Decohering the Fermi Liquid: A dual approach to the Mott transition

D. Mross (MIT)
T. Senthil (MIT)

arxiv July 2011

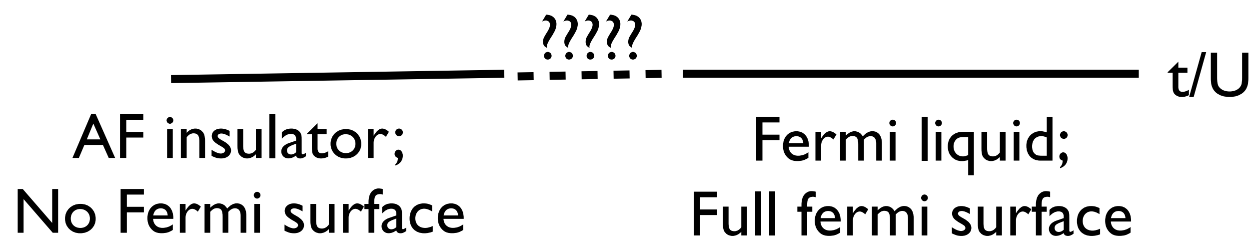
Happy Birthday, Duncan!

The electronic Mott transition

Difficult old problem in quantum many body physics

How does a metal evolve into a Mott insulator?

Prototype: One band Hubbard model at half-filling on non-bipartite lattice



Why hard?

1. No order parameter for the metal-insulator transition

2. Need to deal with gapless Fermi surface on metallic side

3. Complicated interplay between metal-insulator transition and magnetic phase transition

Typically in most materials the Mott transition is first order.

But (at least on frustrated lattices) transition is sometimes only weakly first order - fluctuation effects visible in approach to Mott insulator from metal.

Quantum spin liquids and the Mott transition

Modern condensed matter physics: possibility of quantum
spin liquid Mott insulators with no broken symmetries/conventional
long range order.

Theory: Quantum spin liquids can exist; maturing understanding.

Experiment: Several candidate materials; all of them have some gapless excitations.

Opportunity for progress on the Mott transition:
study metal-insulator transition without complications of magnetism.

Some candidate spin liquid materials

$\kappa - (ET)_2Cu_2(CN)_3$

$EtMe_3Sb[Pd(dmit)_2]_2$

Quasi-2d, approximately isotropic triangular lattice; best studied candidate spin liquids

$Na_4Ir_3O_8$

Three dimensional `hyperkagome' lattice

$ZnCu_3(OH)_6Cl_2$

Volborthtite, ...

2d Kagome lattice (`strong' Mott insulator)

Some candidate materials

$\kappa - (ET)_2Cu_2(CN)_3$

$EtMe_3Sb[Pd(dmit)_2]_2$

Quasi-2d, approximately isotropic triangular lattice; best studied candidate spin liquids

$Na_4Ir_3O_8$

Three dimensional `hyperkagome' lattice

Close to pressure driven Mott transition: `weak' Mott insulators

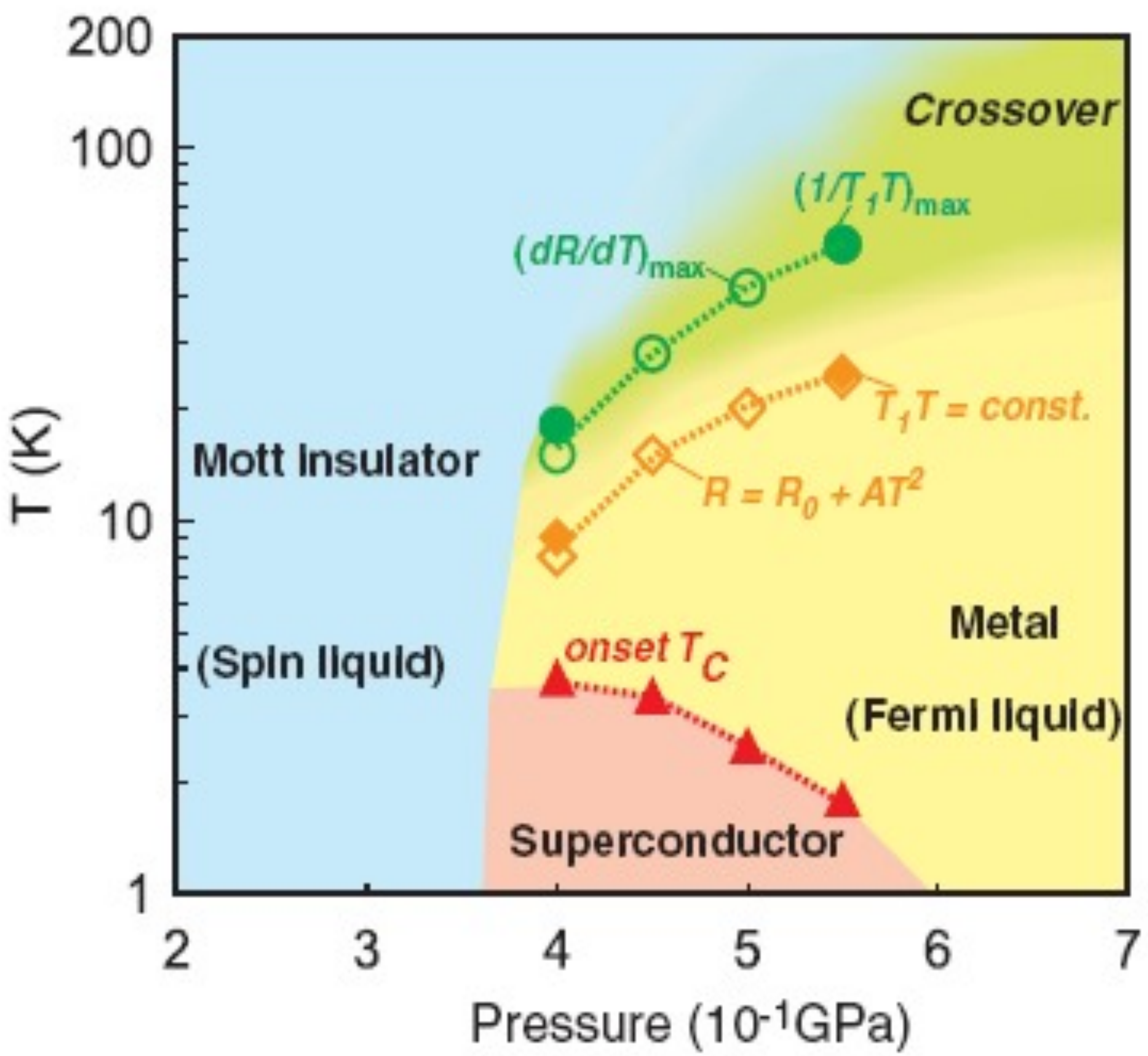
$ZnCu_3(OH)_6Cl_2$

Volborthtite, ...

2d Kagome lattice (`strong' Mott insulator)

Possible experimental realization of a second order Mott transition

Kanoda et al '03 - '08



$\kappa-(ET)_2 Cu_2 (CN)_3$

Under pressure

One band Hubbard model on isotropic Δ lattice

No magnetic order in insulator!

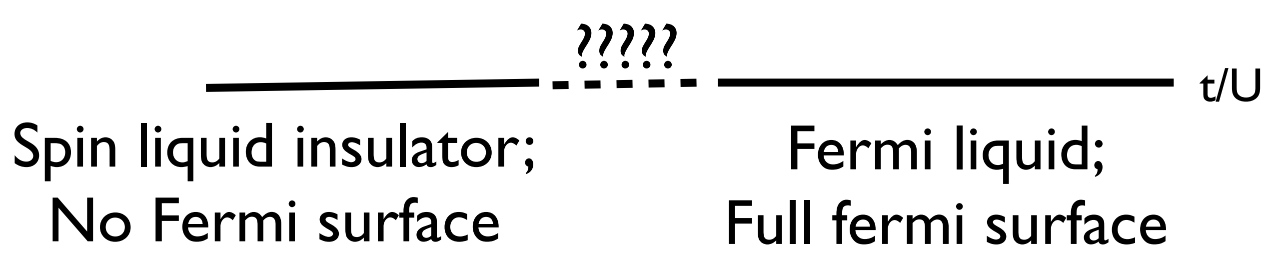
Quantum spin liquids and the Mott transition

Some questions:

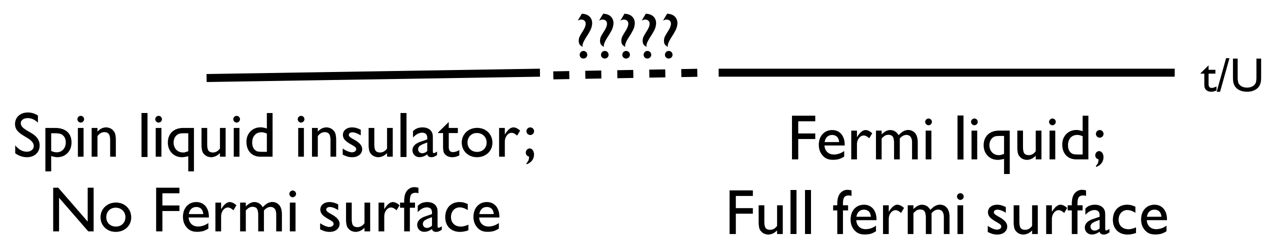
1. How to think about a gapless quantum spin liquid Mott insulator that is proximate to a metal?

2. Can the Mott transition be continuous?

3. Fate of the electronic Fermi surface?



Killing the Fermi surface



At half-filling, through out metallic phase,
Luttinger theorem => size of Fermi surface is fixed.

Approach to Mott insulator: entire Fermi surface must
die while maintaining size (cannot shrink to zero).

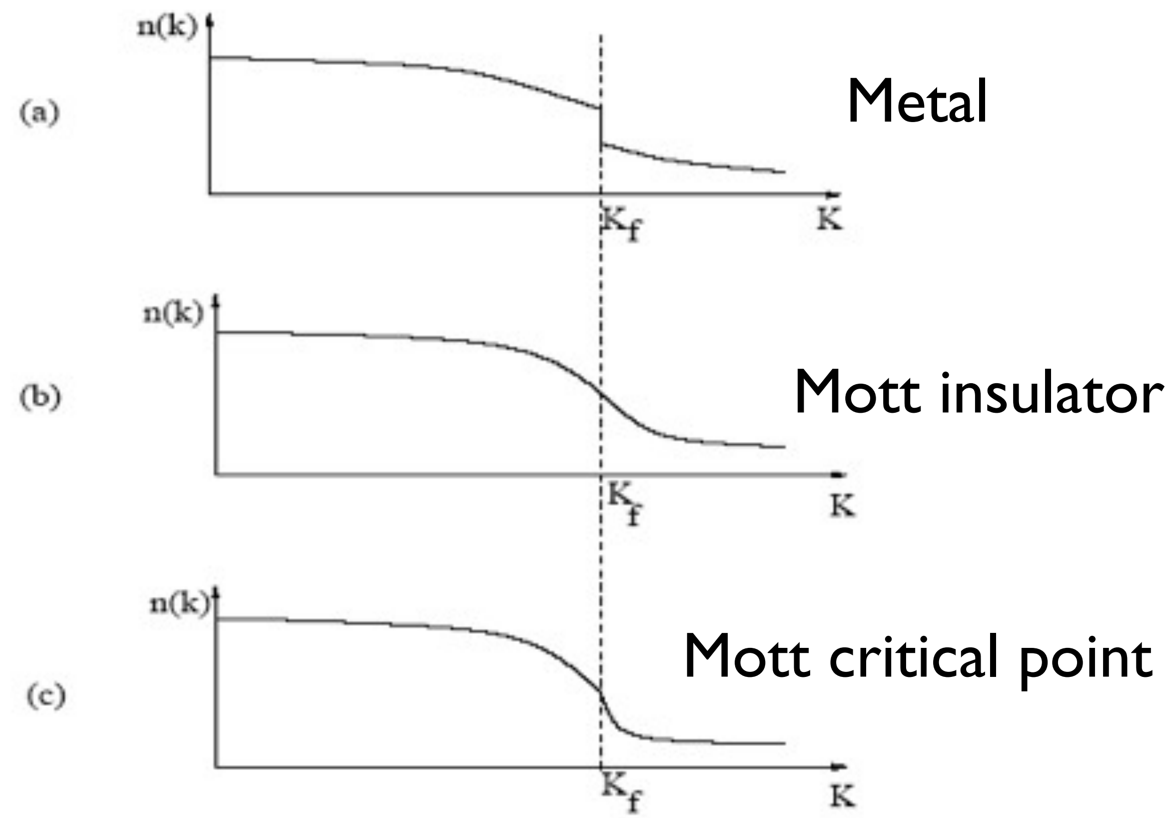
If Mott transition is second order, critical point necessarily very unusual.

``Fermi surface on brink of disappearing" - expect non-Fermi liquid physics.

Similar ``killing of Fermi surface" also at Kondo breakdown transition
in heavy fermion metals, and may be also around optimal doping in cuprates.

How can a Fermi surface die continuously?

Continuous disappearance of Fermi surface if quasiparticle weight Z vanishes continuously everywhere on the Fermi surface (Brinkman, Rice, 1970).



Concrete examples: DMFT in infinite d (Vollhardt, Metzner, Kotliar, Georges 1990s), slave particle theories in d = 2, d = 3 (TS, Vojta, Sachdev 2003, TS 2008)

Electronic structure at a continuous Mott transition

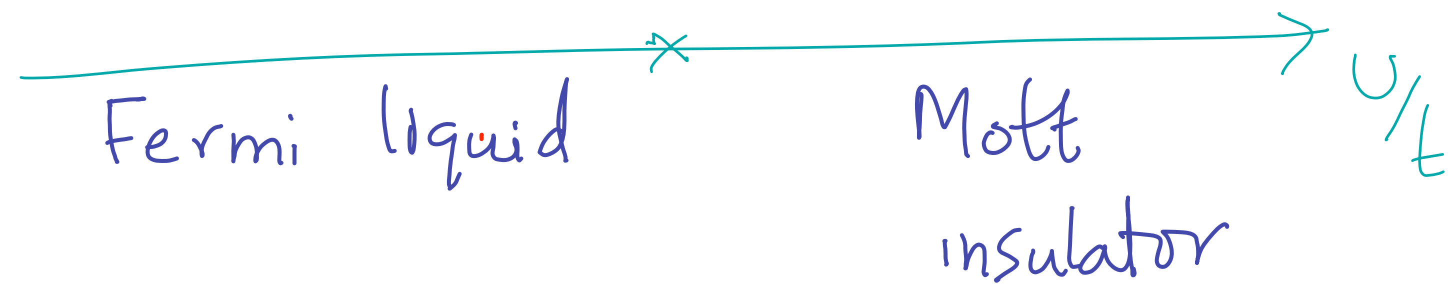
Crucial question: Electronic excitation structure right at Mott critical point when Z has just gone to zero?

Claim: At critical point, Fermi surface remains sharply defined even though there is no Landau quasiparticle (TS, 2008)

``Critical Fermi surface”

Why a critical Fermi surface?

Mott transition



What is gap $\Delta(\vec{K})$ to add an electron at momentum $\vec{K}$?

Fermi liquid : $\Delta(\vec{K} \in FS) = 0$

Mott insulator : Sharp gap $\Delta(\vec{K}) \neq 0$ for all $\vec{K}$

Evolution of single particle gap

Approach from Mott

2^{nd} order transition to metal $\Rightarrow$ expect Mott gap

$\Delta(\vec{K})$ will close continuously

To match to Fermi surface in metal, $\Delta(\vec{K}) \to 0$
for all $\vec{K} \in FS$.

$\Rightarrow$ Fermi surface sharp at critical point.

But as $Z = 0$ no sharp quasiparticle

$\Rightarrow$ Non-fermi liquid with sharp "critical" Fermi surface!

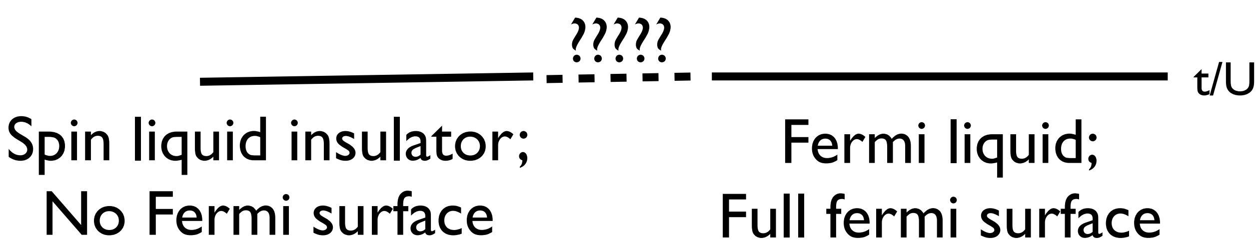
Quantum spin liquids and the Mott transition

Some questions:

1. How to think about a gapless quantum spin liquid Mott insulator that is proximate to a metal?

2. Can the Mott transition be continuous?

3. Fate of the electronic Fermi surface?



Only currently available theoretical framework to answer these questions is slave particle gauge theory.

Slave particle framework

Split electron operator

$$c_{r\sigma}^{\dagger} = b_r^{\dagger} f_{r\alpha}$$

Fermi liquid: $\langle b \rangle \neq 0$

Mott insulator: b_r gapped

Mott transition: b_r critical

In all three cases $f_{r\alpha}$ form a Fermi surface.

Low energy effective theory: Couple b, f to fluctuating $U(1)$ gauge field.

Quantum spin liquids and the Mott transition

Some questions:

1. How to think about a gapless quantum spin liquid Mott insulator that is proximate to a metal?

Gapless spinon Fermi surface coupled to U(1) gage field
(Motrunich, 05, Lee and Lee, 05)

2. Can the Mott transition be continuous?

3. Fate of the electronic Fermi surface?

Concrete tractable theory of a continuous Mott transition;
demonstrate critical Fermi surface at Mott transition;
definite predictions for many quantities (TS, 2008).

Is there an alternate to slave particles?

This talk: a new approach to the Mott transition and proximate quantum spin liquids

Start from the effective low energy theory of the Fermi liquid*;

Freeze out charge density and current fluctuations to obtain a spin liquid Mott insulator

*Most useful for this purpose: a point of view on Fermi liquid developed by Haldane in 1990s.

Landau theory of Fermi liquids

Collisionless Landau kinetic equation for quasiparticle densities

$$\left(\frac{\partial}{\partial t} + \vec{v}_F \cdot \frac{\partial}{\partial \vec{x}}\right) \delta n_{\sigma,\vec{k}} - \frac{\delta(\epsilon^0_{\vec{k}} - \mu)}{V} \sum_{\vec{k}',\sigma'} f^{\sigma,\sigma'}_{\vec{k},\vec{k}'} \vec{v}_F \cdot \frac{\partial \delta n_{\sigma',\vec{k}'}}{\partial \vec{x}} = 0$$

Infinite number of emergent conservation laws in a Fermi liquid:
quasiparticle numbers at each point of Fermi surface.

Corresponding densities are all hydrodynamic modes.

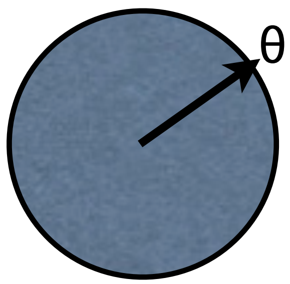
Hydrodynamic theory of Fermi liquids

Hydrodynamic theory: Restrict to small region within $\pm\Lambda_{\|}$ of Fermi surface.

Define concept of θ-mover: quasiparticle at angle θ of Fermi surface of 2d metal.

θ-mover density (of spin σ)

$$\rho_\sigma(\theta, \vec{r}, t) \equiv \int_{-\Lambda_{\|}}^{+\Lambda_{\|}} \frac{dk}{(2\pi)^2} \delta n_{\sigma,\theta,k}(\vec{x}, t).$$



Hydrodynamic equation

$$\partial_t \rho_\sigma(\theta) + v_F \partial_{\|} \rho_\sigma(\theta) + \frac{\partial_{\|}}{(2\pi)^2} \int d\theta' K_F(\theta') f_{\sigma\sigma'}(\theta, \theta') \rho_{\sigma'}(\theta') = 0 \qquad (1)$$

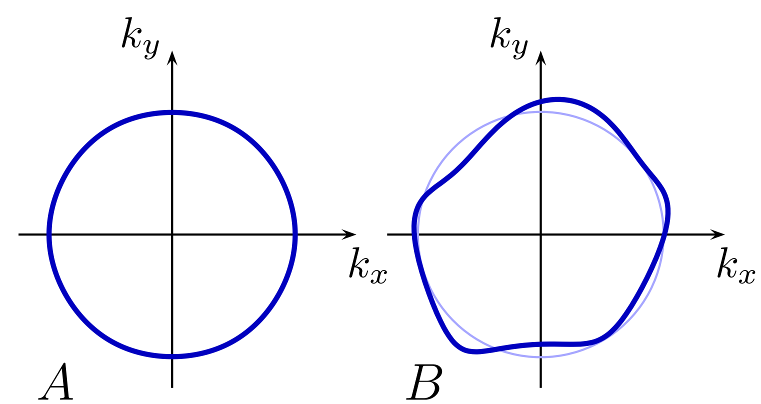
$\partial_{\|} = \hat{v}_F \cdot \vec{\nabla}$ = derivative in direction normal to the Fermi surface.

Note: Deviation of *total* density from mean $\delta\rho_\sigma = \int d\theta K_F(\theta) \rho_\sigma(\theta)$

Fermi surface shape fluctuations

Interesting physical interpretation: assume local version of Luttinger theorem to relate to Fermi surface shape fluctuation.

Slow long wavelength fluctuation of Fermi surface shape => long wavelength disturbance of quasiparticle densities.

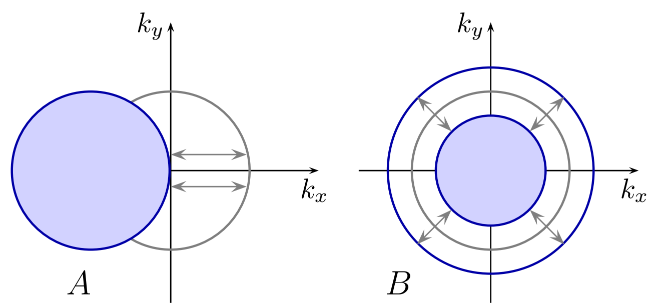


$$K_{F\sigma}(\theta) \to K_{F\sigma}(\theta) + \delta K_{F\sigma}(\vec{r}, \theta, t)$$

$$\rho_\sigma(\vec{r}, \theta, t) = \frac{1}{4\pi^2} \delta K_{F\sigma}(\vec{r}, \theta, t)$$

Approaching the Mott transition: what happens to shape fluctuations of Fermi surface?

Two special Fermi surface shape fluctuations



Current fluctuation:
shift of Fermi surface

Density fluctuation:
`breathing' of Fermi surface

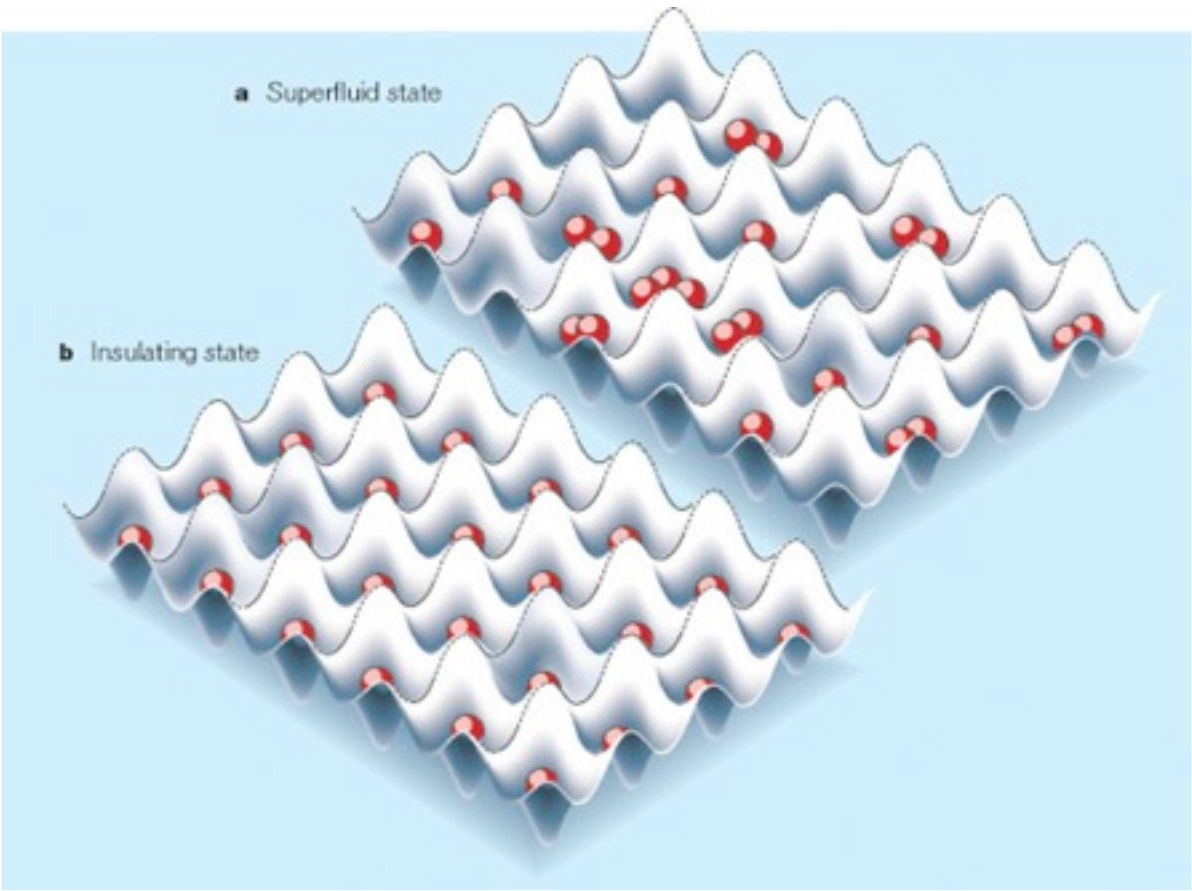
Approach to Mott insulator: density and current fluctuations suppressed

=> clamp down these Fermi surface shape fluctuations but make no other a priori assumptions.

How to implement??

Lessons from a simpler problem: Mott transition of bosons

Superfluid-Mott transition of bosons at integer filling on a lattice: well understood and simple



Cold atoms in an optical lattice: M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Hydrodynamic theory of superfluids

Low energy effective theory of the superfluid phase

$$S_{eff} = \int d^2x d\tau \frac{\rho_s}{2} \left(\partial_\mu \phi\right)^2 + ...$$

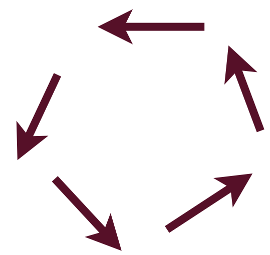
ϕ = phase of superfluid order parameter .

Phase-only theory **incapable** of capturing Mott insulator.

Mott insulation requires quantization of particle number => periodicity of conjugate phase:

$\phi \sim \phi + 2\pi.$

Must incorporate vortices in ϕ to describe Mott insulator.



Vortices and the Mott transition

Incorporating vortices: dual description of bosons.

Dasgupta, Halperin, 1979;
M.P.A. Fisher, D.H. Lee, 1989

Sound mode of superfluid <=> propagating ``photon''

Vortices <=> point source of photons

Dual theory: Vortex field Φ_v minimally coupled to fluctuating $U(1)$ gauge field a_μ.

$$S_{dual} = \int d^2x d\tau \frac{1}{2\rho_s} \left(\vec{\nabla} \times \vec{a}\right)^2 + \left|\left(\partial_\mu - i a_\mu\right)\Phi_v\right|^2 + V\left(|\Phi_v|^2\right) \qquad (1)$$

Superfluid: Φ_v gapped => propagating photon, $i.e$ usual sound mode.
Mott insulator: ϕ_v condensed => photon gapped => boson density, current fluctuations gapped.
Quantization of dual gauge flux => boson number quantization.

Dual vortex theory equivalent to usual $D = 2{+}1$ XY description of superfluid-insulator transition.

Mott transition of fermions: similar program?

To clamp down density/current fluctuations of fermions, perhaps we should condense ``vortices" in ``phase" of fermion ?

What does ``phase" of fermion mean?

How do we define ``vortices" for fermions?

Fermi liquid theory as a phase-only theory: bosonization of the Fermi surface

d = 1: Bosonization describes fermions in terms of their phase.

d > 1: Bosonized representation of Fermi liquid theory (Haldane 92, Castro Neto, Fradkin, 93) enables formulating Fermi surface fluctuations in terms of fermion phase.

Write (by analogy to $d = 1$)

$$\rho_\sigma(\theta) = \frac{1}{2\pi}\partial_\parallel \phi_\sigma(\theta) \tag{1}$$

Equation of motion for $\phi_{\theta\sigma}$

$$(\partial_t + v_F \partial_\parallel)\partial_\parallel \phi_\sigma(\theta) + \int d\theta' K_{F\theta'} f_{\sigma\sigma'}(\theta,\theta')\partial_\parallel \partial'_\parallel \phi_{\theta'\sigma'} = 0 \tag{2}$$

Reproduce from quadratic "bosonized" Lagrangian $\mathcal{L} = \mathcal{L}[\phi_\sigma(\theta)]$

Bosonization of the Fermi surface (cont'd)

Haldane,
Castro Neto, Fradkin,
Houghton, Marston,
Wen, ...

Fermi liquid partition function

$$Z = \int [\mathcal{D}\phi_\sigma(\theta)] e^{-\int d^2x d\tau \mathcal{L}[\phi_\sigma(\theta)]} \qquad (1)$$

Interpretation of $\phi_\sigma(\theta)$: In operator framework, $2\pi\phi_\sigma(\theta)$ is conjugate to θ-mover density.

Further θ-mover creation operator

$$\psi_\sigma(\theta)^\dagger \sim e^{2\pi i \phi_\sigma(\theta)} \qquad (2)$$

=> interpret $2\pi\phi_\sigma(\theta)$ as phase of θ-mover.

Bosonization of the Fermi surface (cont'd)

Define charge and spin bosons

$$\phi_c(\theta) = \frac{\phi_\uparrow + \phi_\downarrow}{2} \quad (1)$$

$$\phi_s(\theta) = \frac{\phi_\uparrow - \phi_\downarrow}{2} \quad (2)$$

$$(3)$$

Lagrangian decouples as $\mathcal{L} = \mathcal{L}[\phi_c] + \mathcal{L}[\phi_s]$.

Sometimes useful to reformulate in terms of "angular momentum" modes

$$\phi_{lc/s} = \int \frac{d\theta}{2\pi} e^{-il\theta} \phi_{c/s}(\theta) \quad (4)$$

Note: Different l modes **do not** decouple in Lagrangian.

$l = 0$ **mode** ϕ_{0c} **is conjugate to total density (*i.e* breathing mode of Fermi surface.**

Plan of attack

Bosonization of Fermi surface has allowed identification of fermion phase that is conjugate to fluctuations of total fermion density.

Follow successful strategy for bosons, incorporate vortices in this phase and condense them to get an incompressible Mott insulator.

How to think about charge vortices in a Fermi liquid?

Consider ring geometry and slowly thread 2π flux through hole.

Transfer momentum 2π/L to each momentum state
=> Fermi surface shifts by 2π/L.

Fermi surface displacement

$$\delta K_{F\sigma}(\theta) = \frac{2\pi}{L} \hat{v}_{F\theta} \cdot \hat{x} \tag{1}$$

which implies

$$\phi_{c\theta} = \frac{x}{L} \tag{2}$$

Therefore there is a vortex in $2\pi\phi_{0c}$.

Incorporating charge vortices in bosonized Fermi liquid

Separate ϕ_{0c} into "smooth" part $\underline{\phi}_{c0}$ and a "vortex" part ϕ^v_{c0}:

$$\phi_{c0} = \underline{\phi}_{c0} + \phi^v_{c0} \tag{1}$$

Smooth part satisfies

$$\epsilon_{\mu\nu\lambda}\partial_\nu\partial_\lambda\underline{\phi}_{c0} = 0 \tag{2}$$

Vortex part is defined in terms of the vortex 3-current $j_{\mu v}$ through the equation

$$\epsilon_{\mu\nu\lambda}\partial_\nu\partial_\lambda\phi^v_{c0} = j_{\mu v} \tag{3}$$

Note: $a_\mu \equiv \partial_\mu\phi^v_{c0}$ actually not a gradient.
Charge boson Lagrangian density

$$\mathcal{L}_c = \mathcal{L}_c[\partial_\mu\underline{\phi}_{c0} + a_\mu, \partial_\mu\phi_{cl\neq 0}] \tag{4}$$

Dual vortex theory

To do: Specify dynamics of vortices and impose condition that vortex current is flux of vector field a_μ.

Introduce a vortex field Φ_v whose three-current is precisely $j_{\mu v}$. Then the modified charge Lagrangian density is

$$\mathcal{L} = \mathcal{L}_c[\partial_\mu \phi_{cl} + a_\mu \delta_{l,0}] + \imath A_\mu \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda + \mathcal{L}[\Phi_v, A_\mu] \qquad (1)$$

A_μ is minimally coupled to Φ_v.

Impose vortex current = flux of a_μ

Important: Thus far effective theory does not know the fermion filling on the lattice which is crucial for existence of Mott insulator.

Fermion filling determines effective action for vortices.

Half-filling => No net Berry phase for the vortex.

Vortex dynamics described by

$$\mathcal{L}[\Phi_v, A_\mu] = |\left(\partial_\mu - \imath A_\mu\right)\Phi_v|^2 + V\left(|\Phi_v|^2\right) + \frac{1}{2e_0^2}\left(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda\right)^2$$

Phases of the dual vortex theory-I
The Fermi liquid revisited

Vortices gapped: Integrate out vortex field.

$$\mathcal{L} = \mathcal{L}_c[\partial_\mu \phi_{cl} + a_\mu \delta_{l,0}] + \imath A_\mu \epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda + \frac{1}{2e^2}\left(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda\right)^2$$

Now integrate out A_μ: ``mass'' term for a_μ

Deep in vortex gapped phase simply recover bosonized Fermi liquid action.

Phases of the dual vortex theory-II
Mott insulator

Condensing the vortex clamps density/current fluctuations to give a Mott insulator.

$\langle \Phi_v \rangle \neq 0 =>$ mass for A_μ.

Integrate out A_μ

$$\mathcal{L} = \mathcal{L}_c[\partial_\mu \phi_{cl} + a_\mu \delta_{l,0}] + \frac{K}{2} \left(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda\right)^2 \tag{1}$$

Together with spin Lagrangian this gives a description of a spin liquid Mott insulator, presumably with gapless excitations.

Simplify through refermionization

Full dual Lagrangian for Mott phase

$$\mathcal{L} = \mathcal{L}_c[\partial_\mu \phi_{cl} + a_\mu \delta_{l,0}] + \frac{K}{2}\left(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda\right)^2 + \mathcal{L}_s[\partial_\mu \phi_{sl}] \tag{1}$$

Precisely the same as the (bosonized) theory of a Fermi surface coupled to a gapless U(1) gauge field!

Refermionize in terms of 'spinon' field

$$f_\sigma^\dagger(\theta) \sim e^{2\pi i(\phi_c(\theta)+\sigma\phi_s(\theta))}$$

Effective Lagrangian

$$\mathcal{L}[f, a_\mu] = \bar{f}_\alpha \left(\partial_\tau - \imath a_0 - \mu - \frac{\left(\vec{\nabla} - \imath \vec{a}\right)}{2m}\right) f_\alpha + \frac{K}{2}\left(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda\right)^2$$

Same as from slave particle approach though we never introduced slave particles.

More on relation to slave particles

Slave particle framework:

$$c_r^\dagger = b_r^\dagger f_{r\sigma}$$

Spin liquid near Mott insulator: b_r gapped; $f_{r\sigma}$ forms gapless Fermi surface. Emergence of $U(1)$ gauge field coupled to spinon Fermi surface.

Bosonized dual vortex theory:

Holon $b_r \sim e^{2\pi i \phi_{c0}}$ => uniformly expand Fermi surface of both spin species

Spinon $f_\sigma^\dagger(\theta) \sim e^{2\pi i(\phi_{\sigma\theta} - \phi_{c0})}$

Magnetic flux of emergent gauge field = vortex density

A posteriori clarification: vortices in the Fermi liquid and the insulator

But do Fermi liquids really support **stable** gapped vortex excitations?

No - the vortex will decay into particle/hole pairs and will not be stable.

Gauge theory language: ``instanton" effects lead to decay of gauge flux.

But at the Mott transition, instanton effects are
irrelevant (Hermele et al, 2004; S.S. Lee, 2008)

=> vortex becomes asymptotically well-defined as Mott transition is approached.

Comments

1. Bosonized dual vortex approach more coarse grained but eventually equivalent to slave particles.

2. Can repeat with spin rather than charge vortices to describe spin gapped charge metal (``algebraic charge liquid") previously also discussed through slave particles.

Higher angular momenta vortex condensates???

Natural question: can we access exotic non-fermi liquid states by condensing vortex condensates with non-zero angular momentum?

No! Conjugates to higher angular momenta charge/spin boson are higher angular momenta Fermi surface shape changes.

These are not quantized in microscopic Hilbert space => vortices in these modes are not legitimate degrees of freedom.

Summary

A new approach to thinking about the electronic Mott transition to a spin liquid Mott insulator.

Start in Fermi liquid, and clamp down charge and current fluctuations.

Implement by condensing vortices in phase conjugate to breathing mode of Fermi surface.

Result identical to that obtained through slave particle methods.

Improved understanding......may be this point of view will be useful to go beyond slave particles in the future.