Competing orders, non-linear sigma models and topological terms in quantum magnets

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TS, MPAF, cond-mat/0510459

TS, Shankar (forthcoming)

Some remarks on topological quantum matter
(or how the actual talk fits in with this program)

Competing orders, non-linear sigma models
and topological terms in quantum magnets

General questions on topological phases of matter

1. Do topological phases exist?

2. Are they common?

3. How do we detect and manipulate them?

Do topological phases exist?

Yes! !

Crowning example – quantum Hall effects

Require special circumstances (d =2, strong B-fields, etc)

Long standing important question in solid state physics:

Are there others?

Theoretical answer: Yes! ! (Work of many people over 15 years)

Topological phases can occur in any dimension and without external B-fields.

(Perhaps a non-quantum Hall realization will eventually be more practical for quantum computation.)

Experimental answer (to date): No!!

Are topological phases common?

Conventional solid state folklore: No!!

Probably right but we don't really know.

Good experimental probes to reveal such phenomena often do not exist (possibly completely new experimental toolbox).

Ferromagnetism (relatively rare)– known for centuries
Antiferromagnetism (much more common) – known only for < 70 years
Had to await development of new probes like neutron scattering

Optimistic view: Perhaps topological phases are very common but we haven't found out yet.

How to detect/manipulate topological phases?

Poorly understood - many new ideas needed in specific experimental contexts.

Some old ideas - use proximity to possible conventional broken symmetry state (TS, MPAF 2000, 2001)

Eg:

Z_2 insulator $\quad g_c \quad$ Superconductor $\longrightarrow g$

Z_2 vortex (vison) $\longrightarrow$ $hc/2e$ vortex as g goes thru' g_c

1. Use SC vortex to create & then detect visons
2. Fractional Josephson oscillations to detect fractional charge

S | I | S

topological insulator

General lesson from topological phases: Emergent non-locality in extended quantum systems

Ground states with excitations that have long range interactions absent in microscopic Hamiltonian

Mathematical description: gauge theory

(In quantum Hall example) excitations with infinitely long ranged statistical interactions

Information encoded in global properties of ground state wavefunction.

(Underlies suggestion to exploit for quantum computation)

Beyond topological phases

Quantum Hallesque topological phases just the tip of the iceberg.

Modern theoretical quantum condensed matter physics:

Non-trivial quantum ground states with similar emergent non-locality exist.

?? Possibility of new "quantum" technologies??

Applications other than quantum computing?

Example 1: Artificial electrodynamics

Quantum (spin) liquids in three dimension with a gapless ``photon" excitation

[Wen '02
Motrunich & TS '02
Hermele, Fisher, Balents '03
Moessner, Sondhi '03
Lee, Lee, '05]

Many different model realizations

Gaplessness of artificial photon is robust – protected against all small perturbations

Artificial electrodynamics (cont'd)

Photon coupled minimally to `artificial' gauge charges

Gauge charges possibly fermionic and gapless
$\Rightarrow$ artificial metal.
Specific microscopic models with the
artificial metal phase exist in d = 3. (TS, Vojta, Sachdev, '04)

??Applications??
??``Electrical" circuits with wires in artificial metal phase??

Example 2: Gapless `deconfined' critical phases/points in d = 2

Deconfined critical phases: (Hermele et al., '04)

Low energy theory - gapless emergent fermions coupled to fluctuating gapless emergent gauge field.

Stable scale invariant quantum phases with no relevant perturbations.

Deconfined critical points: (TS, Vishwanath, Balents, Sachdev, Fisher, '04)

Low energy theory – gapless emergent bosons coupled to fluctuating gapless emergent gauge field.

Scale invariant theory with one relevant perturbation

(often describing a Landau-forbidden quantum phase transition)

Beyond topological phases (cont'd)

- Deconfined criticality not as well understood as topological phases but also has some non-local structure that is apparently captured by the gauge theory.

- Non-locality useful (in the distant future) for some interesting application?

- This talk – focus on some theoretical aspects of these

interesting phenomena.

Insulating quantum antiferromagnets

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + \cdots\cdots$$

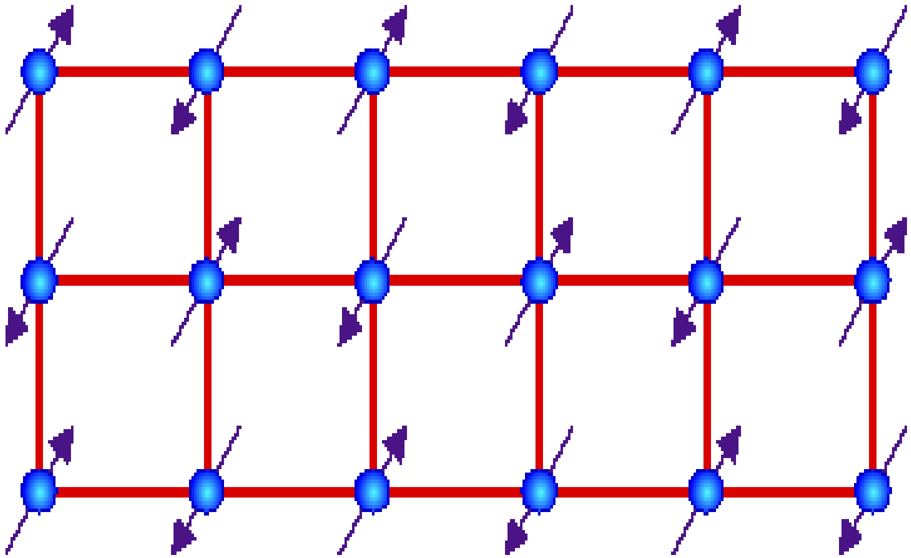
Useful theoretical laboratory to address many central issues in strong correlation physics

(competing orders, quantum criticality, fractional quantum numbers,……..)

Many new materials realizing variety of different spin models on different lattices

Some possible quantum phases

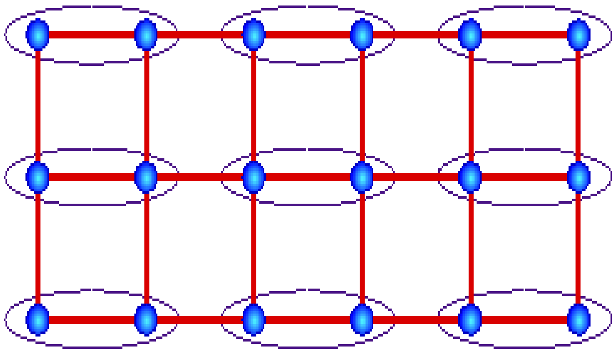
- Neel ordered state



Possible quantum phases (contd)

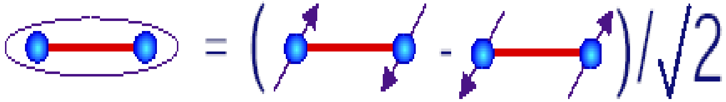
<u>QUANTUM PARAMAGNETS</u>

- Simplest: Valence bond solids.



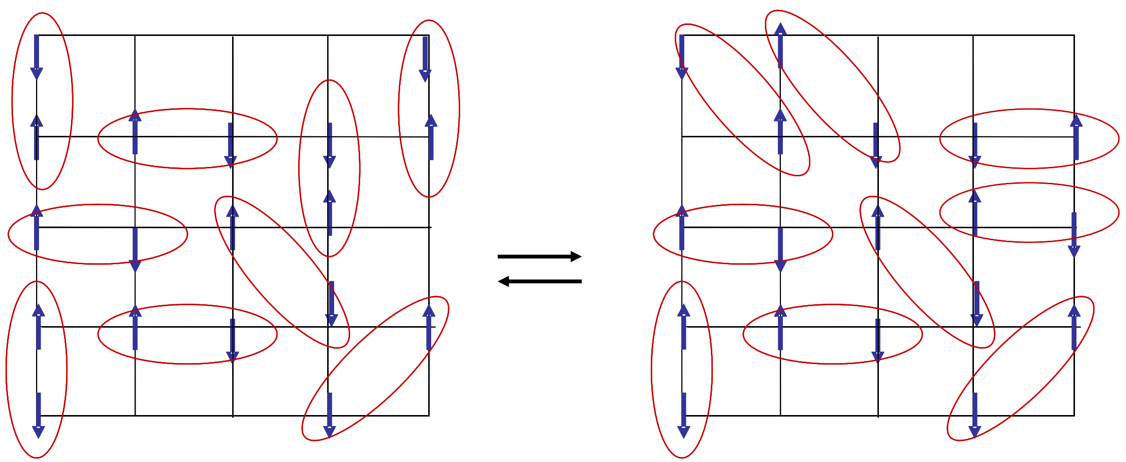
- Ordered pattern of valence bonds breaks lattice translation symmetry.

- Elementary spinful excitations have S = 1 above spin gap.



Possible phases (contd)

- Exotic quantum paramagnets – ``resonating valence bond liquids''.
- Fractional spin excitations, interesting topological structure.



Some recent theoretical advances

1. Landau-forbidden second order quantum phase transitions between 2 phases with different broken symmetry

Eg: Neel-VBS on 2d square lattice.

Critical field theory: Gapless bosonic spinons coupled to fluctuating gauge fields

Slow power law for both Neel and VBS order parameters

``Deconfined quantum critical point”

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M. Fisher, Science 2004

T. Senthil, L. Balents, S. Sachdev, A. Vishwanath and M. Fisher, PR B 2004

Recent theoretical advances (cont'd)

2. Stable critical `spin liquid' phases in 2d

(at least within large-N expansions)

Eg: dRVB/staggered flux phases of spin-1/2 magnets

Low energy theory: gapless fermionic Dirac spinons coupled to gapless gauge field

Slow power law for many competing orders

``Deconfined critical phase" or ``Algebraic spin liquid"

M. Hermele, T. Senthil, M. Fisher, P.A. Lee, N. Nagaosa, X.-G. Wen, PR B 2004

Comment

At deconfined critical points/phases,
spinon-gauge descriptions useful.

But neither spinons nor photon is a good quasiparticle.

Quite possibly no quasiparticle description even exists!

Questions

1. Is a spinon-gauge description necessary at deconfined critical points/phases?

2. Is there any description directly in terms of slow competing order parameters?

(How does Landau-Ginzburg-Wilson theory kill itself?)

3. Why bother with these questions?

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Mostly for improved theoretical understanding at this point.

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This talk - suggest some interesting answers by studying many related questions.

Lessons from d = 1 spin-1/2 chains

(classic example of a gapless algebraic spin liquid)

- Power law phase
- Spinon description of spectrum
- Slow decay of both Neel and VBS correlations with SAME exponents
- Many similarities to 2d deconfined criticality – only much better understood
- Many different equivalent field theoretic descriptions

Field theories of spin-1/2 chains-I
O(3) nonlinear sigma model with topological term

Haldane '83

$$S = \int dx\, d\tau\, \frac{1}{2g} \left(\partial_\mu \hat{n}\right)^2 + i\pi Q$$

$$Q = \frac{1}{4\pi} \int dx\, d\tau\, \hat{n} \cdot \partial_x \hat{n} \times \partial_\tau \hat{n} = \text{"winding \#"}$$

for $S^2 \to S^2$

$\hat{n} \sim$ Neel order parameter

$\hat{n} \cdot \partial_x \hat{n} \times \partial_\tau \hat{n} \sim$ VBS order parameter

Unequal treatment of Neel & VBS

Field theories of spin-1/2 chains –II
$SU(2)_1$ Wess-Zumino-Witten theory

Affleck, Haldane '86

$$S = \int d^2x \, \frac{1}{2g} \, \mathrm{tr}\left(\partial_i U^\dagger \partial_i U\right) + i\Gamma$$

$U \in SU(2)$ defines map $S^2 \to S^3$

$$\Gamma = \text{WZW term} = \frac{1}{2\pi} \left(\frac{\text{volume in } S^3 \text{ bounded by surface traced by } U}{\text{total volume of } S^3} \right).$$

S invariant under $SU_L(2) \times SU_R(2) \simeq SO(4)$

SU(2) WZW Theory (cont'd)

Write $U = \phi_0 + i\vec{\phi}\cdot\vec{\sigma}$

SO(4) rotates 4-vector $(\phi_0, \vec{\phi})$

$\vec{\phi}$ ~ Neel vector, ϕ_0 ~ VBS order parameter

WZW theory – 'superspin' σ model for slow competing orders

but involves topological term

Field theory of spin-1/2 chains-III
QED2

Hosotani, Mudry & Fradkin, Kim & Lee

Fermionic slave spinon representation

$$\vec{S}_i = \tfrac{1}{2} f_i^\dagger \vec{\sigma} f_i \quad \text{with} \quad f_i^\dagger f_i = 1 \ \forall i$$

Mean field - tight binding spinons at ½-filling

Fluctuations - massless 2-component QED_2 ("Schwinger model")

Bosonize $\Leftrightarrow$ $(SU(2))_1$ WZW

Slave spinons/gauge field not part of physical spectrum!

Lessons

(1) "Superspin" field theory in terms of slow competing orders possible but requires topological term.

(2) Slave spinon + gauge fields a possible but not necessary description.

Slave spinons / gauge fields not part of physical spectrum

True spinon of spin chain $\neq$ slave spinon

Outline

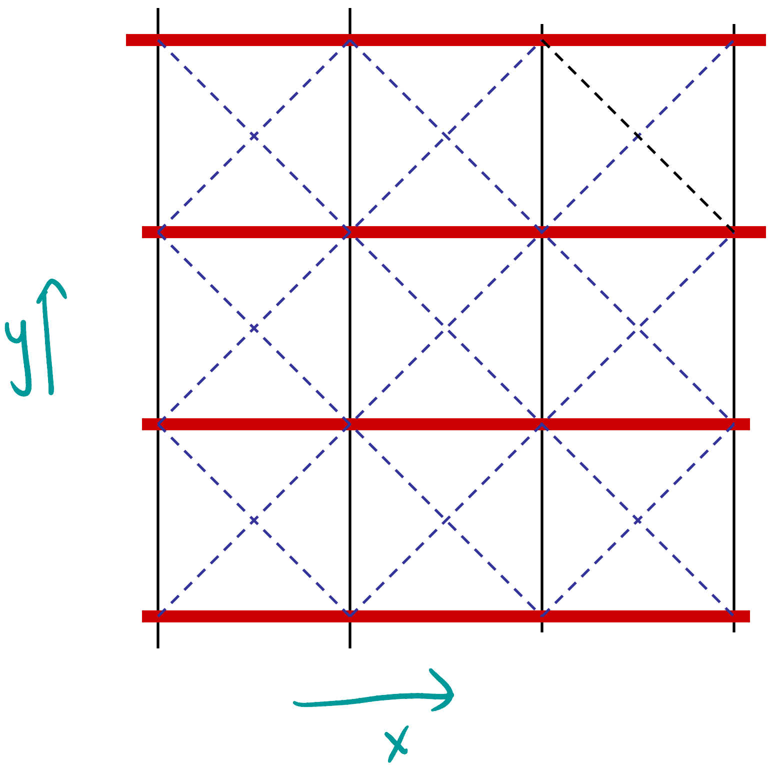
1. Weakly coupled 1d spin chains (two dimensions with rectangular symmetry)

2. Deconfined criticality in 2d square lattice

3. Massless 2-component QED_3

4. Implications/suggestions for other problems

Weakly coupled spin chains (2d with rectangular symmetry)



Weakly frustrated interchain coupling: Neel at (π, π)

Strong frustration:

Often VBS at $(\pi, 0)$

(Starykh, Balents)

Neel-VBS competition - "conventional" picture: Haldane, Read-Sachdev

2+1D O(3) σ-model with Berry phases for hedgehogs

Berry phases ⇒ VBS order in hedgehog proliferated paramagnet

Hedgehogs doubled after coarse graining; Generic 1st order transition?

NOTE - Unequal treatment of Neel & VBS orders.

Weakly coupled spin chains
``Superspin" sigma model description

Start with decoupled chains

$$S = \sum_y \underbrace{S[U(x,\tau,y)]}_{(SU(2))_1 \text{ WZW theory}}$$

Include interchain couplings for slow modes

$$S_{int} = \int dx\, d\tau \sum_y \Big[u\, \vec{\phi}(x\,\tau\,y) \cdot \vec{\phi}(x\,\tau\,y+1) - v\, \phi_0(x\,\tau\,y)\, \phi_0(x\,\tau\,y+1) \Big]$$

AntiFerro coupling between Neel

Ferro " " VBS

Superspin description (cont'd)

$u = v$: Extra global $SO(4)$ symmetry

Perturb around this limit

At $u = v$ write $S_{int} = -\frac{u}{2} \int_{x,\tau} \sum_{y} \mathrm{tr}\left(U(y+1) U(y) \right)$

Continuum limit : write

$$U(y) = g(y) \qquad y = 2n$$

$$= g^{+}(y) \qquad y = 2n+1$$

Take continuum limit with g varying slowly in both directions

Superspin description (cont'd)

$$S \simeq \frac{1}{t} \int d^2x \, d\tau \, \mathrm{tr}\left(\partial_i g^\dagger \partial_i g\right) + \sum_y (-1)^y \, i \, \Gamma\left[g(y)\right]$$

Alternating sum

$\sim \frac{1}{2}$ (volume of S^3 swept out by $g(x\,y\,\tau)$)

(analagous to old Haldane calculation for spin chains)

$$\Rightarrow \quad \frac{i}{24\pi} \int dx\, dy\, d\tau \, \epsilon_{ijk} \, \mathrm{tr}\left(g^{-1}\partial_i g \, g^{-1}\partial_j g \, g^{-1}\partial_k g\right)$$

Anisotropic O(4) model with topological term

Full action for $g = \phi_0 + i\vec{\phi}\cdot\vec{\sigma}$

$$S = \int d^2x\, d\tau \, \frac{1}{2t} \left(\partial_i \hat{\phi} \right)^2 + i\pi Q + \text{anisotropy}^*$$

Q = "winding #" for $\pi_3(S^3) = \mathbb{Z}$

$\phi_0 \sim (\pi, 0)$ VBS order parameter

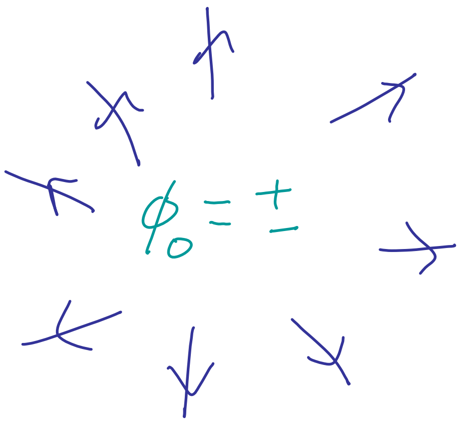
$\vec{\phi} \sim (\pi, \pi)$ Neel "

$\Rightarrow$ Required "superspin" σ-model but with topological term

*Anisotropy breaks $SO(4)$ to $SO(3) \times \mathbb{Z}_2$

Effective O(3) model

Defects of O(3) field $\vec{\phi}$ ~ hedgehogs.



$\hat{\phi}$ along ϕ_0 in core

2 kinds of hedgehogs with
$\mathrm{sgn}(\phi_0) = \pm$ in core

"meron-hedgehogs"

Each meron hedgehog ~ ½ (point defect of O(4) model with Q = 1)

$\theta = \pi \Rightarrow$ Phase $e^{\pm i\pi/2}$ for each meron-hedgehog

Coulomb gas formulation

Neel state $\langle \vec{\varphi} \rangle \neq 0$: hedgehogs unimportant

Paramagnetic state : Coulomb gas of hedgehogs

$$S = u \sum_r \left(m_{+r}^2 + m_{-r}^2 \right) + i\pi/2 \sum_r \left(m_{+r} - m_{-r} \right)$$
$$+ \sum_{rr'} m_r V(r-r') m_{r'}$$

$m_\pm$ = integer hedgehog #

$m_r = m_{+r} + m_{-r}$

$V(r-r') \sim \frac{1}{|r-r'|}$; $u \sim$ hedgehog core energy

Sine Gordon theory

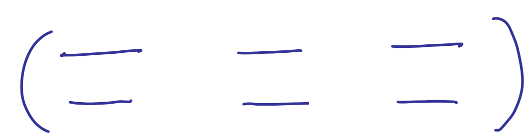
Decouple interaction with potential χ and sum over $m_\pm$

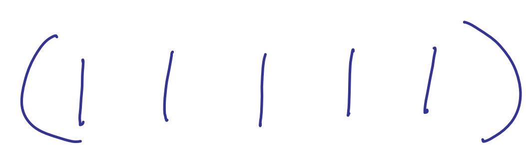
$$S = \int d^3x \; K\left(\nabla \chi\right)^2 - \sum_n \lambda_n \cos(2n\chi)$$

$e^{i\chi} \sim$ strength-1 hedgehog

$\Rightarrow$ Doubled hedgehogs proliferate

$\Rightarrow$ 2-fold degenerate ground state

[$\lambda_1 \ll 0$: $(\pi, 0)$ VBS state 

$\lambda_1 \gg 0$: $(0, \pi)$ " ]

Subconclusion

Anisotropic O(4) model at $\theta = \pi$

correctly describes $S = 1/2$ quantum

magnets on rectangular lattice!

"Superspin" theory requires including

topological term!

Neel & VBS treated on equal footing!

Deconfined criticality in 2d square lattice

2^{nd} order "Landau-forbidden" transition between (π,π) Neel and columnar/plaquette VBS with 4-fold degenerate ground state

"Conventional" descriptions:

(TS, AV, LB, SS, MPAF, 2004)

Attack from Neel: $O(3)$ σ-model with Berry phases for hedgehogs

Attack from VBS: XY model with spinons as vortices (Levin, TS 2005)

Critical theory – "non-compact CP^1" (NCCP^1)

$$S = \int d^2x\, d\tau \; |(\partial_\mu - i a_\mu) z|^2 + r|z|^2 + u|z|^4 + (\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2$$

$z = (z_\uparrow, z_\downarrow)$, $a \sim$ non-compact $U(1)$ gauge field

Easy plane magnets – "Motrunich-Vishwanath self-duality" (Motrunich-Vishwanath 2004)

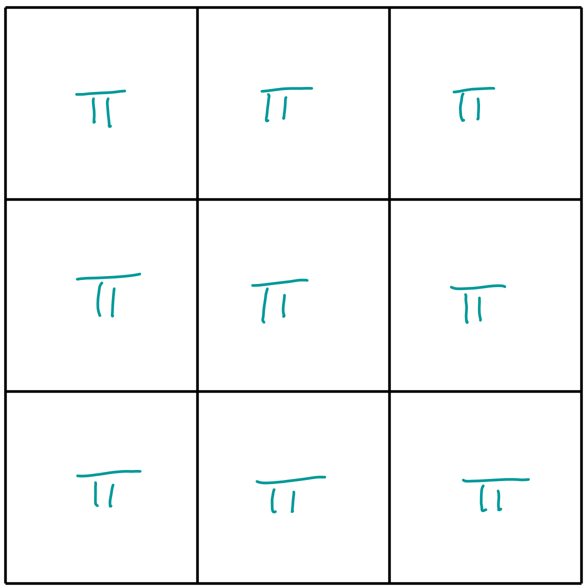
NCCP^1 with easy plane has self-dual 2^{nd} order critical point.

``Superspin" sigma model

(adapted from Tanaka-Hu PRL 05)

Start with Hubbard model on square lattice with π-flux

$$H = -\sum_{\langle ij \rangle} t_{ij} \left(c_i^{\dagger} c_j + h.c. \right) + U \sum_i \frac{n_i (n_i - 1)}{2} + \ldots$$



Band structure:
4 Fermi points at $(\pm \pi/2, \pm \pi/2)$

Low energy theory: 2-component massless Dirac theory

Weak-U: irrelevant ⇒ stable semi-metal

Strong-U: Insulator, typically with Neel / VBS order

Theory of ordered insulators

Neel order parameter $N^a \sim \bar{\psi}\Gamma^a\psi$, $a = 1, 2, 3$

VBS " $V_x \sim \bar{\psi}\Gamma^4\psi$

$V_y \sim \bar{\psi}\Gamma^5\psi$

$\{\vec{\Gamma}\}$: 5 4×4 matrices satisfying $\{\Gamma_i, \Gamma_j\} = 2\delta_{ij}$

$$\Gamma_5 = -\Gamma_1\Gamma_2\Gamma_3\Gamma_4$$

Hartree-Fock : $S = S_{\text{Dirac}} + im\int d^3x\, \bar{\psi}\, \hat{\phi}\cdot\vec{\Gamma}\,\psi$

mean field

Beyond mean field : $\hat{\phi}$ fluctuating

Integrate out fermions in insulator

$$S_{eff}[\hat{\phi}] = \operatorname{tr}\ln(-i\cancel{\partial} + im\hat{\phi}\cdot\vec{\Gamma})$$

$$\underset{\text{large } m}{\approx} \int d^3x \, \frac{1}{2g}(\partial_i\hat{\phi})^2 - 2\pi i \Gamma[\hat{\phi}]$$

(Abanov, Wiegmann 2000)

$$\Gamma[\hat{\phi}] = \text{WZW term for } \hat{\phi} : S^3 \to S^4$$

$$= \frac{\text{Volume of } S^4 \text{ bounded by hypersurface traced by } \hat{\phi}}{\text{total volume of } S^4}$$

Comments

① Required superspin σ-model

$$\hat{\phi} = (\underbrace{N_z, N_x, N_y}_{\text{Neel}}, \underbrace{V_x, V_y}_{\text{VBS}})$$

② Must include anisotropy to break $SO(5) \to SO(3) \times U(1)$

③ First derivation: Tanaka, Hu PRL '05

from slightly different point of view.

Equivalence to gauge theory of deconfined critical point

Easy plane anisotropy for Neel

$$\hat{\phi} \approx (0, \hat{\pi}) = (0, \vec{N}_\perp, v_x, v_y)$$

$S_{WZW} = -i\pi Q$, Q = winding # for $\pi_3(S^3) = Z$

$\Rightarrow S_{eff}[\hat{\pi}] =$ $O(4)$ σ-model at $\theta = \pi$

now with $O(2) \times O(2)$ anisotropy

Practice with Q

Q invariant under smooth deformations of $\hat{\pi}$

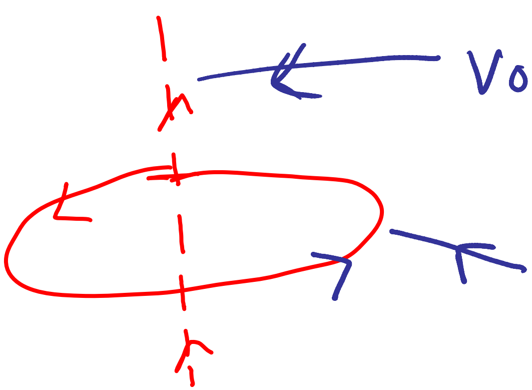
$$\hat{\pi} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \; ; \; z_{1,2} \text{ complex}, \; |z_1|^2 + |z_2|^2 = 1$$

Q = 0 unless there are vortices in both z_1 and z_2.

Configuration with non-zero Q.

$$\hat{\pi} = (\cos\alpha(r), \sin\alpha(r)\,\hat{e}_r), \quad \alpha(0) = 0, \; \alpha(\infty) = \pi$$

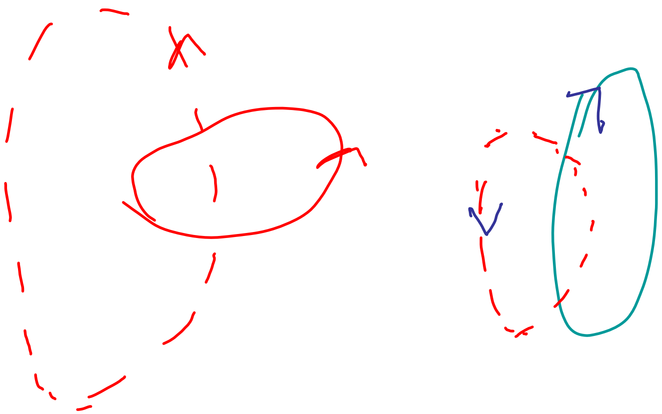
$$(Q = 1)$$



Vortex line in z_2

vortex line in z_1

Mutual non-locality of vortices



In general

Q = linking # between oriented vortex loops in z_1 and z_2

$\Rightarrow$ For $\theta = \pi$ model, z_1-vortex picks up phase π on encircling z_2-vortex & vice versa.

Non-trivial mutual statistics for vortices

Lattice action for vortices

$e^{i\phi_{1,2}}$: vortices in $z_{1,2}$

$$S = -t \sum \sigma_{ij} \cos(\vec{\nabla}\phi_1 - \vec{a}_1) + \mu_{ij} \cos(\vec{\nabla}\phi_2 - \vec{a}_2)$$

$$+ K \sum (\nabla \times a_1)^2 + (\nabla \times a_2)^2 + \underbrace{\frac{i\pi}{4} \sum (1-\mu_{ij})(1-\prod_{ij} \sigma)}_{\text{Ising mutual Chern-Simons term}}$$

Standard duality on one vortex species

$\rightarrow$ easy plane NCCP^1 action

Comments

① σ-model has $O(2) \times O(2) \times Z_2$ symmetry

Z_2 = interchange of two $O(2)$'s

NCCP^1 : One $O(2)$ – global charge conservation

Another $O(2)$ – "topological", gauge flux conservation

Z_2 – "Motrunich-Vishwanath" self-duality

Self-duality realized as ordinary symmetry in σ-model

Comments

(2) 'Superspin' description in terms of competing orders also possible at deconfined critical point but again needs inclusion of topological terms.

(3) Possible (useful?) alternate to gauge theory.

Massless QED3

$$S = \int d^3x \; \bar{\psi}\left(-i\not{\partial} - \not{a}\right)\psi + \frac{1}{2e^2}\left(\vec{\nabla}\times a\right)^2$$

a : non-compact $U(1)$ gauge field

ψ : N 2-component Dirac fermion

$N = 4$: Low energy theory of dRVB spin liquid

N large enough : Stable "critical" phase
(algebraic spin liquid)

Slow power law for gauge-invariant fermion bilinears

N = 2 version

ψ : spinors under global SU(2)

Expect $\bar{\psi}\vec{\sigma}\psi$ to have slow correlations

$\therefore$ Study $S = \int d^3x \; \bar{\psi}(-i\cancel{\partial} - \cancel{a})\psi + (\nabla \times a)^2$

$+ i m \, \hat{n} \cdot \bar{\psi}\vec{\sigma}\psi$

with integration over $(\psi, a, \hat{n})$

Large m limit: integrate out fermions

Fermion det $= e^{-S}$

Abanov, Wiegmann, 2000

$$S \simeq \int d^3x \; \frac{1}{g} (\partial_i \hat{n})^2 + i \vec{J} \cdot \vec{a} - i\pi H[\hat{n}]$$

$$J_i = \frac{1}{8\pi} \epsilon_{ijk} \hat{n} \cdot \partial_j \hat{n} \times \partial_k \hat{n}$$ = skyrmion current density

$H[\hat{n}]$ = "Hopf invariant" = integer classifying $\pi_3(S^2) = Z$

$$= \frac{1}{24\pi^2} \int d^3x \; \epsilon_{ijk} \mathrm{tr}\left(U^{-1} \partial_i U U^{-1} \partial_j U U^{-1} \partial_k U\right)$$

$$U = \begin{bmatrix} z_1 & -z_2^* \\ z_2 & z_1^* \end{bmatrix}, \quad \hat{n} = z^\dagger \vec{\sigma} z$$

Integrate out gauge fluctuations

CP^1 representation:

$$S = \int d^3x \ \frac{1}{g} |(\partial - iA)z|^2 + \frac{i}{2\pi} \nabla \times A \cdot a + (\nabla \times a)^2$$

$$- i\pi H$$

Integrate out a:

$$S = \int d^3x \ \frac{1}{g} |(\partial - iA)z|^2 + \frac{e^2}{8\pi^2} |A|^2 - i\pi H$$

A massive $\approx$ so drop

$$\Rightarrow S \approx \int d^3x \ 1/g \ |\partial z|^2 - i\pi H$$

Global O(4) model

Now H exactly same as winding # Q of $\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$ (for $\pi_3(S^3) = \mathbb{Z}$)

$\Rightarrow$ $S_{eff} = O(4)$ at $\theta = \pi$.

$\Rightarrow$ $N=2$ massless $QED_3 \Longleftrightarrow O(4)$ at $\theta = \pi$

Comments

① O(4) broken phase - 3 Goldstone modes

In QED_3 $\langle \hat{n} \rangle \neq 0$: 2 spin waves

+ 1 gapless photon

② Recent conjecture (Alicea et al 2005)

$N=2$ QED_3 $\Longleftrightarrow$ usual critical O(4) model

Derivation here - partial support but reveals

extra topological term for O(4) model.

Summary

- Sigma model descriptions in terms of slow competing order parameters possible in 2d quantum magnets provided topological terms are included.

- Possible alternate to gauge theory descriptions

(Interesting? Useful?)

- O(4) model with theta term – interesting phase structure/transitions

Questions

- Similar sigma models for fermionic algebraic spin liquid phases?

- Isotropic O(4) model with theta term in D = 3: nature of phase transitions?

- Topological terms in superspin descriptions of other competing orders (eg: SO(5) theory of SC/AF in half-filled extended Hubbard models)?
- Similar considerations in d = 3: Interesting conjecture on

  low energy physics of strongly coupled SU(2) gauge theory with theta term based on current understanding of 3d quantum magnets

(TS, Shankar)