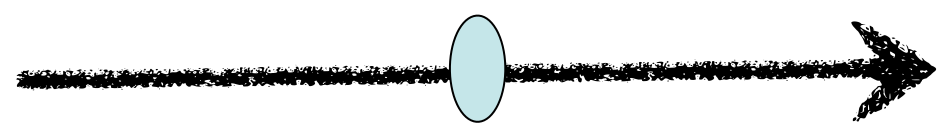
LRO, no LRE — LRE, no LRO

Eg: AF - quantum spin liquid



LRO, no LRE — LRO', no LRE

Eg: AF - VBS in spin-1/2 square lattice



LRE — LRE'

Eg: FQHE plateau transitions

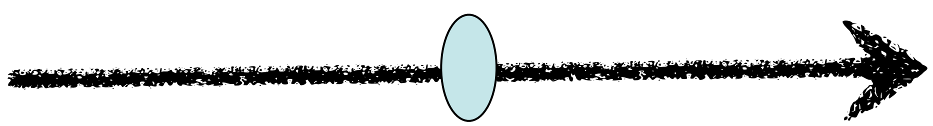
Note: quantum critical points have Long Range Entanglement

Landau-forbidden phase transitions between Landau-allowed phases

TS, Vishwanath, Balents, Fisher, Sachdev, 2004

Naive expectation: Breakdown of LGW paradigm at QCP when one of the two proximate phases has non-Landau order.

Very interesting that LGW can also break down at critical point between two Landau-allowed phases.



LRO, no LRE LRO', no LRE

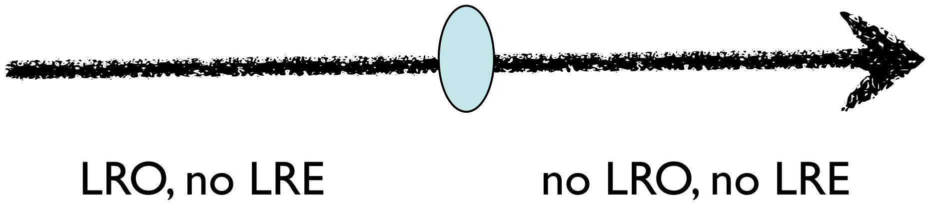
Eg: AF - VBS in spin-1/2 square lattice

Landau-forbidden critical point: Critical theory described in terms of emergent gapless spinons coupled to emergent gauge fields.

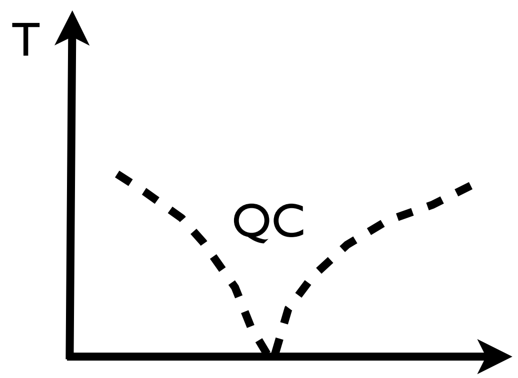
``Deconfined Quantum Critical Points''

Many other proposed examples by now.

Theories of quantum phase transitions: Landau transitions - field theory/numerics



Prototype: quantum Ising/O(N) models



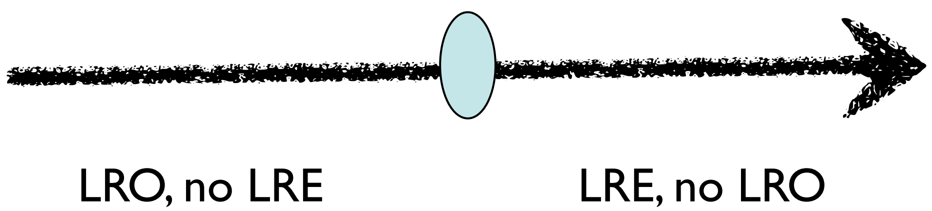
Extremely well understood (see Sachdev book).

Field theory: Continuum LGW field theory analyzable by epsilon/1/N expansions
Scaling structure clear.

Numerics: sign-problem free models in right universality class typically exist.

Most important utility: benchmark for modern numerical approaches (eg: real space RG algorithms etc).

Non-Landau criticality: a simple example



Eg: Boson superfluid - fractionalized Mott insulator

Boson Hubbard model at integer/half integer filling on a 2d lattice.

Simple Fractionalized Mott insulator

Excitation spectrum: gapped charge-1/2 boson (b) and gapped Z_2 vortex (vison) with mutual semion statistics.

Deconfined phase of Z_2 gauge theory with topological order.

Transition to superconductivity when charge-1/2 bosons condense.

In terms of b, transition is in 2+1- XY universality class.

XY* transition (not XY)

Physical superfluid order parameter $\sim b^2$.

=> superfluid order parameter has large anomalous dimension $\eta \approx 1.4$ (compare with 0.03 for XY).

Only physical operators with critical correlations are Z_2 gauge invariant combinations of b

=> only a subset of local operators of XY fixed point are allowed.

New XY^* fixed point.

Exponents v, z same as for XY but η, and hence β very different.
Universal critical boson conductivity smaller by factor of 4.

Despite appearances a non-Landau transition with unusual but simply derived properties.

Other similar transitions

1. AF - quantum spin liquid: spinon condensation (Chubukov, TS, Sachdev, 94; Isakov, TS, Kim, 06; Ghaemi, TS, 06)

2. Valence Bond Solid with broken lattice symmetry - gapped quantum spin liquid:

Describe as vison condensation in a dual LGW theory (Blankshtein et al 1980s, Jalabert, Sachdev, 91, Moessner, Sondhi, 99)

Despite appearances this is actually an LGW* not LGW transition.

Physical VBS order parameter is a composite of basic vison field
=> has large anomalous dimension;

Spinon gap of spin liquid finite at transition.

Models and numerics for the XY* transition

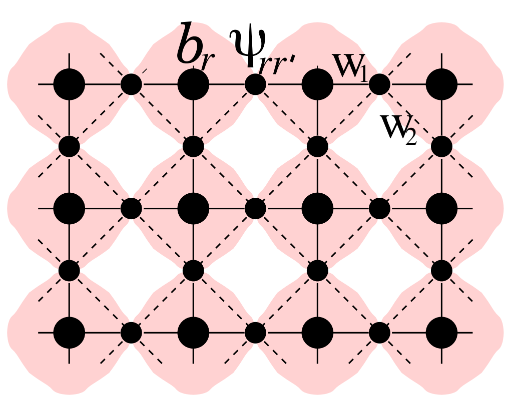
$$H = -t \sum_{\langle ij \rangle} [b_i^\dagger b_j + b_i b_j^\dagger] + V \sum_{\hexagon} (n_{\hexagon})^2$$

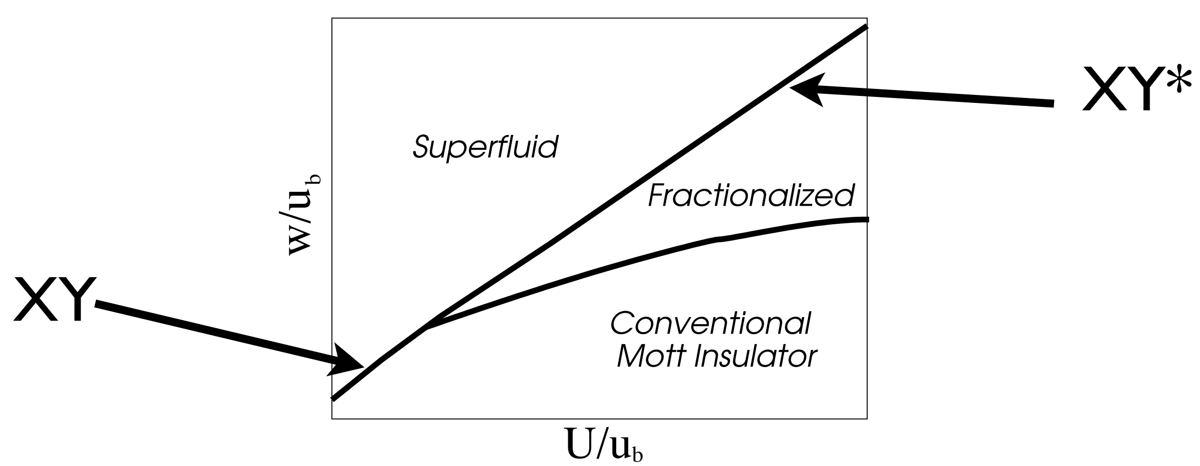
A Kagome Bose Hubbard model (Isakov, Melko, Hastings 2011)

Closely related model: Balents, Fisher, Girvin, 2002.

$t \gg V$: superfluid
$V \gg t$: fractionalized Mott insulator
Numerics at transition consistent with ν, z of $3D$ XY.
Future: distinguish between XY^* and XY.

O(2) quantum rotor models with cluster charging: Motrunich, TS, 2002.





Schematic phase diagram

Others: Dimer liquid - Dimer solid (VBS) in 2d non-bipartite quantum dimer models.
Can large eta of solid order be demonstrated?

Comments

Despite its simplicity the XY* and related transitions provide valuable examples of non-Landau transitions that can be studied quite completely.

Interplay of **gaplessness** and **non-trivial topological structure.**

Readily generalized to 3d to provide example of a non-Landau 3+1-d QCP with large non-zero anomalous dimensions.

Comments (cont'd)

Platform to make progress on otherwise difficult questions in quantum criticality.

Two examples:

1. Tractable models of strongly coupled quantum criticality in a metallic environment (Grover, TS, 10; Mross, TS, to appear)

2. Understanding how to characterize entanglement structure of non-Landau QCPs (Swingle, TS)

Detour: quantum criticality in metals

Many difficult problems associated with quantum criticality in metals, particularly in two dimensions.

Interplay between Fermi surface and criticality

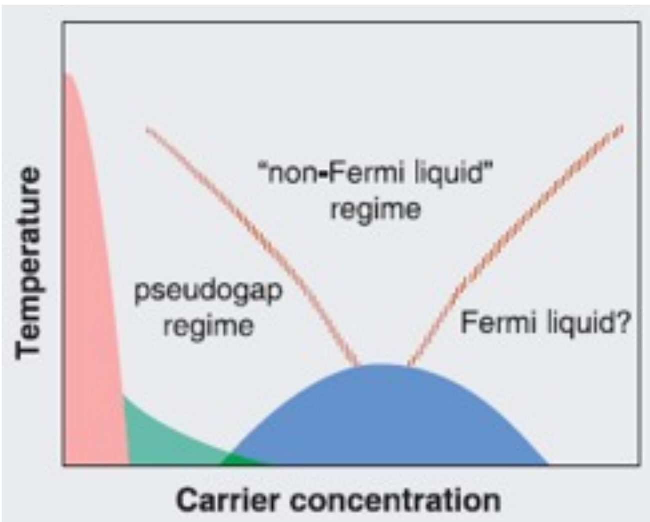
Most (interesting) experiments on quantum criticality are in metallic systems - heavy fermion critical points, criticality in cuprates, the Mott transition, etc.

This talk: a simple tractable example of a non-Landau quantum critical point associated with a continuous melting transition of stripes in a correlated metal.

Many interesting phenomena:

1. Strongly coupled quantum criticality in a metal
2. Reconstruction of Fermi surface across the transition.

Why study stripe criticality?



Stripes seem remarkably common in almost all families of underdoped cuprates
(most recently evidenced in YBCO in high field NMR (Julien et al, 11)).

Stripes absent in overdoped cuprates.

Many theories invoke fermi surface reconstruction by stripe ordering to explain low-T quantum oscillations in high field in underdoped cuprates.

Very popular idea: stripe criticality and associated Fermi surface reconstruction responsible for observed strange metal phenomenology.

Challenge: work out theory of stripe ordering quantum phase transition in a metal

Theories of stripe criticality

Weak coupling approach: couple Fermi surface to stripe order parameter.

Dynamics of stripe order parameter dominated by Landau damping of p/h pairs of Fermi surface at low energies.

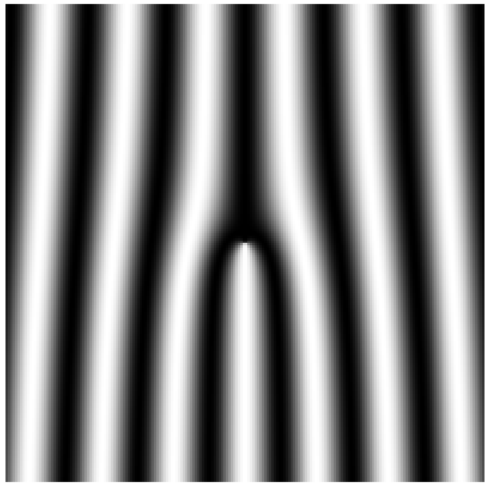
Old theory: ``Hertz-Millis", $z = 2$ criticality, upper critical dimension 2

Modern developments (Abanov, Chubukov, Metlitski, Sachdev): theory is strongly coupled in 2d, standard 1/N expansion methods spiral out of control.

No reliable description of low energy physics.

Alternate approach

Strong coupling point of view: start from the stripe ordered phase and describe quantum melting of stripe order.



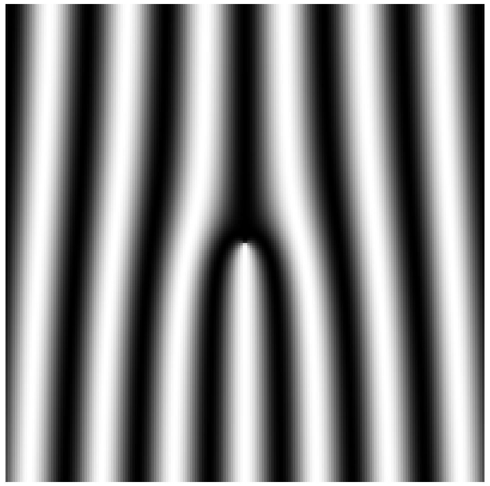
Concrete example: charge stripes in a layered orthorhombic metallic crystal with period 4 lattice spacings.

Stripe ordering reconstructs band structure `large' Fermi surface.

Melt stripes by proliferating topological defects = dislocations in stripe pattern.

Proliferate single dislocations: conventional large Fermi surface FL metal; theory of transition not understood.

Alternate approach



Proliferate doubled dislocations to melt stripe order

Restore Landau quasiparticles at large Fermi surface
(visible in dHvA, ARPES, transport,.....)

but stripe fluctuations
are ``fractionalized'' (Zaanen, 1999, Sachdev, Morinari, 2002).

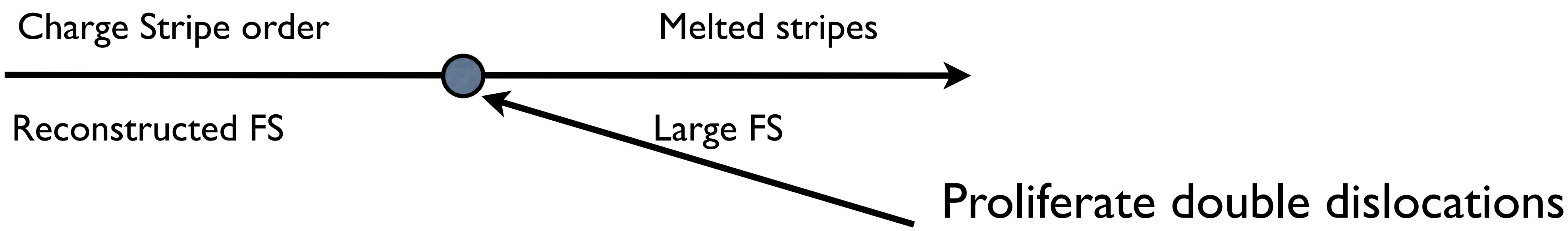
Stripe fractionalized metal:

Very subtle distinction from ordinary FL
(surely not visible to any current experimental probe).

Might this be the overdoped cuprate metal???

Theory of a continuous stripe melting transition

Mross, TS, to appear



Stripe melting by doubled dislocation proliferation is an XY* transition*.

Stripe order parameter has large anomalous dimension.

* No ``chemical potential" term for XY order parameter due to lattice reflection symmetry.

Theory of a continuous stripe melting transition

Mross, TS, to appear

Consequences of large anomalous dimension :

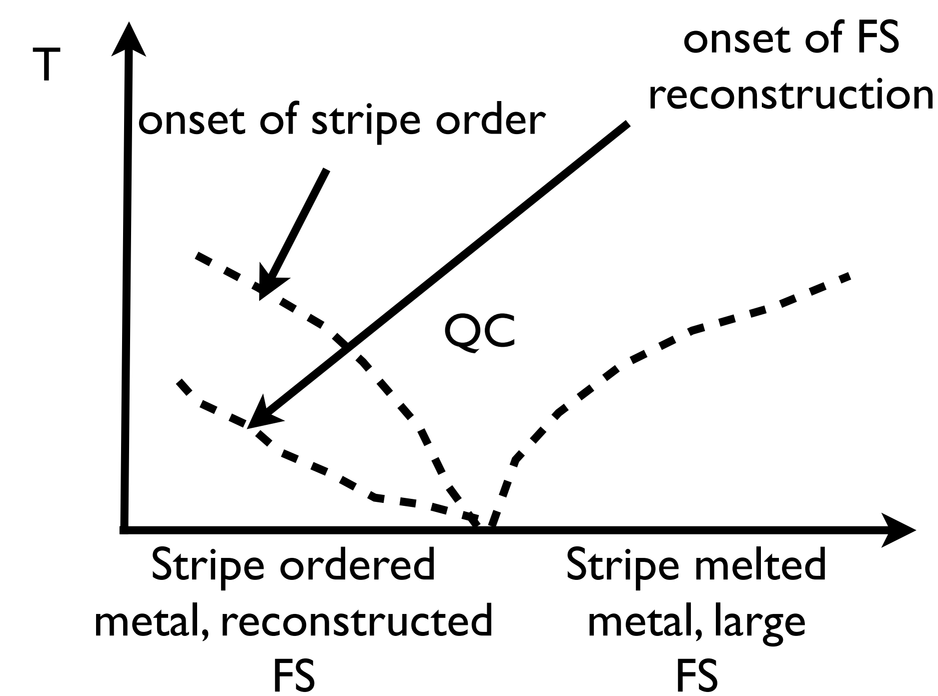
1. Lattice pinning of stripe order parameter irrelevant at critical point.

2. Landau damping and other coupling to Fermi surface also irrelevant at criticality!

Critical stripe fluctuations are strongly coupled but tractable.

Coupling to Fermi surface dangerously irrelevant: in stripe ordered state, scale of stripe ordering parametrically different from scale of Fermi surface reconstruction.

Single electron physics Fermi liquid like even at critical point.



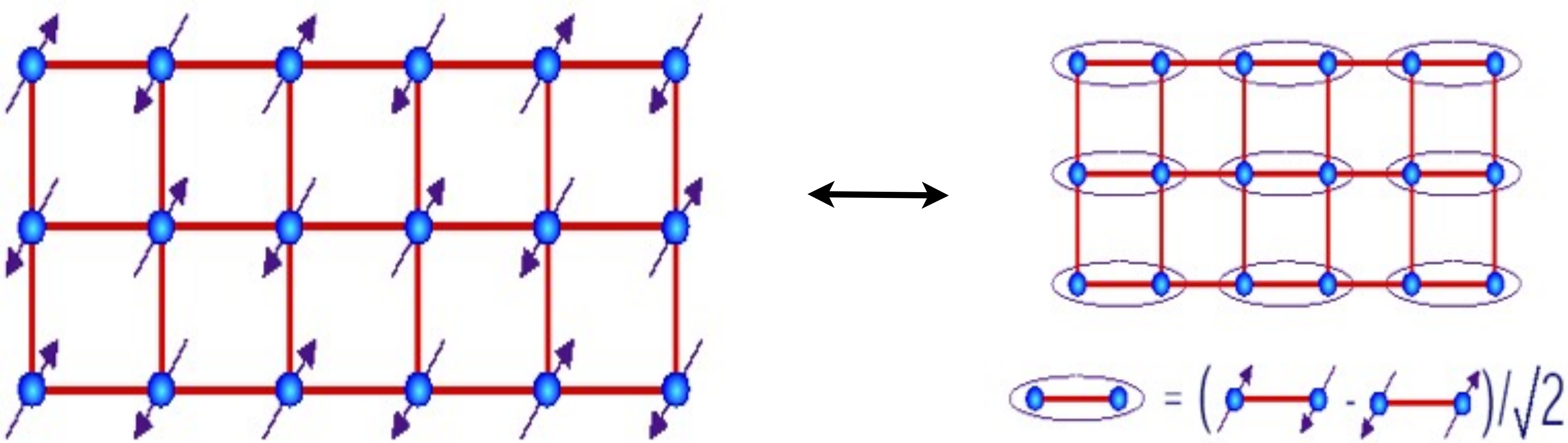
As far as I know, the ONLY understood example of stripe quantum criticality in a 2d metal.

Can generalize to tetragonal symmetry, other period stripes etc.

End of detour

Deconfined criticality and the Neel-VBS transition: lightning review

Landau-forbidden continuous transitions between Landau allowed phases



Topological defects of either order parameter carry non-trivial quantum numbers.

Many other proposed examples:

1. Bosons at fractional filling on various lattices (Balents et al, 05,06)
2. Spin nematic - VBS of spin-1 magnets (Harada, Kawashima, Troyer, 06; Grover, TS, 07)
3. Spontaneous spin Hall Mott - SC on honeycomb (Grover, TS, 08)
4. Staggered VBS - magnetic order (Xu, Balents, 11),

and ....

Critical theory for Neel-VBS
``Non-compact CP_1 model"

$$S = \int d^2x d\tau \, |(\partial_\mu - i a_\mu) z|^2 + r|z|^2 + u|z|^4$$
$$+ (\varepsilon_{\mu\nu\lambda} \partial_\nu a_\lambda)^2$$

z = two-component spin-1/2 spinon field
a_μ = non-compact U(1) gauge field.
Distinct from usual O(3) or Z_4 critical theories*.

Theory not in terms of usual order parameter fields but involve fractional spin objects and gauge fields.

*Distinction with usual O(3) fixed point due to non-compact gauge field (Motrunich,Vishwanath, '03)

Deconfined criticality and the Neel-VBS transition: microscopic models and numerics

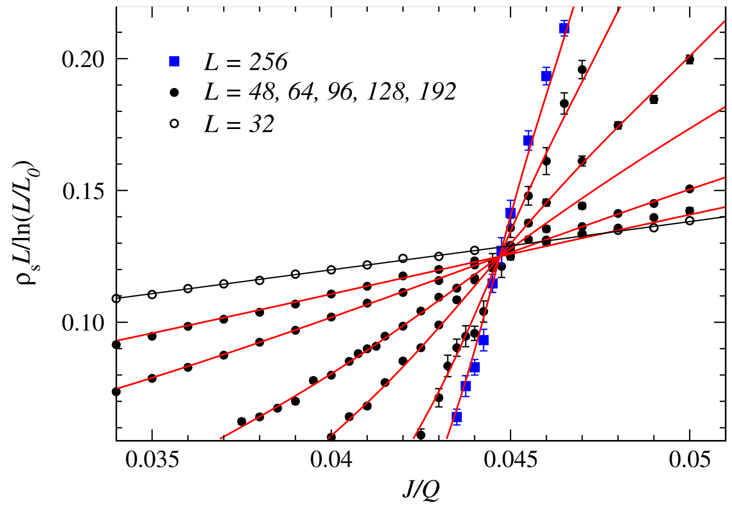
Sandvik J-Q model $$H = -J\sum_{\langle ij \rangle} C_{ij} - Q\sum_{\langle ijkl \rangle} C_{ij}C_{kl},$$ $$C_{ij} = \frac{1}{4} - \vec{S}_i.\vec{S}_j$$

Sign problem free: large scale QMC possible.
Important contributions from many people (Sandvik, Melko, Kaul, Prokofiev, Svistinov, Wiese, Chandrasekharan, Kawashima, Damle, Alet, ...)

Early controversy on order of transition.

Emerging picture from most extensive recent simulations:

Second order but with unusual logarithmic correction to scaling for spin stiffness. (Sandvik 10, Bannerjee, Damle, Alet, 2010)



Sandvik 2010

Deconfined criticality and the Neel-VBS transition: microscopic models and numerics

Field theory: controlled calculation possible only in 1/N expansion; no log scaling corrections visible.

Numerics: study SU(N) J-Q models to understand correction to scaling (Lou, Sandvik, Kawashima, 09; Kaul, 10; Bannerjee, Damle, Alet, 10).

Transition second order for N = 3, 4 but a significant correction to scaling persists.

Full picture remains to be clarified.................

Characterizing non-Landau critical points

Common theme to all non-Landau criticality: emergent non-locality captured by field theory in terms of gauge fields, fractional degrees of freedom etc.

Q: Should one expect this to show up as non-trivial Long Range Entanglement at non-Landau critical ground states?

Problem: **All** quantum critical points have LRE.

Long range correlations at a generic QCP necessarily imply LRE.

Characterizing non-Landau critical points: refine the question

Refined Q: Do non-Landau QCPs have ``stronger'' LRE than Landau QCPs?

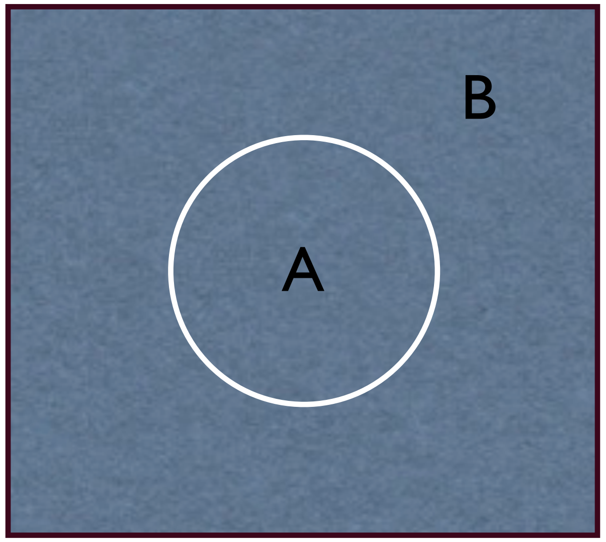
More refined: Do non-Landau QCPs have stronger LRE than is simply dictated by the long range correlations of local operators?

Can we sharpen these questions?

Motivates general study of entanglement structure of non-Landau QCPs.

Formulate simpler answerable questions

Does the deconfined critical point for the Neel-VBS transition have stronger Long Range Entanglement than the O(3) critical point?



Suggestive evidence that answer is yes!

Characterize entanglement by universal contribution to Entanglement Entropy (EE) of a spatial bipartition.

$S_A = Tr_B|\psi >< \psi|$
$S_A = \alpha L - a \; (L = \text{area of boundary})$
a universal but depends on shape of spatial region.

Conjecture: (Myers, Sinha, 10; related old work by Cardy):

For any Conformal Field Theory (CFT), put system on spatial sphere and choose A to be one hemisphere.

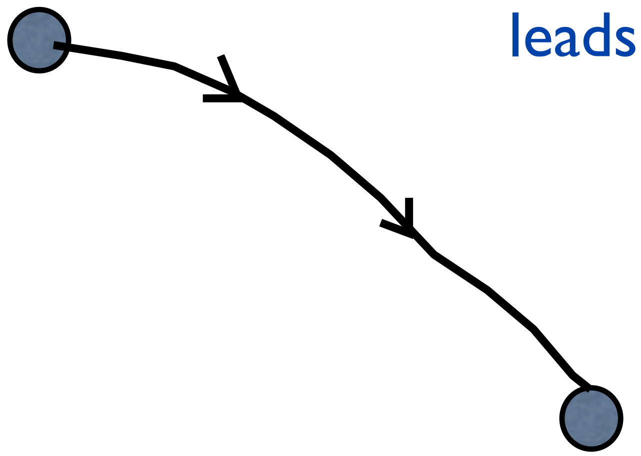
Then universal piece in EE decreases under RG flow.

(A generalization of c-theorem of 1+1 CFT to any d)

Does the deconfined critical point for the Neel-VBS transition have stronger Long Range Entanglement than the O(3) critical point?

Deconfined critical point:
Non-compact CP^1 CFT

Perturbing NCCP1 by single monopole operator leads to relevant flow to O(3) fixed point.



O(3) CFT = compact CP^1

General CFT conjecture of Myers and Sinha

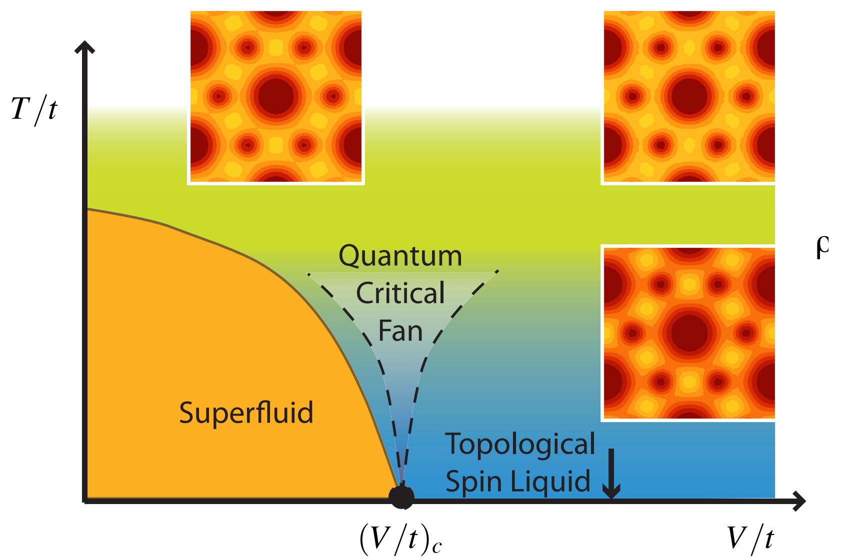
=> universal piece of EE between two hemispheres larger for deconfined critical point than O(3) critical point.

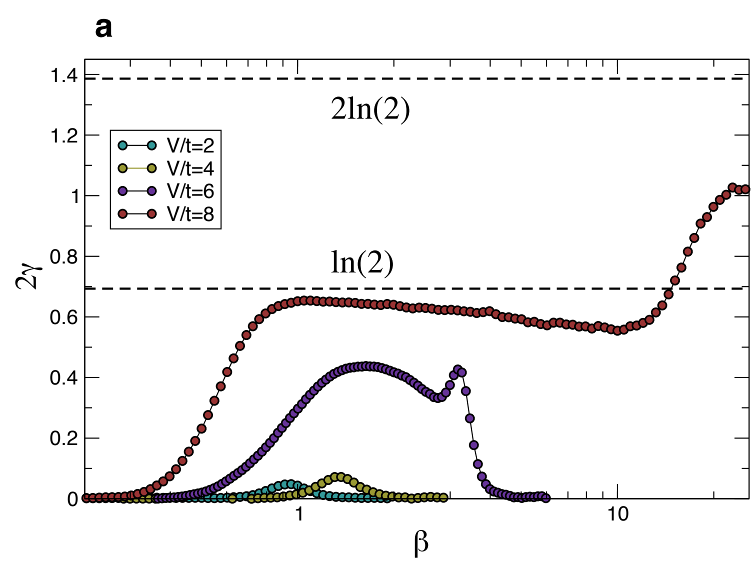
Entanglement entropy at the XY* transition

Ideal case study to separate out effects of long range correlation and global topological structure on entanglement.

Fractionalized Mott insulator has Topological Entanglement Entropy: what is its fate at the critical point of transition to superfluid?

Further inspiration: recent numerics (Isakov, Hastings, Melko, Nat Phys11) on Kagome Bose-Hubbard model



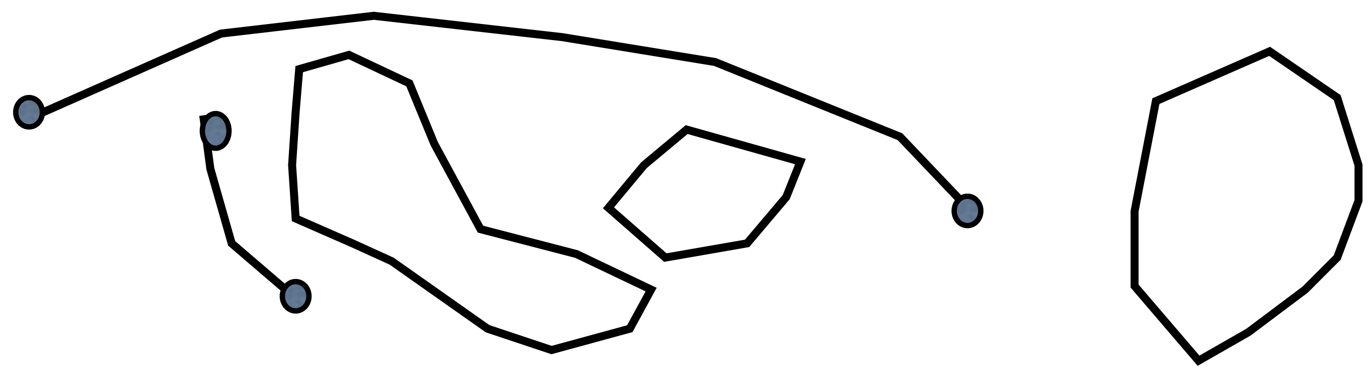


Useful effective model for XY* transition

Set vison gap $= \infty$.

Charge-1/2 hard-core boson (chargon) condensation with static zero Z_2 gauge flux.

Alternate 'electric field' picture: ∞ vison gap: tensionless electric field lines (strings). Chargons are end points of open strings.



Ground state wavefunction

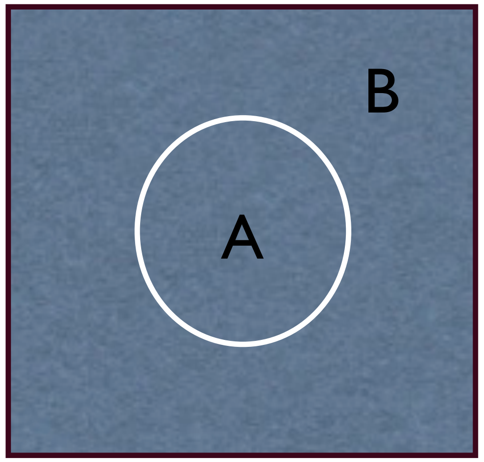
$|\psi >= \sum_R \phi(R)|R\rangle|X(R)\rangle$

$|X(R)\rangle$ = sum of all string configurations with open ends at R.

$R = (r_1, r_2, ..., r_N)$ = set of chargon locations.

Swingle, TS, forthcoming

Entanglement entropy of a simply connected region



AB boundary has n sites.

Decompose wavefunction:

Partition chargon sites into R_A, R_B.

For each partition sum over all string configurations in A and glue to sum over all string configurations in B.

Total 2^{n-1} possible boundary string configurations.

Easy to show

$S_A = S_A^{XY} + (n-1)ln2.$

Universal piece

$S_{univ}^{XY*} = S_{univ}^{XY} + ln2.$

Apparent clean separation between gapless and topological contributions.

Under scrutiny: multiply connected partitions of the kind required in Levin-Wen protocol for extracting Topological EE in fractionalized phase.

Other general issues

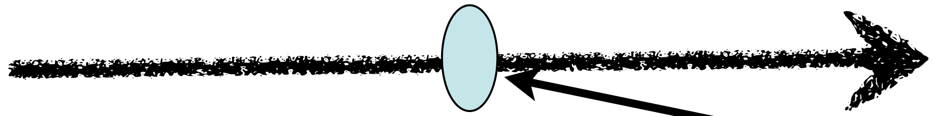
1. A clean protocol for measuring universal quantum critical contribution to EE in a lattice numerical calculation?

Beware of corners!

2. Shape dependence? How universal?

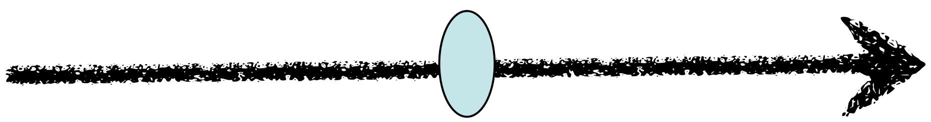
3. (More general) is there a better probe of Long Range Entanglement than the various entropies?

Summary: Landscape of quantum phase transitions



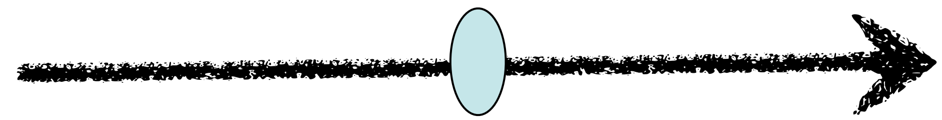
LRO, no LRE | no LRO, no LRE

Landau-Ginzburg-Wilson paradigm only works here!



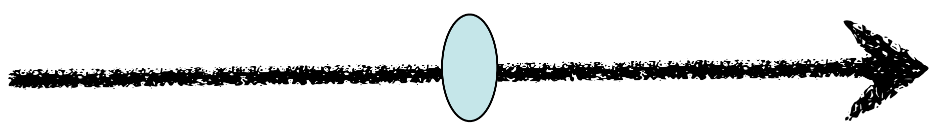
LRE, no LRO | no LRO, no LRE

Eg: Topological quantum spin liquid
- trivial paramagnet



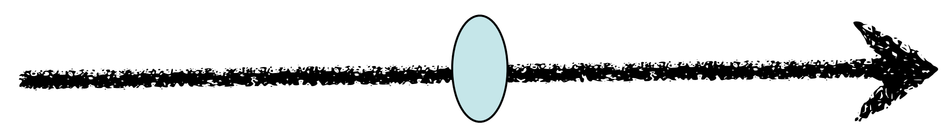
LRO, no LRE | LRE, no LRO

Eg: AF - quantum spin liquid



LRO, no LRE | LRO’, no LRE

Eg: AF - VBS in spin-1/2 square lattice



LRE | LRE’

Eg: FQHE plateau transitions

Note: typically quantum critical points have Long Range Entanglement