Dualities in condensed matter physics:

Quantum Hall phenomena, topological insulator surfaces, and quantum magnetism

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Chong Wang, Adam Nahum, Max Metlitski, Cenke, Xu, TS, arXiv (today)

Wang, TS, PR X 15, PR X 16, PR B 16

Seiberg, TS, Wang, Witten, Annals of Physics, 16.

Related papers: Metlitski, Vishwanath, PR B 16; Metlitski, 1510.05663
Son, PR X 15

Karch, Tong, PR X 16; Murugan, Nastase, 1606.01912; Benini, Hsin, Seiberg, 17

Plan

Two contemporary topics in condensed matter physics

1. Phase transitions beyond the Landau-Ginzburg-Wilson paradigm in quantum magnets

2. Composite fermions and the half-filled Landau level

Dualities will be useful (but with a perspective that is slightly different from what may be familiar in high energy physics).

Many connections with themes of both CFT and QHE workshops.

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Two contemporary topics in condensed matter physics

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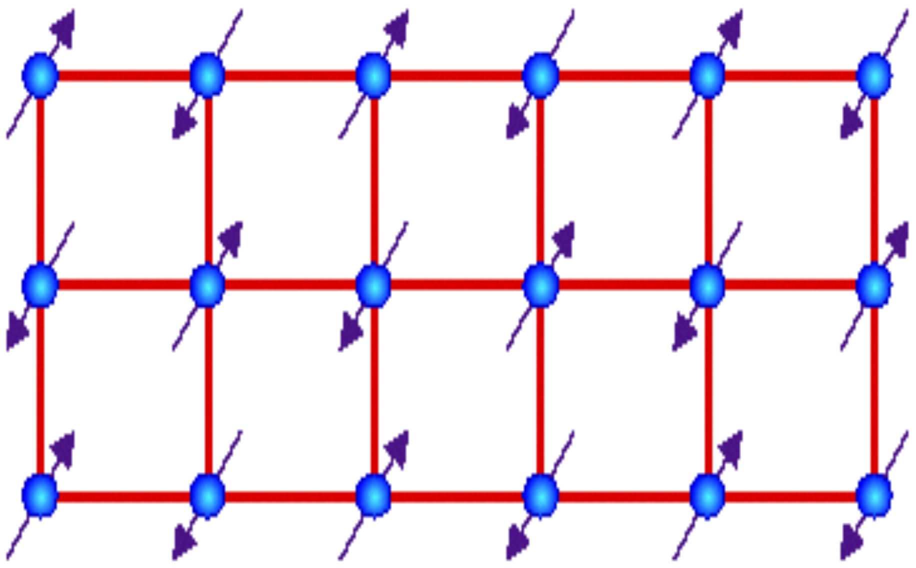
Phase transitions in quantum magnets

Spin-1/2 magnetic moments on a square lattice

Model Hamiltonian

$$H_0 = J \sum_{<rr'>} \vec{S}_r \cdot \vec{S}_{r'} + \cdots$$

$\cdots$ = additional interactions to tune quantum phase transitions



Usual fate: Neel antiferromagnetic order

Breaks SO(3) spin rotation symmetry.

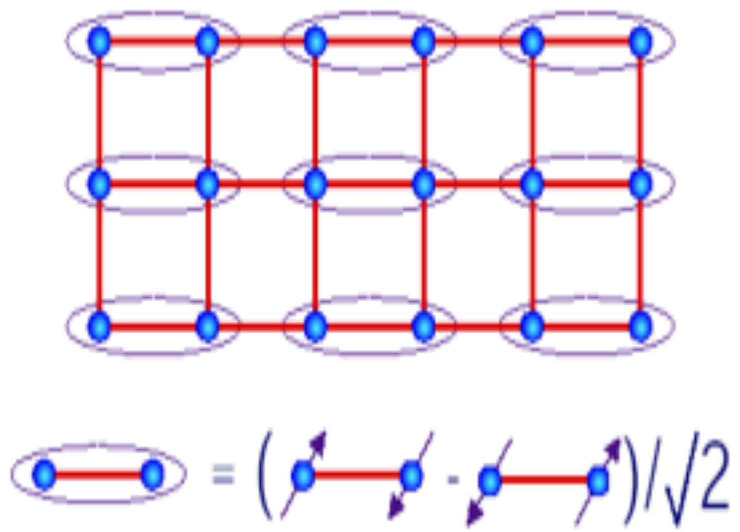
Neel order parameter: SO(3) vector

An SO(3) symmetry preserving phase (``quantum paramagnet")

With suitable additional interactions, obtain other phases that preserve spin rotation symmetry.

Here I will focus on a particular such phase called a Valence Bond Solid (VBS) that breaks lattice symmetries.

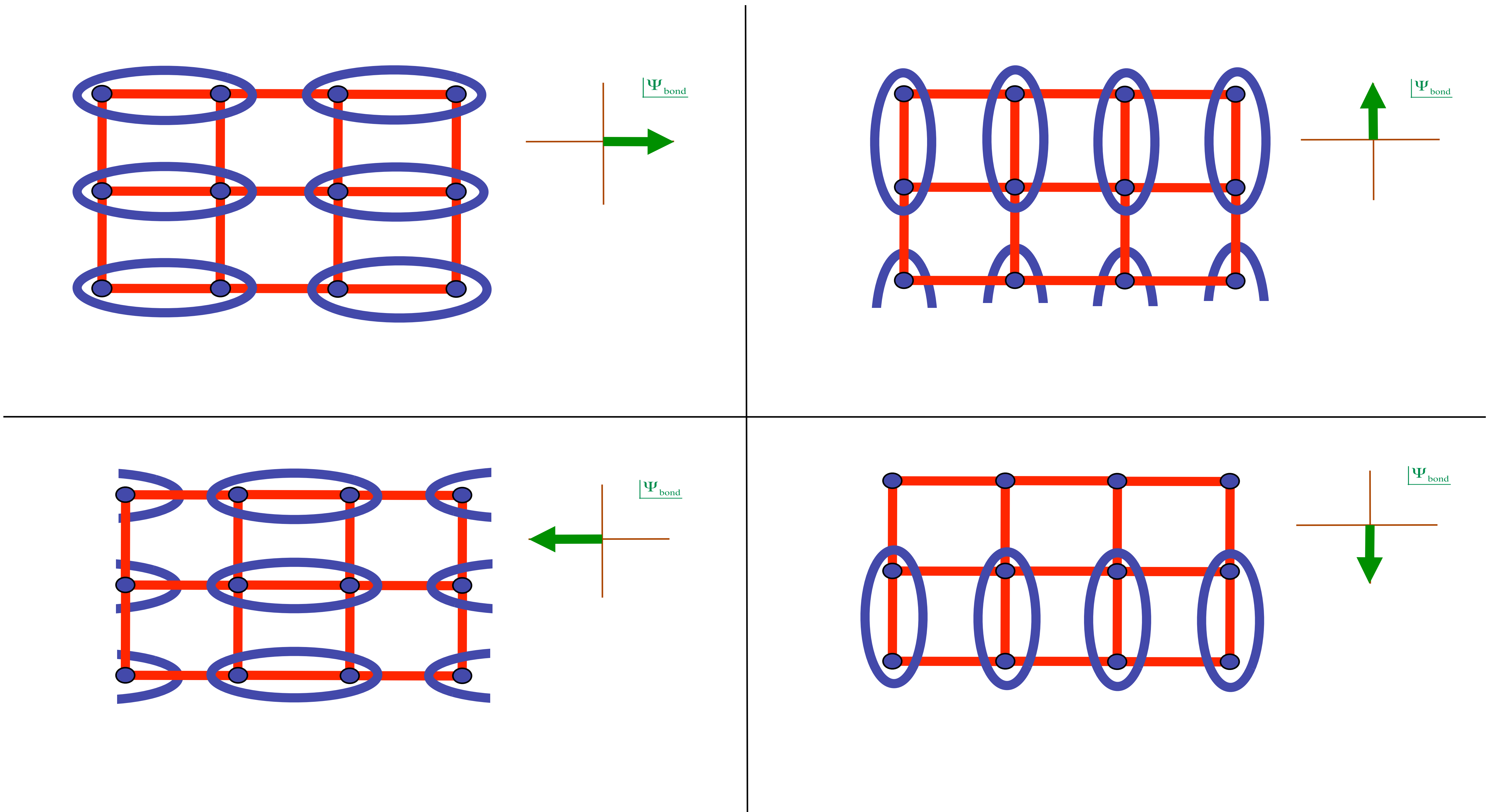
Z_4 order parameter associated with four patterns of VBS ordering.



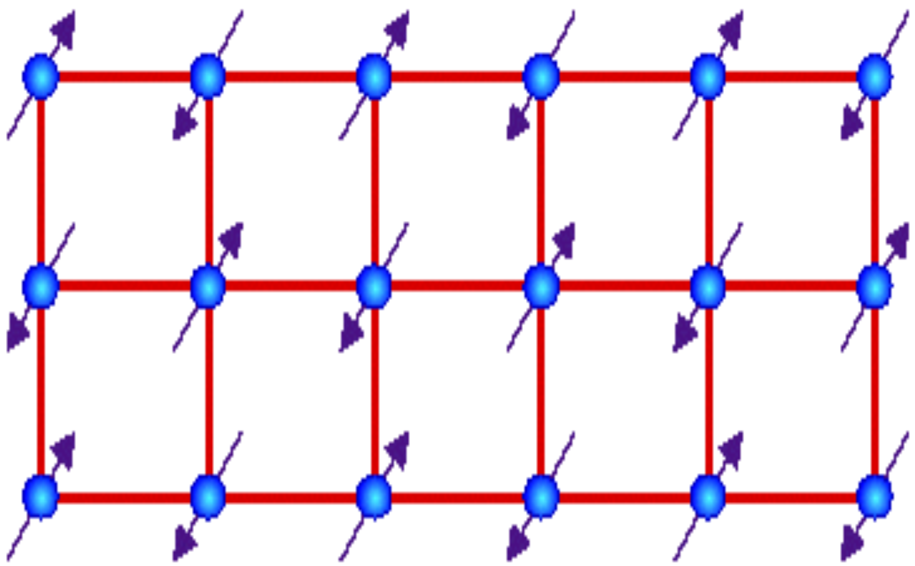
(Read, Sachdev, 89-91; many lattice models, eg, Sandvik 06)
* (VBS a.k. a ``Spin-Peierls")

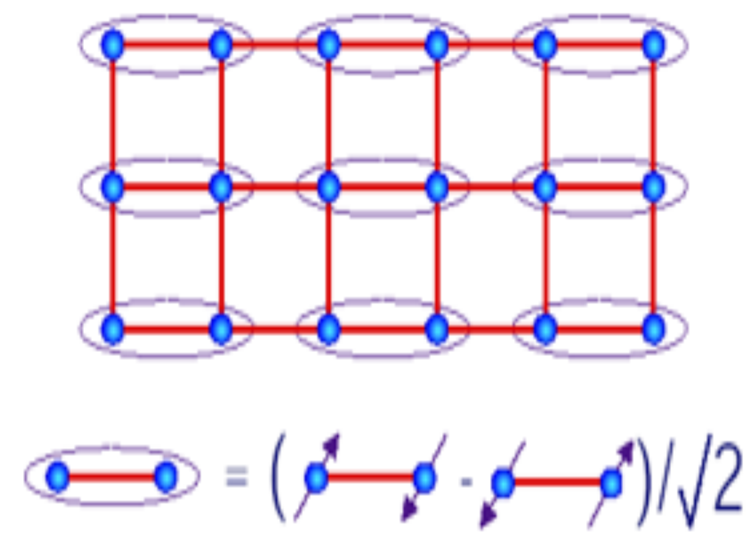
VBS Order Parameter

- Associate a Complex Number Ψ_{bond}



The Neel-VBS quantum phase transition: A lightning review





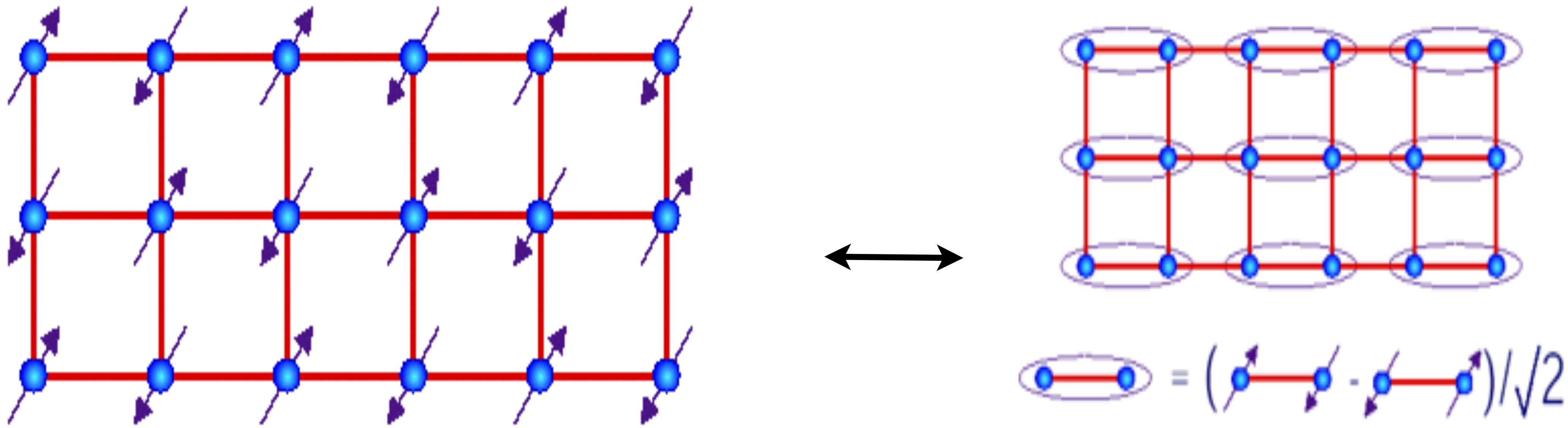
Naive Landau expectation: Two independent order parameters - no generic direct second order transition.

Naive expectation is incorrect: Possibility of a continuous Landau-forbidden phase transition between Landau allowed phases

The Neel-VBS transition: A lightning review

TS, Vishwanath, Balents, Sachdev, Fisher 2004

Possible Landau-forbidden continuous transitions between Landau allowed phases



Field theoretic framework:

$$\mathcal{L} = \sum_{\alpha=1,2} |D_b z_\alpha|^2 + V(|z|^2) + \cdots$$

z_α: $SU(2)$ doublet (“spinon”)

b: dynamical $U(1)$ gauge field.

$\cdots$: all allowed local operators consistent with symmetries of lattice magnet.

Comments

$$\mathcal{L} = \sum_{\alpha=1,2} |D_b z_\alpha|^2 + V(|z|^2) + \cdots$$

- Theory known as "Non-compact CP^1 model" $(NCCP^1)$

  Monopole operators in b not added to action

- Neel order parameter $\vec{N} = z^\dagger \vec{\sigma} z$

  VBS order parameter $\psi_{VBS} = \mathcal{M}_b$ (monopole operator)

Read, Sachdev, 89; Haldane 88

Theory not in terms of natural order parameters but in terms of \`fractional spin' fields z + gauge fields.

\`\`Deconfined quantum critical point"

TS, Vishwanath, Balents, Sachdev, Fisher 2004

Physical mechanism for non-Landau transition

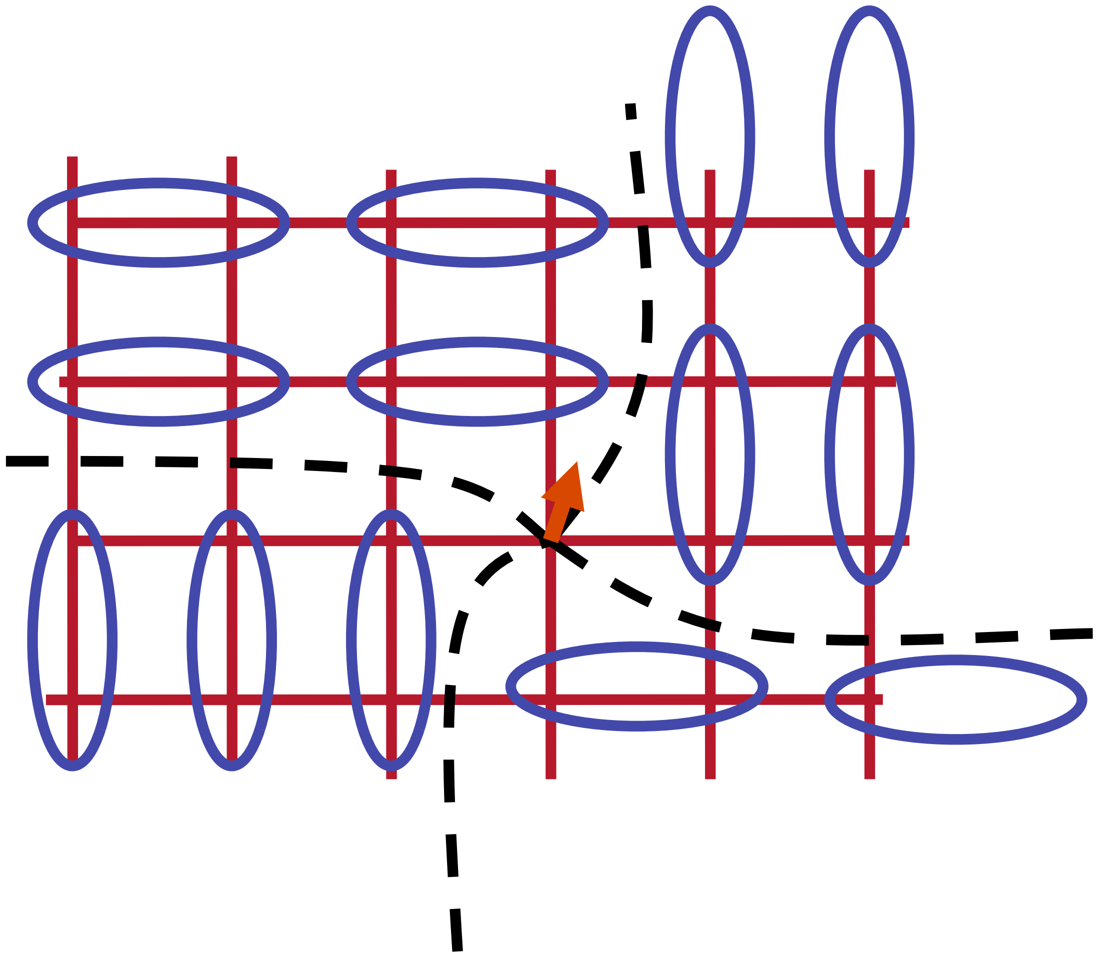
Topological defects carry non-trivial quantum numbers.

Eg: VBS phase with a Z_4 order parameter:

Z_4 vortices where four elementary domain walls meet hosts spin-1/2 moment.

Transition to Neel phase:

Proliferate these vortices.



Levin, TS 04

Putative critical theory (using usual charge-vortex duality):
$NCCP^1$ (+ anisotropies)

Identify z_α with the VBS vortex.

A formal description of Neel/VBS competition

Tanaka, Hu 06
TS, Fisher, 06

Form 5-component unit vector $\in S^4$

$$\hat{n} = (n^1, n^2, n^3, n^4, n^5), \quad \hat{n}^2 = 1$$

$\psi_{VBS} = n_1 + in_2$, Neel vector $= n^{3,4,5}$
Neel-VBS competition:

$$S = \frac{1}{2g}\int d^3x\,(\partial n^a)^2 + 2\pi\Gamma_{WZW}\,[n^a] + \cdots .$$

$\cdots =$ anisotropies demanded by microscopic symmetries.

Wess-Zumino-Witten term Γ_{WZW} crucial to capture non-trivial structure of topological defects.

Vortex in any 2 components (breaks $SO(5)$ to $SO(2) \times SO(3)$):
$SO(2)$ charge 0 but $SO(3)$ spinor.

Is the Neel/VBS transition continuous?

Good evidence it is described by $NCCP^1$ but does $NCCP^1$ have a second order transition?

Apparently yes but ultimate fate not yet clear.

(i) Indirect support: $NCCP^N$ in $1/N$ flows to CFT.

(ii) Direct support: Numerics (Sandvik, Kaul, Melko, Damle, Alet,……., Nahum et al, 06- present)

Simulations of variety of lattice models in same expected universality class: apparent continuous transition with roughly consistent properties and with theory.

Caveat: Drift of critical exponents with system size

Recent numerics: emergence of SO(5) rotating Neel into VBS (Nahum et al, 15)

(But putative SO(5) CFT in tension with bootstrap (Nakayama 16; Simmons-Duffin, unpublished))

Some questions

1. How to think about emergence of SO(5)?

2. Field theoretic formulation with manifest SO(5) symmetry?

Some questions

1. How to think about emergence of SO(5)?
Proposal: duality web for $NCCP^1$ which implies SO(5) as a ``quantum symmetry''.

2. Field theoretic formulation with manifest SO(5) symmetry?

Wang, Nahum, Metlitski, Xu, TS, 17

Duality web for NCCP[1]: A proposal

A first member: Self-duality

$$NCCP^1 \longleftrightarrow \widehat{NCCP}^1$$

$$\mathcal{L} = |(\partial_\mu - i b_\mu) z_\alpha|^2 + \cdots \qquad \hat{\mathcal{L}} = |(\partial_\mu - i \hat{b}_\mu) w_\alpha|^2 + \cdots$$

$$z_1^* z_2 \longleftrightarrow \mathcal{M}_{\hat{b}} \text{ (dual monopole)}$$

$$\text{(Monopole) } \mathcal{M}_b \longleftrightarrow w_1^* w_2$$

$$z^\dagger \sigma^z z \longleftrightarrow -w^\dagger \sigma^z w$$

Can match other local operators.

Consequences of self-duality - I

Emergent SO(5) symmetry:
Either theory only has manifest SO(3) x U(1) continuous symmetry.

For symmetries to agree, must be that either theory has hidden SO(5) symmetry

$$
\begin{aligned}
(n_1, n_2, n_3, n_4, n_5) \quad &\sim \quad (2\,\mathrm{Re}\mathcal{M}_b, 2\,\mathrm{Im}\mathcal{M}_b, z^\dagger\sigma_x z, z^\dagger\sigma_y z, z^\dagger\sigma_z z) \\
&\sim \quad (w^\dagger\sigma_x w, -w^\dagger\sigma_y w, 2\,\mathrm{Re}\mathcal{M}_{\tilde{b}}, -2\,\mathrm{Im}\mathcal{M}_{\tilde{b}}, w^\dagger\sigma_z w)
\end{aligned}
$$

Observed SO(5) in numerics explained if self-duality is correct.

Conversely, the observed SO(5) supports (but not prove) the self-duality.

Consequences of self-duality - II

Easy plane deformation:

Perturb both sides by the same operator $\mathcal{O} \sim (z^\dagger \sigma^z z)^2 \sim (w^\dagger \sigma^z w)^2$

Symmetry of either theory is reduced to O(2) x O(2):

Self-duality of original theory => self-duality of the perturbed theories.

Can be derived directly using the standard charge-vortex duality (Peskin-Dasgupta-Halperin) and has long been known to be true (Motrunich, Vishwanath 04).

Useful consistency check ✓

Other members of the duality web:
Fermionic QED3-Gross Neveau

$$
\begin{tikzcd}
NCCP^1 \arrow[rr, leftrightarrow, "\text{self-dual}"] \arrow[d, leftrightarrow] & & \widehat{NCCP}^1 \arrow[d, leftrightarrow] \\
QED_3 - GN \arrow[rr, leftrightarrow, "\text{self-dual}"] & & \widehat{QED_3 - GN}
\end{tikzcd}
$$

$$\mathcal{L} = \sum_{j=1,2} \bar{\psi}_j \gamma^\mu (-i\partial_\mu - a_\mu)\psi_j + \phi \sum_{j=1,2} \bar{\psi}_j \psi_j + V(\phi)$$

$$\mathcal{L} = \sum_{j=1,2} \bar{\chi}_j \gamma^\mu (-i\partial_\mu - \hat{a}_\mu)\chi_j + \hat{\phi} \sum_{j=1,2} \bar{\chi}_j \chi_j + \hat{V}(\hat{\phi})$$

Passes simple consistency checks.
<u>**Many predictions for numerics that can tested!**</u>

Conversely fermionic formulation allows new numerical (and analytical?) handle on deconfined criticality theory.

Some questions

1. How to think about emergence of SO(5)?

2. *Field theoretic formulation with manifest SO(5) symmetry?*

SO(5) is realized anomalously: can be viewed as surface theory of 3+1-D bosonic Symmetry Protected Topological paramagnet.

N_f = 2 QCD3 has same anomalous symmetry, and could have same IR physics.

Wang, Nahum, Metlitski, Xu, TS, 17

Anomalous SO(5) symmetry

Couple in a background $SO(5)$ gauge field A^5, and study instanton associated with $\Pi_1(SO(5)) = Z_2$.

Example $A^5 = A^{mon}(x)T^1$

Usual monopole potential

SO(5) generator

(Breaks continuous symmetry from SO(5) to SO(3) x SO(2))

Instanton creates a vortex in 2 components of the SO(5)-vector.

=> has zero SO(2) charge but **is an SO(3) spinor.**

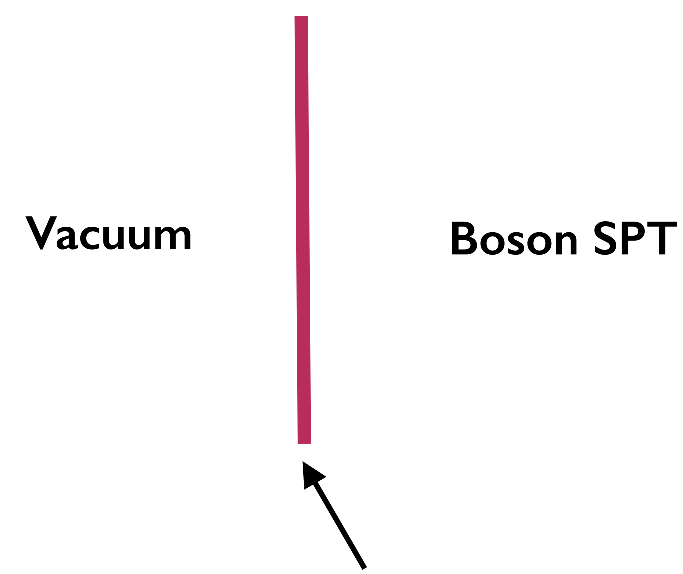
=> anomalous SO(5) symmetry.

Surface state of a a 3+1-D bosonic SPT state

A 3+1-D boson SPT with SO(5) x T can be constructed such that

Response to background SO(5) gauge field characterized by a **``discrete theta term''** (Aharony, Seiberg, Tachikawa, 13)

Modifies SO(5) monopole in precisely the right way to match the known instanton structure of the boundary theory.



putative 2+1-D theory with anomalous SO(5) x T symmetry.

A manifestly SO(5) x T invariant boundary theory

Massless fermionic $N_f = 2$ QCD_3 with an SU(2) gauge field(*)

(Start with 8 massless Majorana fermions with SO(8) symmetry and gauge an SU(2) subgroup).

Alternate formulation of Neel-VBS competition that can be tuned to have manifest anomalous SO(5) x T.

(*) In condensed matter this is familiar as the theory of the ``pi-flux" state of the antiferromagnet.

Comments

1. IR fate of N_f = 2 QCD_3?

Can show anomaly implies either SO(5) x T symmetry is broken, or theory is a CFT.
``Symmetry enforced gapless”

2. Same local operators, and anomaly as putative Neel/VBS critical point.

Useful to compare numerics on QCD3 with Neel/VBS.

3. Alternate to sigma model + WZW formulation which also has manifest symmetry (better suited for calculations/formal manipulations; a renormalizeable field theory).

Summary of Part I

New progress on old problem of deconfined critical points;
better understanding of possible emergent symmetries and dualities.

Many new predictions for tests of the dualities, etc in numerics.

Combined input from field theory + numerics + bootstrap will be great!

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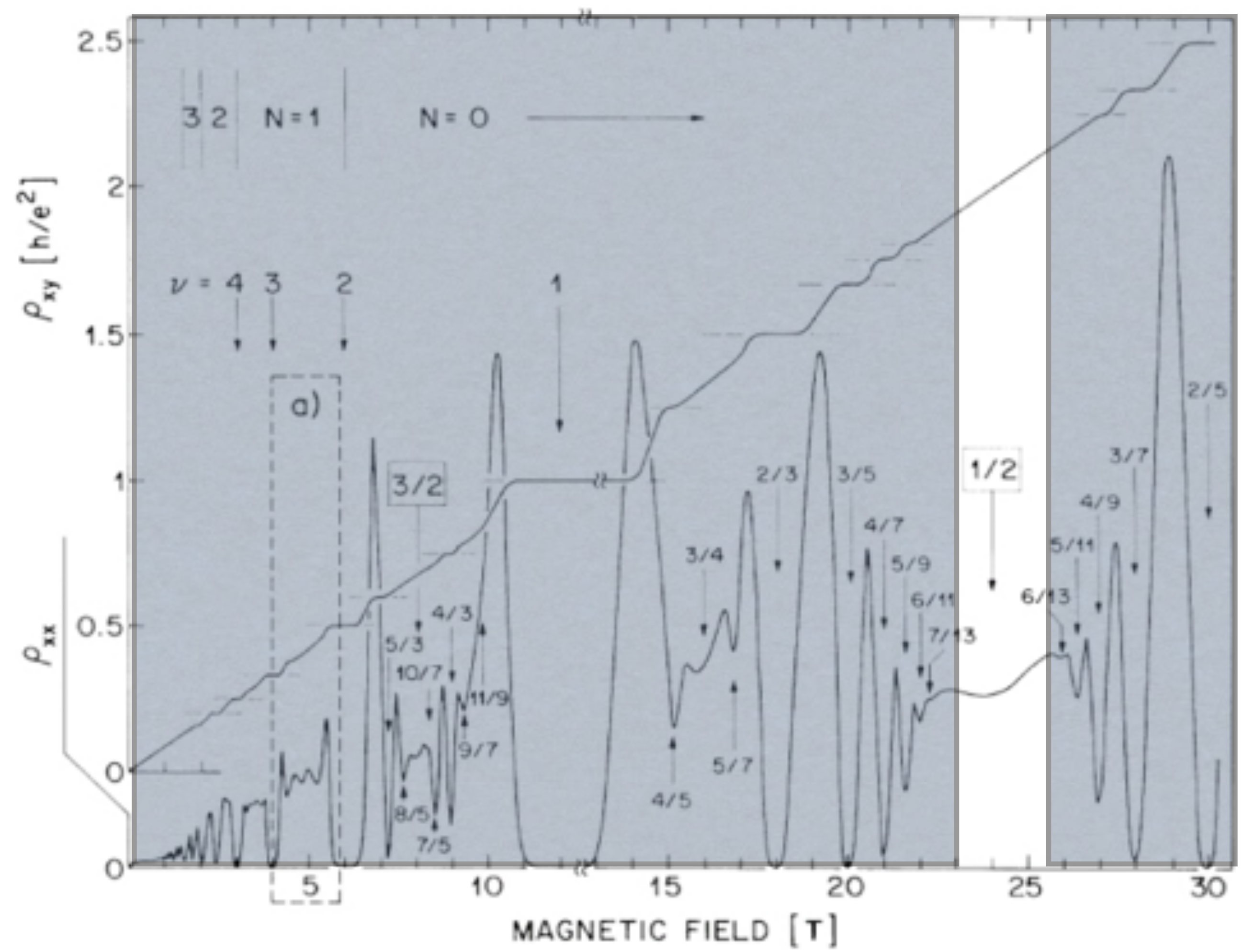
1/2-filled Landau level: the problem

What happens at ν = 1/2 ???

Experiment: Metal

``Unquantized quantum Hall effect"

How do interactions in the half-filled Landau level produce a metal?



Old <u>successful</u> theory (Halperin. Lee, Read 93): Fermi surface of composite fermions + Chern-Simons gauge field.

Recent re-examination: Role of Landau level particle/hole symmetry ignored in the old work

p/h symmetric LL as a surface of a 3d fermionic topological insulator: Preliminaries

Consider (initially free) fermions with ``weird" action of time-reversal (denote C):

$C \rho C^{-1} = - \rho$

ρ = conserved ``charge" density.

Full symmetry = U(1) x C

p/h symmetric LL as a surface of 3d fermion SPT (cont'd)

Surface: Single massless Dirac fermion

C symmetry guarantees that surface Dirac cone is exactly at neutrality.

$$\mathcal{L} = \bar{\psi}\left(-i\partial\!\!\!/ + A\!\!\!/\right)\psi + \ldots.$$

external probe gauge field

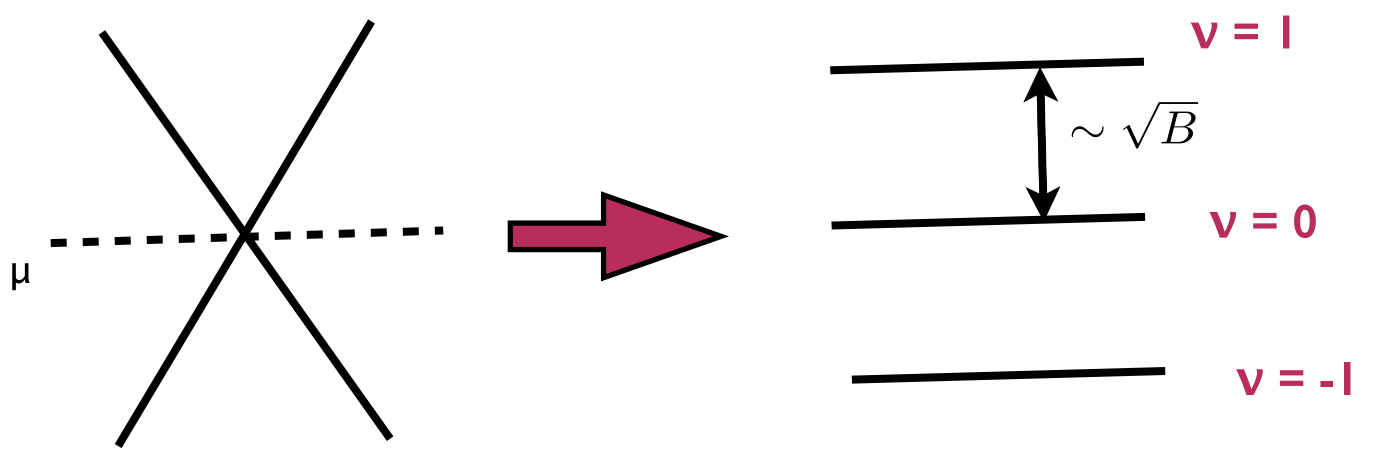
2-component fermion

p/h symmetric LL as a surface of 3d fermion SPT (cont'd)

ρ is odd under C => `electric current' is even.

External E-fields are odd but external B-fields are even.

=> Can perturb surface Dirac cone with external B-field.



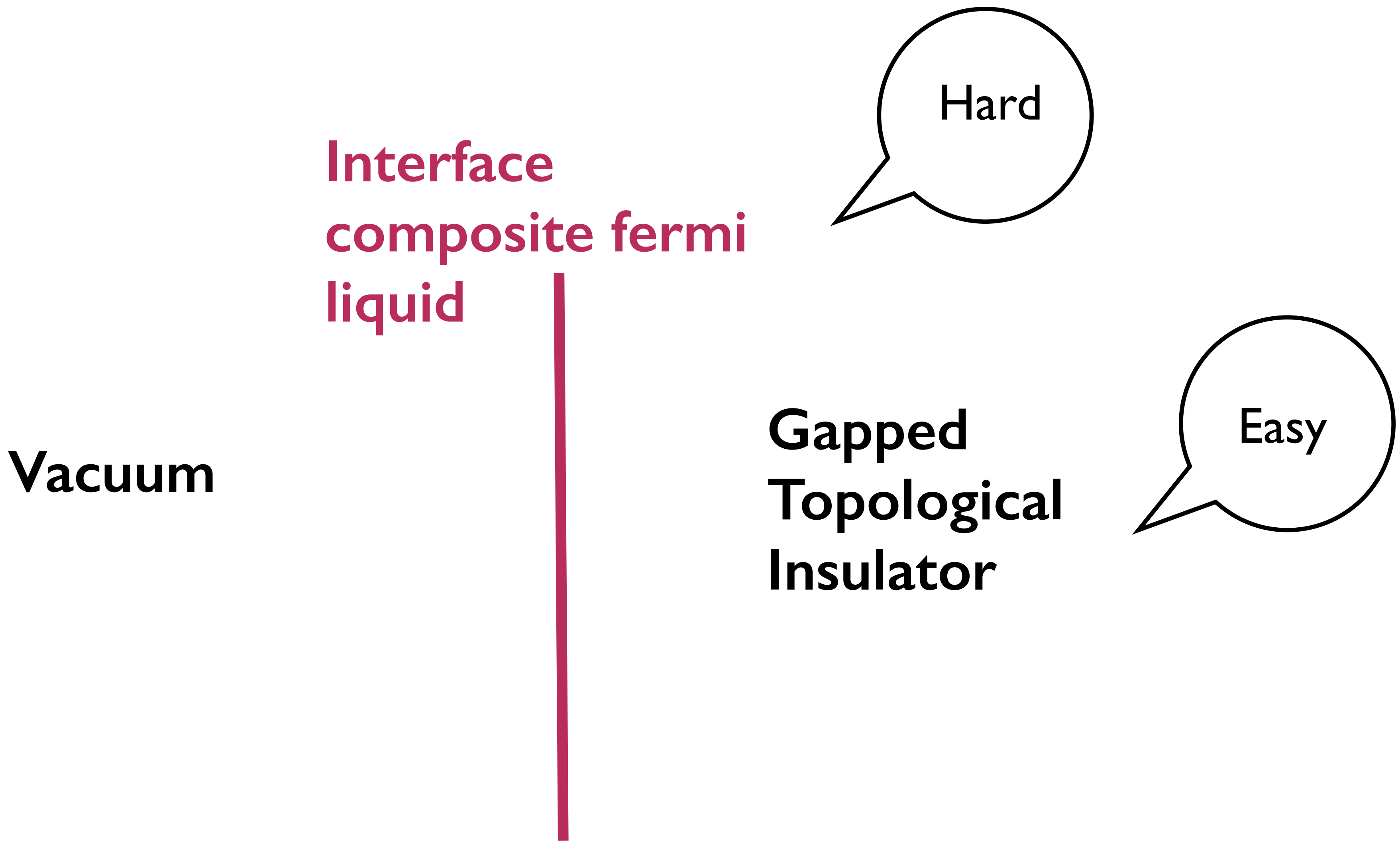
C-symmetry: ν = 0 LL is exactly half-filled.

Low energy physics: project to 0LL

With interactions => map to usual half-filled LL

Comments

Implication: Study p/h symmetric half-filled LL level by studying correlated surface states of such 3d fermion topological insulators.



Exploit understanding of relatively trivial bulk TI to learn about non-trivial correlated surface state.

Theories of TI surface

Charge-vortex duality for Dirac fermions in 2+1-D

Two different surface theories for the topological insulator:

Standard surface theory:

$$\mathcal{L} = \bar{\psi} \left(-i \not{\partial} + \not{A} \right) \psi + \ldots.$$

external probe gauge field

2-component fermion

Wang, TS 15
Metlitski, Vishwanath 15
Mross et al 15
More precise version:
(Seiberg, TS, Wang, Witten 2016)

Dual surface theory:

$$``\mathcal{L} = \bar{\psi}_v \left(-i \not{\partial} - \not{a} \right) \psi_v + \frac{1}{4\pi} \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda "$$

dynamical gauge field

Justification of Dirac composite fermion theory of 1/2-filled Landau level

1/2-filled Landau level: Obtain by turing on B-field in standard surface Dirac cone.

Dual description:

B/(2h/e) = density of dual fermions

=> Theory of 1/2-filled Landau level: = dual Dirac fermions at finite density + U(1) gauge field but no Chern-Simons term.
(exactly as proposed by Son (2015))

Compare with **Composite Fermi liquid of bosons at $\nu = 1$**

LLL approach: write Hamiltonian in terms of density operators projected to LLL.

Pasquier-Haldane (98): Represent density in terms of auxiliary fermions.

Read(98): Mean field + gauge fluctuations for CFL state.

Effective action different from HLR!

$$\begin{aligned}\mathcal{L}[\psi, a_\mu] = \quad & \mathcal{L}[\psi, a_\mu] \\ & - \frac{1}{2\pi}\epsilon^{\mu\nu\lambda} a_\mu \partial_\nu A_\lambda + \frac{1}{4\pi}\epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda \end{aligned} \tag{1}$$

No Chern-Simons term in action.
Theory of neutral fermionic vortex

Comments

1. Similar formal structure to p/h symmetric theory for electrons at **ν = 1/2**
Eg, CF density = flux density (and not charge density)

2. **Relation to HLR for bosons at ν = 1: postulate Fermi surface Berry phase -2 π**

3. Read's theory has an emergent anti-unitary particle hole symmetry (Wang, TS 16).

$$\psi \to \psi$$

$$a_i \to -a_i$$

Same theory with exact p/h obtained at surface of 3d boson topological insulator (studied previously by Vishwanath, TS, 13)

Composite fermi liquids as vortex metals

HLR/Jain composite fermion: Charge - flux composites

Particle-hole symmetric composite fermion: Neutral vortex

Describe CFL as a vortex liquid metal formed by neutral fermionic vortices.

Vortex metal description:

- Simple understanding of transport
(similar to other 2d quantum vortex metals, eg, in Galitski, Refael, Fisher, TS, 06)

- Extensions to CFLs away from $\nu = 1/2$

Transport in the CFL

1. Longitudinal electrical conductivity $\propto$ composite fermion resistivity (natural from vortex liquid point of view)

   Hall conductivity $= \frac{e^2}{2h}$(exactly)

2. Longitudinal thermal conductivity = composite fermion thermal conductivity

   Wiedemann-Franz violation (Wang, TS, 15)

$$\frac{\kappa_{xx}}{L_0 T \sigma_{xx}} = \left(\frac{\rho_{xy}}{\rho_{xx}}\right)^2 < 10^3$$

   L_0: free electron Lorenz number

   (Also actually in HLR)

3. Thermoelectric transport:

   Vortex metal: Nernst effect from mobile vortices - unlike HLR? (Potter et al, 15).

Comments/summary

1. Old issue of p/h symmetry in half-filled Landau level: simple, elegant answer

General viewpoint (even for Composite Fermi Liquids (CFL) at other fillings): Regard LLL composite fermi liquid as a quantum metal of neutral fermionic vortices with Fermi surface Berry phases.

2. Surprising, powerful connection of CFL theory to correlated 3d TI surfaces

- many new insights.