How might a Fermi surface die?

Unconventional quantum criticality in metals

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Fermi Liquid Theory (FLT) of conventional metals

- Electron retains integrity at low energies as a `quasiparticle'
- Sharp Fermi surface satisfying Luttinger sum rule
- Infinite number of emergent conservation laws: quasiparticle number at each point of Fermi surface
- Landau FL: hydrodynamic theory of low energy conserved densities
- FLT beyond Landau: Universal short distance physics (2Kf singularities)

Size and shape of Fermi surface: important defining feature of a Fermi liquid

Luttinger's theorem for Fermi liquids

In a Fermi liquid, volume V_F of Fermi surface is set by electron density n independent of interaction strength.

$V_F = (2\pi)^d n/2$ (mod Brillouin zone volume).

Perturbative proof: Luttinger
Non-perturbative topological arguments:
Yamanaka, Oshikawa, Affleck (d = 1), Oshikawa (d > 1).

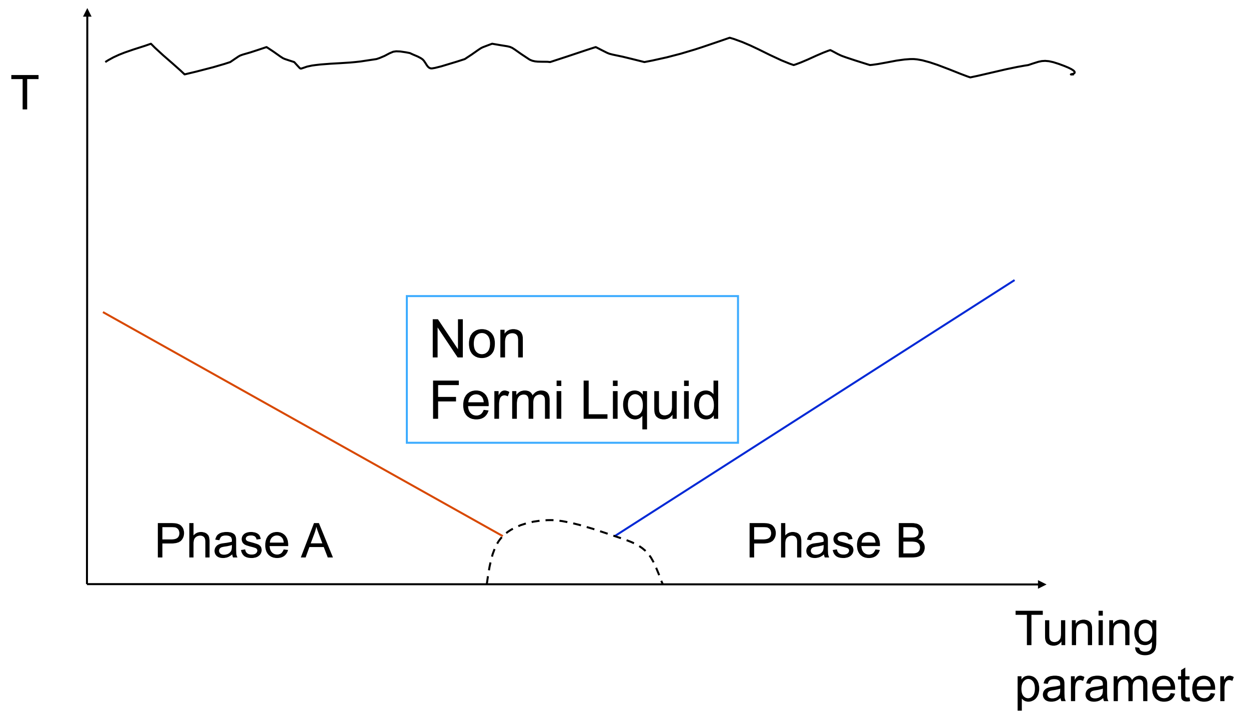
Oshikawa: Regard as ``topological quantization''.

Focus of this talk: strange metals

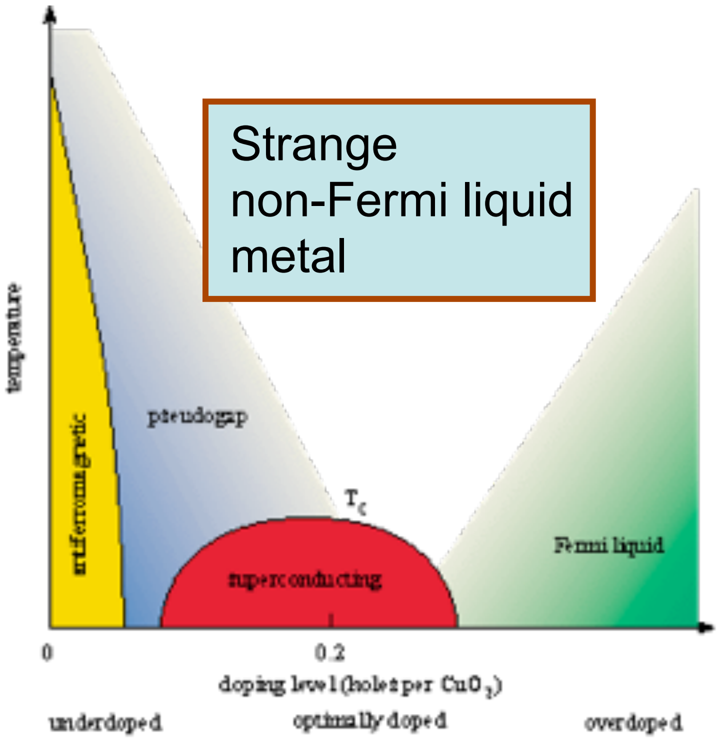
Many interesting metals where Landau's Fermi Liquid Theory breaks down.

``Non-Fermi Liquid" metals: Very little theoretical understanding though many interesting scattered ideas exist

A common phase diagram



Example 1: high temperature superconductors

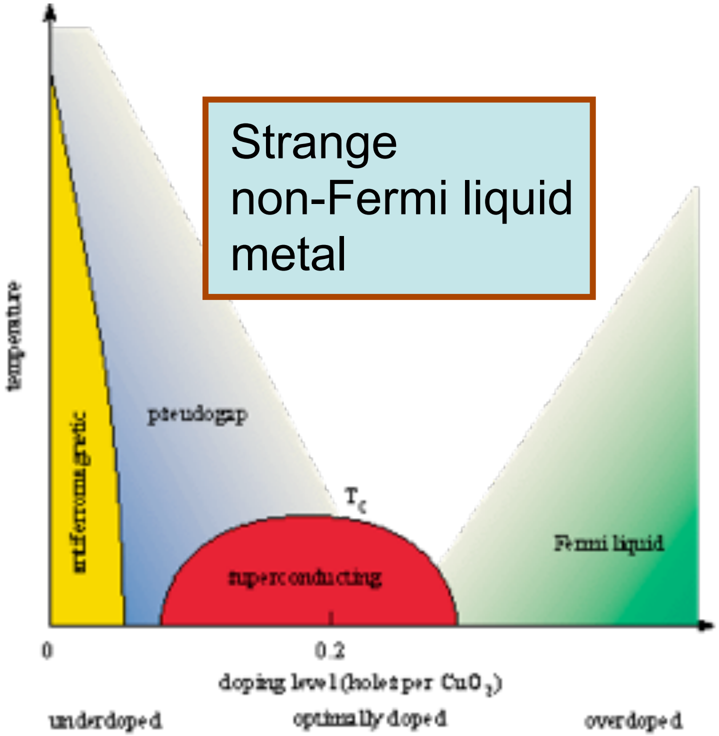


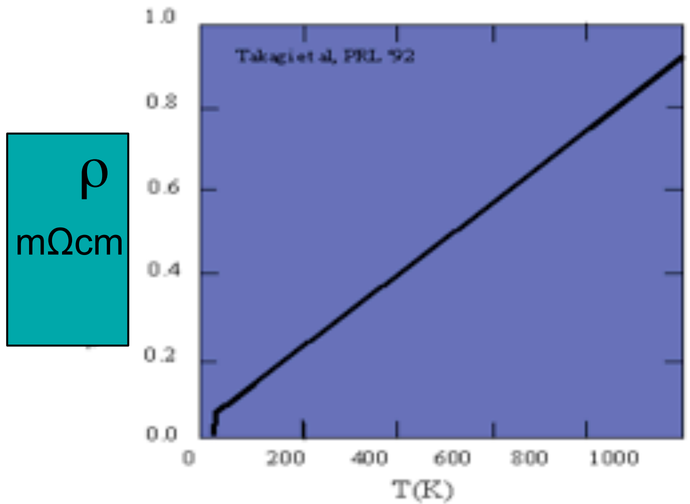
Strange metal: most mysterious!

Eg: Resistivity $\rho(T) \sim T$

Compare with Fermi Liquid $\rho(T) \sim T^2$

Example 1: high temperature superconductors





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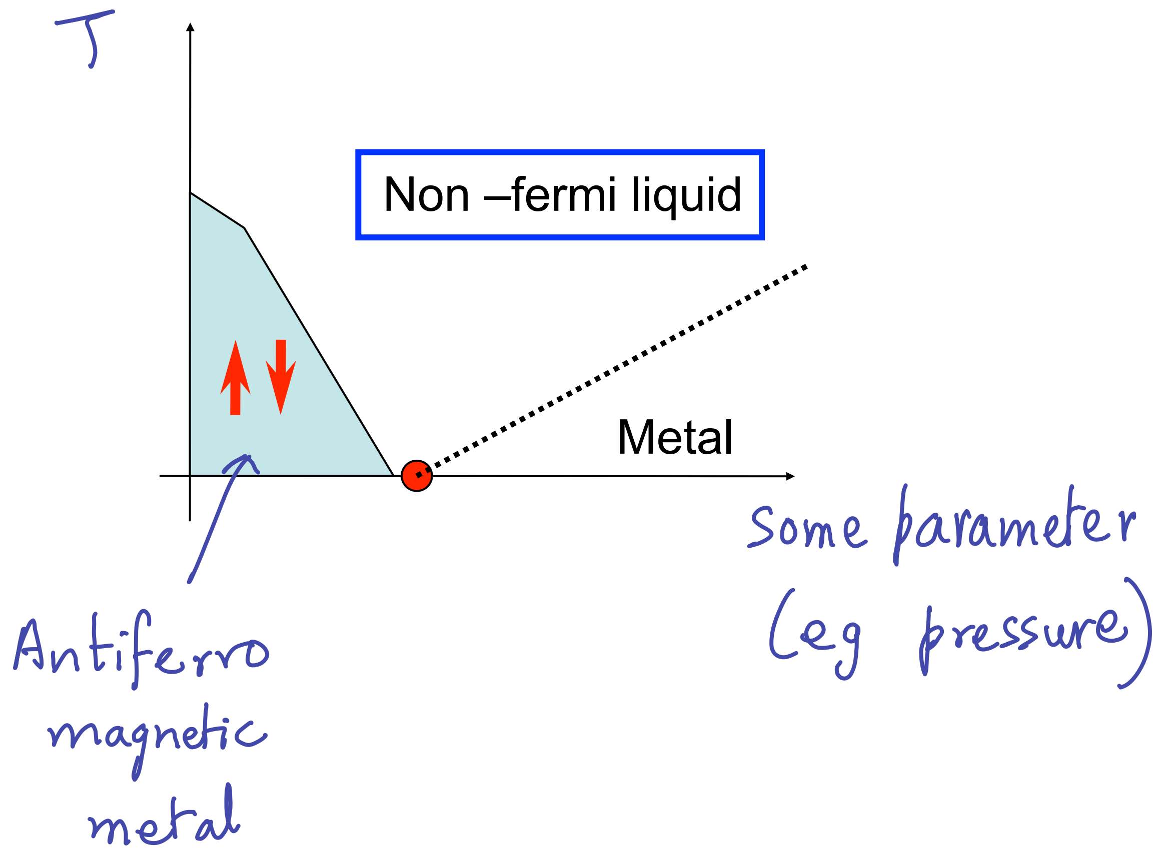
Eg: Résistivity $\rho(T) \sim T$

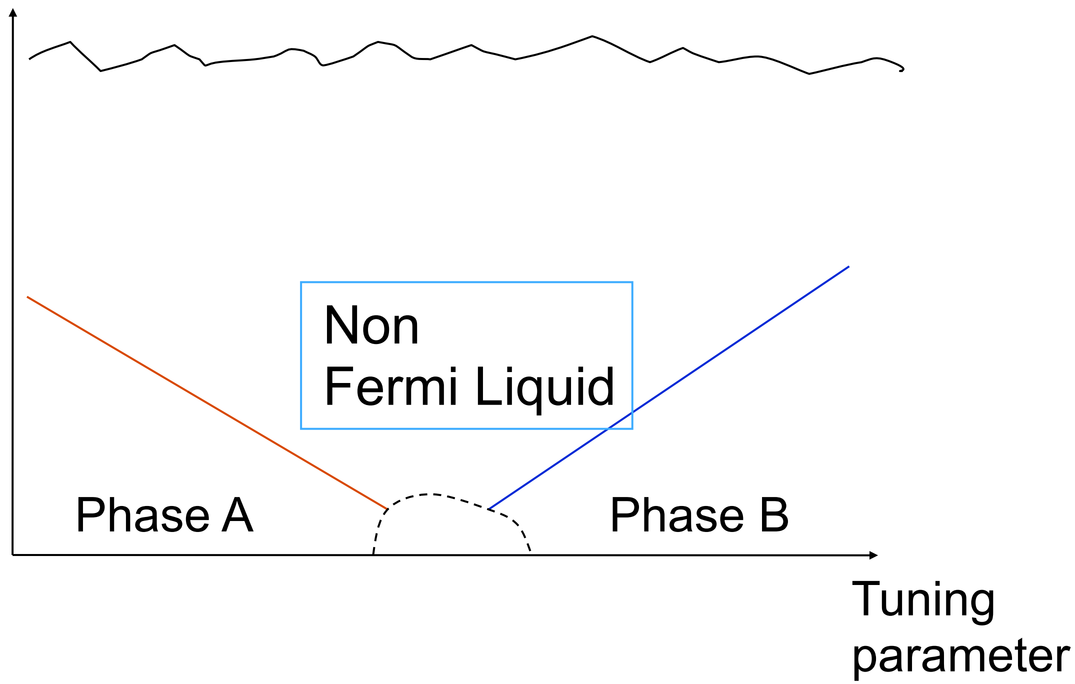
Compare with Fermi Liquid $\rho(T) \sim T^2$

Example 2: Magnetic ordering in heavy fermion alloys

$CePd_2Si_2$, $CeCu_{6-x}Au_x$, $YbRh_2Si_2$,……



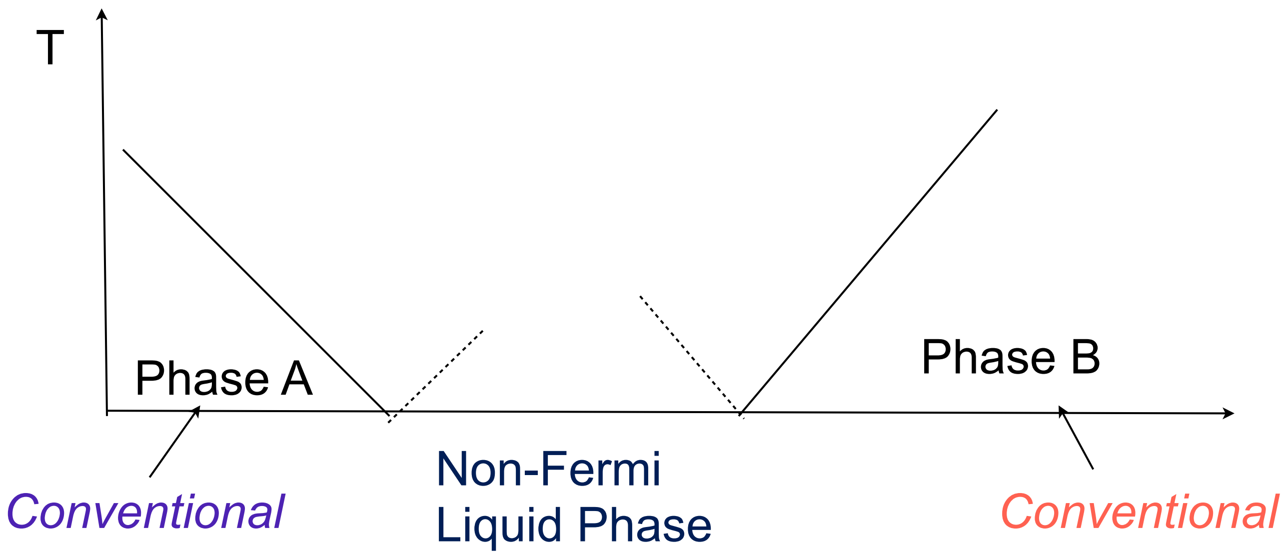
Origin of Non-Fermi Liquid (NFL) physics?



Perhaps whatever causes phase transition between phases A & B also underlies non-fermi liquid physics

NFL: universal singularities of putative quantum critical point between phases A and B

Alternate (less common?) phase diagram



Certainly possible theoretically.

Experiment?

Some tantalizing examples: beta-YbAlB4, may be also with Ir substitution for Rh in YbRh2Si2, MnSi?, hiTc?

Approach from Fermi liquid

Crucial question:

Fate of the Fermi surface as a Fermi liquid metal undergoes a quantum phase transition?

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Fate of the Fermi surface as a Fermi liquid metal undergoes a quantum phase transition?

Two general possibilities

Mutilate

Fermi surface evolves continuously but is distorted in some way.
(Ferromagnet, nematic, SDW, CDW,.....)

Kill

Original Fermi surface completely disappears.
(Mott transition, Kondo breakdown,.....)

Mutilate versus Kill

Mutilate: Typically associated with development of broken symmetry characterized by Landau order parameter.

Model: Fermi surface + X
(X = gapless bosons associated with singular order parameter fluctuations).

Kill: No Landau order parameter; physics beyond Landau-Ginzburg-Wilson paradigm.

Outline

1. Killing the Fermi surface: concept of critical FS, scaling theory, and some calculations

2. New results on quantum critical points where FS is mutilated

(i) Controlled expansion for nematic and other similar (i.e Pomeranchuk) quantum critical points in 2d

(ii) QPT between an antiferromagnet and a spin liquid in a metal - non-trivial scaling at an itinerant QCP.

Killing a Fermi surface

At certain such T = 0 phase transitions in metals, an entire Fermi surface may disappear.

Eg: (i) Heavy fermion `Kondo breakdown'

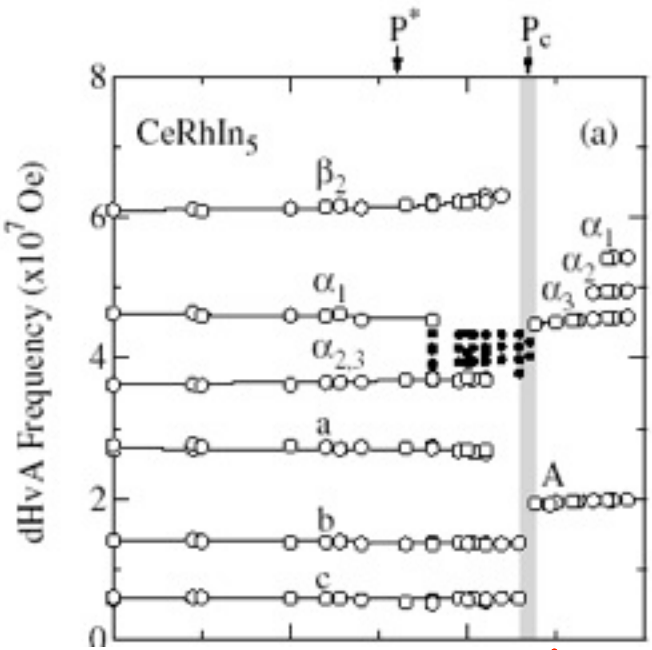
(ii) Transition from metal to (Mott) insulator

(iii) High-Tc cuprates as function of doping?

IF second order, non-fermi liquid very natural!

Example: Evolution of Fermi surface across the magnetic phase transition in CeRhIn5

Area of extremal Fermi surface orbits in some direction



Magnetic phase transition

H. Shishido, R. Settai, H. Harima, & Y. Onuki, JPSJ 74, 1103 (2005)

Entire sheets of Fermi surface disappear at the phase transi--tion

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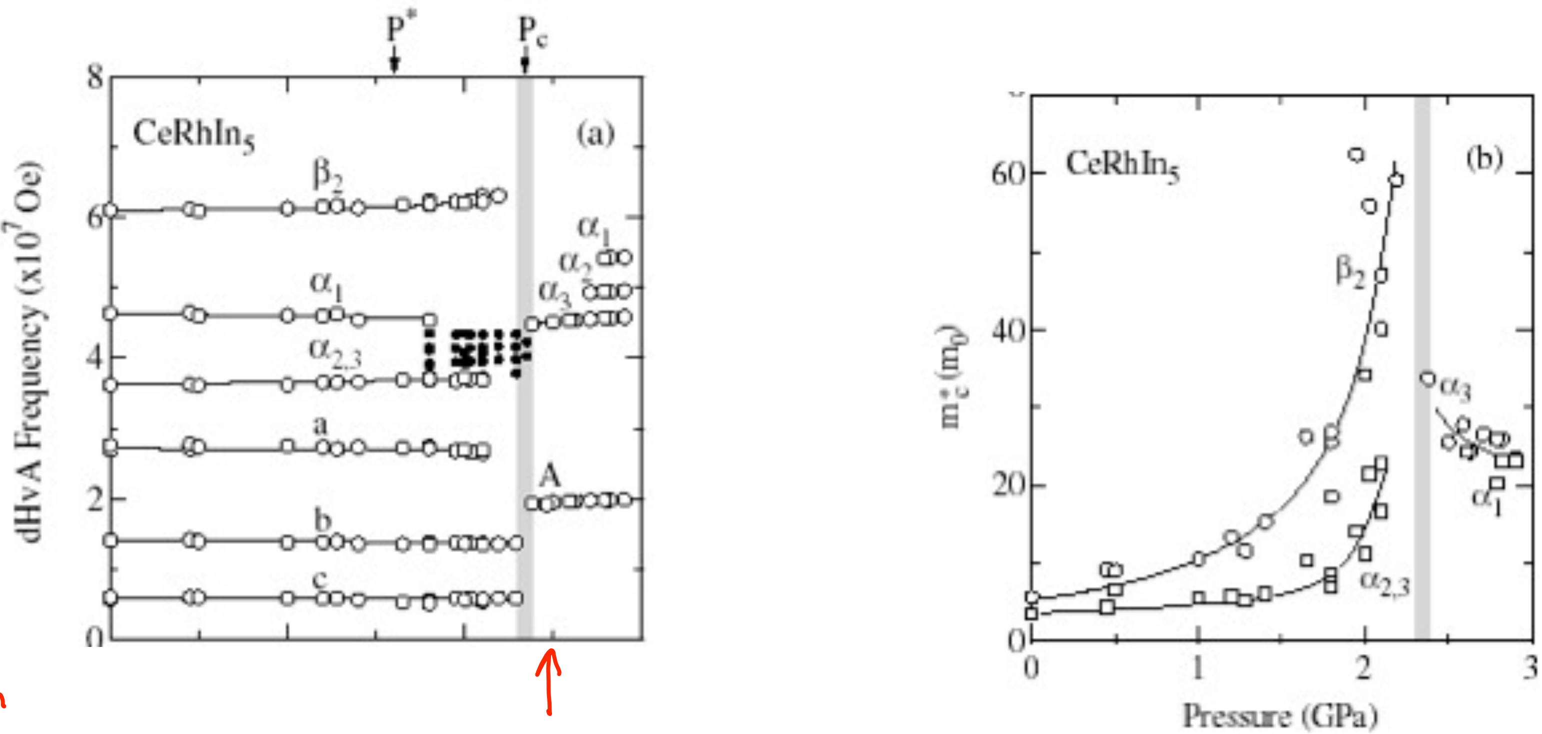

Magnetic phase transition

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Entire sheets of Fermi surface disappear at the phase transi- -tion

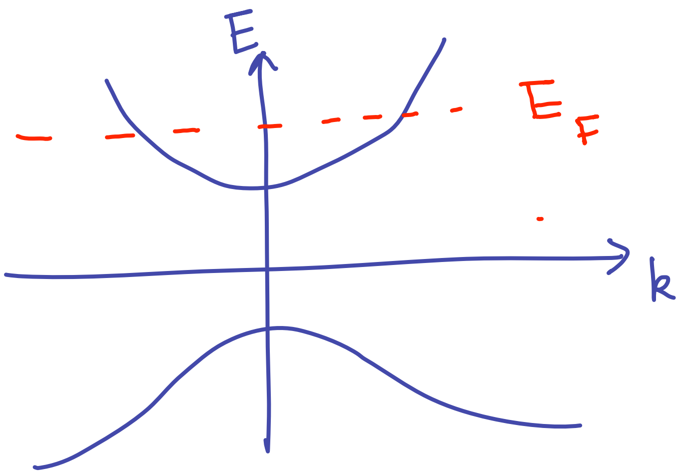
A simpler example: the metal- insulator transition

Insulators do not have Fermi surfaces!

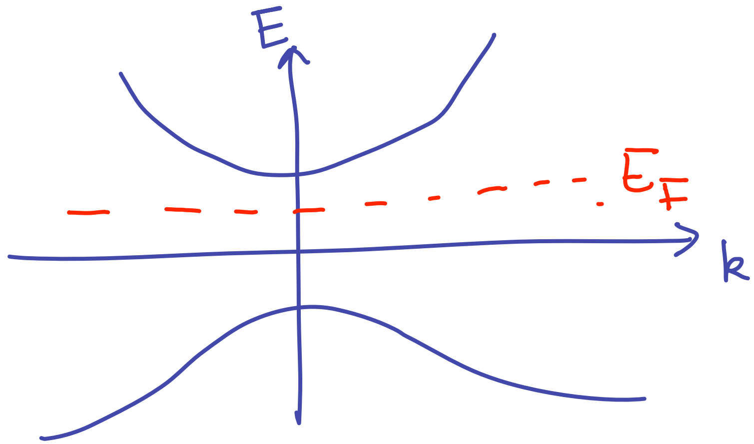
When a metal evolves into an insulator (eg by carrier doping, or by tuning pressure), it must lose its Fermi surface.

How does Fermi surface die when a metal evolves into an insulator?

Simplest possibility: Fermi surface shrinks in size and disappears.



Metal



Band insulator

Question more interesting if insulation is due to Coulomb repulsion, i.e, a `Mott' insulator

How does Fermi surface die when a metal evolves into a Mott insulator?

Not fully understood……

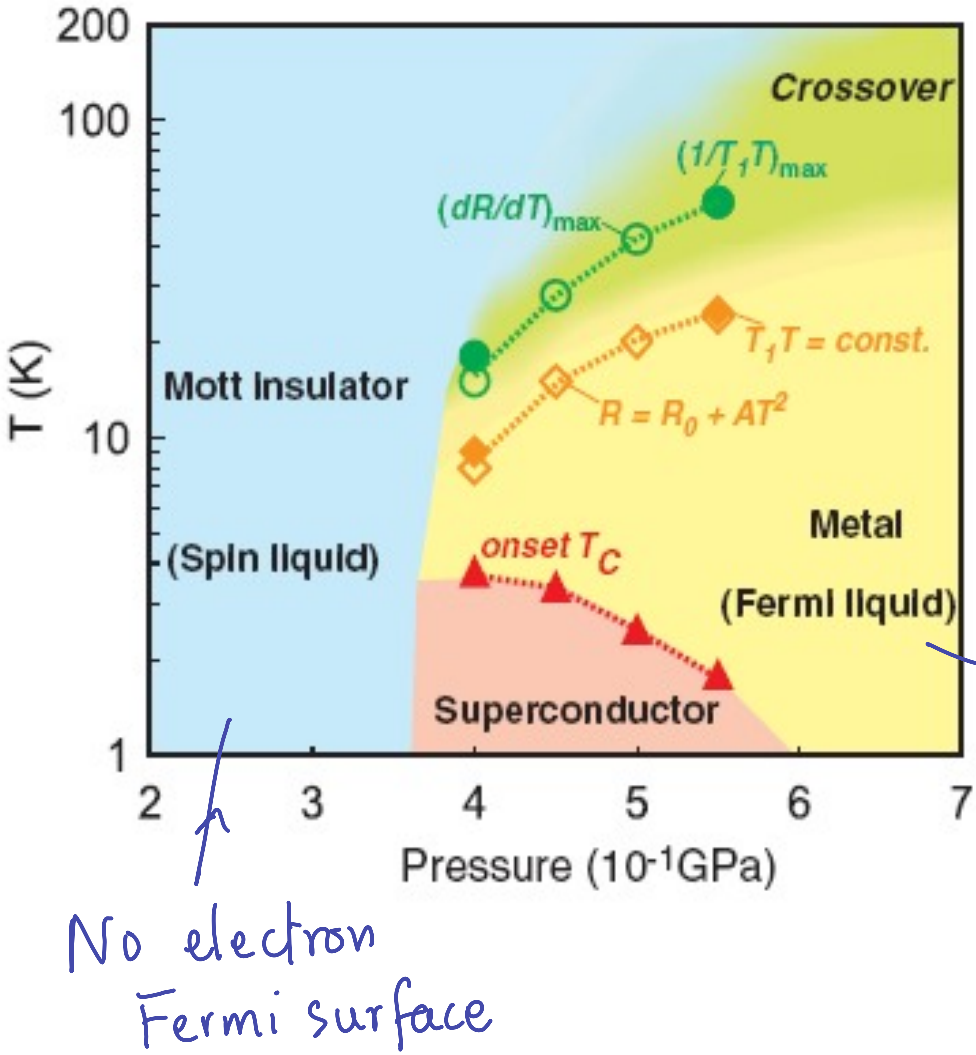
Central to some of the most mysterious phenomena in quantum condensed matter physics.

Mott insulator: simple in real space (electrons are particles)

Metal with Fermi surface: simple in momentum space (electrons are waves)

Vicinity of Mott metal-insulator transition: neither wave nor particle points of view superior.

Possible experimental realization of a second order Mott transition

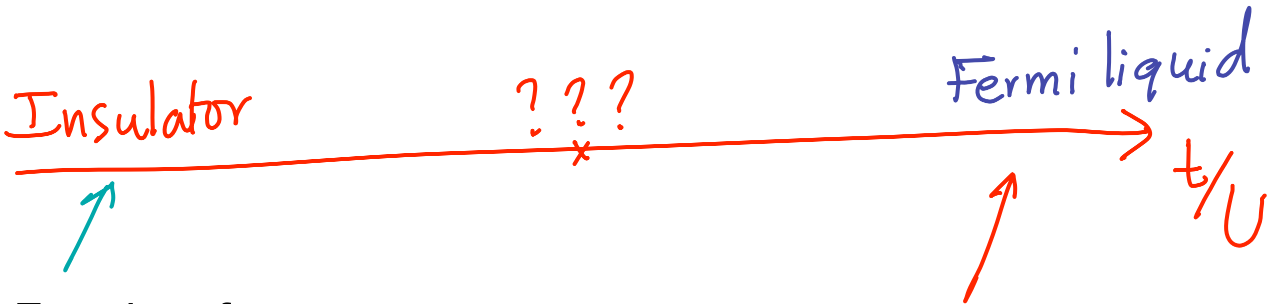


κ-(ET)$_2$Cu$_2$(CN)$_3$

under pressure

Electron
Fermi surface
with 1 e⁻/unit cell

Evolution from metal to insulator

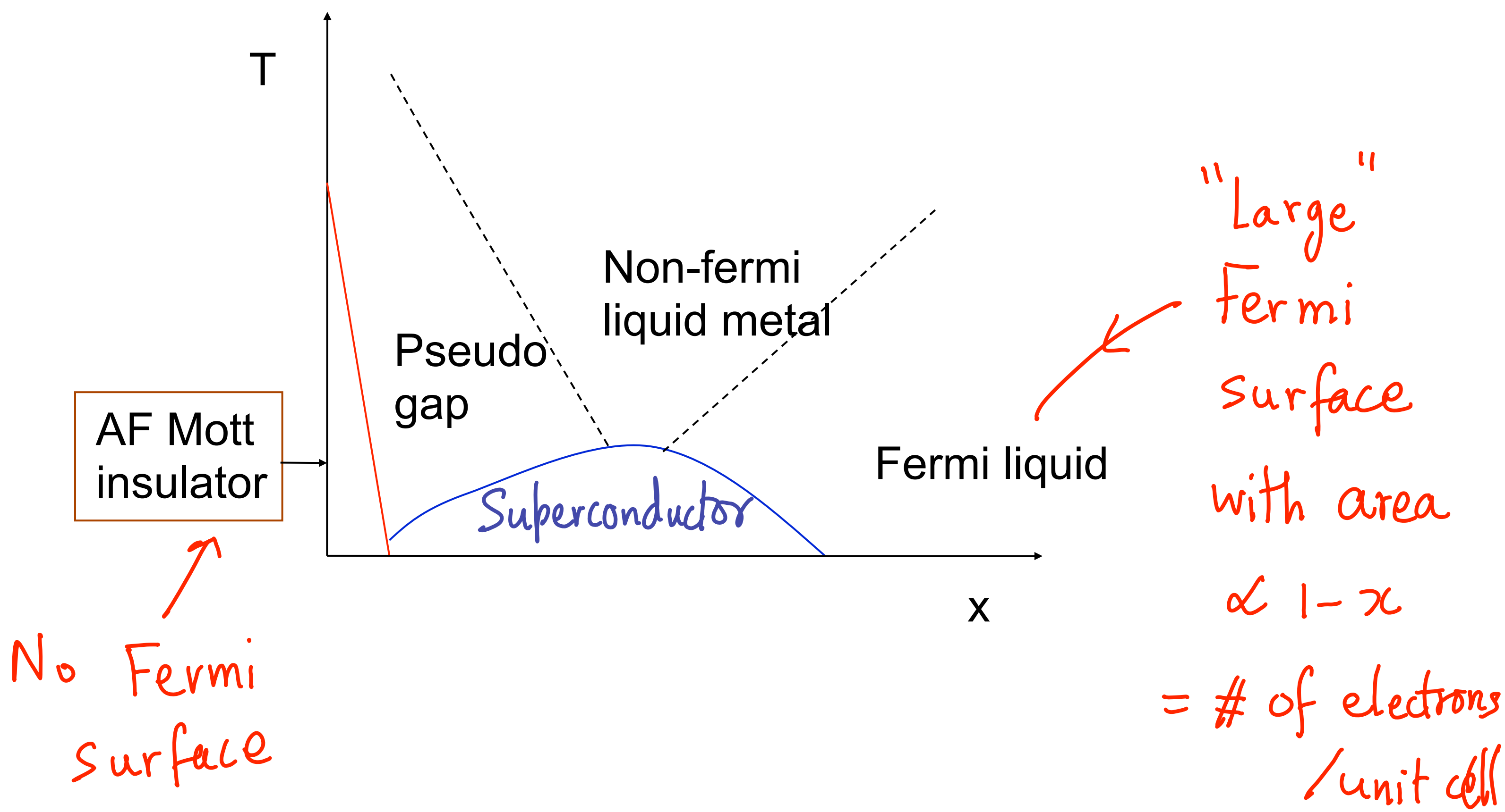


No Fermi surface

Fermi surface with size fixed by fixed electron density (1 per lattice site)

In evolving from metal to insulator, entire Fermi surface of metal needs to disappear.

Another example: High temperature superconducting materials



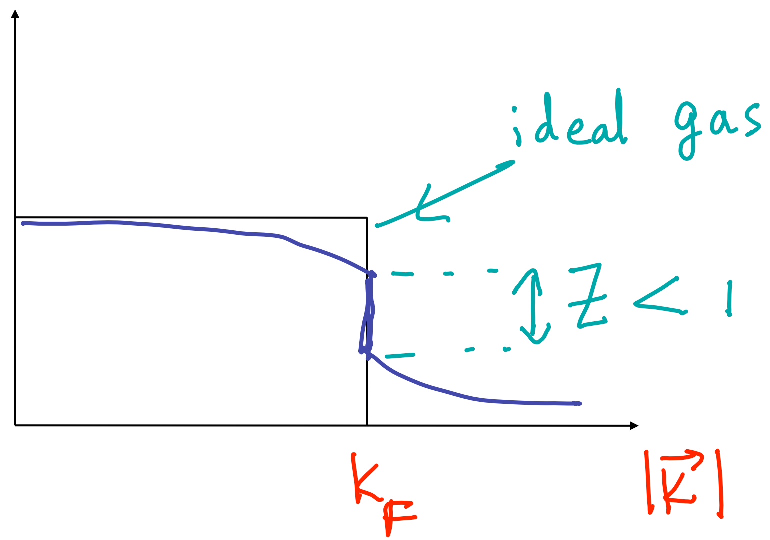
Basic question for theory

How can an entire Fermi surface disappear continuously?

Even more basic: What is the Fermi surface?

Interacting Landau Fermi liquid

Momentum occupation $n(\vec{k}) = \langle c^\dagger_{\vec{k}} c_{\vec{k}} \rangle$



Sharp jump discontinuity Z in $n(\vec{k})$ at k_F.

Z = extent to which e^- overlaps with Landau quasiparticle

How might the Fermi surface die?

When Fermi surface has disappeared, $n(\vec{k})$ is smooth at k_F

Disappearance of Fermi surface thru' continuous transition if Z vanishes continuously and everywhere on Fermi surface!

Brinkman, Rice, 1970

$Z \searrow 0$ at critical point

Concrete examples in dimensions d = 2,3: TS, Vojta, Sachdev, 2004; TS 2008

Electronic structure at criticality: ``Critical Fermi surface"

Crucial question : Nature of electronic excitations right at quantum critical point when $Z = 0$?

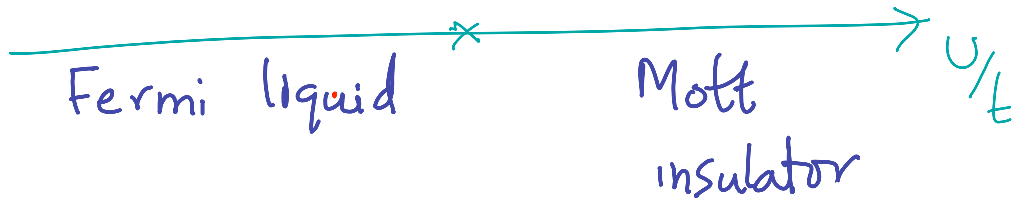
Claim : At critical point, Fermi surface remains sharply defined even though there is no Landau quasiparticle

TS, 2008

"Critical Fermi surface"

Why a critical Fermi surface?

Mott transition example:



What is gap $\Delta(\vec{k})$ to add an electron at momentum $\vec{k}$?

Fermi liquid: $\Delta(\vec{k} \in FS) = 0$

Mott insulator: Sharp gap $\Delta(\vec{k}) \neq 0$ for all $\vec{k}$

Evolution of single particle gap

Approach from Mott

2^{nd} order transition to metal $\Rightarrow$ expect Mott gap

$\Delta(\vec{K})$ will close continuously

To match to Fermi surface in metal, $\Delta(\vec{K}) \to 0$
for all $\vec{K} \in FS$.

$\Rightarrow$ Fermi surface sharp at critical point.

But as $Z = 0$ no sharp quasiparticle

$\Rightarrow$ Non-fermi liquid with sharp "critical" Fermi surface!

Why a critical Fermi surface?

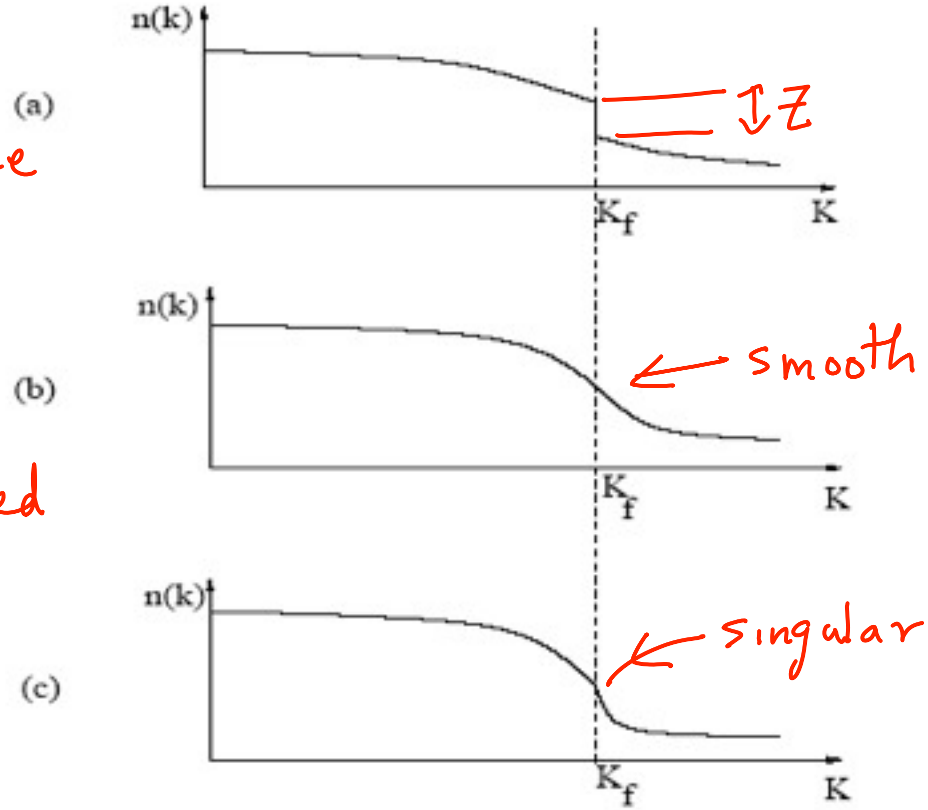
Evolution of momentum distribution

Metal with Fermi surface

Phase where Fermi surface has disappeared

Critical point

$n(K)$ continuous at K_F but is singular



Killing a Fermi surface

Disappearance of Fermi surface through a continuous transition

At critical point

(a) $Z = 0$

(b) Fermi surface sharp

Some obvious consequences/questions

Critical Fermi surface $\Rightarrow$ unusual criticality with phenomena different from familiar critical points

1. Structure of universal singularities/scaling phenomena?

2. Calculational framework?

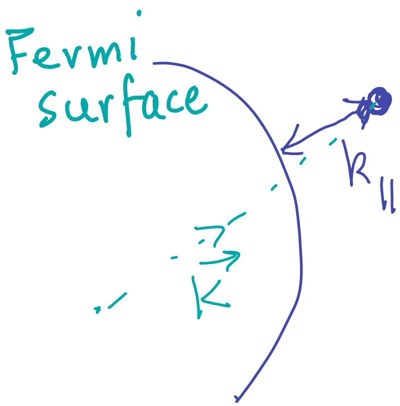
Scaling phenomenology at a quantum critical point with a critical Fermi surface? TS, 2008

Focus initially on electron density of states

$$A(\vec{K}, \omega) = \sum_n |\langle n| c_{\vec{K}} |gd\rangle|^2 \, \delta(\omega - (E_n - E_{gd}))$$

Critical Fermi surface: scaling for single particle physics

Right at critical point expect universal scale invariant singularity in $A_c(\vec{K}, \omega)$ for small ω, $k_{\parallel}$



Scaling ansatz :

For every point θ on FS

$$A_c(\vec{K}, \omega, T) \sim \frac{1}{|\omega|^{\alpha/z}} F\left(\frac{\omega}{|k_{\parallel}|^z}, \frac{\omega}{T}\right)$$

New possibility: angle dependent exponents

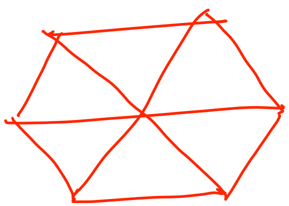
A priori must allow angle dependent exponents:

$$z = z(\theta), \quad \alpha = \alpha(\theta)$$

consistent with lattice symmetries

Eg: Triangular lattice $z(\theta + \pi/3) = z(\theta)$

$$\alpha(\theta + \pi/3) = \alpha(\theta)$$



Can expand $z(\theta) = \sum_n z_n \cos(6n\theta)$, ...

Leaving the critical point

Expect scale invariant spectrum is cut off

at $k_\parallel \sim \frac{1}{\xi}$, $\omega \sim \frac{1}{\xi^z}$ so that

$$A_c(\vec{k}, \omega) \sim \frac{1}{|\omega|^{\alpha/z}} F_1\left(\frac{\omega}{k_\parallel^z}, k_\parallel \xi\right)$$

Expect $\xi \sim |g - g_c|^{-\nu}$ but again

a priori must let $\nu = \nu(\theta)$

Scaling theory can be developed for singular behavior of various other quantities.

Many phenomenological differences with ordinary criticality

Eg: Specific heat at critical point $C_v \sim \int_{FS} d\theta \, T^{1/z(\theta)}$

= integral over Fermi surface

$T \to 0$: dominate by one portion of Fermi surface

Implications of angle dependent exponents

(i) Different properties dominated by different portions of Fermi surface

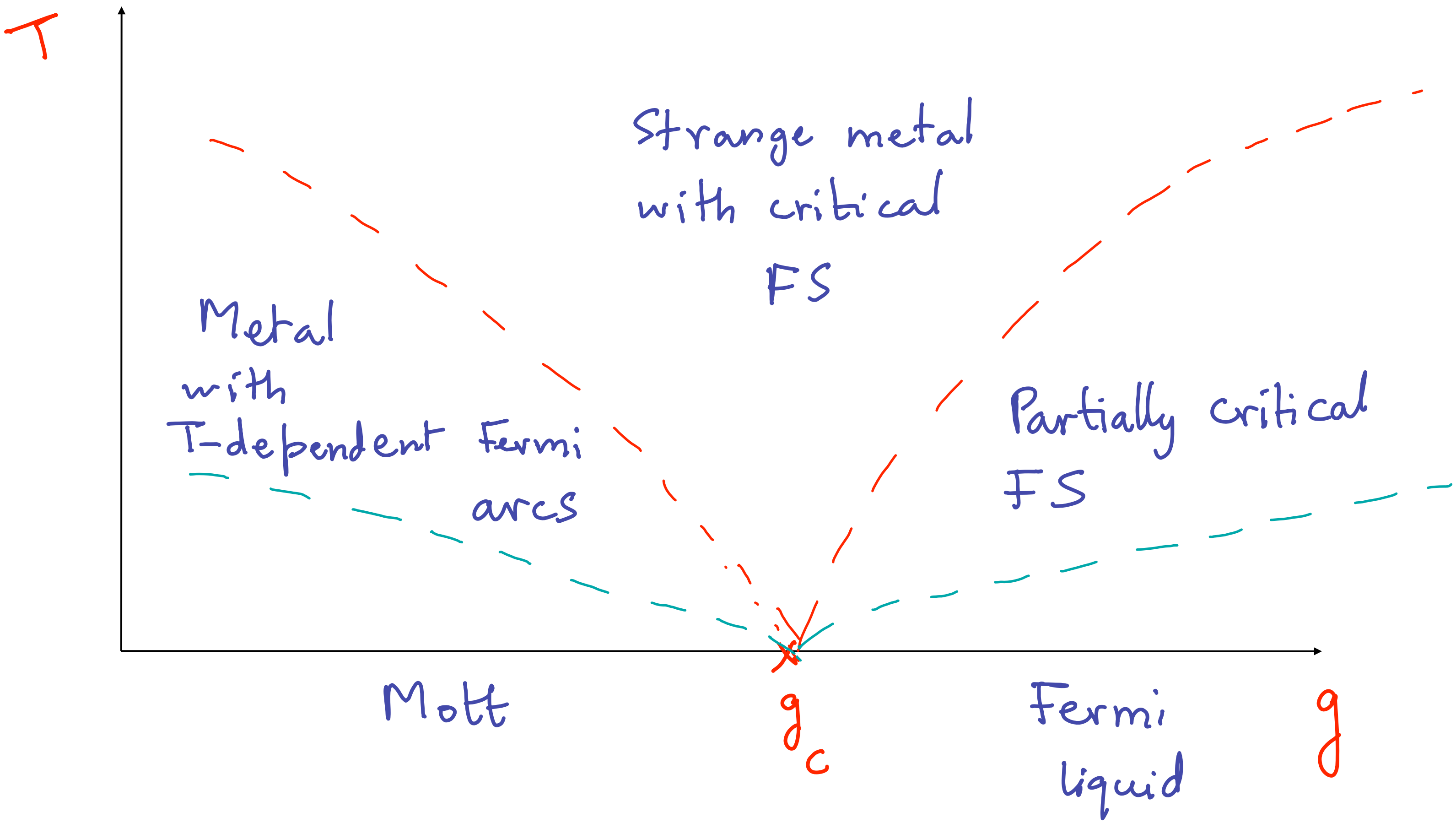
(ii) Different portions of Fermi surface will emerge out of criticality at different energy scales

Example: At Mott transition

Mott gap $\Delta(\theta) \sim |\delta g|^{z(\theta)\nu(\theta)}$

$\Rightarrow$ Finite-T xovers richer than usual

Finite T crossovers



(Similarity to some phenomena in hiTc materials)

? Calculational framework ?

1. Slave particle methods

View electron as composite of `slave' particles with fractional quantum numbers

Reformulate electron model in terms of slave particles interacting through gauge forces.

Provides concrete examples of phase transitions where an entire Fermi surface disappears continuously.

Successes: Demonstrate critical Fermi surface, emergence of non-fermi liquids (TS, 2008)

Important as proof of principle, application to experiment with caution.

2. A loooooooong shot: `Dual gravity' calculations AdS/CMT (Faulkner, Liu, McGreevy, Vegh, 2009)

Mutilating a Fermi surface

Onset of electronic nematic order from a Fermi liquid

Electronic nematic: break lattice rotational symmetry without breaking translational symmetry.

Growing number of examples in experiments. (Review: Fradkin et al, arXiv:0910.4166)

In a metal this leads to distortion of Fermi surface

Order parameter $O = \sum_{\vec{K}} (CosK_x - CosK_y)\, c^{\dagger}_{K} c_{K}$.

Right at the quantum phase transition, Fermi surface of electrons are coupled to critical fluctuations of nematic order parameter.

Generic model action

$$S = S_f + S_{int} + S_a \quad (1)$$

$$S_f = \int_{\vec{k},\omega} \bar{f}_{k\alpha}\left(-i\omega - \mu_f + \epsilon_{\vec{k}}\right) f_{k\alpha} \quad (2)$$

$$S_{int} = \int_{\vec{k},\omega} a(\vec{k},\omega) O(-\vec{k},-\omega) \quad (3)$$

$$S_a = \int_{\vec{k},\omega} \frac{1}{e^2} k^2 |a(k,\omega)|^2 \quad (4)$$

Nematic quantum criticality: O = nematic order parameter.

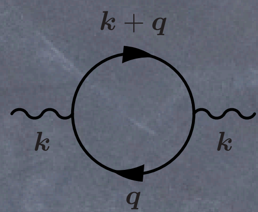
Exactly same structure in **different** problem - Fermi surface coupled to transverse gauge field: O = transverse current density

For nematic criticality mass term for a prohibited by gauge invariance.
For gauge model mass term tuned away by going to critical point.

Preliminary look: Random Phase Approximation

Many papers: '89 - present

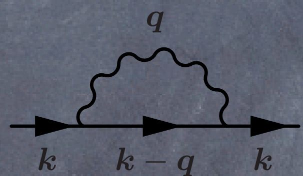
Boson self energy



$$\sim \frac{|\omega|}{|k|}$$

Landau damping; overdamped boson

Fermion self energy



$$\sim -i|\omega|^{\frac{2}{3}} sgn(\omega)$$

Fermi liquid destroyed!

Momentum independent

Beyond RPA

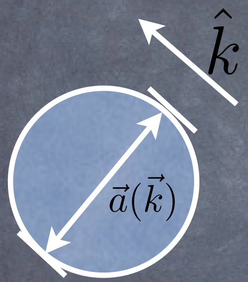
- Controlled expansions to access legitimacy of RPA and otjer approaches?
- Does scale invariant structure persist with same/modified exponents?

Large-N?

- Generalize to model with N fermion species- attempt to develop systematic 1/N expansion (Polchinski '94, Altshuler, Ioffe, Millis '94, ...)
- Hope that RPA can be formally justified at infinite N.

A useful observation

Bosons with momentum $\vec{k}$ primarily couple with patches of Fermi surface that are tangent to $\vec{k}$.



A scattering off such a boson keeps the fermion close to the Fermi surface.

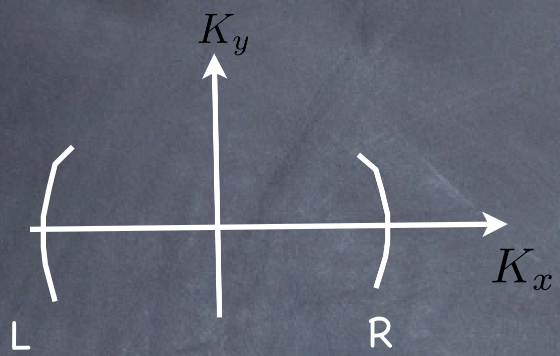
True even for a non-isotropic Fermi surface

Divide and conquer: The patch construction

Polchinski, '94
Altshuler et al, '94
Motrunich, Fisher, '07

- Divide Fermi surface into patches; in the low energy limit only patches with parallel normals are strongly coupled together.
- Check a posteriori that short range four fermion interactions that couple different patches are irrelevant at low energies.
- Simple nearly circular Fermi surface: focus on two antipodal patches of Fermi surface.

Patch action



Focus on R/L patches: s = +1 for R, -1 for L

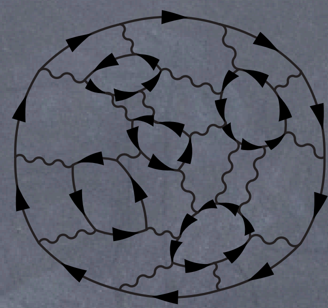
$$
\begin{aligned}
S &= S_f + S_{int} + S_a \\
S_f &= \int d^2x d\tau \sum_{s\alpha} \bar{f}_{s\alpha} \left(\eta \partial_\tau - is\partial_x - \partial_y^2 \right) f_{s\alpha} \\
S_{int} &= \int d^2x d\tau \frac{s}{\sqrt{N}} a \bar{f}_{s\alpha} f_{s\alpha}
\end{aligned}
$$

$$S_a = \int_{\vec{k},\omega} |k_y|^2 |a(\vec{k},\omega)|^2$$

For the nematic phase transition, the fermion-boson interaction does not have the factor of s => some differences in physics with gauge model, particularly in 2Kf and Cooper response

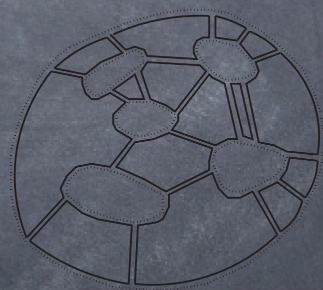
Difficulties with large-N: study just one patch

S.S. Lee (2009) argues that the theory remains strongly coupled at large-N.



Infinite number of diagrams contribute at each order.

Book-keeping device: a ``double" line notation for boson propagator.

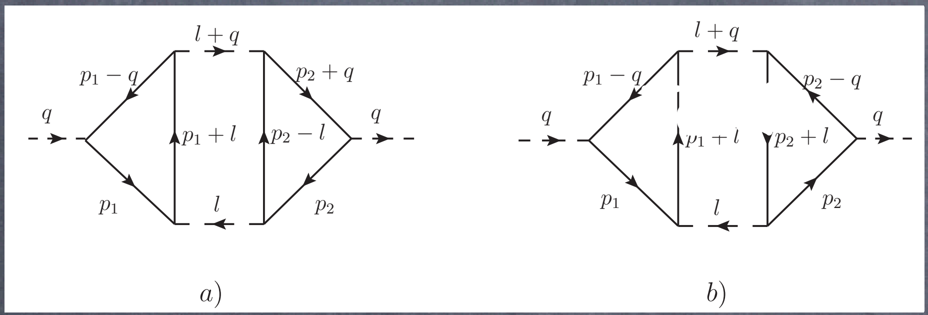


All `planar' diagrams survive in large-N limit.

What is the correct low energy physics?

Further difficulties with large-N: 2-patch theory

Metlitski, Sachdev (arxiv:1001.1153) find three loop graphs that contribute o($N^{3/2}$) to boson propagator => failure of large-N expansion?



Also at three loops, singular momentum dependence of fermion self energy: qualitative difference with RPA

Other controlled approaches?

- An epsilon expansion: Nayak, Wilczek '94 - leading order answers consistent with RPA; higher orders difficult?
- This talk: combine 1/N with small epsilon cures difficulties with large-N; non-perturbative in epsilon.

Mross, McGreevy. Liu, TS, arXiv:1003.0894

Controlled calculations of singular scaling structure of all physical quantities.

Examples: Boson, fermion Greens functions; universal 2Kf singularities.

A family of models

Replace original `bare' boson action by

$$S_a = \int_{\vec{k},\omega} \frac{|\vec{k}|^{z_b-1}}{e^2}|a(\vec{k},\omega)|^2$$

$z_b = 3$: original model.

$z_b = 2$ arises in some contexts.
Examples

- Theory of half-filled Landau level with long range $1/r$ Coulomb repulsion (Halperin, Lee, Read 1993)
- Theory of bandwidth tuned continuous Mott transition on a 2d lattice (TS, 2008).

Physics at z_b = 2

Within RPA gauge propagator

$$D(\vec{k},\omega) = \frac{1}{\gamma\frac{|\omega|}{|k|} + |k|} \tag{1}$$

Fermion self energy

$$\Sigma \sim -i\omega ln\frac{1}{|\omega|} \tag{2}$$

Fermi liquid preserved by a log ("marginal Fermi liquid").
Actually holds beyond RPA (many papers in the 90s)
Justify through detailed diagrammatics or through perturbative RG (see later)

Small epsilon: one loop

Gauge propagator $D(\vec{k},\omega) = \frac{1}{\gamma\frac{|\omega|}{|k_y|} + \frac{|k|^{z_b-1}}{e^2}}$ $(\gamma = \frac{1}{4\pi})$

Fermion self energy $\Sigma = -i\frac{1}{\lambda N}\mathrm{sgn}(\omega)|\omega|^{\frac{2}{z_b}}$ $(\lambda \propto z_b - 2)$

Suggests using $\epsilon = z_b - 2$ as a control parameter to approach $z_b = 3$.

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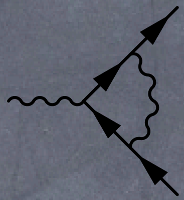
Fixed N, small epsilon: Nayak-Wilczek RG

Perturbative 1-loop RG for coupling constant

$$\frac{de^2}{dl} = \frac{\epsilon e^2}{2} - \frac{e^4}{4\pi^2 N}$$

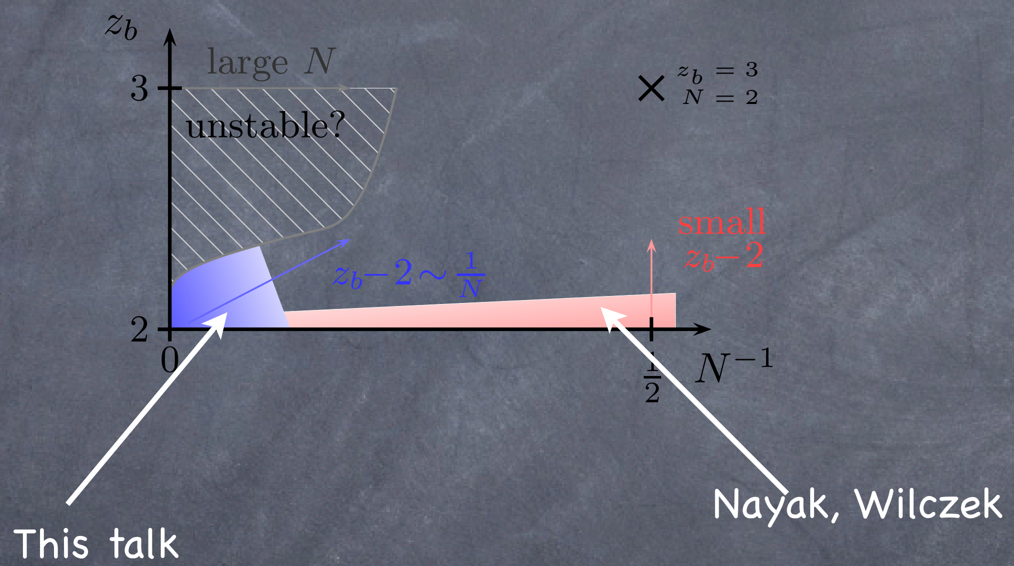


e^2 marginally irrelevant for $z_b = 2$: log correction to fermion self energy.
For $\epsilon > 0$, fixed point at $e_*^2 = 2\pi^2 N\epsilon$. Resulting fermion self energy same as in RPA.

Higher orders in epsilon? Necessary for some phenomena.
Other singularities?

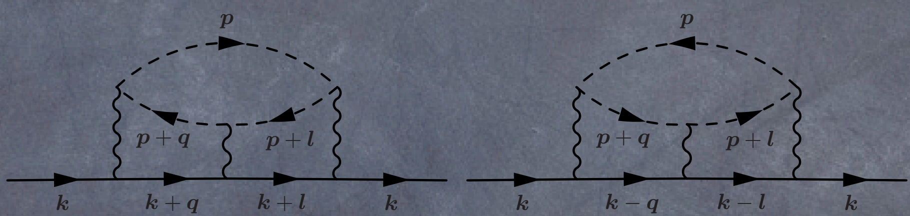
Large N at small epsilon

A "phase diagram"



Boson and fermion propagators

RPA answers exact at large-N.
Boson propagator retains RPA structure upto at least $o(\frac{1}{N^2})$.
Fermion self energy acquires singular momentum dependence at $o(\frac{1}{N^2})$.



For the right moving fermion

$$\delta\Sigma(\vec{p}, \omega = 0) = \pm\frac{4}{3N^2} J(\epsilon N)(p_x + p_y^2) \ln\left(\frac{\Lambda}{\left(p_x + p_y^2\right)^{\frac{z_b}{2}}}\right) \quad (1)$$

$+$ sign for nematic, $-$ for gauge model and $J(x)$ a known function.
Leading term of a singular contribution

$$(p_x + p_y^2)^{1 \mp \frac{4}{3N^2} J(\epsilon N)} \quad (2)$$

Similar ω dependent contribution.

Fermion Greens function and scaling

Gauge model: Singular $1/N^2$ contribution subdominant to bare terms in inverse fermion Green function.

RPA form unchanged to this order.

Nematic criticality: Singular $1/N^2$ contribution dominates over bare terms; scaling structure modified from RPA.

$$G(\vec{K},\omega) \sim \frac{c_0}{|\omega|^{\frac{\alpha}{z}}} g_0\left(\frac{c_1\omega}{k_{\parallel}^{z}}\right) \tag{1}$$

$\alpha = 1 - \eta_f$ with $\eta_f > 0$. Extrapolate to $z_b = 3, N = 2$, $\eta_f \approx 0.3$.

Fermion dynamical critical exponent $z = \frac{z_b}{2}$.

Nematic quantum criticality: implications of deviation from RPA

Power law suppression of tunneling density of states

$$N(\omega) = \int \frac{d^2\vec{K}}{(2\pi)^2} A(\vec{K}, \omega) \tag{1}$$

$$A(\vec{K}, \omega) = -\frac{1}{\pi} \mathrm{Im} G(\vec{K}, i\omega \to \omega + i0^+) \tag{2}$$

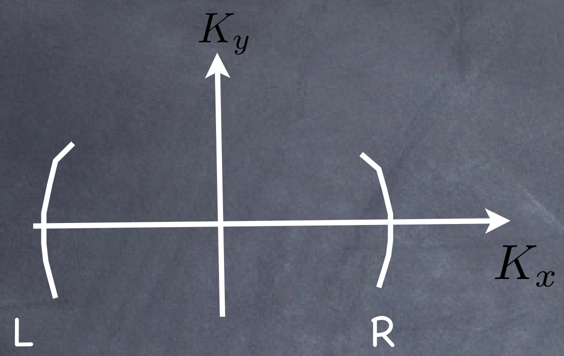
$\vec{K}$ = full momentum (not linearized near the Fermi surface).

At nematic criticality

$$N(\omega) \sim |\omega|^{\frac{\eta_f}{z}} \tag{3}$$

Contrast with RPA where $\eta_f = 0$.

2Kf and Cooper singularities: Amperean interaction



Expect singular non-Fermi liquid structure of 2Kf density/spin correlations (Altshuler et al, 1994)

$\rho_{2K_f} \sim \bar{f}_L f_R$: particle-hole pair formed from opposite sides of Fermi surface.

Two different effects:

(i) Loss of Landau quasiparticle tends to weaken $2K_f$ singularity

(ii) Amperean interaction: In gauge model such a particle and hole carry parallel currents; so they attract => tend to enhance $2K_f$ singularity.

Competition determines whether $2K_f$ enhanced or not compared to Fermi liquid.

Nematic criticality: R/L particle-hole repel => both effects tend to suppress $2K_f$; so definitely expect suppression.

Cooper singularity: situation reversed, enhanced for nematic, suppressed for gauge.

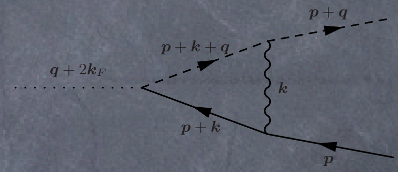
2Kf singularity exponents

Singular part of $2K_f$ density correlation function $C_{2K_f}(x,y,\tau) = \langle \rho^*_{2K_f}(x,y,\tau)\rho_{2K_f}(0,0,0)\rangle$.

Scaling form of Fourier transform

$$C_{2K_f}(p_x,p_y,\omega) = \omega^{\phi}\mathcal{C}\left(\frac{\omega}{|p_y|^{z_b}}, \frac{p_x}{p_y^2}\right) \quad (1)$$

$p_x, p_y =$ *deviation* of the full momentum from $2K_f\hat{x}$.



Large-N, small ϵ:

$$\phi = \frac{1}{2} + \frac{\epsilon}{4}\left(1 - \frac{g_2(2\pi^2\epsilon N)}{\pi\epsilon N}\right) \quad (2)$$

$g_2(v)$ a known function.

$2K_f$ singularity enhanced compared to FL for $\epsilon N \to 0$ but suppressed for $\epsilon N \to \infty$.

Fixed N, small ϵ:

$$\phi = \frac{1}{2} + \frac{\epsilon}{4}\left(1 - 4ln\left(\frac{2}{\pi\epsilon N}\right)\right) \quad (3)$$

Enhanced compared to FL.

Extrapolation to $z_b = 3, N = 2$ gives supression in both expansions.

Physics of this non-Fermi liquid-I

Gauge model: Useful point of view (Kim, Lee, Wen, '95, Stern, Halperin '95): smooth versus rough shape fluctuations of Fermi surface.
Smooth shape fluctuations(eg, Long wavelength density/current response)Fermi liquid like.
Rough fluctuations (eg, 2Kf response, single particle response) non-Femi liquid like.

Patch construction for universal low energy physics

Smooth FS shape fluctuation $\sim \int d\theta u(\theta) \bar{f}_\theta f_\theta$ ($\theta =$ angle around Fermi surface, $u(\theta)$ parametrizes shape).

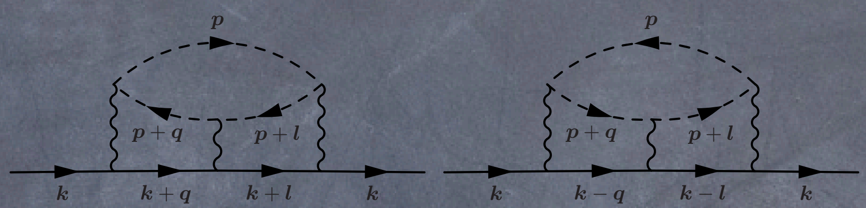
Within single patch $\bar{f}_\theta f_\theta$ susceptibility non-singular due to emergent low energy gauge structure(rotate phases of f_θ and $f_{\theta+\pi}$ with opposite phases).

=> Smooth FS fluctuations non-singular susceptibility like in a Fermi liquid.

Nematic criticality similar except (of course) for order parameter l = 2 channel;
but beware mixing with l = 0 response, i.e, compressibility (M. Metlitski)

Physics of this non-Fermi liquid -II

Fermion self energy at o(1/N^2): understand in terms of two-particle scattering amplitudes in 2Kf/Cooper channels



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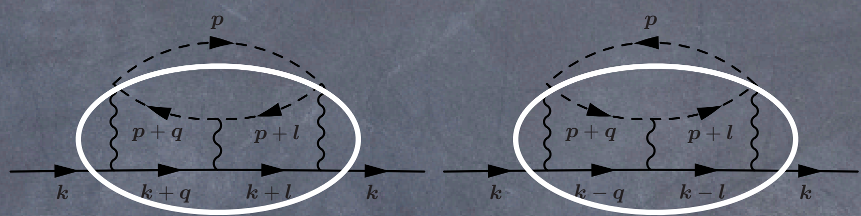

2Kf

Cooper

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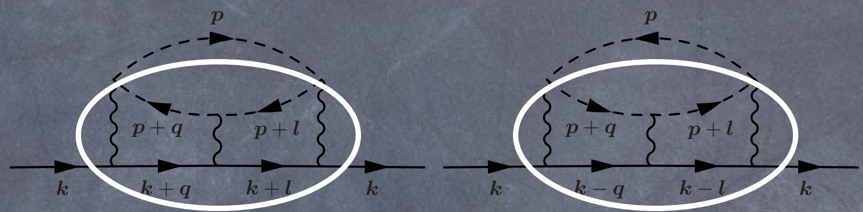
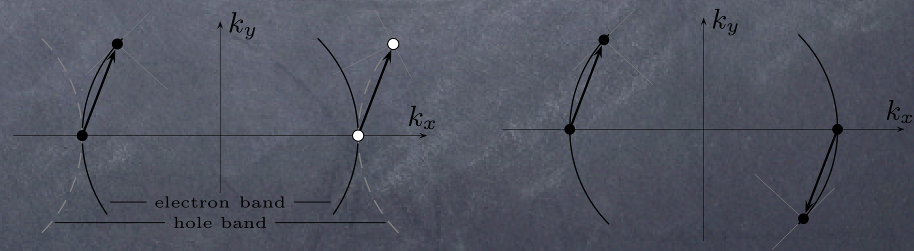


2Kf Cooper

Signs determined by Amperean rule but Cooper always dominates in magnitude due to perfect `nesting' in Cooper but not in 2Kf.



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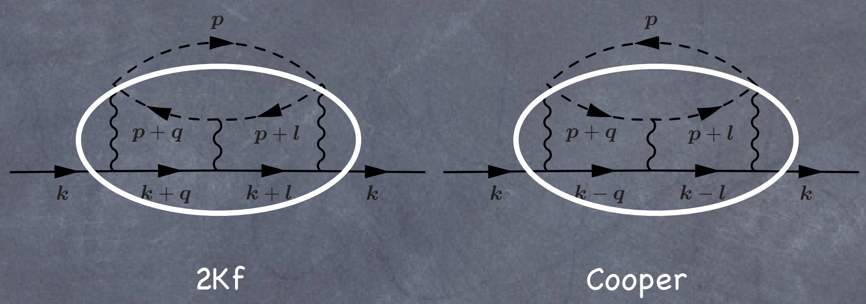

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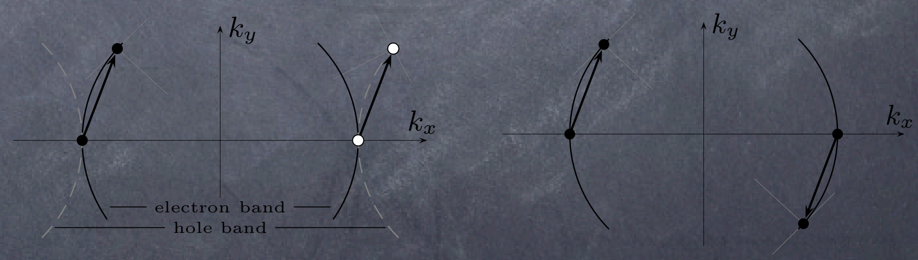
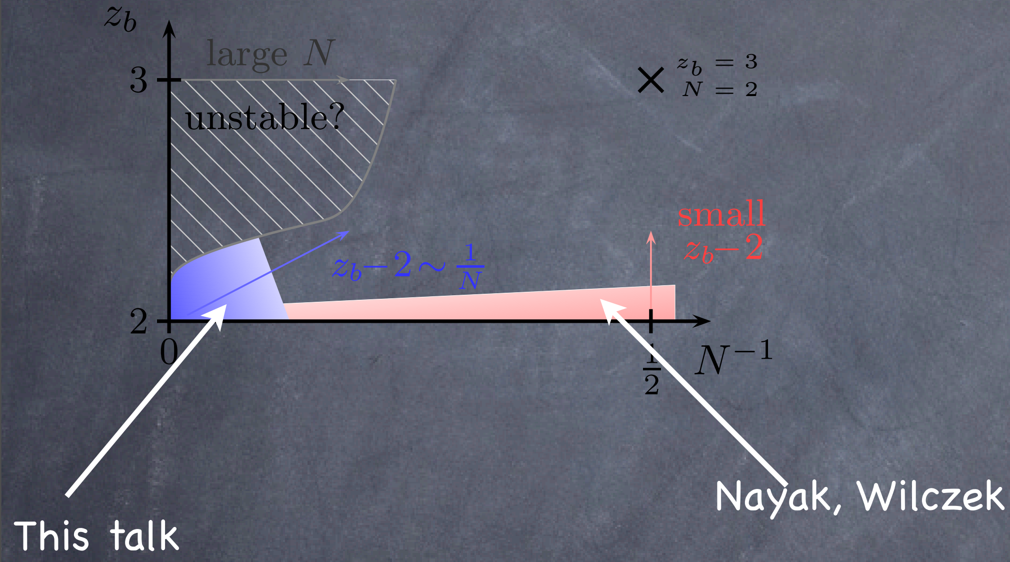


Net sign determined by Cooper => fermion Greens function singularity enhanced for nematic beyond RPA.
Suppression of tunneling density of states natural in terms of enhanced pairing fluctuations.

What about z_b = 3 at large-N?



Nematic: We suggest theory is unstable toward breaking translation and other symmetries
=> no direct second order nematic transition.

Evidence:

Direct 3-loop calculation (Metlitski, Sachdev, '10) of static boson polarizability gives

$$\int_k N\left(1 - c\sqrt{N}\right)|k_y|^2|a(\vec{k}, \omega = 0)|^2 \tag{1}$$

For N large enough get negative coefficient of k_y^2: instability toward broken translation symmetry.

Similar fate for gauge model? Hope that N = 2, z_b = 3 is not part of unstable region!

Summary/comments on non-Fermi liquid theory

Fermi surface + X : simple, reasonably tractable, model for a non-fermi liquid - control by expanding in dynamical exponent and 1/N.
Electronic spectrum has `critical Fermi surface` with no sharp Landau quasiparticle.

Similar critical Fermi surface general feature(TS, 2008) of other quantum phase transitions associated with the death of a Fermi surface (Mott criticality, heavy electron critical points, possibly cuprates).

Existing theoretical models for such phase transitions (TS, 2008, TS, Vojta, Sachdev, 2004) demonstrate critical Fermi surfaces.

Cannot be described as Fermi surface +X.
Need fractionalized degrees of freedom; have stronger destruction of Landau quasiparticle at critical point.