

Quantum phase transitions out of the heavy fermi liquid

T. Senthil (MIT)

S. Sachdev (Yale)

M. Vojta (Karlsruhe)

Collaborators on related issues: Levin, Motrunich,
Vishwanath, Balents, Fisher.

Luttinger's theorem for Fermi liquids

In a **Fermi liquid**, volume V_F of Fermi surface is set by **electron density n independent** of interaction strength.

$$V_F = (2\pi)^d n/2 \quad (\text{mod Brillouin zone volume}).$$

Perturbative proof: Luttinger

Non-perturbative topological arguments:

Yamanaka, Oshikawa, Affleck ($d = 1$), Oshikawa ($d > 1$).

Oshikawa: Regard as ``**topological quantization**''.

Heavy fermion liquids

- Heavy fermion materials: Typically rare earth intermetallic compounds with **strongly correlated half-filled f-band** weakly hybridized with **conduction electron band**.
- Describe by **Anderson model**

$$H = \sum_k \varepsilon_k c_k^\dagger c_k + \sum_k (\varepsilon_k^f - \mu_f) f_k^\dagger f_k \\ + V \sum_r (c_r^\dagger f_r + f_r^\dagger c_r) + U \sum_r (f_r^\dagger f_r)^2$$

or in large - U limit by Kondo lattice model

$$H = \sum_k \varepsilon_k c_k^\dagger c_k + \frac{J_K}{2} \sum_r \vec{S}_r \cdot c_r^\dagger \vec{\sigma} c_r$$

Luttinger's theorem in Kondo lattices

- **Kondo lattice model** admits a **Fermi liquid** phase.
(Denote ``Kondo liquid’’).
- To satisfy Luttinger's theorem must **include local moments in the count of conduction electron density**
``Large'' Fermi surface
- Fermi liquid adiabatically connected to small U Anderson model.
- Understand through (i) **slave particle mean field calculation**
(Read et al, Millis et al)
(ii) Oshikawa topological argument.

Other known metallic states of Kondo lattice

- **Magnetically ordered** (typically antiferromagnetic) **metal** due to RKKY

Favored at small J_K (Doniach).

There are actually two distinct kinds of antiferromagnetic metals.

- Other **broken symmetry** states (SC,)

Two kinds of antiferromagnetic metals in Kondo lattices

(A): ``Local moment magnetic metal''

Ordering of local moments (due to intermoment exchange).

c-electrons \approx decoupled from local moments.

Typically large (staggered) moment.

``f-electrons do not participate in Fermi surface''.

(B): ``Spin density wave metal''

SDW instability of heavy fermi liquid with large fermi surface.

c- and f-electrons strongly coupled.

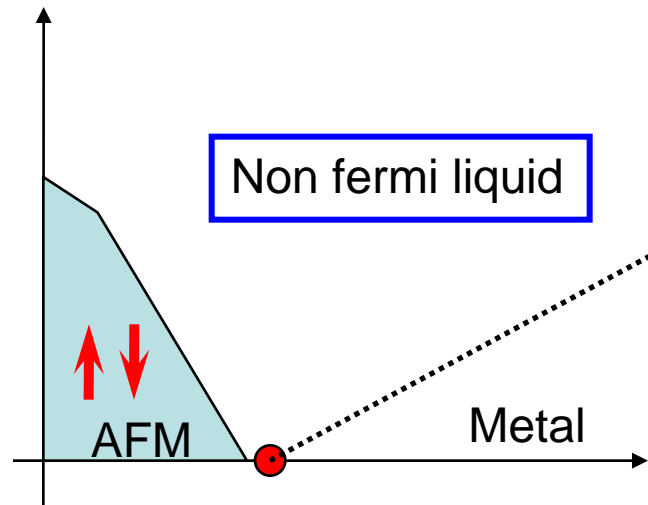
Typically weak moment.

``f-electrons participate in Fermi surface''.

- Two kinds of magnetic metals often sharply distinct!!
- Distinction in topology of Fermi surface –
Evolution from one to other occurs through a quantum phase transition where the Fermi surface topology changes
(Lifshitz transition)

Magnetic ordering in heavy electron systems

CePd₂Si₂, CeCu_{6-x}Au_x, YbRh₂Si₂,.....



Non fermi liquid: diverging γ coefficient, T dependence of resistivity, scaling of spin fluctuation spectrum with ω/T ,

“Classical” assumption

1. NFL: Universal physics associated with quantum critical point between heavy fermi liquid and magnetic metal
2. Landau: Universal critical singularities ~ fluctuations of natural magnetic order parameter for transition

Try to play Landau versus Landau.

Which magnetic metal?

1. Heavy fermi liquid – SDW metal:

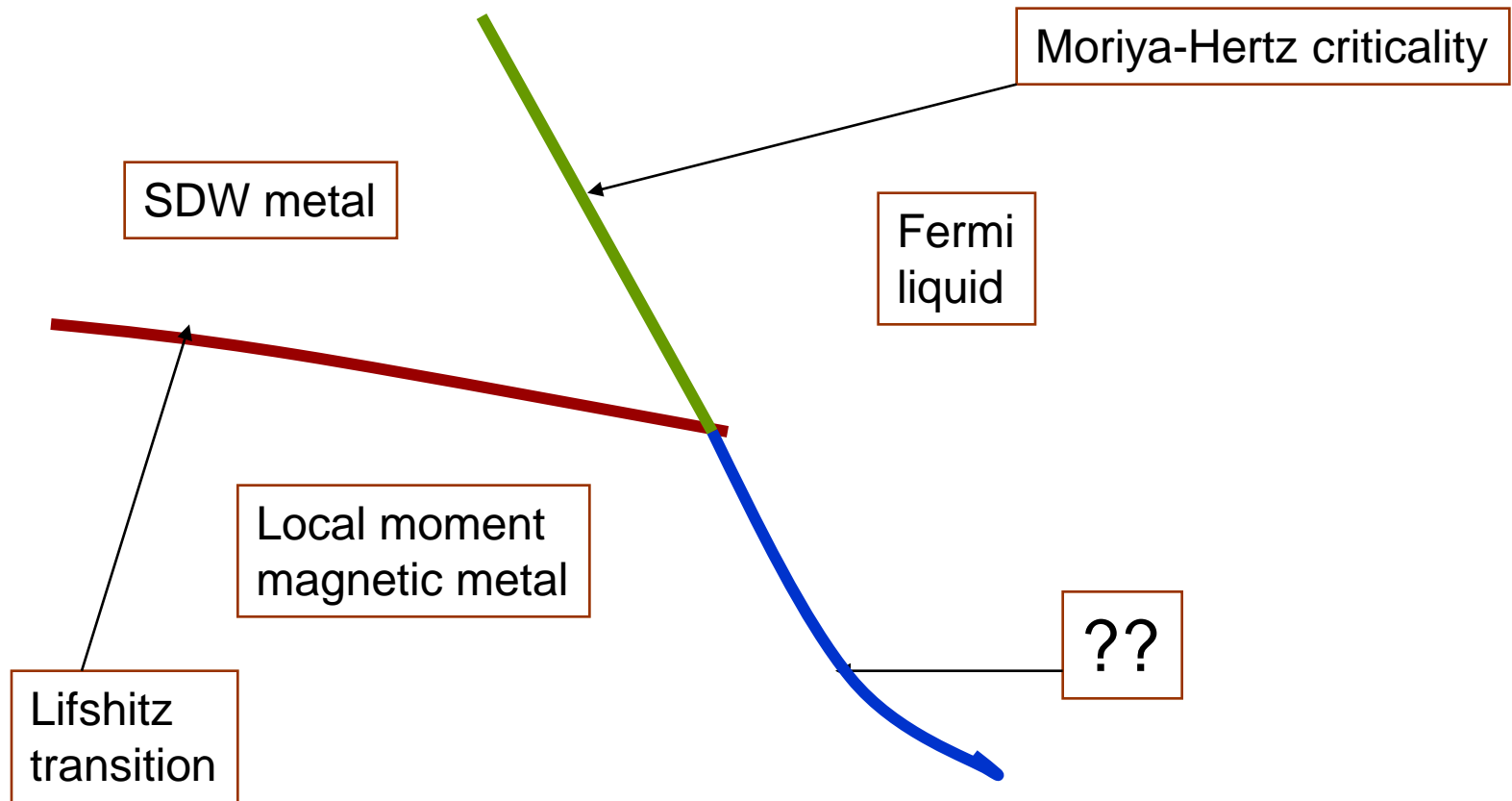
Fluctuations of magnetic order parameter with damping
due to fermionic quasiparticles (Moriya-Hertz-Millis)

- Fail to reproduce observed NFL physics.

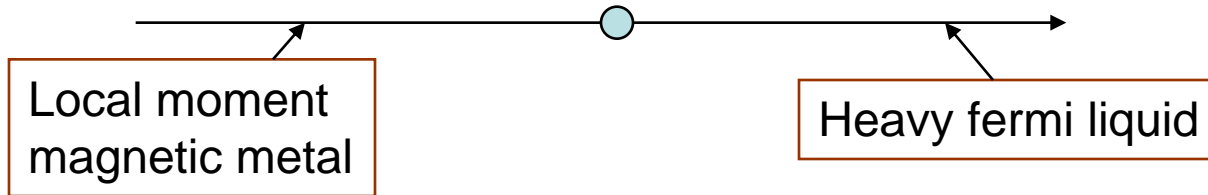
2. Explore alternate possibility:

Transition between heavy fermi liquid and local moment
magnetic metal.

Schematic phase structure



Questions



1. Is there a generic second order quantum phase transition between the two phases?
(Loss of magnetic order happens at same point as onset of ``Kondo'' order)
2. Theoretical description?
3. Will it reproduce observed non-fermi liquid behaviour?

Answers not known!!

This talk: describe some ideas I am pursuing with various collaborators.

Other ideas/points of view: Q. Si et al, ,
Coleman, Pepin,

General observations

- f-moments drop out of Fermi surface (\Leftrightarrow change of electronic structure)

Associated time scale t_e .

- Onset of magnetic order

Associated time scale t_m .

Both time scales diverge if there is a critical point.

General observations

- f-moments drop out of Fermi surface (\Leftrightarrow change of electronic structure)

Associated time scale t_e .

- Onset of magnetic order

Associated time scale t_m .

Both time scales diverge if there is a critical point.

Suggestion: t_m diverges faster than t_e .

(electronic structure change first, magnetic order comes later)

Separation between two competing orders as a function of scale (rather than tuning parameter) might make second order transition possible.

Some implications

- “Underlying” transition: **loss of participation of the f-electrons** in forming the heavy fermi liquid.
(View as a Mott “metal-insulator” transition of f-band).
- **Magnetic order**: “secondary” effect – a low energy complication once Kondo effect is suppressed.
- Non-fermi liquid due to fluctuations associated with change of electronic structure rather than those of magnetic order parameter.

⇒ PHYSICS BEYOND LANDAU-GINZBURG-WILSON PARADIGM FOR PHASE TRANSITIONS.

(Natural magnetic order parameter is a distraction).

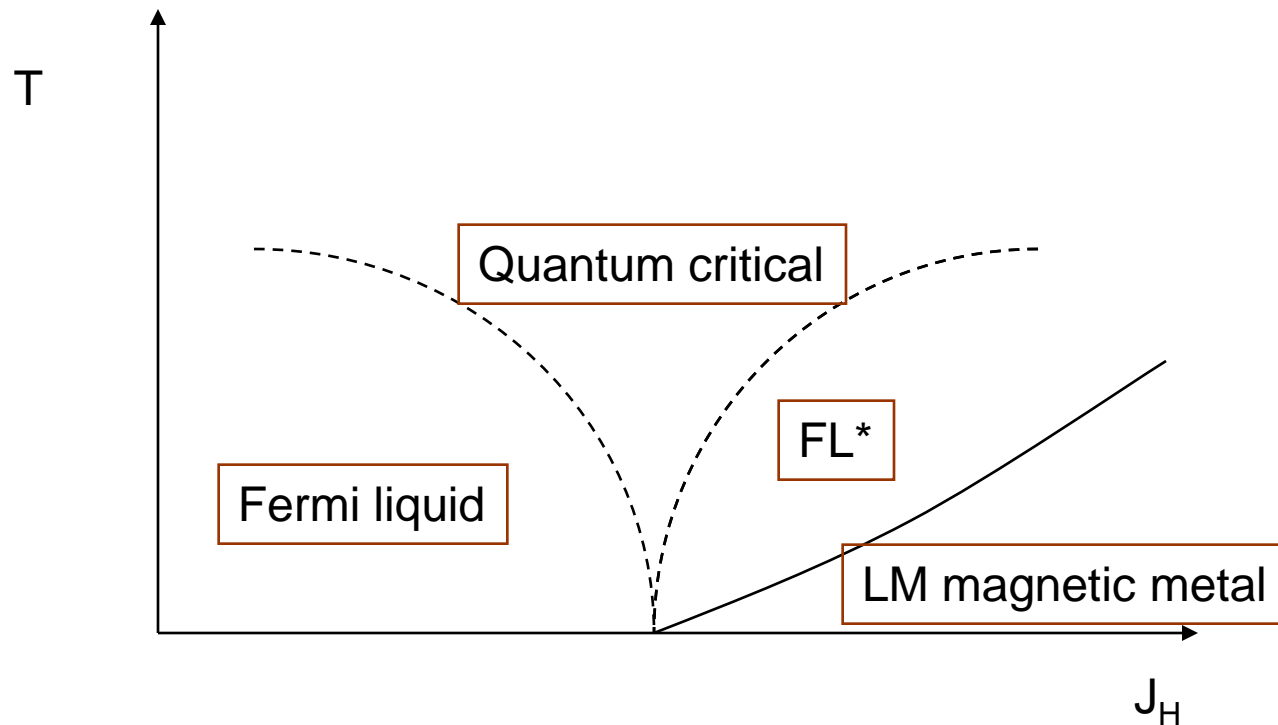
Intermediate time scale physics

- f-moments drop out of Fermi surface but continue to form singlet bonds with each other

Resulting state: spin liquid of f-moments coexisting with small Fermi surface of conduction electrons
(a ``fractionalized Fermi liquid’’)

Magnetism: low energy instability of such a small Fermi surface state.

Suggested phase diagram and crossovers



? HOW TO PUT MEAT INTO THIS PICTURE ?

Study tractable simpler questions

1. Effects of loss of ``Kondo'' order?

Study second order quantum transitions associated with loss of Kondo screening.

[TS, Vojta, Sachdev;

Related precursor: Burdin, Grempel, Georges]

Worry about magnetism later

2. Do similar theoretical phenomena (eg: breakdown of Landau paradigm) happen in other contexts?

Yes! (study quantum phase transitions in insulating magnets)

(TS, Vishwanath, Balents, Sachdev, M. Fisher)

Kondo Heisenberg models

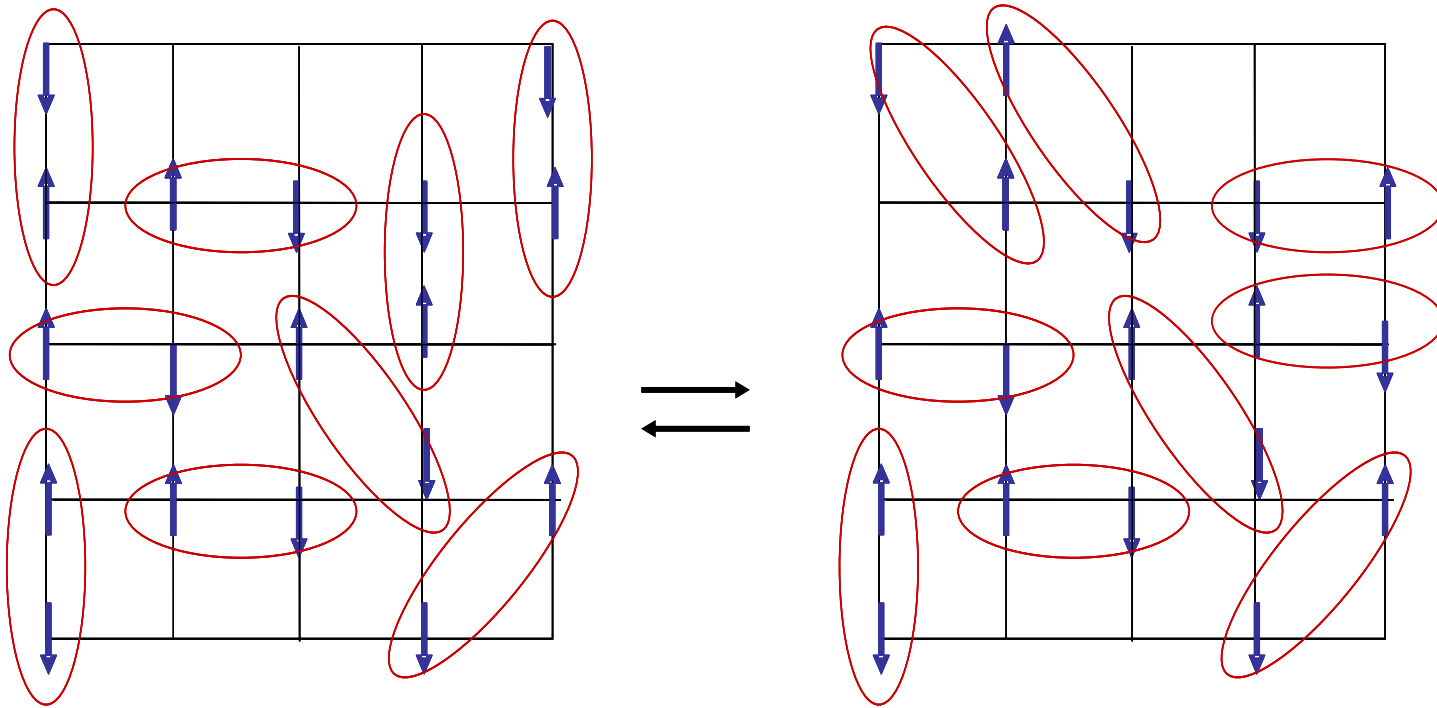
$$H = \sum_k \varepsilon_k c_k^\dagger c_k + \frac{J_K}{2} \sum_r \vec{S}_r \cdot c_r^\dagger \vec{\sigma} c_r + \sum_{rr'} J_H(r, r') \vec{S}_r \cdot \vec{S}_{r'}$$

- $J_K = 0$: Conduction electrons are decoupled from local moments and have small Fermi surface.
- Non-magnetic ground states of spin system
 - (i) **Spin –Peierls**: break translational symmetry
 - (ii) **Fractionalized**: can preserve translational symmetry

Focus on (ii) to discuss small Fermi surface state.

Fractionalization in $d > 1$

- Anderson: RVB spin liquid state for quantum spin models.



Couple spin liquids to conduction electrons

- Small non-zero J_K : Perturb in J_K
- Emergent gauge structure of local moment system survives; conduction electrons stay sharp on a small Fermi surface^{*}.
 - advertised small fermi surface state.
- A fermi liquid in peaceful coexistence with fractionalization
 “Fractionalized fermi liquid” (denote FL^{*})
- Large J_K : Recover large Fermi surface Kondo liquid.

(* Possible pairing instability at low T).

Physics of fractionalized fermi liquid (FL*) state

- Each local moment forms singlet with another local moment.
- Weak Kondo coupling can't break singlets:

Local moments and conduction electrons essentially stay decoupled.

Mean field theory

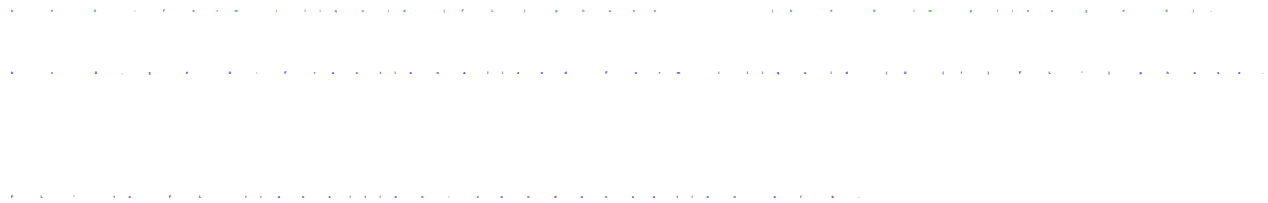
Kondo-Heisenberg model

$$H = -t \sum_{\langle rr' \rangle} c_r^+ c_{r'} - \mu \sum_r c_r^+ c_r + J_K \sum_r \vec{S}_r \cdot c_r^+ \frac{\vec{\sigma}}{2} c_r + J_H \sum_{\langle rr' \rangle} \vec{S}_r \cdot \vec{S}_{r'}$$

$$\vec{S}_r = f_r^+ \frac{\vec{\sigma}}{2} f_r \text{ with } f_r^+ f_r = 1.$$

Decouple Kondo with $b \sim c^\dagger f$; Heisenberg with $\chi \sim f_r^\dagger f_{r'}$

Treat b and χ in mean field.



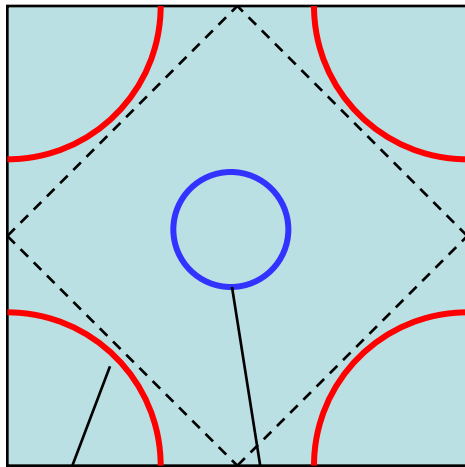
Direct fermi volume changing transition

- Condensation of hybridization amplitude b drives direct Fermi volume changing transition.
- Transition can be second order despite jump in fermi surface volume!
(Z goes to zero).
- Critical point is clearly a non fermi liquid.

Mean field fermi surface evolution

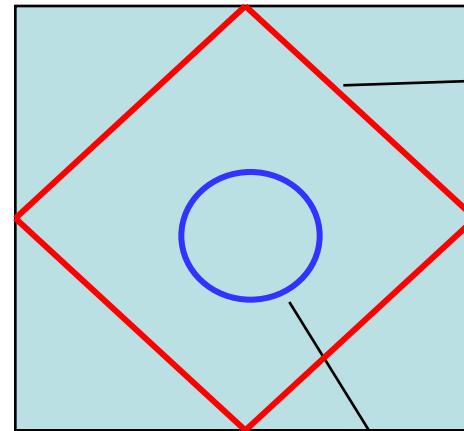
Fermi liquid

Fractionalized fermi liquid



``Hot''
Fermi
surface

``Cold''
Fermi
surface



Spinon
Fermi
surface

Electron
Fermi
surface

At transition $Z \sim b^2 \rightarrow 0$ on hot Fermi surface.

Fluctuations

FL*: spinons coupled to gapless U(1) gauge field⁺

- Near critical point: (slave) bosons and spinons coupled to a gapless U(1) gauge field

Transition driven by condensation of slave boson.

Non-fermi liquid critical point:

Eg: Specific heat $C \approx T \ln T$ ($d = 3$), $T^{2/3}$ ($d = 2$),
singular χ_{spin} along lines in k-space ($d = 2$),
conductivity $\approx \ln(1/T)$,.....

(Similar to gauge theories of optimally doped cuprates but bosons are at fixed chemical potential rather than fixed density).

+other possibilities such as a Z_2 gauge field also exist.

How to get magnetism?

- Gauge field can confine spinons leading to magnetic long range order (particularly in $d = 2$) – low energy instability of FL^* to local moment magnetic metal.
 - Interesting possibility: confinement effective only in FL^* phase but not at critical point
- ⇒ Direct second order transition between heavy Fermi liquid and local moment magnetic metal but with interesting 'deconfined' critical point described in terms of spinons and gauge fields.

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Admittedly speculative but.....

Evidence from a simpler context – insulating quantum magnets

- Highlights: Clear demonstration of such theoretical phenomena at (certain) quantum transitions
- Emergence of ‘fractional’ charge and gauge fields near quantum critical points between two CONVENTIONAL phases.
- “Deconfined quantum criticality” (made more precise later).
- Many lessons for competing order physics in correlated electron systems.

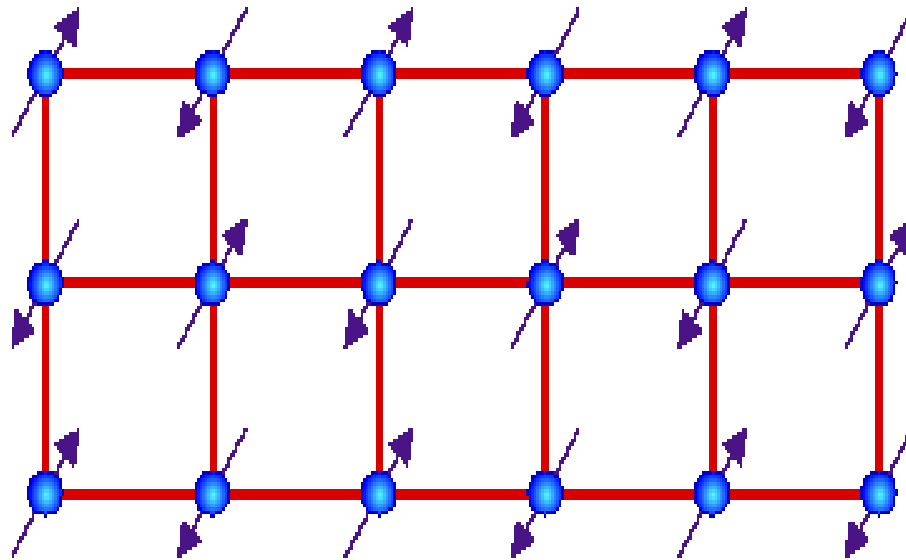
Phase transitions in quantum magnetism

$$H = J \sum_{\langle rr' \rangle} \vec{S}_r \cdot \vec{S}_{r'} + \dots\dots\dots$$

- Spin-1/2 quantum antiferromagnets on a square lattice.
 - “.....” represent frustrating interactions that can be tuned to drive phase transitions.
- (Eg: Next near neighbour exchange, ring exchange,.....).

Possible quantum phases

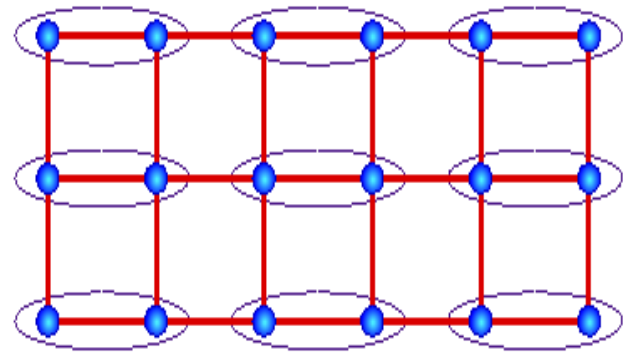
- Neel ordered state



Possible quantum phases (contd)

QUANTUM PARAMAGNETS

- Simplest: Valence bond solids.
- Ordered pattern of valence bonds breaks lattice translation symmetry.
- Elementary spinful excitations have $S = 1$ above spin gap.

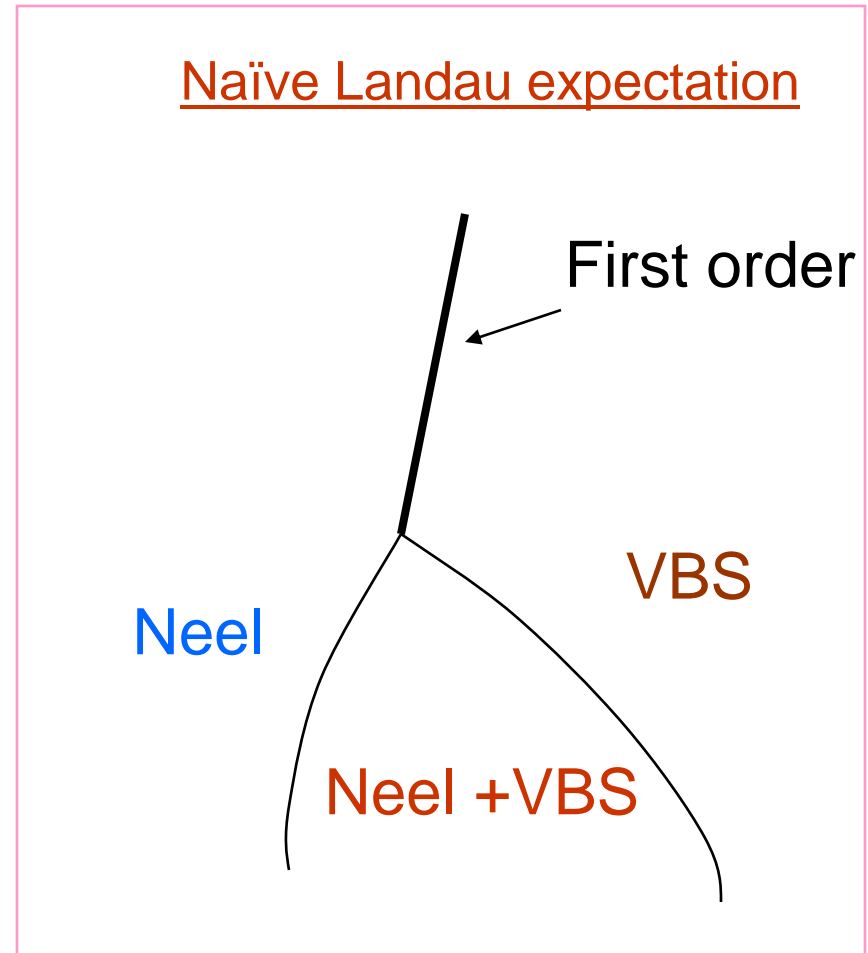


$$\text{Diagram of two atoms in a bond} = \left(\begin{array}{c} \nearrow \\ \bullet \end{array} - \begin{array}{c} \nwarrow \\ \bullet \end{array} \right) / \sqrt{2}$$

Neel-valence bond solid(VBS) transition

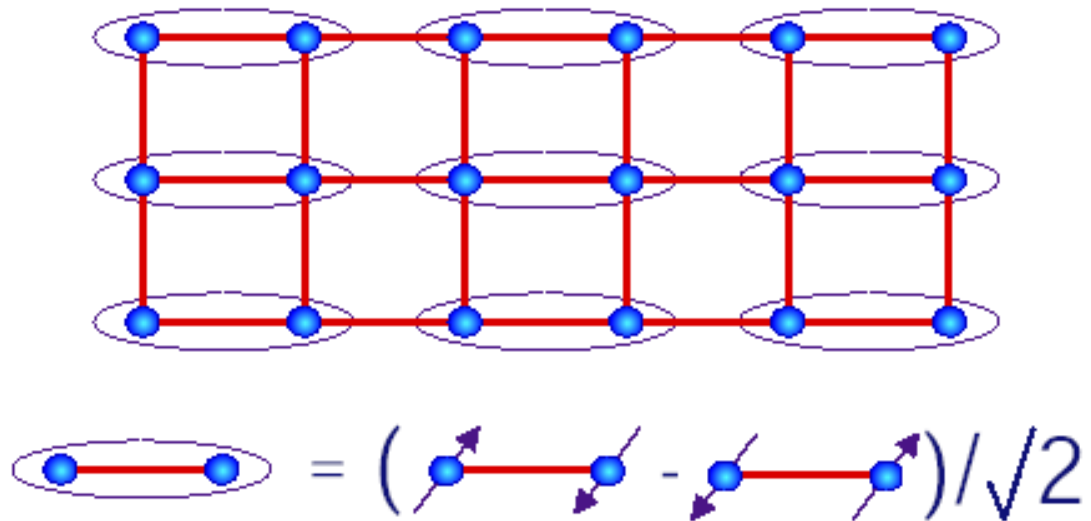
- Neel: Broken spin symmetry
- VBS: Broken lattice symmetry.
- Landau – Two independent order parameters.
 - no generic direct second order transition.
 - either first order or phase coexistence.

This talk: Direct second order transition but with description not in terms of natural order parameter fields.

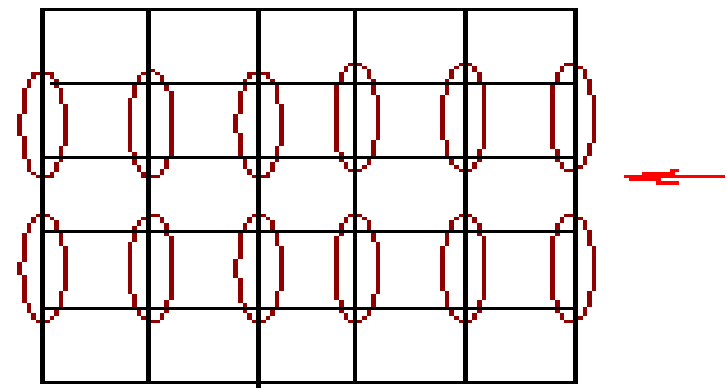
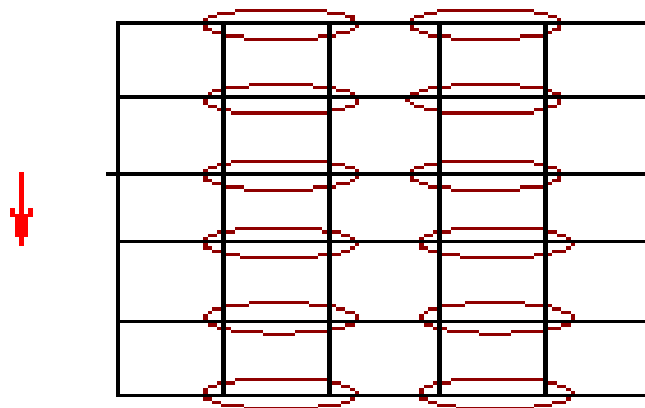
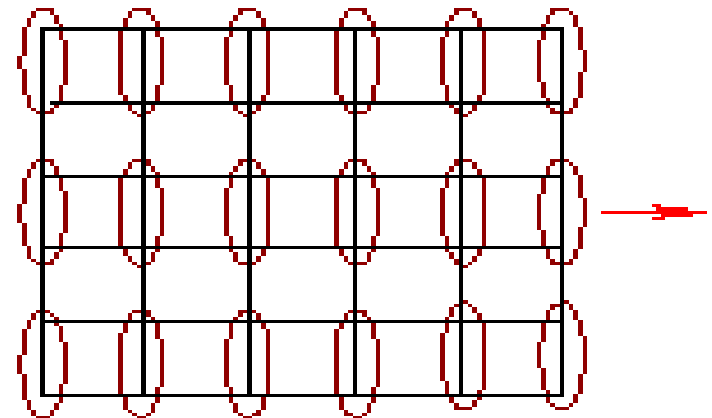
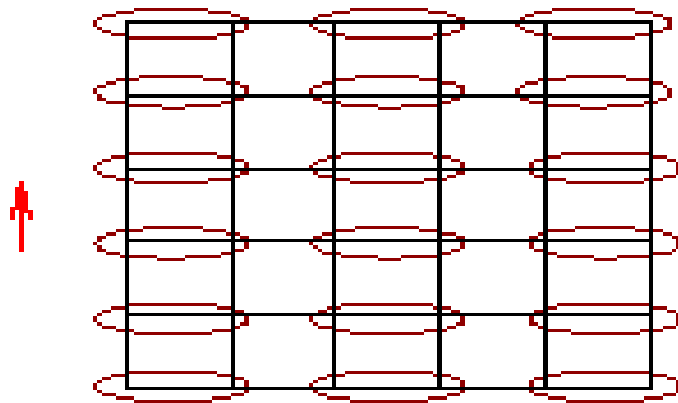


Broken symmetry in the valence bond solid(VBS) phase

Valence bond solid with spin gap.



Discrete Z_4 order parameter



Neel-Valence Bond Solid transition

- Naïve approaches fail

Attack from Neel \neq Usual $O(3)$ transition in $D = 3$

Attack from VBS \neq Usual Z_4 transition in $D = 3$

(= XY universality class).

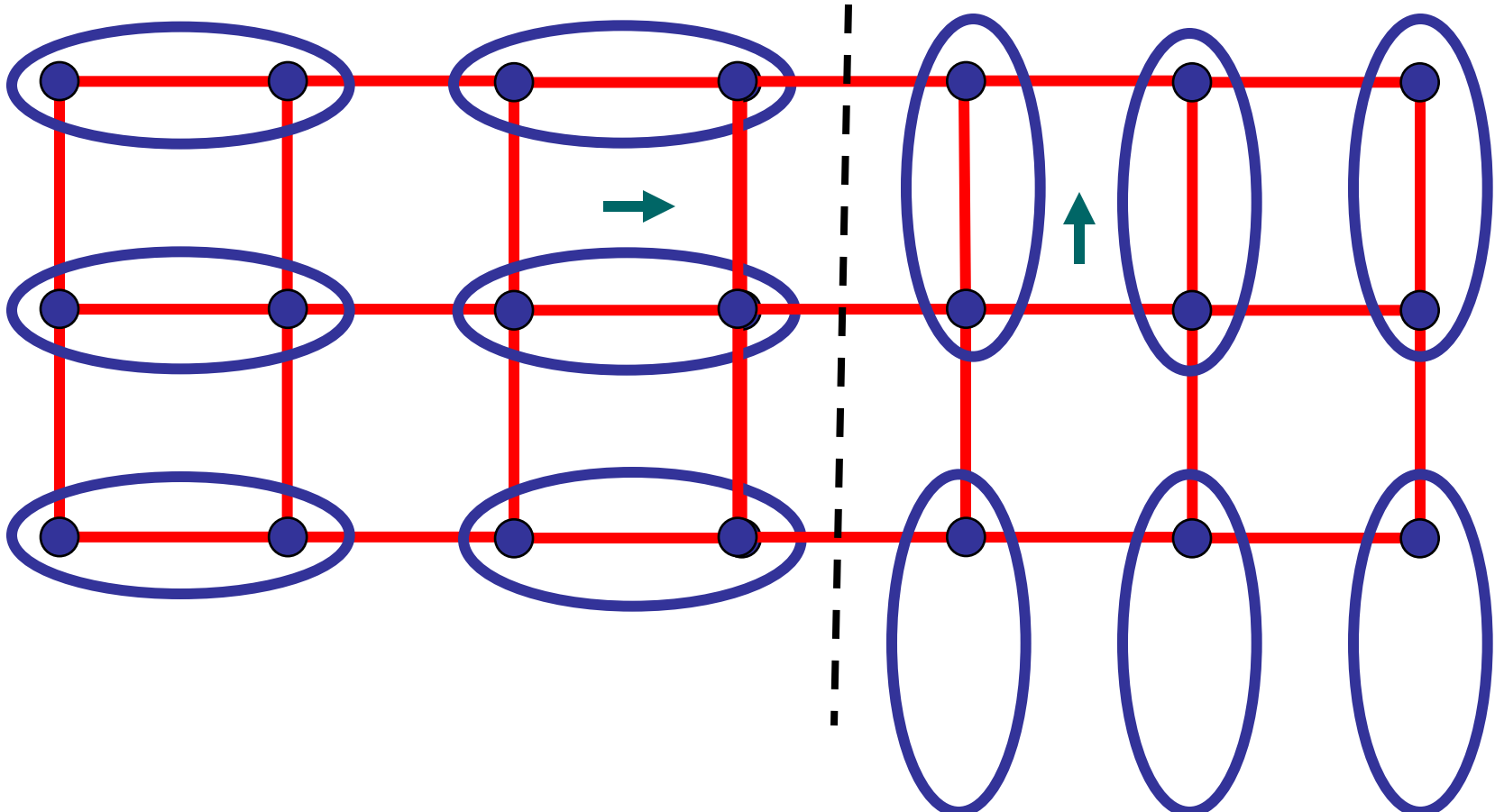
Why do these fail?

Topological defects carry non-trivial quantum numbers!

This talk: attack from VBS (Levin, TS, [cond-mat/0405702](#))

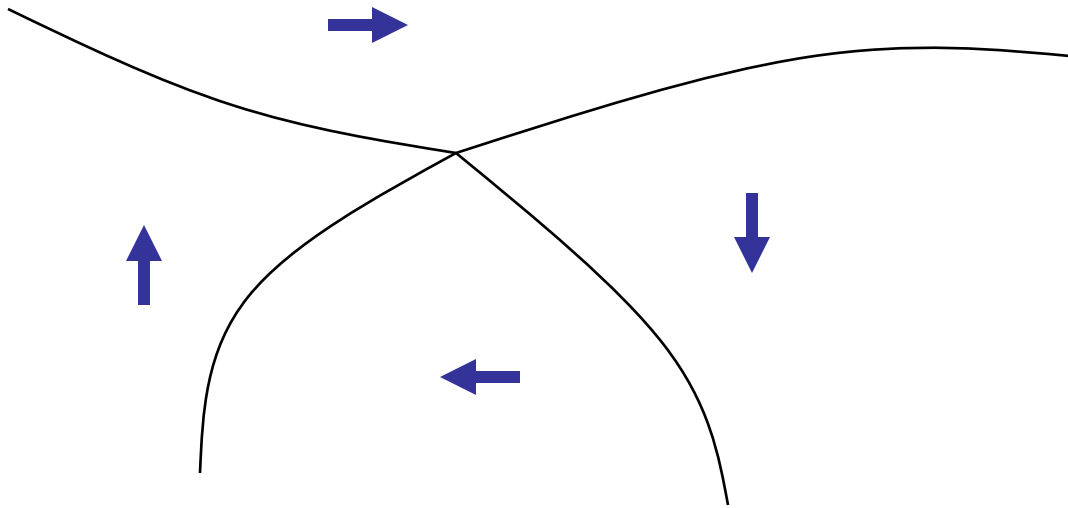
Topological defects in Z_4 order parameter

- Domain walls – elementary wall has $\pi/2$ shift of clock angle



Z_4 domain walls and vortices

- Walls can be oriented; four such walls can end at point.
- End-points are Z_4 vortices.

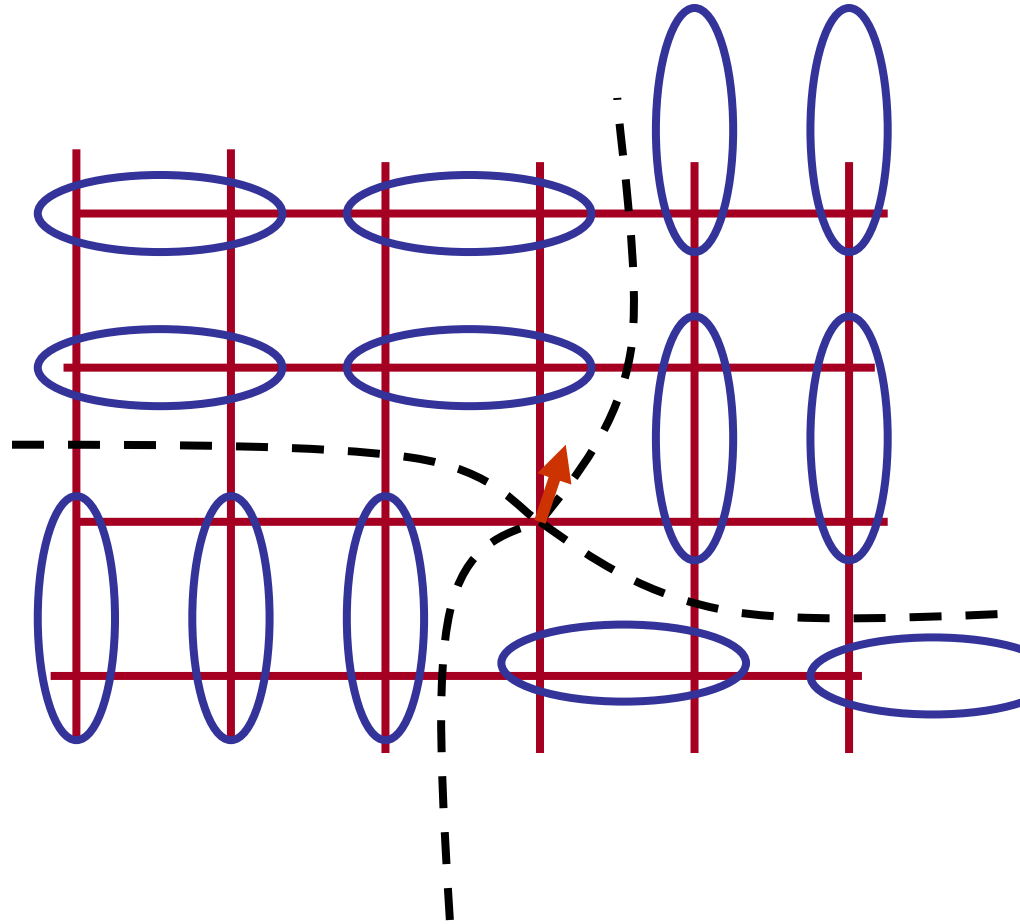


Z_4 vortices in VBS phase

Vortex core has an unpaired spin-1/2 moment!!

Z_4 vortices are ``spinons”.

Domain wall energy confines them in VBS phase.



Disordering VBS order

- If Z_4 vortices proliferate and condense, cannot sustain VBS order.
- Vortices carry spin => develop Neel order

Z_4 disordering transition to Neel state

- As for usual (quantum) Z_4 transition, expect clock anisotropy is irrelevant.

(confirm in various limits).

Critical theory: (Quantum) XY but with vortices that carry physical spin-1/2 (= spinons).

Alternate (dual) view

- Duality for usual XY model (Dasgupta-Halperin)

Phase mode - ``photon''

Vortices – gauge charges coupled to photon.

Neel-VBS transition: Vortices are spinons

=> Critical spinons minimally coupled to fluctuating $U(1)$ gauge field*.

*non-compact

Proposed critical theory “Non-compact CP₁ model”

$$S = \int d^2x d\tau \left[(\partial_\mu - ia_\mu)z \right]^2 + r |z|^2 + u |z|^4 \\ + (\varepsilon_{\mu\nu\lambda} \partial_\nu a_\lambda)^2$$

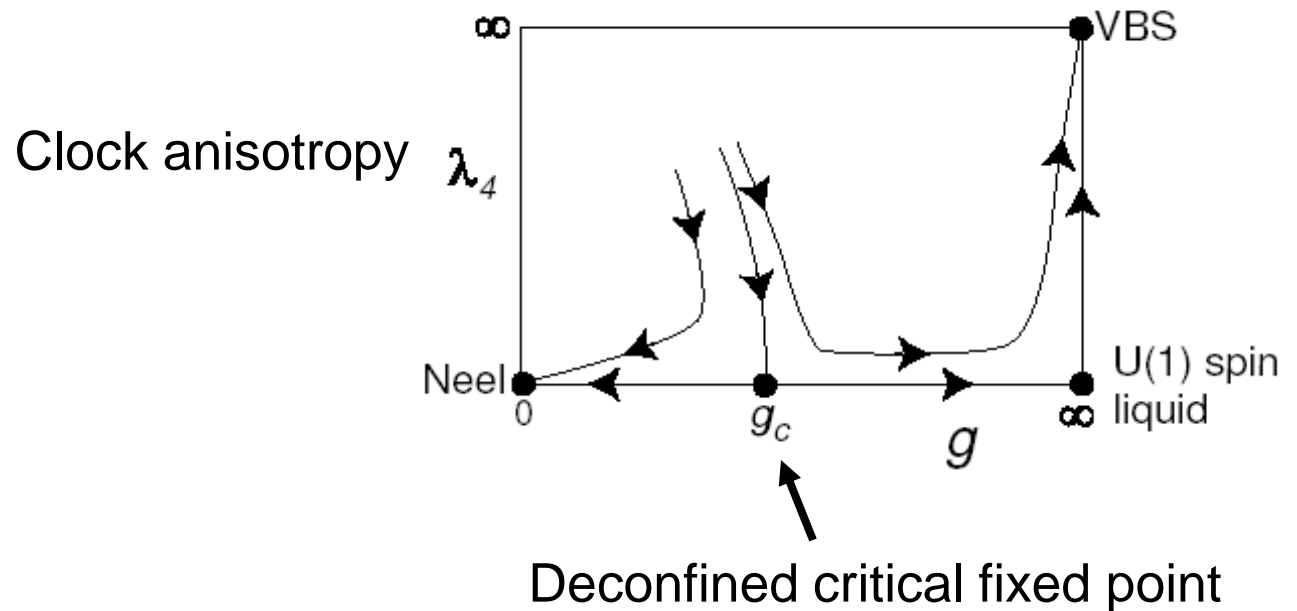
z = two-component spin-1/2 spinon field

a_μ = non-compact U(1) gauge field.

Distinct from usual O(3) or Z₄ critical theories.

Theory not in terms of usual order parameter fields
but involve spinons and gauge fields.

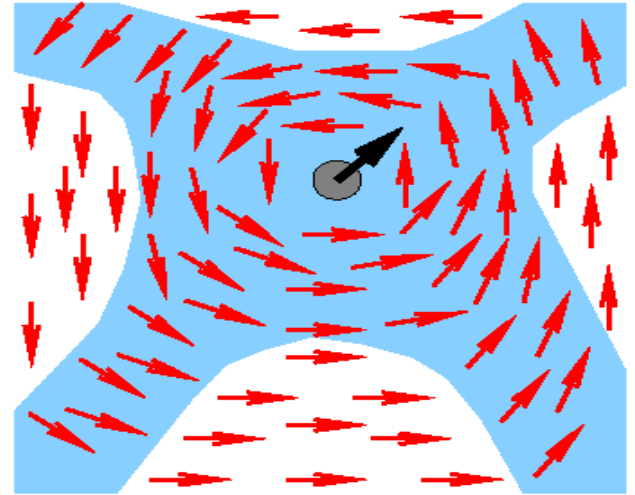
Renormalization group flows



Clock anisotropy is “dangerously irrelevant”.

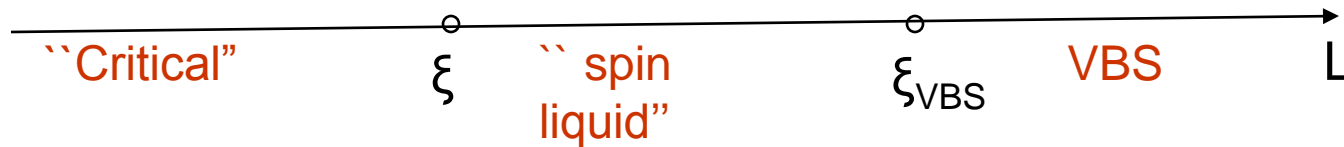
Precise meaning of deconfinement

- Z_4 symmetry gets enlarged to XY
- \Rightarrow Domain walls get very thick and very cheap near the transition.
- \Rightarrow Domain wall energy not effective in confining Z_4 vortices (= spinons)



Formal: Extra global $U(1)$ symmetry
not present in microscopic model :

Two diverging length scales in paramagnet

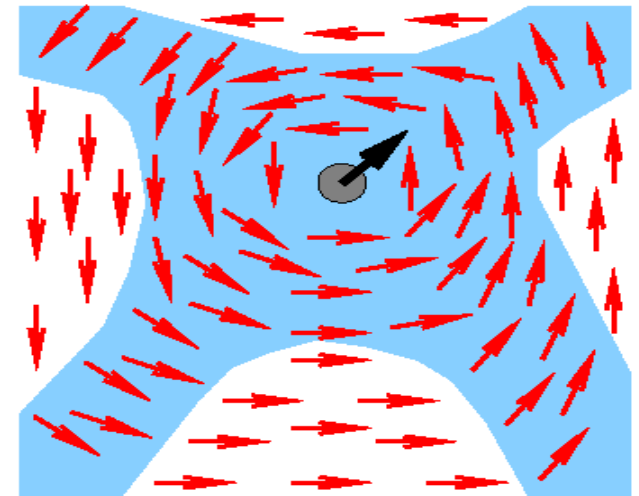


ξ : spin correlation length

ξ_{VBS} : Domain wall thickness.

$\xi_{VBS} \sim \xi^K$ diverges faster than ξ

Spinons confined in either phase
but 'confinement scale' diverges
at transition.



Other examples

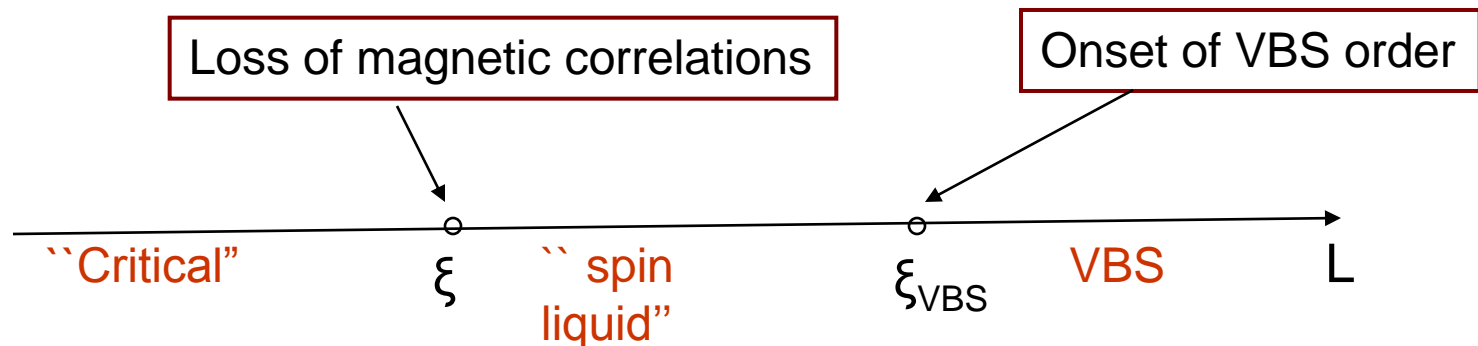
- Similar phenomena at other quantum transitions of spin-1/2 moments in $d = 2$

(VBS- spin liquid, VBS-VBS, Neel – spin liquid, ...)

Apparently fairly common

Some lessons-I

Separation between the two competing orders
not as a function of tuning parameter but as a function of
(length or time) scale (exactly as suggested near heavy fermion critical point)



Some lessons-II

- Striking “non-fermi liquid” (morally) physics at critical point between two competing orders.

Eg: At Neel-VBS, magnon spectral function is anomalously broad (roughly due to decay into spinons) as compared to usual critical points.

Most important lesson:

Failure of Landau paradigm – order parameter fluctuations do not capture true critical physics.

Caricature of phenomena suggested near heavy fermion critical points.

Experiments: Are there really two distinct time/length scales at heavy fermion critical points?