

# Deconfined quantum criticality

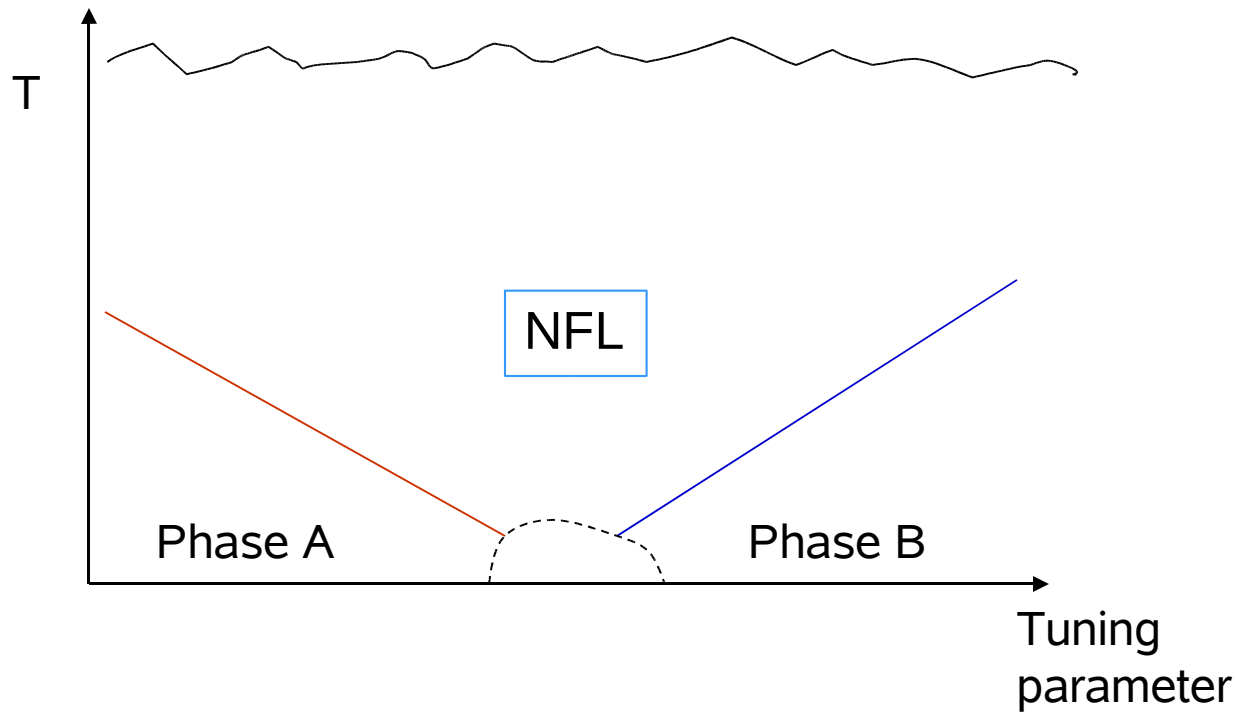
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# Competing orders and non-fermi liquids(NFL) in correlated systems

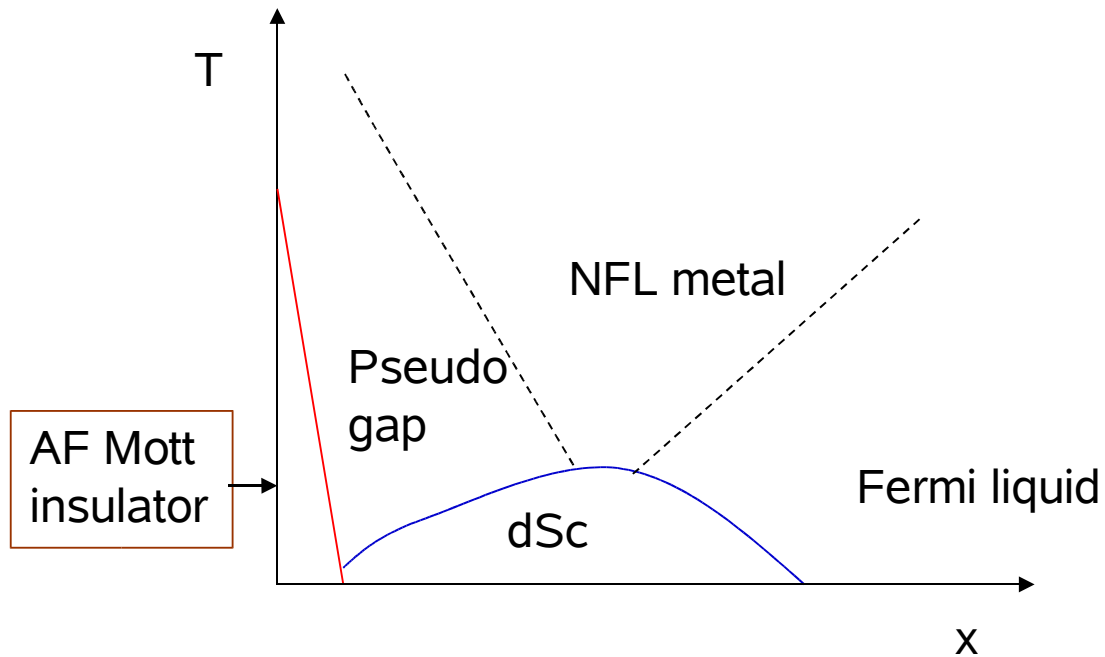


# “Classical” assumptions

1. NFL: Universal physics associated with quantum critical point between phases A and B.
3. Landau: Universal critical singularities ~ fluctuations of order parameter for transition between phases A and B.

Try to play Landau versus Landau.

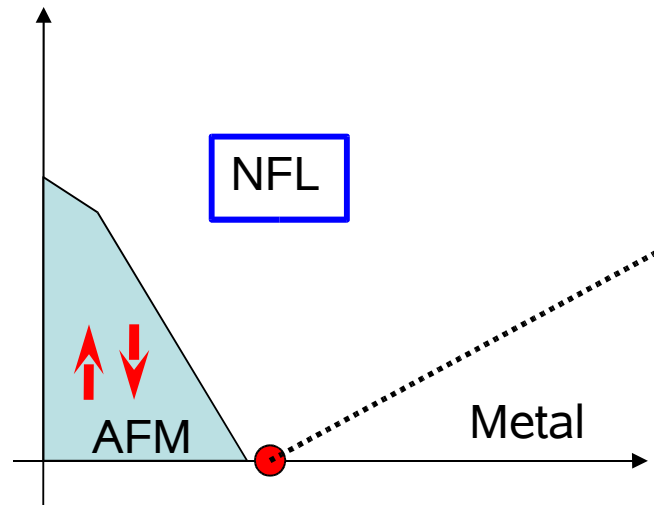
# Example 1: Cuprates



## Example 2: Magnetic ordering in heavy electron systems

$\text{CePd}_2\text{Si}_2$ ,  $\text{CeCu}_{6-x}\text{Au}_x$ ,  $\text{YbRh}_2\text{Si}_2$ ,.....

Model as Kondo lattice: local moments coupled to conduction electrons

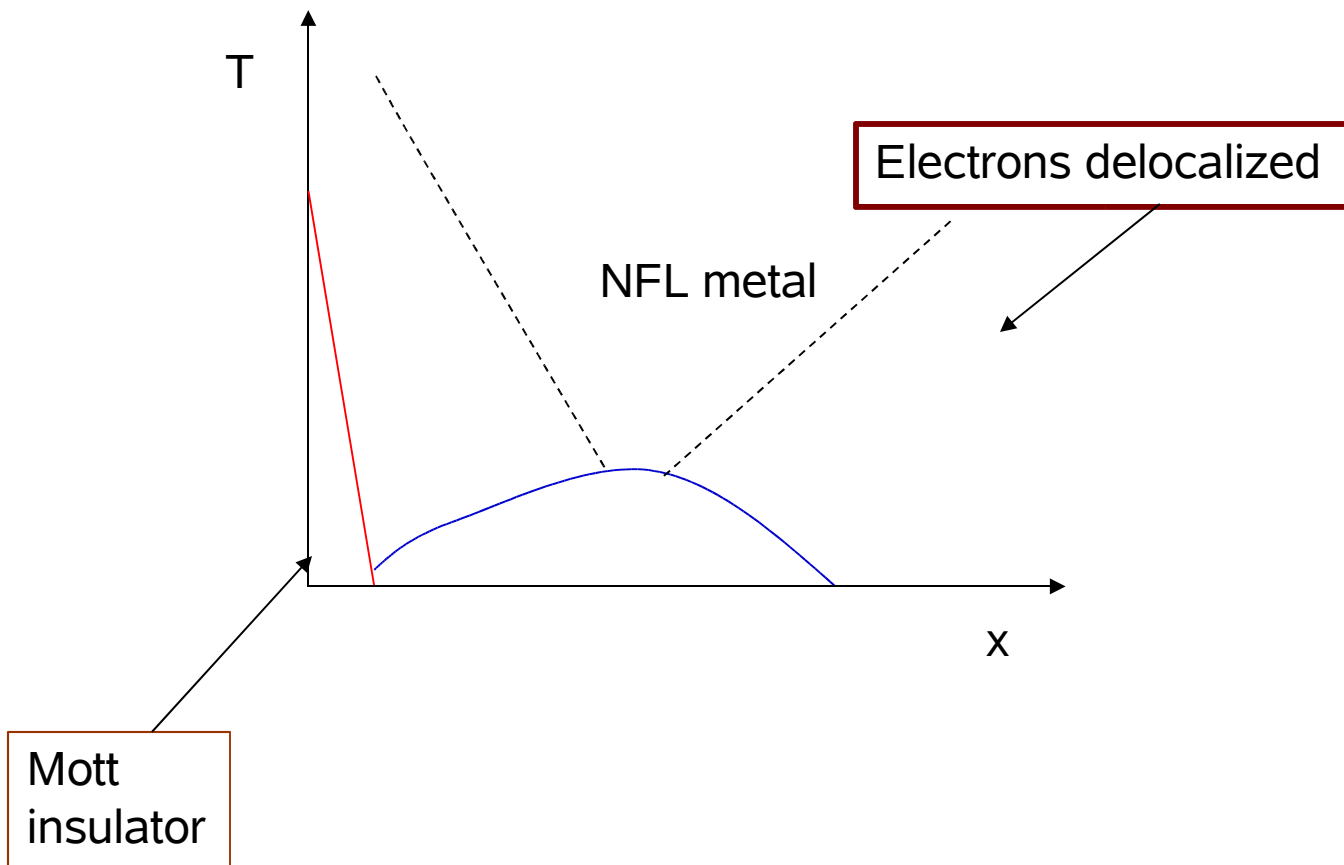


“Classical” assumptions have difficulty with producing NFL at quantum critical points

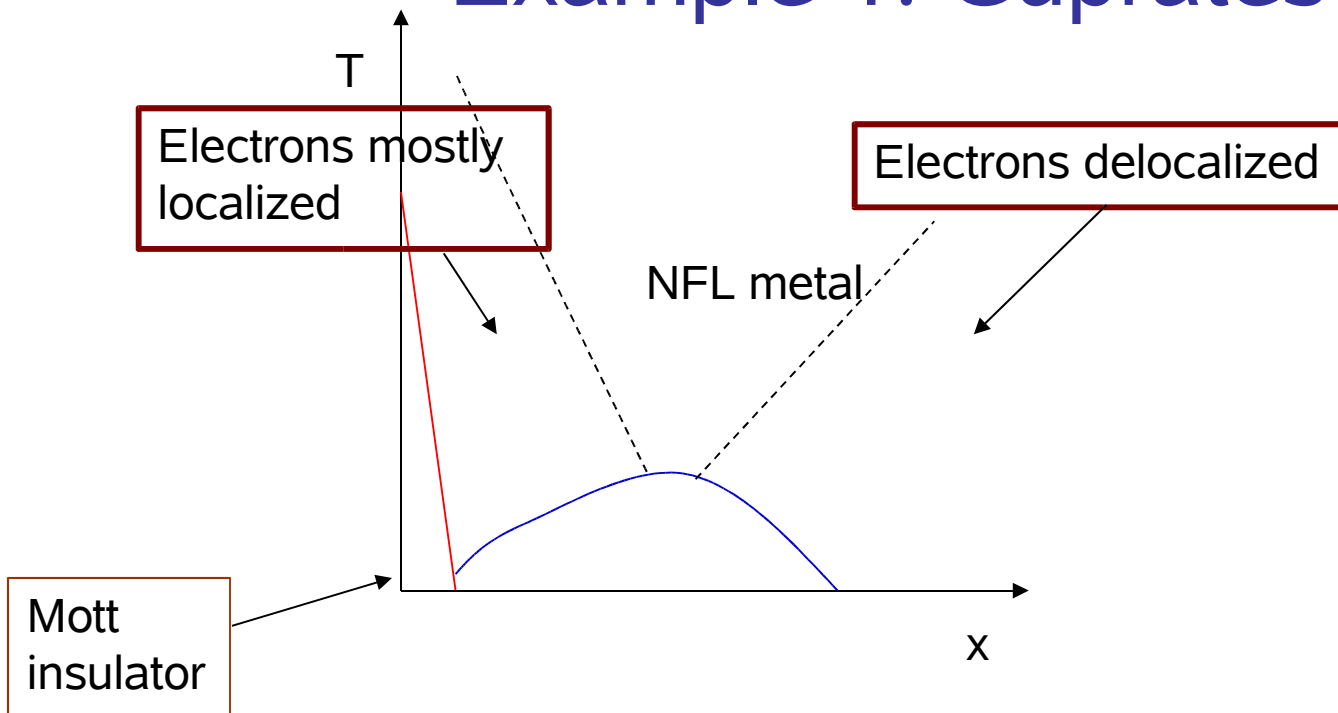
## (Radical) alternate to classical assumptions

- Universal singularity at some quantum critical points: Not due to fluctuations of natural order parameter but due to some other competing effects.
  - Order parameters/broken symmetries of phases A and B mask this basic competition.
- => Physics beyond Landau-Ginzburg-Wilson paradigm of phase transitions.

# Example 1: Cuprates

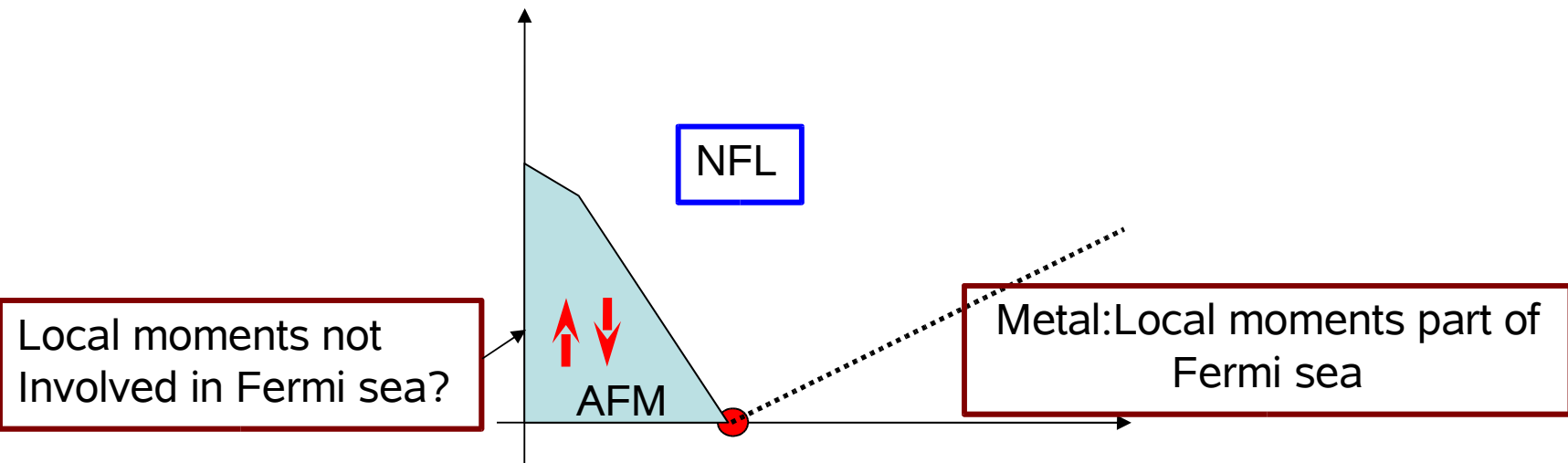


# Example 1: Cuprates



- Competition between Fermi liquid and Mott insulator
- Low energy order parameters (AF, SC, ...) mask this competition.

# Similar possibility in heavy electron systems



Critical NFL physics: fluctuations of loss of local moments from Fermi sea?  
Magnetic order – a distraction??

## This talk – more modest goal

- Are there any clearly demonstrable theoretical instances of such strong breakdown of Landau-Ginzburg-Wilson ideas at quantum phase transitions?

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- Are there any clearly demonstrable theoretical instances of such strong breakdown of Landau-Ginzburg-Wilson ideas at quantum phase transitions?

Study phase transitions in insulating quantum magnets

- Good theoretical laboratory for physics of phase transitions/competing orders.

# Highlights

- Failure of Landau paradigm at (certain) quantum transitions
- Emergence of `fractional' charge and gauge fields near quantum critical points between two CONVENTIONAL phases.
  - ``Deconfined quantum criticality'' (made more precise later).
- Many lessons for competing order physics in correlated electron systems.

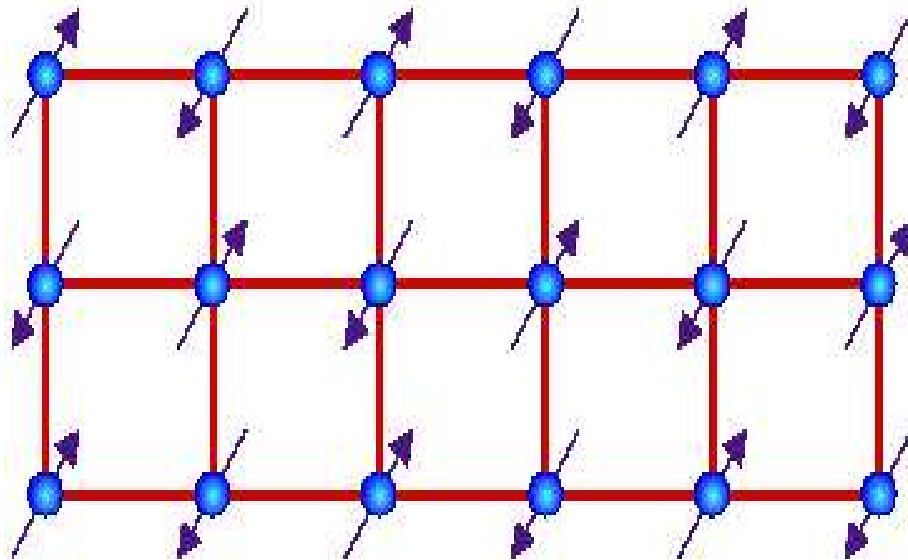
# Phase transitions in quantum magnetism

$$H = J \sum_{\langle rr' \rangle} S_r \cdot S_{r'} + \dots$$

- Spin-1/2 quantum antiferromagnets on a square lattice.
- “.....” represent frustrating interactions that can be tuned to drive phase transitions.  
(Eg: Next near neighbour exchange, ring exchange,.....).

# Possible quantum phases

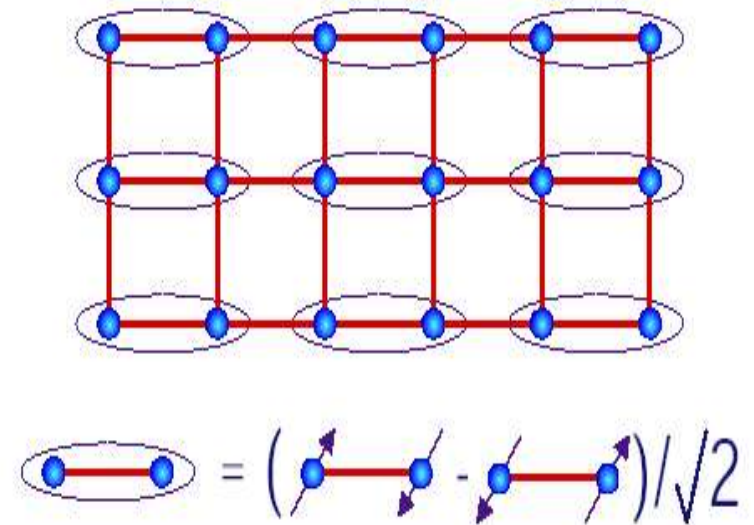
- Neel ordered state



# Possible quantum phases (contd)

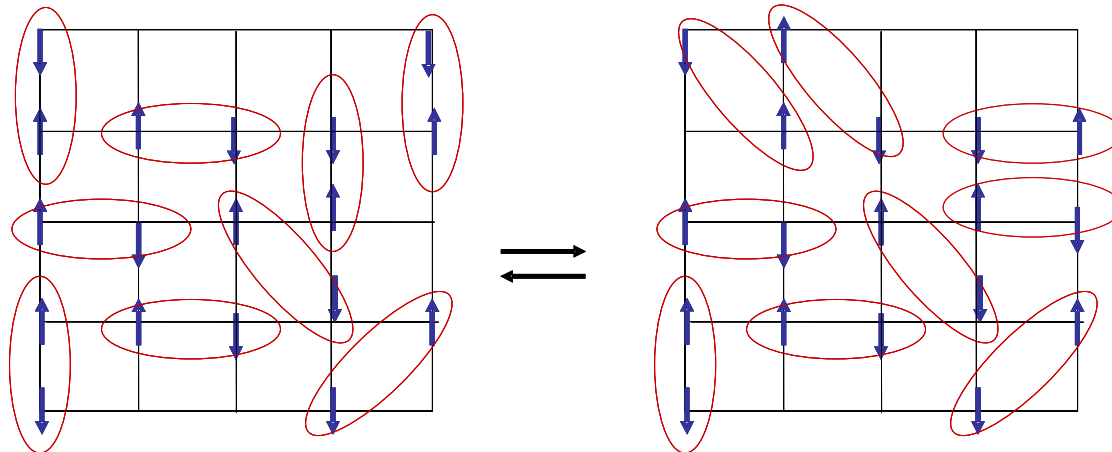
## QUANTUM PARAMAGNETS

- Simplest: Valence bond solids.
- Ordered pattern of valence bonds **breaks** lattice translation symmetry.
- Elementary spinful excitations have  $S = 1$  above spin gap.



# Possible phases (contd)

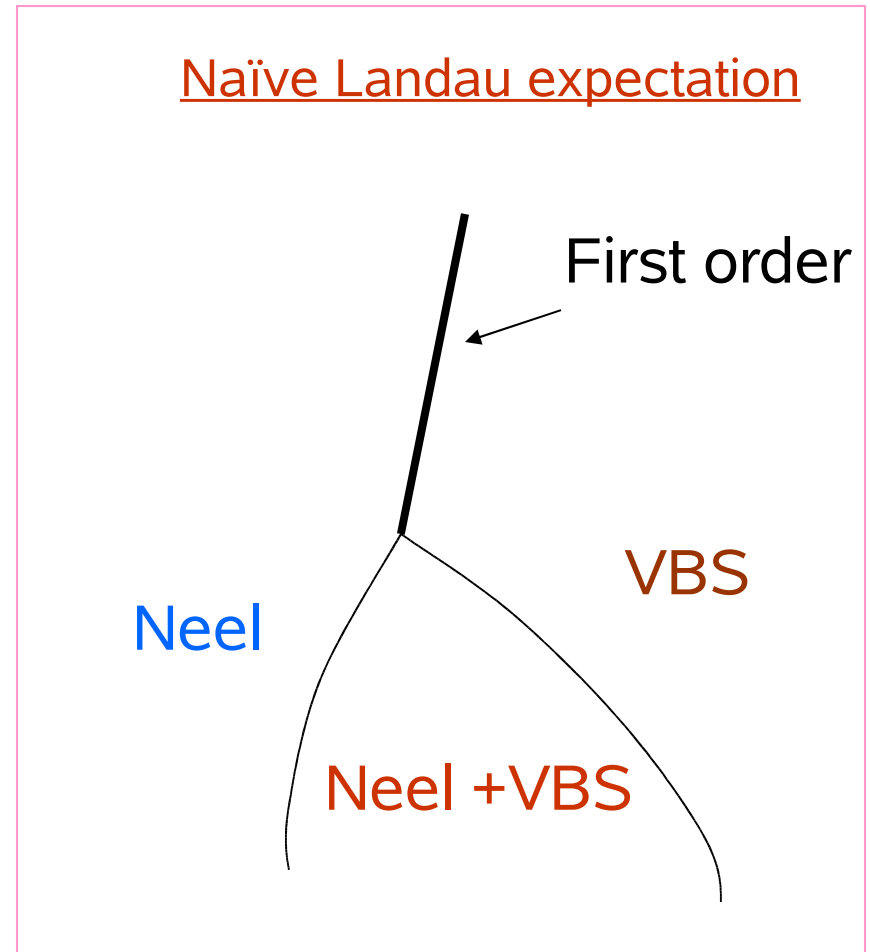
- Exotic quantum paramagnets – “resonating valence bond liquids”.
- Fractional spin excitations, interesting topological structure.



# Neel-valence bond solid(VBS) transition

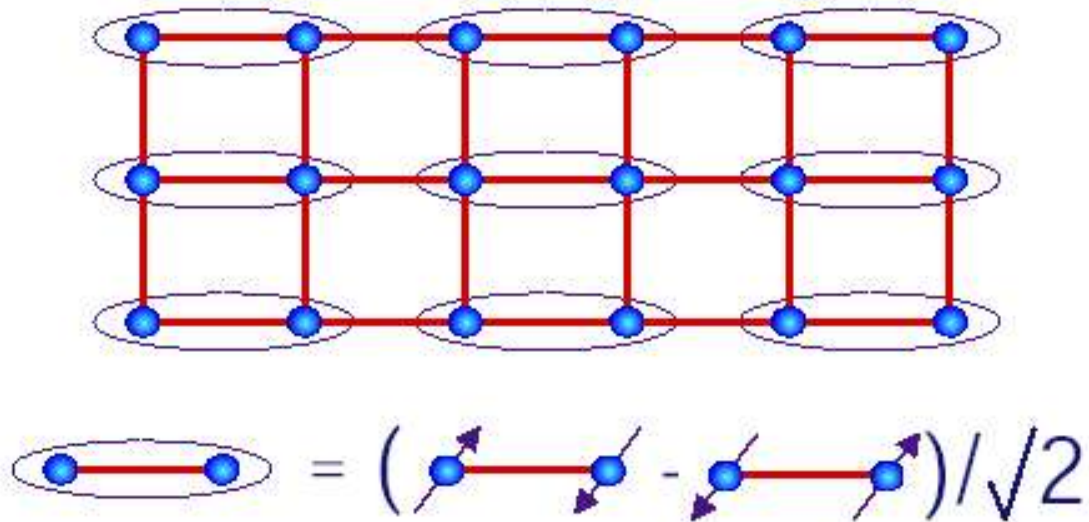
- Neel: Broken spin symmetry
- VBS: Broken lattice symmetry.
- Landau – Two independent order parameters.
- no generic direct second order transition.
- either first order or phase coexistence.

This talk: Direct second order transition but with description not in terms of natural order parameter fields.

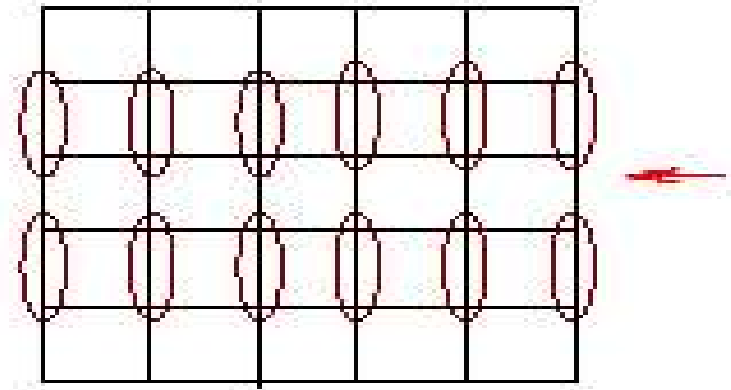
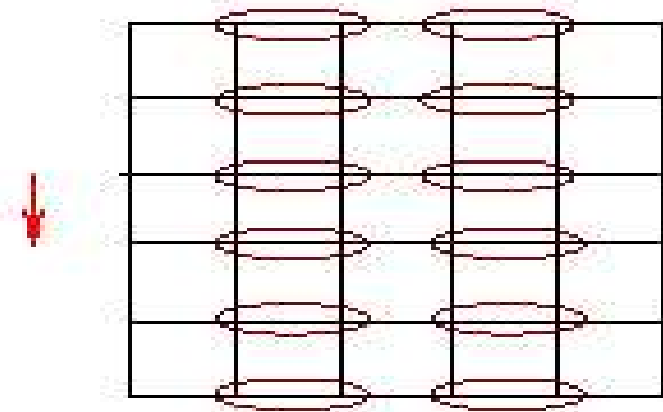
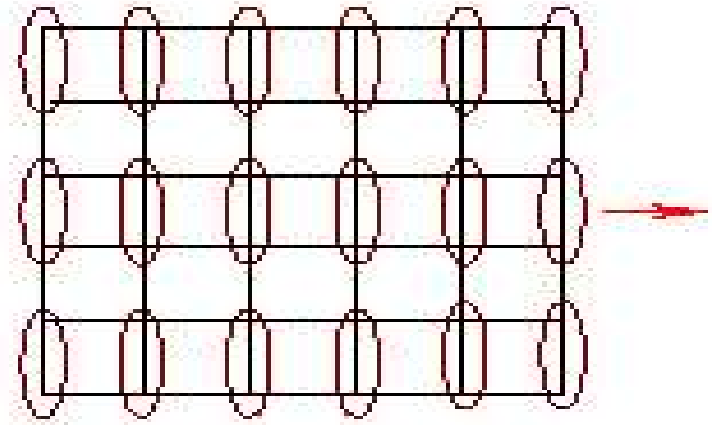
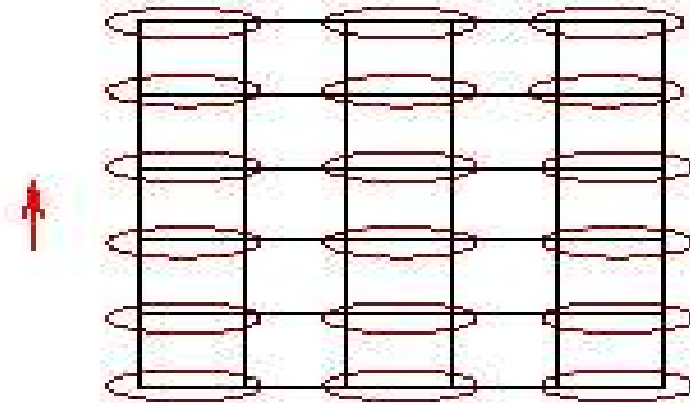


# Broken symmetry in the valence bond solid(VBS) phase

Valence bond solid with spin gap.



# Discrete $Z_4$ order parameter



# Neel-Valence Bond Solid transition

- Naïve approaches fail

Attack from Neel  $\neq$  Usual  $O(3)$  transition in  $D = 3$

Attack from VBS  $\neq$  Usual  $Z_4$  transition in  $D = 3$

(= XY universality class).

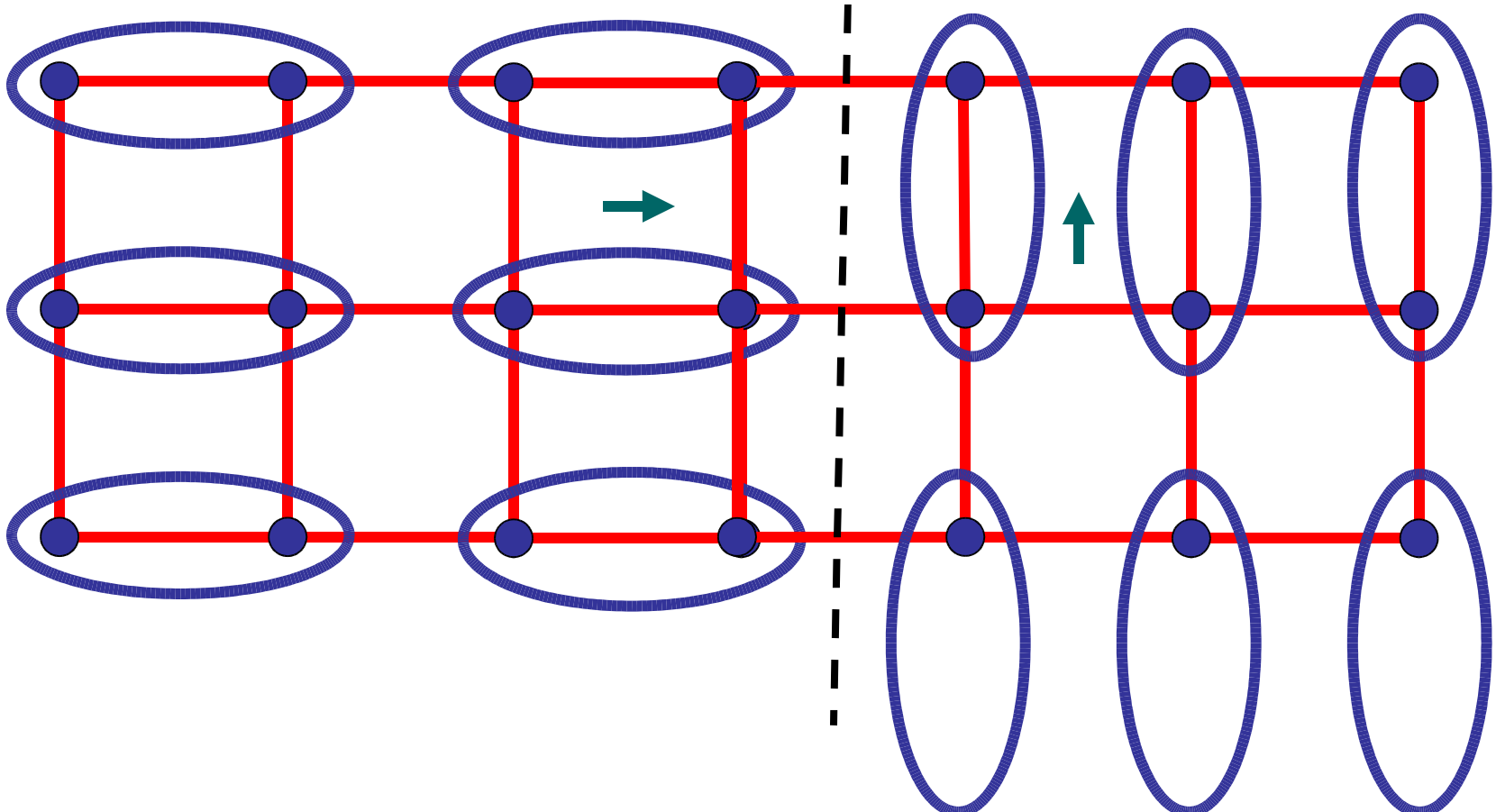
Why do these fail?

Topological defects carry non-trivial quantum numbers!

This talk: attack from VBS (Levin, TS, cond-mat/0405702)

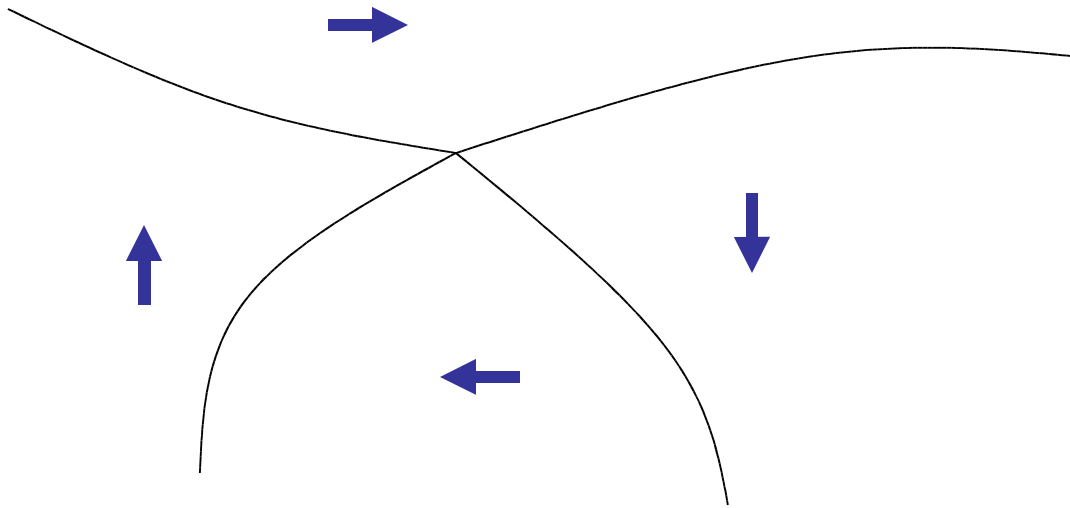
# Topological defects in $Z_4$ order parameter

- Domain walls – elementary wall has  $\pi/2$  shift of clock angle



# $Z_4$ domain walls and vortices

- Walls can be oriented; four such walls can end at point.
- End-points are  $Z_4$  vortices.

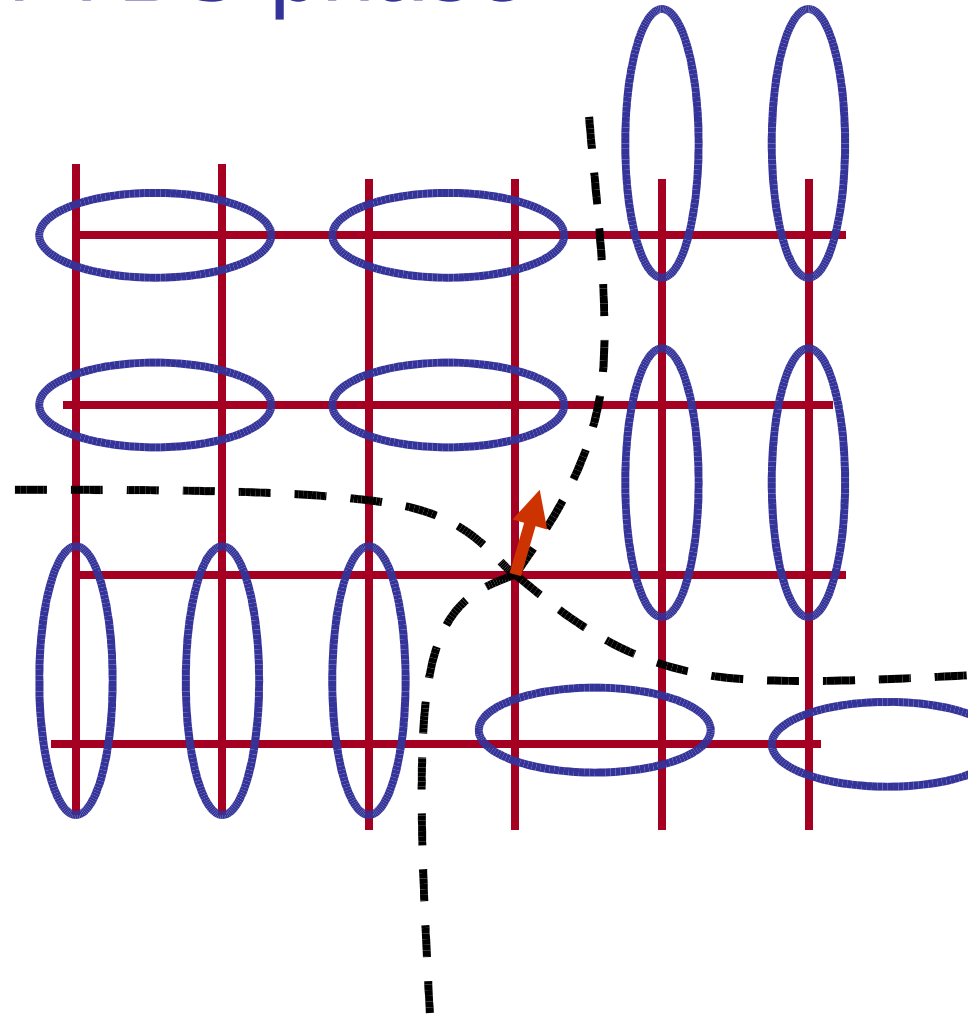


# $Z_4$ vortices in VBS phase

Vortex core has an unpaired spin-1/2 moment!!

$Z_4$  vortices are “spinons”.

Domain wall energy confines them in VBS phase.



# Disordering VBS order

- If  $Z_4$  vortices proliferate and condense, cannot sustain VBS order.
- Vortices carry spin =>develop Neel order

# $Z_4$ disordering transition to Neel state

- As for usual (quantum)  $Z_4$  transition, expect clock anisotropy is irrelevant.

(confirm in various limits).

Critical theory: (Quantum) XY but with vortices that carry physical spin-1/2 (= spinons).

# Alternate (dual) view

- Duality for usual XY model (Dasgupta-Halperin)  
Phase mode - ``photon''

Vortices – gauge charges coupled to photon.

Neel-VBS transition: Vortices are spinons

=> Critical spinons minimally coupled to fluctuating U(1) gauge field\*.

\*non-compact

# Proposed critical theory “Non-compact $CP_1$ model”

$$S = \int d^2x d\tau \left[ |(\partial_\mu - ia_\mu)z|^2 + r|z|^2 + u|z|^4 \right. \\ \left. + (\epsilon_{\mu\nu} \partial_\nu a_\lambda)^2 \right]$$

$z$  = two-component spin-1/2 spinon field

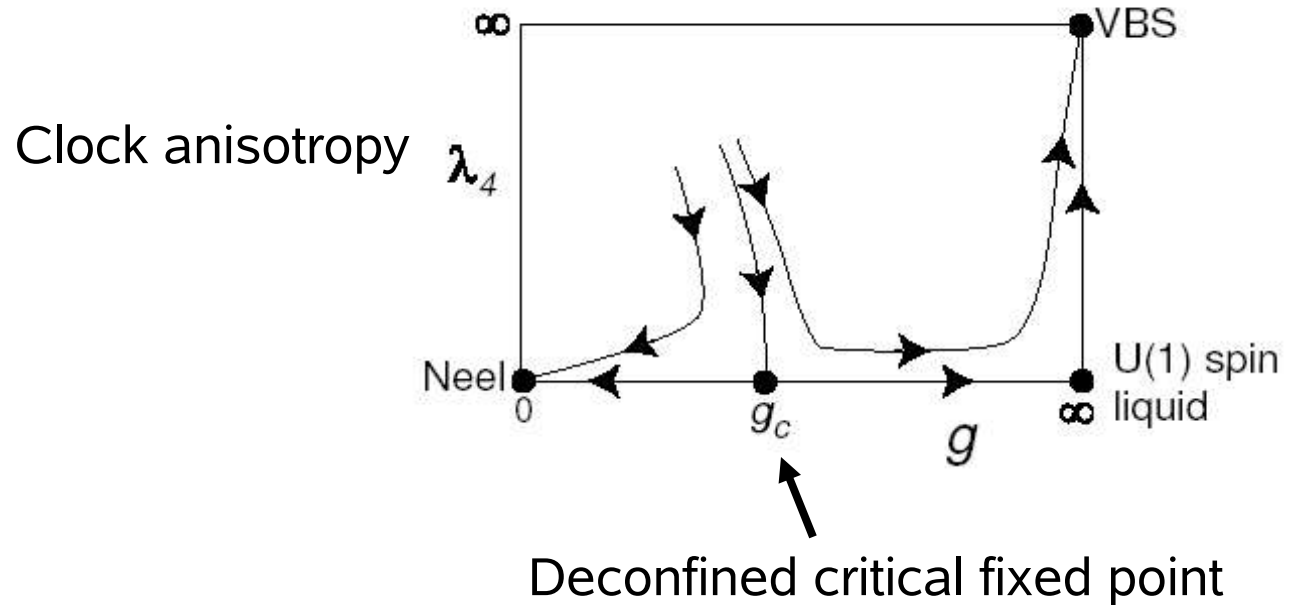
$a_\mu$  = non-compact U(1) gauge field.

Distinct from usual O(3) or  $Z_4$  critical theories\*.

Theory not in terms of usual order parameter fields  
but involve spinons and gauge fields.

\*Distinction with usual O(3) fixed point definitively established by detailed numerics (Motrunich, Vishwanath, '03)

# Renormalization group flows



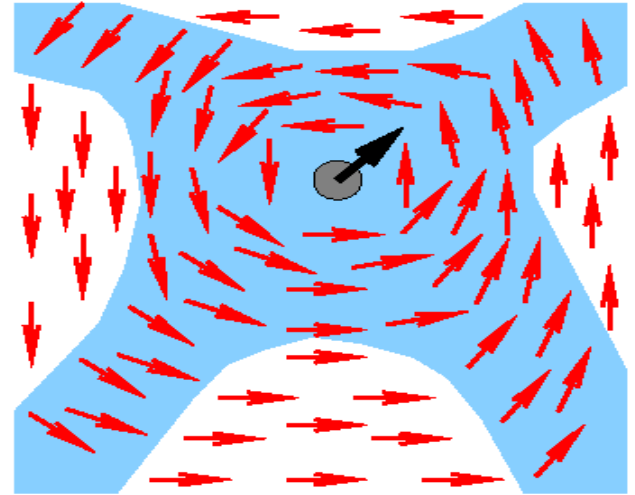
Clock anisotropy is “dangerously irrelevant”.

# Precise meaning of deconfinement

- $Z_4$  symmetry gets enlarged to XY

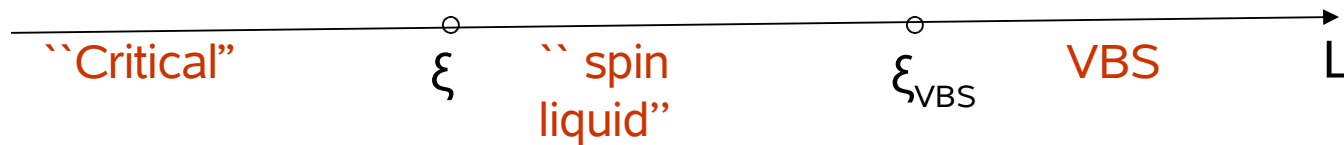
⇒ Domain walls get very thick and very cheap near the transition.

⇒ Domain wall energy not effective in confining  $Z_4$  vortices (= spinons)



Formal: Extra global U(1) symmetry  
not present in microscopic model :

# Two diverging length scales in paramagnet

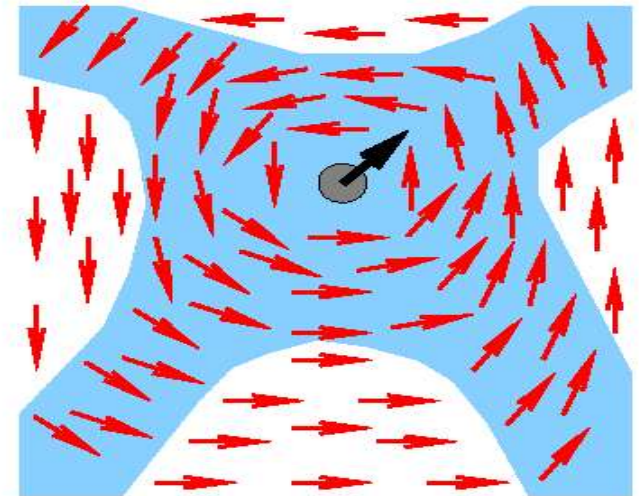


$\xi$ : spin correlation length

$\xi_{\text{VBS}}$ : Domain wall thickness.

$\xi_{\text{VBS}} \sim \xi^{\kappa}$  diverges faster than  $\xi$

Spinons confined in either phase  
but 'confinement scale' diverges at  
transition.



# Extensions/generalizations

- Similar phenomena at other quantum transitions of spin-1/2 moments in  $d = 2$
- 2. VBS- spin liquid (Senthil, Balents, Sachdev, Vishwanath, Fisher, '03)
- 2. Neel –spin liquid (Ghaemi, Senthil)
- 3. Certain VBS-VBS (Vishwanath, Balents, Senthil, '03; Fradkin, Huse, Moessner, Oganessian, Sondhi, '03)

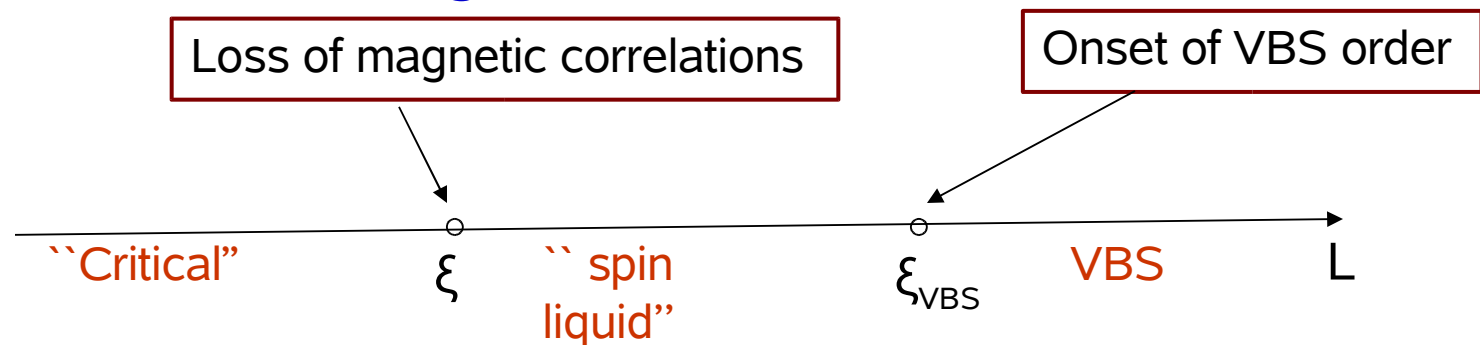
## Apparently fairly common

- Deconfined critical phases with gapless fermions coupled to gauge fields also exist in 2d quantum magnets (Hermele, Senthil, Fisher, Lee, Nagaosa, Wen, '04)
- interesting applications to cuprate theory.

# Summary and some lessons-I

- Direct 2<sup>nd</sup> order quantum transition between two phases with different competing orders possible (eg: between different broken symmetries) even if simple order parameters distinguish the two phases

Separation between the two competing orders not as a function of tuning parameter but as a function of (length or time) scale



# Summary and some lessons-II

- Striking “non-fermi liquid” (morally) physics at critical point between two competing orders.

Eg: At Neel-VBS, magnon spectral function is anomalously broad ( $\eta \sim 0.6$ )- roughly due to decay into spinons- as compared to usual critical points.

Most important lesson:

Failure of Landau paradigm – order parameter fluctuations do not capture true critical physics even if natural order parameters exist.

Strong impetus to radical approaches to NFL physics at heavy electron critical points (and to optimally doped cuprates).