Interacting electronic topological insulators in 3 dimensions

T. Senthil (MIT)

Chong Wang, Andrew C. Potter, and T. Senthil, Science (2014).

Chong Wang, T. Senthil, arxiv:1401.1142

Collaborators

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Other related collaborations: A. Vishwanath (Berkeley), Cenke Xu (UCSB), Michael Levin (Chicago), N. Regnualt (Princeton), Adam Nahum (MIT post-doc)

Some questions about interacting topological insulators

1. Are there new phases that have no non-interacting counterpart? *

2. Physical properties?

3. Experimental realization?

*Focus on ``short range entangled'' phases without fractionalization/ intrinsic topological order (``Symmetry Protected Topological'' (SPT) phases)

Plan of talk

1. Lightning review of free fermion topological insulators in 3d

2. Lightning summary of bosonic topological insulators in 3d

3. 3d interacting electron TIs

- new Z_2^3 classification

- description of the new interacting TIs

4. 3d fermionic interacting TI/TSc with other symmetries: Beyond the 10-fold way.

5. Toward materials

Review: free fermion 3d topological insulators

Characterize by

1. presence/absence of non-trivial surface states

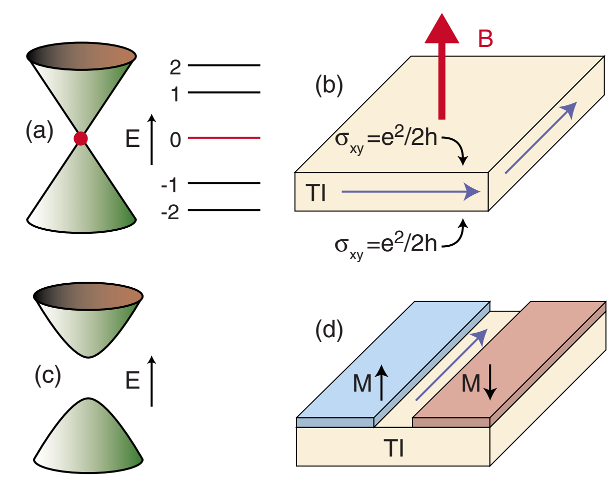
2. EM response

Surface states: Odd number of Dirac cones

Trivial gapped/localized insulator not possible at surface so long as T-reversal is preserved (even with disorder)

Review: free fermion topological insulators

EM response: Surface quantum Hall Effect



If surface gapped by B-field/proximity to magnetic insulator, surface Hall conductance

$$\sigma_{xy} = \left(n + \frac{1}{2}\right)\frac{e^2}{h}$$

Domain wall between opposite T-breaking regions: chiral edge mode of 2d fermion IQHE

Review: Free fermion topological insulators
Axion Electrodynamics

Qi, Hughes, Zhang, 09
Essin, Moore, Vanderbilt, 09

EM response of *any* 3d insulator

$$\mathcal{L}_{eff} = \mathcal{L}_{Max} + \mathcal{L}_{\theta} \quad (1)$$
$$\mathcal{L}_{\theta} = \frac{\theta}{4\pi^2}\vec{E}.\vec{B}$$

Under $\mathcal{T}$-reversal, $\theta \to -\theta$.
Periodicity $\theta \to \theta + 2\pi$: only $\theta = n\pi$ consistent with $\mathcal{T}$-reversal.
Domain wall with $\theta = 0$ insulator: Surface quantum Hall effect

$$\sigma_{xy} = \frac{\theta}{2\pi}$$

Free fermion TI: $\theta = \pi$.
Interpretation of periodicity:
$\theta \to \theta + 2\pi$: deposit a 2d fermion IQHE at surface.
Not a distinct state.

Consequences of axion response: Witten effect

External magnetic monopole in EM field:

θ term => monopole has electric charge θ/2π.

(``Witten dyons").

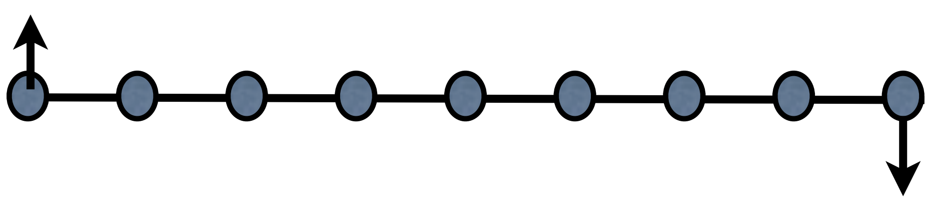
$$
\begin{aligned}
\mathcal{L}_\theta &= \frac{\theta}{4\pi^2}\vec{E}.\vec{B} \\
&= -\frac{\theta}{4\pi^2}\vec{\nabla}A_0.\vec{B} + \ldots. \\
&= \frac{\theta}{4\pi^2}A_0\vec{\nabla}.\vec{B}
\end{aligned}
$$

A very useful detour: Bosonic topological insulators

Useful stepping stone to interacting fermionic TIs.

Many new and useful non-perturbative ideas to discuss topological insulator/SPT states.

Old example: Haldane spin-1 chain
(Symmetry protected dangling spin-1/2 edge states).



d > 1: Progress in classification
1. Group Cohomology (Chen, Gu. Liu, Wen, 2011)
2. Chern-Simons approach in d =2 (Lu, Vishwanath, 2012).

Here we will need some physical ideas on d = 3 boson TI/SPTs.

Physics of 3d boson topological insulators

Vishwanath, TS, 2012

1. Quantized magneto-electric effect (eg: axion angle $\theta = 2\pi, 0$)

2. Emergent exotic (eg: fermionic, Kramers or both) vortices at surface, ...

3. Related exotic bulk monopole of external EM field (fermion, Kramers, or both) (Wang, TS, 2013; Metlitski, Kane, Fisher, 2013).

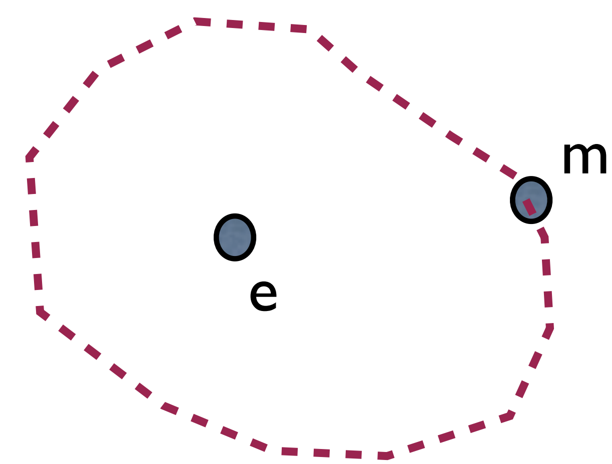
Explicit construction in systems of coupled layers: Wang, TS (2013).

Surface topological order of 3d SPTs

3d SPT surface can have intrinsic topological order though bulk does not (Vishwanath, TS, 2012).

Resulting symmetry preserving gapped surface state realizes symmetry `anomalously' (cannot be realized in strict 2d; requires 3d bulk).

Targetting such `anomalous' surface topological ordered state has been fruitful in microscopic constructions of boson TIs
(Wang, TS, 2013; Burnell, Chen, Fidkowski, Vishwanath, 2013)



Phase of π

Electronic topological insulators
(Chong Wang, A.Potter, TS 2013)

The problem

Electronic insulators with no bulk topological order/fractionalization (``short range entangled'').

Realistic Symmetry: charge conservation, T-reversal (strong spin orbit => spin not conserved)*

Band theory: Z_2 classification

Beyond band theory: Strong correlations + strong spin-orbit

How many such phases are there with strong interactions?

What are their physical properties?

* Symmetry group $U(1) \rtimes Z_2^T$

The answer

3d electronic insulators with charge conservation/T-reversal classified by Z_2^3 (corresponding to total of 8 distinct phases).

3 `root' phases:
Familiar topological band insulator, two new phases obtained as electron Mott insulators where spins form a spin-SPT (topological paramagnets).

Topological Insulator	Representative surface state	$\mathcal{T}$-breaking transport signature	$\mathcal{T}$-invariant gapless super-conductor
Free fermion TI	Single Dirac cone	$\sigma_{xy} = \frac{\kappa_{xy}}{\kappa_0} = \pm 1/2$	None
Topological paramagnet I ($eTmT$)	$\mathbb{Z}_2$ spin liquid with Kramers doublet spinon(e) and vison(m)	$\sigma_{xy} = \kappa_{xy} = 0$	$N = 8$ Majorana cones
Topological paramagnet II ($e_f m_f$)	$\mathbb{Z}_2$ spin liquid with Fermionic spinon(e) and vison(m)	$\sigma_{xy} = 0; \frac{\kappa_{xy}}{\kappa_0} = \pm 4$	$N = 8$ Majorana cones

Obtain all 8 phases by taking combinations of root phases.

Proof

Bulk EM response

Start with EM response of *any* 3d insulator

$$\mathcal{L}_{eff} = \mathcal{L}_{Max} + \mathcal{L}_\theta \tag{1}$$
$$\mathcal{L}_\theta = \frac{\theta}{4\pi^2}\vec{E}.\vec{B}$$

Under $\mathcal{T}$-reversal, $\theta \to -\theta$.
Periodicity $\theta \to \theta + 2\pi$: only $\theta = 0, \pi$ consistent with $\mathcal{T}$-reversal.

If there are 2 distinct insulators with $\theta = \pi$, can combine to make $\theta = 0$ insulator.
=> To look for new insulators, sufficient to restrict to $\theta = 0$.

Bulk magnetic monopole

Witten effect => monopole charge $\frac{\theta}{2\pi}$.
At $\theta = 0$, monopole has charge 0.
Time reversal:

$$\begin{aligned} \mathcal{T}^{-1} m \mathcal{T} &= m^\dagger \\ \mathcal{T}^{-1} m^\dagger \mathcal{T} &= m \end{aligned}$$

=> Monopole transforms trivially under T-reversal.
Symmetries of monopole fixed.

Remaining possibilities:
Bosonic versus fermionic statistics of the monopole.

Claim

Bosonic monopole:

Only allow for topological paramagnets* as new root states (simple proof next few slides)

Fermionic monopole:

Impossible in strictly 3d interacting topological insulators (somewhat difficult proof, see Wang, Potter, TS Science paper Supplement).

*Reminder: Topological paramagnet = electronic Mott insulator where spins form a bosonic SPT phase.

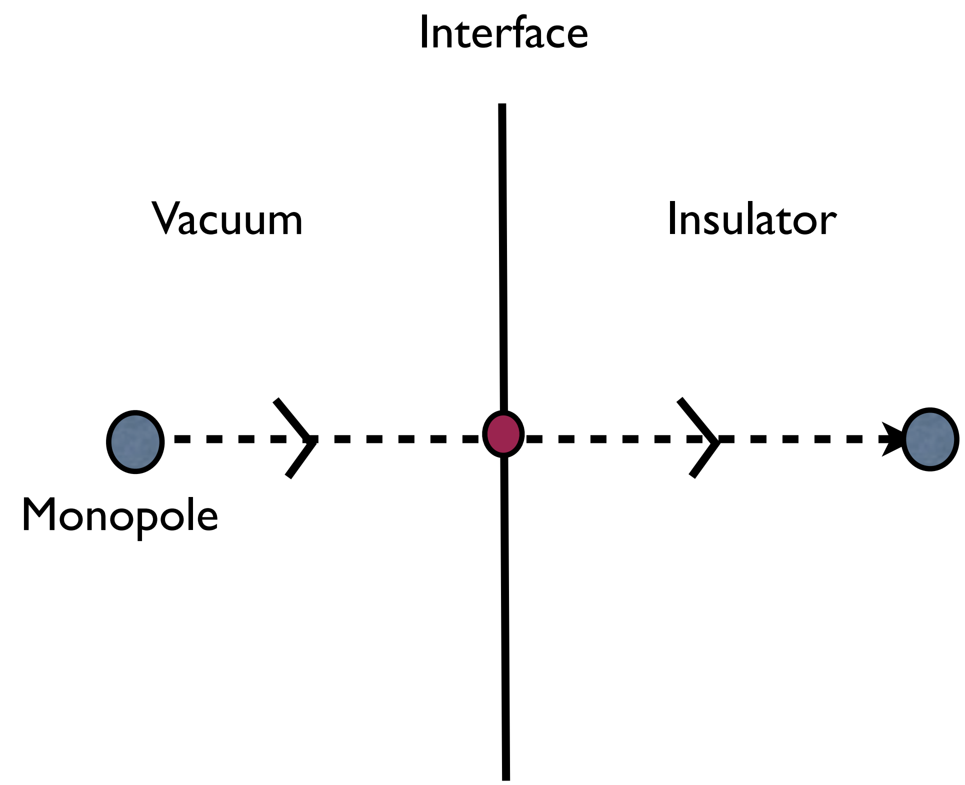
Bosonic magnetic monopole: implications for surface effective theory

Tunnel monopole from vacuum into bulk

Tunneling event leaves behind surface excitation which has charge-0 and is a boson.

A convenient surface termination- a surface superconductor*

Monopole tunneling leaves behind hc/e vortex which is a boson (and transforms trivially into a hc/e antivortex under T-reversal).



*More details : see Appendix of Wang, Potter, TS, Science 2014.

Symmetry preserving surface

Disorder the superconducting surface:

Condense the bosonic hc/e vortex.

Result: symmetry preserving insulating surface with distinct topological sectors.

hc/e vortex condensate => Charge quantized in units of e.

=> Surface theory: every topological sector can be made neutral
(integer charge => bind physical electrons to make neutral).

Surface theory = $(1, \varepsilon, ...) \times (1,c)$ = (Neutral boson theory) $\times (1,c)$

=> Bulk SPT order is same as for neutral boson SPT (supplemented by physical electron).

Neutral boson SPTs in electronic systems

From electrons form neutral composites which are bosons, and let these bosons form a boson SPT.

Describe as electron Mott insulators where spins form an SPT.

Strong spin-orbit => spin system only has T-reversal symmetry.

``Topological paramagnets" protected by time reversal symmetry.

Classification of 3d electron TI:

(band insulator classification) x (such topological paramagnets classification)

= Z_2 x (such topological paramagnets classification)

Time reversal invariant Topological paramagnets

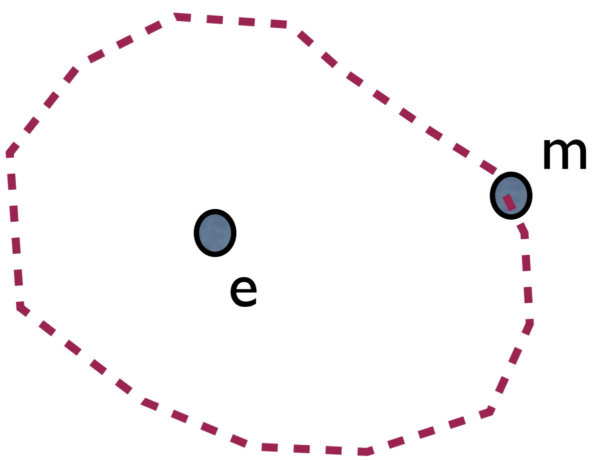
Classified by Z_2^2 with two root states conveniently characterized by surface topological order (Vishwanath, TS, 2012, Wang, TS, 2013) of deconfined Z_2 gauge theory.

1. Topological paramagnet-I

Z_2 topological order where both e and m are Kramers

2. Topological paramagnet-II (beyond `cohomology')

``All fermion'' Z_2 topological order where all topological particles are fermions



Phase of π

Classification of 3d electron TI

(Band insulator classification) x (T-reversal symmetric topological paramagnets classification)

= Z_2 x (such topological paramagnets classification)

= $Z_2 \times Z_2^2$ = Z_2^3

Explicit construction of 3d time reversal invariant Topological Paramagnets

C. Wang, TS, 2013

Strategy: stack layers of Z_2 topological ordered (``toric code") phases with symmetry allowed in strict 2d.

Transition to confine all bulk topological quasiparticles but leave deconfined surface topological order.

Engineer surface topological order specific to SPT surface.

Topological Paramagnet I

AV, TS 12
C. Wang, TS, 13

Surface topological order:

Z_2 gauge theory (``toric code") where both e and m are Kramers doublets (not possible in 2d with T-reversal).

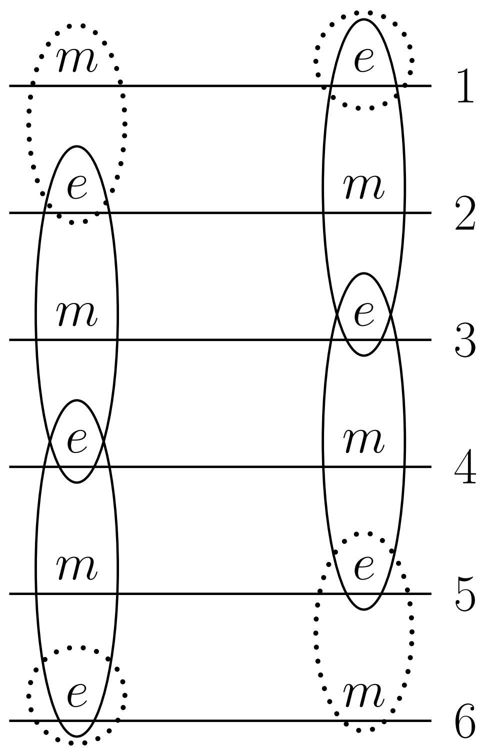
Coupled layer construction:
1. Each layer: conventional 2d Z_2 spin liquid where e_i is Kramers doublet, m_i is singlet.

2. Condense $e_i\ m_{i+1}\ e_{i+2}$ (all self and mutual bosons)

=> confine all bulk quasiparticles

3. Surface: e_1, m_1e_2 survive (at top surface)

These are mutual semions, and self-bosons
=> surface Z_2 topological order, both e_1, m_1e_2 are Kramers.



Topological Paramagnet-II

AV, TS 12
C. Wang, TS, 13
Burnell, Fidkowski, Chen, AV 13

Surface topological order:
T-reversal invariant `all fermion' Z2 gauge theory where all three topological quasiparticles are fermions

Not possible in strict 2d with T-reversal (has chiral central charge 4 mod 8).

Coupled layer construction: Start with trivial realization of T-reversal in each 2d layer, condense $\varepsilon_i m_{i+1} \varepsilon_{i+2}$ (ε_i = fermion quasiparticle in layer i)

T-broken `confined' surface: quantized thermal Hall effect (in units of quantum of thermal conductance)

$$\kappa_{xy} = \pm 4.$$

Physical characterization of the 8 interacting 3d TIs

4 insulators with $\theta = 0$: Trivial insulator + 3 topological paramagnets

4 insulators with $\theta = \pi$: Topological band insulator + 3 topological paramagnets

How to tell in experiments?

Symmetry preserving surface topological order?

Conceptually powerful characterization of surface but not very practical

Alternate: Break symmetry at surface to produce a simple state without topological order (eg: deposit ferromagnet or superconductor)

Unique experimental fingerprint!

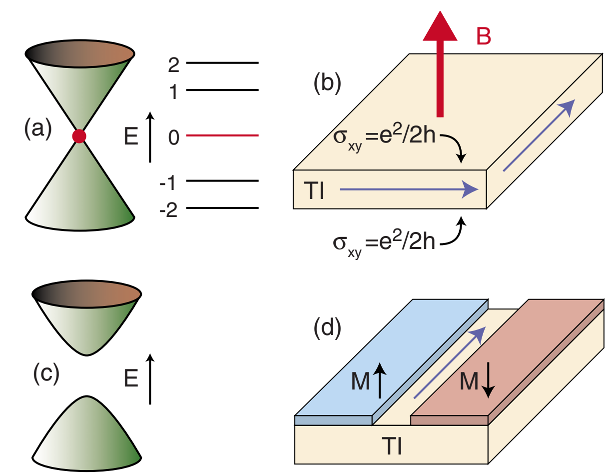
Hall transport signatures of interacting TIs

Break T explicitly to get a `trivial' surface:

Surface electrical Hall conductivity:

Band TI: $\sigma_{xy} = 1/2$ (related to $\theta = \pi$)

Topological paramagnets: $\sigma_{xy} = 0$



Surface thermal Hall conductivity*:

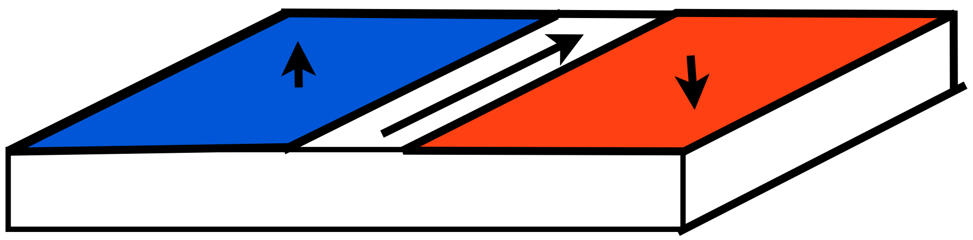
Band TI: $\kappa_{xy} = 1/2$

Topological Paramagnet-I: $\kappa_{xy} = 0$

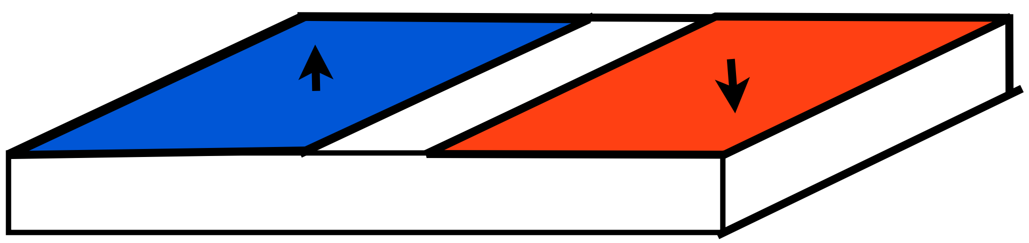
Topological Paramagnet-II: $\kappa_{xy} = 4$

*(mod 8)

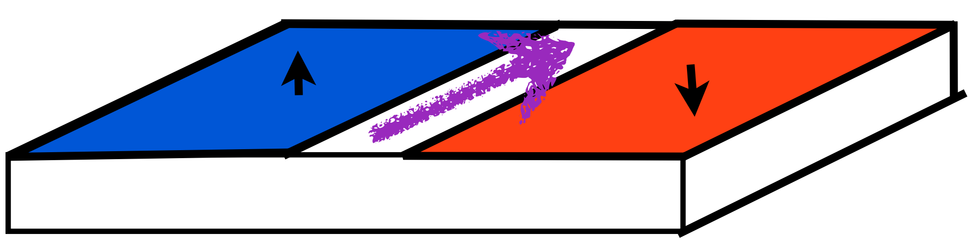
Depositing a ferromagnet: domain wall structure



Topological band insulator: Charged one-way mode
(= edge of 2d electron IQHE)



Topological paramagnet-I: Gapped domain wall



Topological paramagnet-II: Neutral one-way modes
(= edge of 2d E_8 bosonic IQHE)

Depositing s-wave superconductor: induced quasiparticle nodes

Topological band insulator: depositing s-wave SC leads to a **gapped** surface SC with interesting Majorana zero modes on vortices (Fu, Kane 08)

Topological paramagnets (both I and II):

Deposit s-wave SC => get a **gapless** nodal SC with 8 gapless Majorana cones protected by time reversal symmetry! (Wang, Potter, TS 2013)

Probe by Angle Resolved Photoemission (and tunneling, etc)

Understanding induced quasiparticle nodes

Wang, Potter, TS, Science 2014
Wang, TS, arxiv 2014

Argue in reverse:

Start with surface SC theory with 4 Dirac nodes:

$$\mathcal{L}_{free} = \sum_{i=1}^{4} \psi_i^\dagger (p_x \sigma^x + p_y \sigma^z) \psi_i, \tag{1}$$

with time-reversal acting as

$$\mathcal{T} \psi_i \mathcal{T}^{-1} = i\sigma_y \psi_i^\dagger. \tag{2}$$

Time reversal symmetric free fermion perturbations cannot gap the nodes (same as surface of certain free fermion topological SC).

But interactions can induce a gap while preserving symmetry.

Understanding induced quasiparticle nodes

Enlarge symmetry to $U(1) \times \mathcal{T}$ with $U(1)$ acting as

$$U_\theta \psi U_\theta^{-1} = e^{i\theta}\psi \tag{1}$$

Consider pairing mass that breaks $U(1)$ and $\mathcal{T}$ but preserves a combination:

$$\mathcal{L}_{gap} = i\Delta \sum_{i=1}^{4} \psi_i \sigma_y \psi_i + h.c. \tag{2}$$

Now attempt to disorder the broken symmetry => proliferate vortices in the pairing order parameter.

However vortices have zero modes which restrict which kinds can condense.

Understanding induced quasiparticle nodes

For N = 4 Dirac nodes, can condense strength-2 vortices.

Result: Z_2 topological order where e and m are both Kramers (eTmT) = surface topological order of Topological Paramagnet-I.

(Can now get rid of auxiliary $U(1)$).

Closely related to a result using other methods: Classification of interacting topological SC with time reversal by Fidkowski, Chen, Vishwanath (2013)

Other symmetries: Beyond the 10-fold way

Ideas discussed above enable us to determine stability to interactions of all the 3d free fermion topological insulators/SC represented in ``10-fold way" and in many cases to get the full classification. (Wang, TS, arxiv:1401:1142)

SC with time reversal (class D III): Free fermion classification: Z

Interactions reduce this to Z_{16} (`Walker-Wang' methods: Fidkowski, Chen, Vishwanath (2013)).

Our arguments give elementary understanding of this result, and to generalize to other symmetries.

Results for other symmetries (Wang, TS, arxiv:1401:1142)

Symmetry class	Reduction of free fermion states	Distinct boson SPT	Complete classification
$U(1)$ only (A)	0	0	0
$U(1) \rtimes \mathbb{Z}_2^T$ with $\mathcal{T}^2 = -1$ (AII)	$\mathbb{Z}_2 \to \mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^3$
$U(1) \rtimes \mathbb{Z}_2^T$ with $\mathcal{T}^2 = 1$ (AI)	0	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$
$U(1) \times \mathbb{Z}_2^T$ (AIII)	$\mathbb{Z} \to \mathbb{Z}_8$	$\mathbb{Z}_2$	$\mathbb{Z}_8 \times \mathbb{Z}_2$
$U(1) \rtimes (\mathbb{Z}_2^T \times \mathbb{Z}_2^C)$ (CII)	$\mathbb{Z}_2 \to \mathbb{Z}_2$	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^5$
$(U(1) \rtimes \mathbb{Z}_2^T) \times SU(2)$	0	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^4$
$\mathbb{Z}_2^T$ with $\mathcal{T}^2 = -1$ (DIII)	$\mathbb{Z} \to \mathbb{Z}_{16}$	0	Z_{16} (?)
$SU(2) \times \mathbb{Z}_2^T$ (CI)	$\mathbb{Z} \to \mathbb{Z}_4$	$\mathbb{Z}_2$	$Z_4 \times Z_2$ (?)

Example: SC with spin U(1) and T-reversal (class A III)
Z classification in free fermion becomes Z_8 x Z_2 with interactions.

Results for other symmetries

(Wang, TS, arxiv:1401:1142)

Spin-orbit coupled insulators

Symmetry class	Reduction of free fermion states	Distinct boson SPT	Complete classification
$U(1)$ only (A)	0	0	0
$U(1) \rtimes \mathbb{Z}_2^T$ with $\mathcal{T}^2 = -1$ (AII)	$\mathbb{Z}_2 \to \mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^3$
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$(U(1) \rtimes \mathbb{Z}_2^T) \times SU(2)$	0	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^4$
$\mathbb{Z}_2^T$ with $\mathcal{T}^2 = -1$ (DIII)	$\mathbb{Z} \to \mathbb{Z}_{16}$	0	Z_{16} (?)
$SU(2) \times \mathbb{Z}_2^T$ (CI)	$\mathbb{Z} \to \mathbb{Z}_4$	$\mathbb{Z}_2$	$Z_4 \times Z_2$ (?)

Results for other symmetries

(Wang, TS, arxiv:1401:1142)

Spin-orbit coupled insulators

Spin-rotation invariant insulators

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$U(1) \rtimes \mathbb{Z}_2^T$ with $\mathcal{T}^2 = -1$ (AII)	$\mathbb{Z}_2 \to \mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^3$
$U(1) \rtimes \mathbb{Z}_2^T$ with $\mathcal{T}^2 = 1$ (AI)	0	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$
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Results for other symmetries

(Wang, TS, arxiv:1401:1142)

Spin-orbit coupled insulators

Spin-rotation invariant insulators

Topological SC

Topological SC with spin SU(2)

Symmetry class	Reduction of free fermion states	Distinct boson SPT	Complete classification
$U(1)$ only (A)	0	0	0
$U(1) \rtimes \mathbb{Z}_2^T$ with $\mathcal{T}^2 = -1$ (AII)	$\mathbb{Z}_2 \to \mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^3$
$U(1) \rtimes \mathbb{Z}_2^T$ with $\mathcal{T}^2 = 1$ (AI)	0	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$
$U(1) \times \mathbb{Z}_2^T$ (AIII)	$\mathbb{Z} \to \mathbb{Z}_8$	$\mathbb{Z}_2$	$\mathbb{Z}_8 \times \mathbb{Z}_2$
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$(U(1) \rtimes \mathbb{Z}_2^T) \times SU(2)$	0	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^4$
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Results for other symmetries

(Wang, TS, arxiv:1401:1142)

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$SU(2) \times \mathbb{Z}_2^T$ (CI)	$\mathbb{Z} \to \mathbb{Z}_4$	$\mathbb{Z}_2$	$Z_4 \times Z_2$ (?)

Many new insights:

1. In some cases there is no symmetry preserving surface topological order (minimal SC with spin SU(2) and T-reversal): ``symmetry-enforced gaplessness.

2. new parton constructions of topological paramagnets suggesting possible `realistic' model, and may be material identifications (in progress).

Symmetry enforced gaplessness: Topological SC with spin SU(2), time reversal.

C. Wang, TS, 2014

Free fermions: classified by Z (collapses to Z_4 with interactions).

Surface has n gapless Dirac cones per spin

n = 1: Gauging SU(2) => SU(2) gauge theory at $\theta = \pi$.

Stable to interactions.

n = 2:

2 gapless Dirac cones per spin.

Interactions => boson SPT formed out of spins alone (and protected by just time reversal).

Symmetry enforced gaplessness: Proof by contradiction

C. Wang, TS, 2014

Assume n = 1 state admits symmetric Surface Topological Order (STO)

TQFT = (1, X, Y, ...) x (1, c_α)

where X, Y, .. are SU(2) singlets and c_α is the electron.

(1, X, Y,....): consistent topological theory of spin singlets that is closed under time reversal

=> STO of some boson SPT protected by T-reversal.

But 2 copies of this is also a boson SPT protected by T-reversal.

T-reversal symmetric boson SPTs do not admit a `square- root'.

=> Minimal topological superconductor in class C1 does not have symmetric surface topological order!

Towards realistic models/materials

C. Wang, Adam Nahum, TS, in progress.

Frustrated spin-1 magnets in 3d

Possible platform for Topological Paramagnet-I (*)

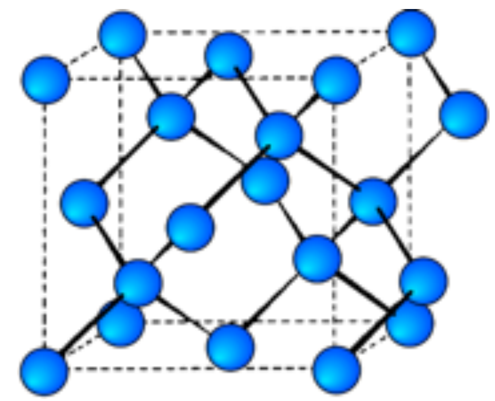
1. Simple picture for ground state wavefunction

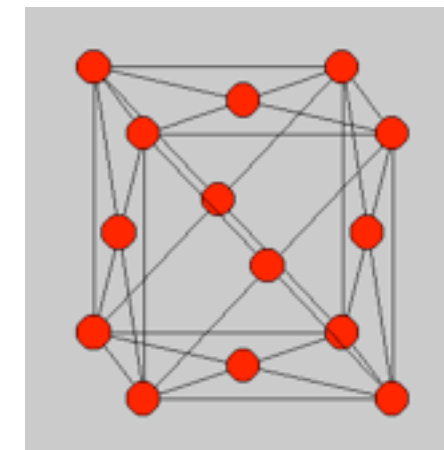
2. Natural parton construction

*Surface topological order is Z2 gauge theory with both e and m particles Kramers.

Spin-1 model wavefunction

Diamond lattice with frustrating interactions.



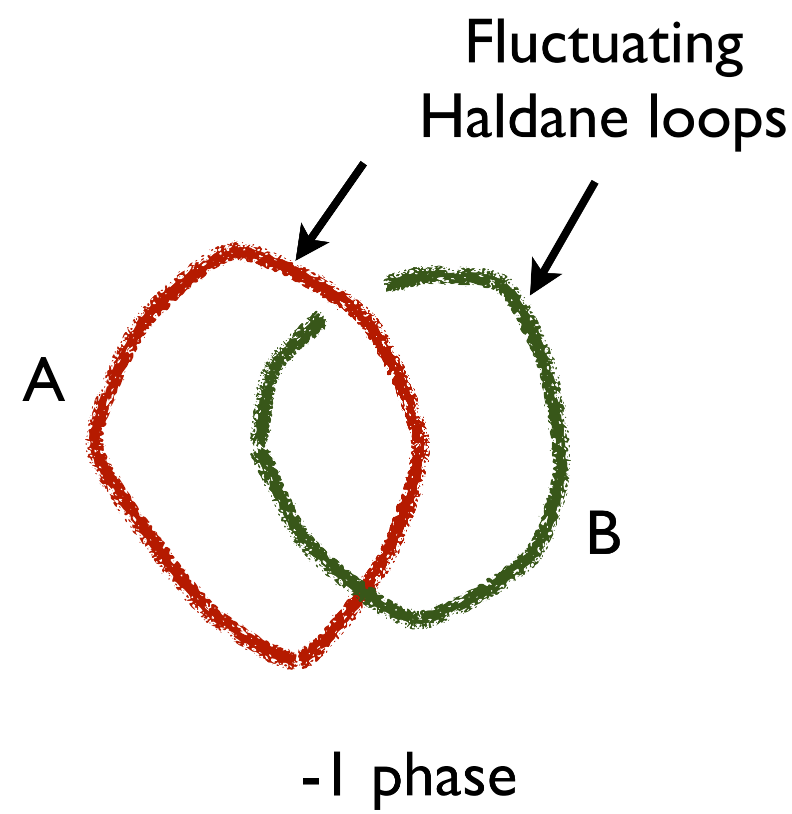


Diamond = two interpenetrating FCC lattices (label A and B sublattices)

Close pack each FCC lattice with loops.

For each loop let the spin-1 form a Haldane chain.

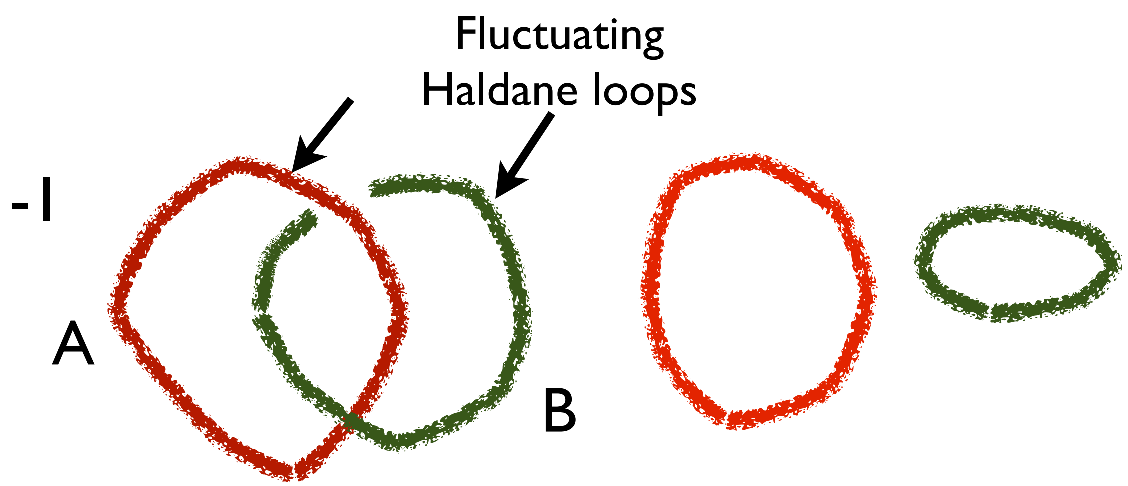
Sum over all loop configurations with phase factors.



$$|\psi\rangle \sim \sum_{L_A, L_B} (-1)^{l(L_A, L_B)} |L_A, L_B\rangle$$

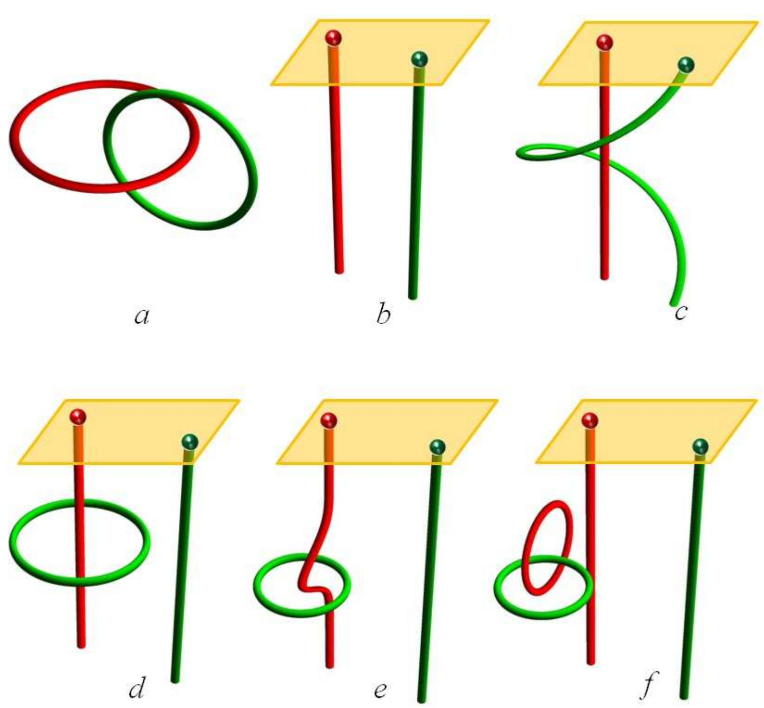
$l(L_A, L_B) =$ linking number of A loops with B loops.

Physics of the wavefunction



Spin-1 ``dimer liquid”:

(-1) linking phase confines all non-trivial bulk excitations but what about surface?



End points at surface of both A and B loops are Kramers doublets.
These end points are mutual semions

=> surface is Z2 gauge theory where both e and m are Kramers

(= surface of Topological Paramagnet-I).

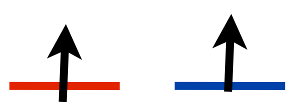
Closely related to Walker-Wang wavefunctions

Slide particle (`Parton') construction

C. Xu et al, 2012

Two orbital fermionic parton description of spin-1

$$\vec{S}_i = \frac{1}{2} f^\dagger_{ia\alpha} \vec{\sigma}_{\alpha\beta} f_{ia\beta}$$



a: color index = 1,2.

α, β: spin index = $\uparrow, \downarrow$.

4 complex fermions, i.e 8 Majorana fermions per site leading to Sp(4) gauge redundancy.

Parton construction of topological paramagnet

Strategy: Parton mean field ansatz

1. Put partons in a mean field that leaves an $SU_g(2)$ gauge group unbroken.

2. Mean field band structure: n = 2 topological superconductor for $SU_g(2) \times T$.

Surface theory: 2 Dirac cones/spin

With interactions this can be reduced to Z2 gauge theory where e and m are Kramers (and are spin-0 under $SU_g(2)$)

+ gapped excitations that are not singlet under $SU_g(2)$.

Parton construction of topological paramagnet

Fluctuations:

Bulk: $SU_g(2)$ fluctuations confine the partons - `trivial' paramagnet.

Surface: Retain surface topological order of Topological Paramagnet-I but confine all gapped non-singlet (under $SU_g(2)$) excitations.

=> Confinement produces Topological Paramagnet-I state.

$$|\psi\rangle = P_G|n = 2 \;\; TSc\rangle$$

This approach: potentially useful in evaluating energetics of state in realistic spin-1 models.

Magnetic materials with S > 1/2 and with strong frustration known in 3d.
(eg: ``A-site spinel oxides'' $Co_2 AlO_4$, (S = 3/2), Mn_2AlO_4 (S = 5/2): frustrated diamond lattice magnets; S = 1 examples?).

Summary

1. Interacting electron TIs in 3d have a Z_2^3 classification

- apart from trivial and topological band insulators, 6 new TI phases with no non-interacting counterpart.

2. Unique experimental fingerprint:

Deposit ferromagnet at surface: Quantum `anomalous' ***electrical*** and ***thermal*** Hall effect of surface.
Deposit s-wave SC at surface: Presence/absence of 4 induced gapless Dirac nodes.

3. Progress in understanding interacting 3d fermion SPTs with many symmetries.

3. Hints to important open question:

what kinds of real electronic insulators may be these new 3d TIs?