# Charge-vortex duality for fermions: a new window into strong correlation problems

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# Collaborators

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# Driving progress on SCES problems

## Invest in

- materials development
- new and better experimental probes
- computational methods on microscopic models
- conceptual advances in theory

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This talk

# Effective field theory in condensed matter physics

Microscopic models (e.g, Hubbard, lattice spin Hamiltonians, etc)

`Low energy' effective field theory

`Low energy' experiments/
phenomenology

In this talk I will describe some recent advances in our understanding of some very basic (effective field) theories of correlated electrons.

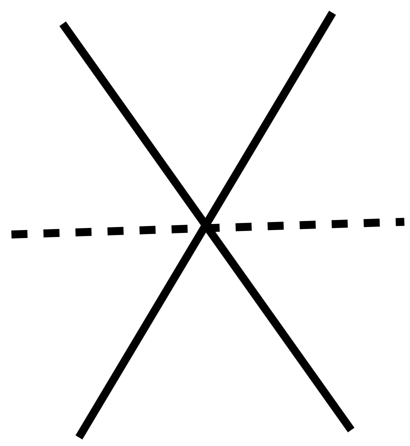
These advances go by the name of ``duality”:

Much more on this later but briefly it gives a new way of thinking about the fermion system that is potentially very useful.

# Dirac fermions

A particularly simple and basic theory:

Single massless Dirac fermion in 2d



$$\mathcal{L} = \bar{\psi} \left(-i\gamma^\mu \partial_\mu\right) \psi$$

Realized, eg,  at surface of 3d topological insulator.

(More Dirac nodes: graphene, d-wave SC, etc)

I will describe this simple theory from a very different  \`dual' point of view that hides the simplicity and show why it is powerful.

# Duality in condensed matter physics

Two equivalent descriptions of the same theory but from different points of view.

Very powerful *non-perturbative* insights into strongly interacting theories.

Origins: classical statistical mechanics of 2d Ising model (Kramers, Wannier 1941)

Many profound developments since in both (quantum) condensed matter and in quantum field theory.

# Classic example I: the 2d Ising model

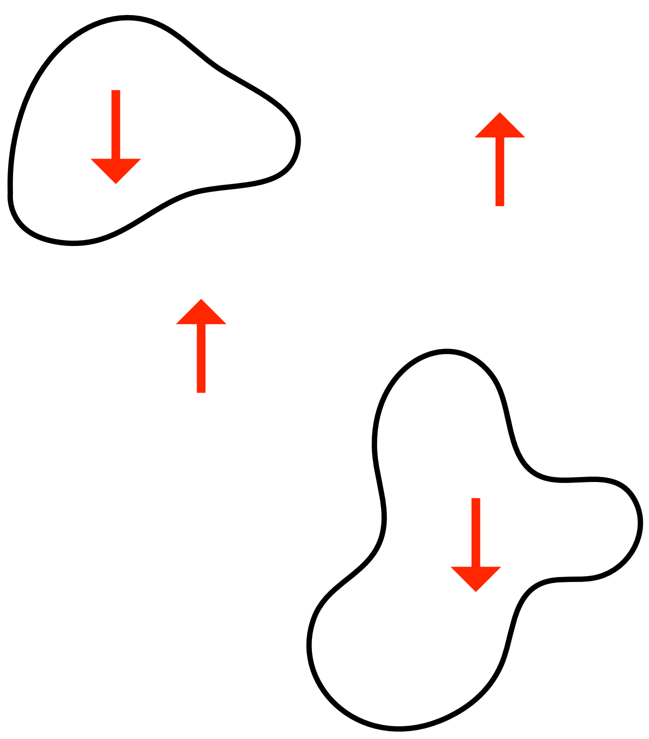
2d Ising model

Low T phase $\Longleftrightarrow$ High T phase

Physics: Describe either terms of Ising spins or in terms of domain walls (= topological defects of Ising ferromagnet).

Low-T phase: Spins ordered but domain walls costly

High-T phase: Spins disordered; domain walls have proliferated.



# Classic example II: Berezinsky-Kosterlitz-Thouless theory of 2d XY systems

Physics Nobel 2016 to K and T

Phase transition driven by vortices in XY order parameter

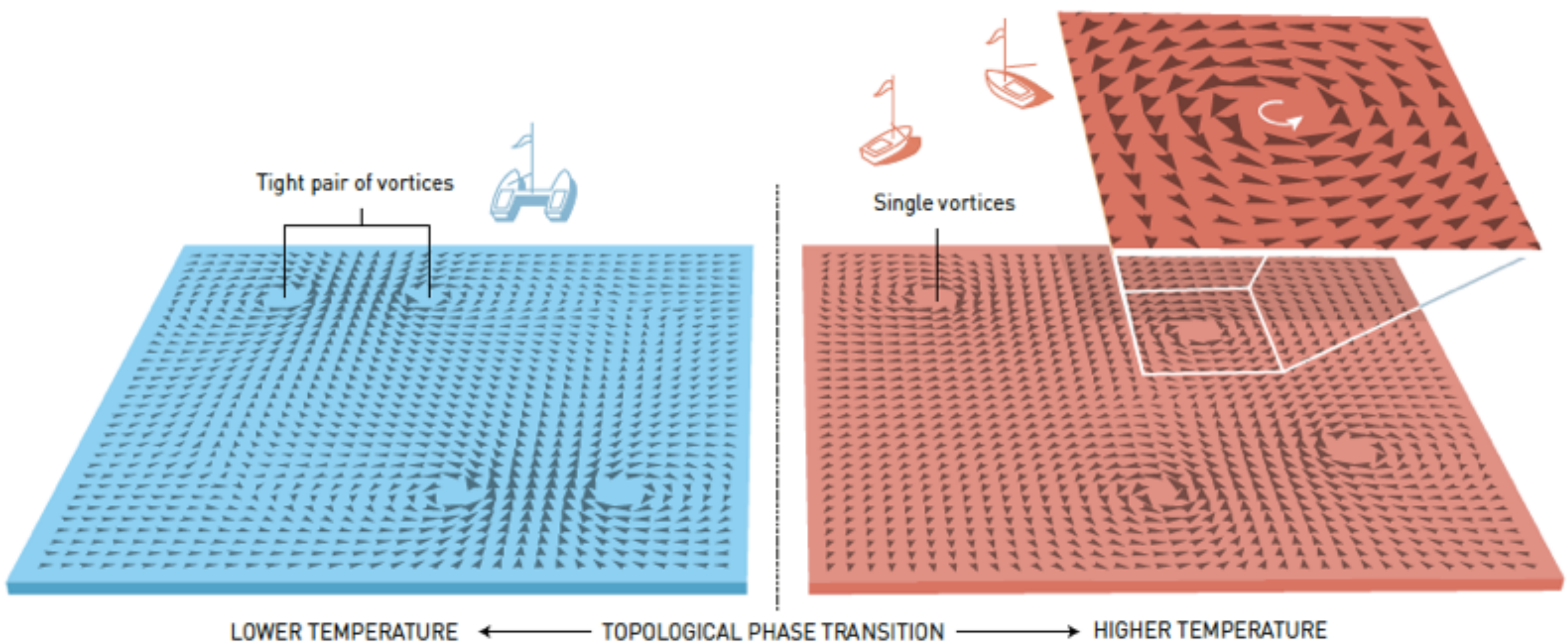



Illustration: ©Johan Jarnestad/The Royal Swedish Academy of Sciences

# BKT theory of 2d XY systems

Low-T phase: vortices cost log energy (= Coulomb potential in 2d).

High-T phase: vortices proliferate.

Duality of 2d XY model:

Reformulate as gas of +/-  charges interacting through 2d Coulomb potential.

In the context of ***quantum*** many body physics in 1d, these dualities are tremendously powerful and are part of the standard theoretical toolbox.

Is there a generalization to 2d quantum matter?

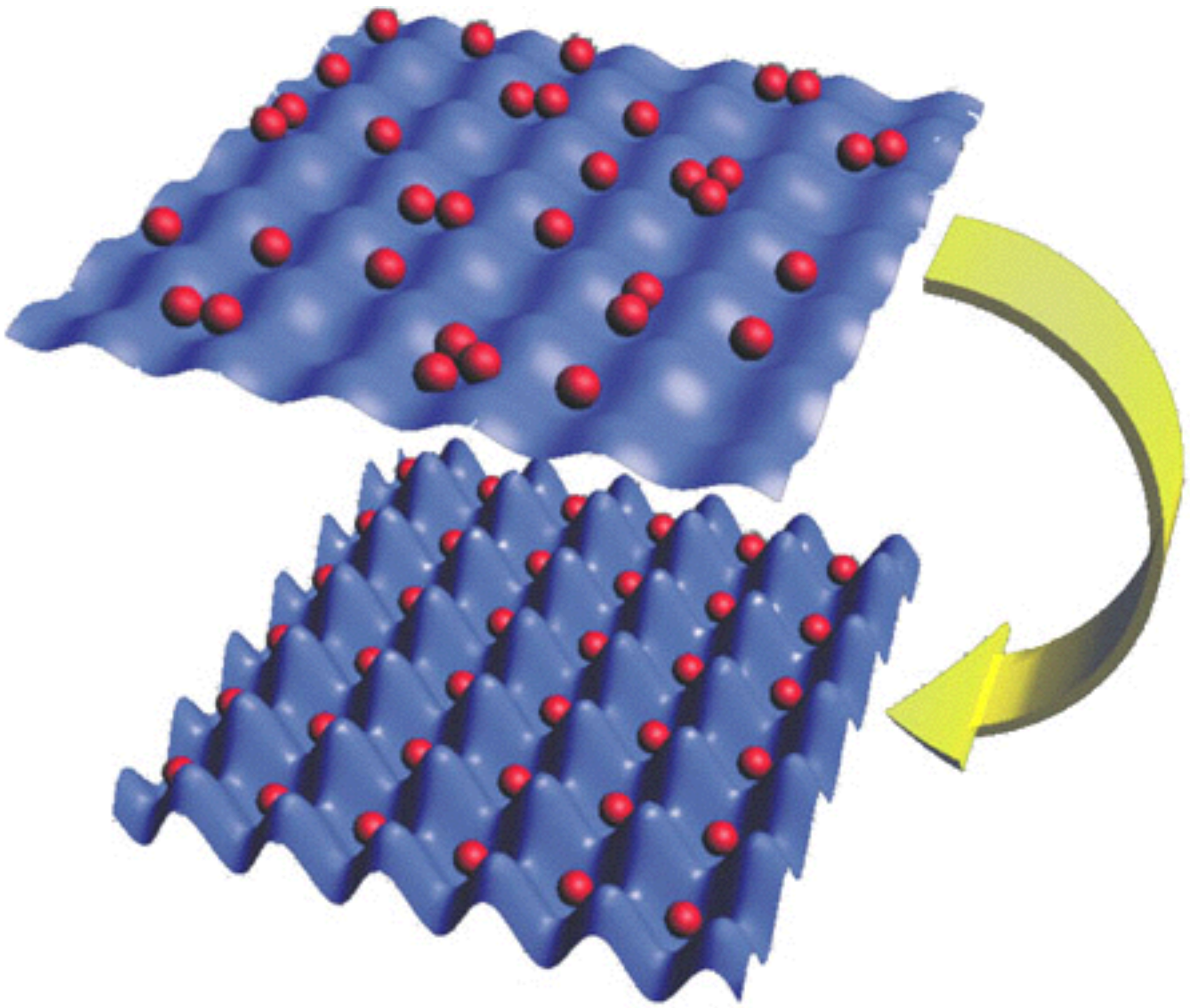
An old answer: yes (for strongly correlated bosons in 2d).

``Charge-vortex duality" for bosons

Dasgupta, Halperin 1981;
Peskin 1978;
Fisher, Lee, 1989

# Strongly correlated boson systems in 2d

Paradigmatic model: Boson Hubbard model
Specialize to integer filling/site.



$$H = -t \sum_{<ij>} \phi_i^\dagger \phi_j + h.c + U \sum_i n_i(n_i - 1)$$

t >> U: superfluid
U >> t: Boson Mott insulator

# Superfluid-insulator transition: Continuum effective theory

$$\mathcal{L} = |\partial_\mu \phi|^2 + r|\phi|^2 + u|\phi|^4$$

Global $U(1)$ symmetry: $\phi(x) \to e^{i\alpha}\phi(x)$.

Two phases:
$\langle\phi\rangle \neq 0$: superfluid order

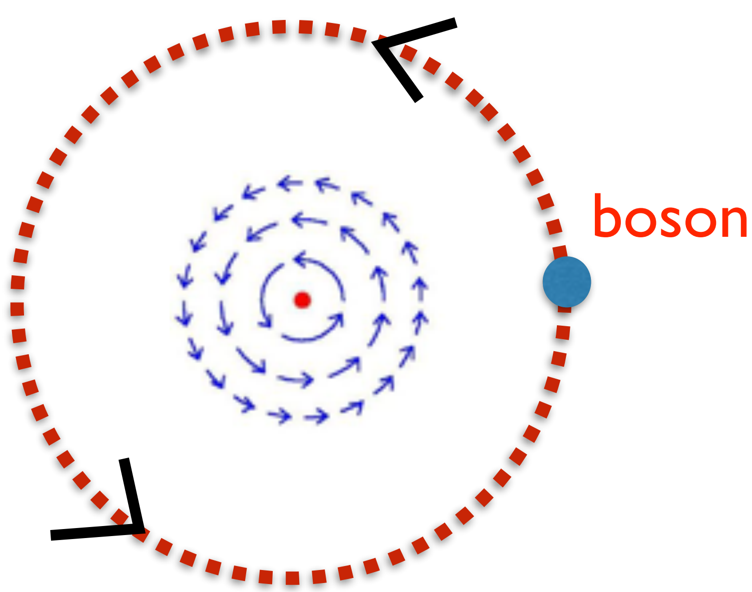
$\langle\phi\rangle = 0$ Mott insulator

Quantum critical point: extremely well understood, see, eg, Sachdev book

# Charge-vortex duality for bosons

Alternate description of phases/phase transitions in terms of vortices  of XY order parameter.

In 2d: vortices are point defects.




2π phase

# Charge-vortex duality for bosons

A vortex sees

(i) particle density as effective magnetic field $b = 2\pi\rho$

(ii) particle current as effective electric field $\vec{e} = 2\pi(\hat{z} \times \vec{j})$

Fluctuating particle density/current => vortices see fluctuating effective ``electromagnetic field" (represent in terms of effective scalar/vector potentials).

Dual vortex description: Vortices + U(1) gauge field.

# Charge-vortex duality of bosons

$$\mathcal{L} = |\partial_\mu \phi|^2 + r|\phi|^2 + u|\phi|^4$$

Physical boson

$$\mathcal{L}_v = |(\partial_\mu - i a_\mu)\phi_v|^2 + r_d|\phi_v|^2 + u_d|\phi_v|^4 + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2$$

Vortex field

Internal gauge field

# Dual description of phases

Boson superfluid:

Charge condensed $\Longleftrightarrow$ Vortices costly (energy gap)

Boson Mott Insulator

Charge gapped $\Longleftrightarrow$ Vortices condensed

# Applications of duality of bosons

This basic charge-vortex duality of bosons underlies
our understanding of many novel correlated boson phenomena.

Examples:

1. Fractional quantum Hall heirarchy (D.-H. Lee, M. P.A. Fisher 1989)

2. Fractional charge (spin) in 2d boson insulators (quantum magnets)
(TS, Fisher 2000)

3. Non-Landau quantum criticality in boson/spin systems
(TS et al 2004)

Other applications: a possible theory of SC-insulator transition in thin films (Fisher 1992)

# Is there a similar duality for fermions in 2d?

Yes! After > 35 years, in 2015, there is a generalization to fermions.

Wang, TS 2015; Metlitski, Vishwanath 2015
(Inspiration from Son 2015 ideas on composite fermions in quantum Hall effect)

Seiberg, TS, Wang, Witten 2016;
Karch, Tong 2016.

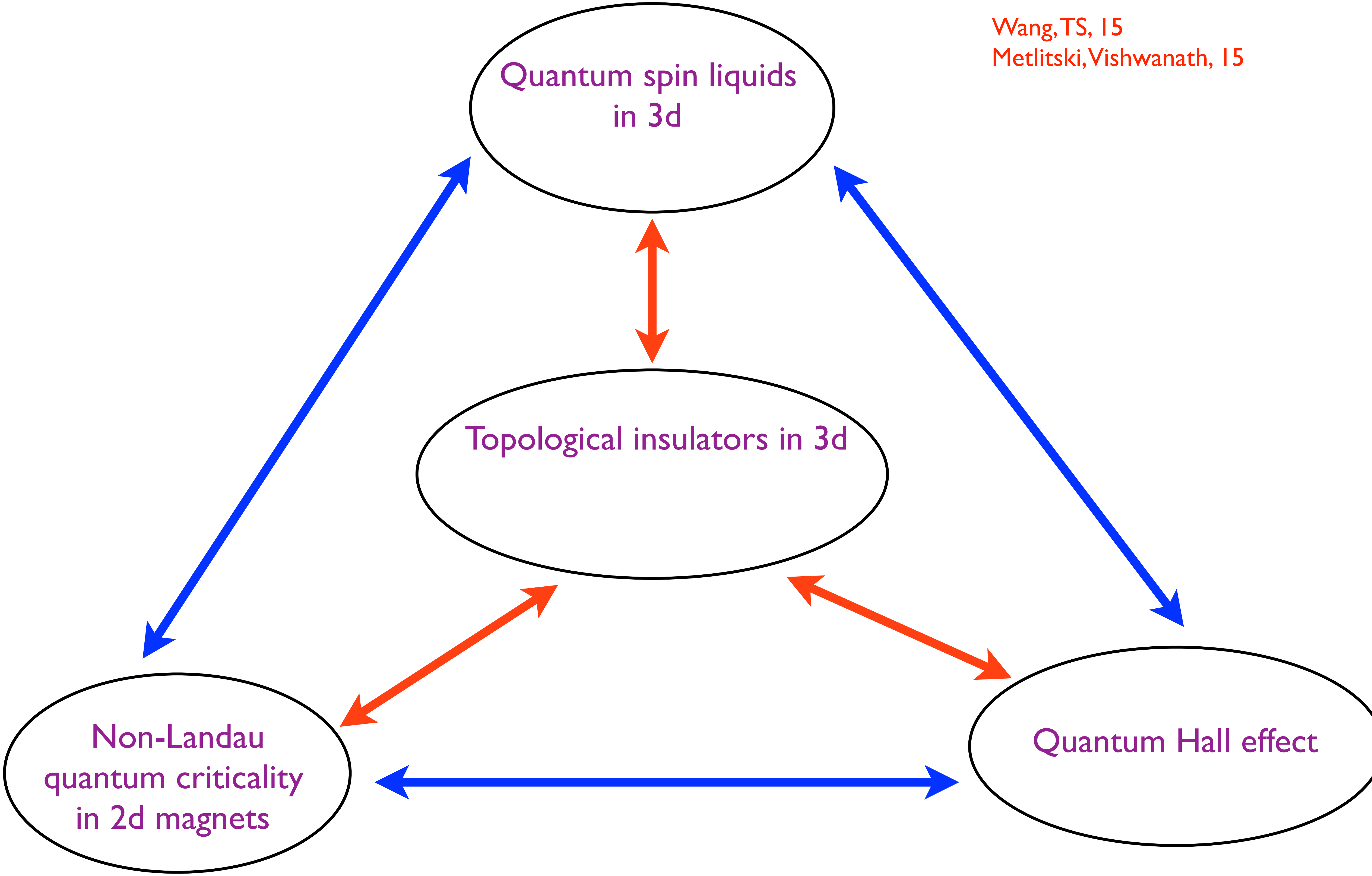
# Charge-vortex duality for fermions

New non-perturbative window into many correlated fermion problems.

Many  powerful applications expected - several already explored.

This duality is an  outgrowth of deep and surprising connections between many different topics in  correlated quantum matter.

# Deep connections between many apparently different problems

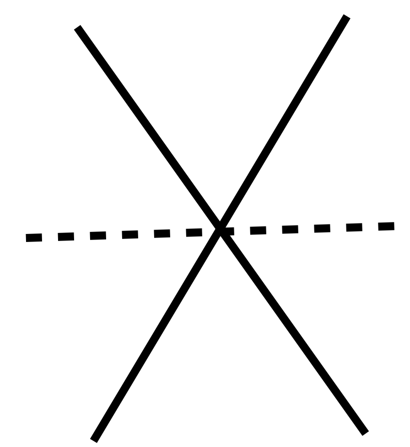


# Charge-vortex duality for fermions

Wang, TS, 15
Metlitski, Vishwanath, 15

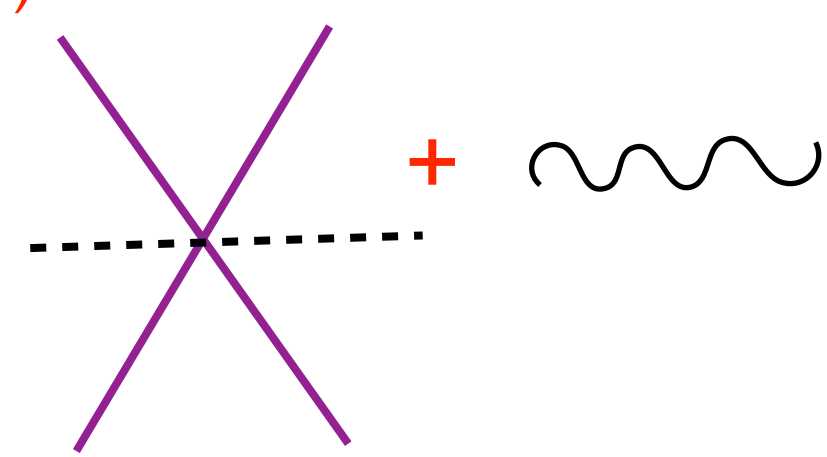
Theory A: Single massless Dirac fermion

$$\mathcal{L} = \bar{\psi}\left(-i\gamma^{\mu}\partial_{\mu}\right)\psi$$



Theory B: Dual massless Dirac fermion + U(1) gauge fields (*)

$$\mathcal{L}_v = \bar{\psi}_v\left(-i\gamma^{\mu}(\partial_{\mu} - ia_{\mu})\right)\psi_v + \frac{1}{2e^2}\left(\epsilon_{\mu\nu\lambda}\partial_{\nu}a_{\lambda}\right)^2$$



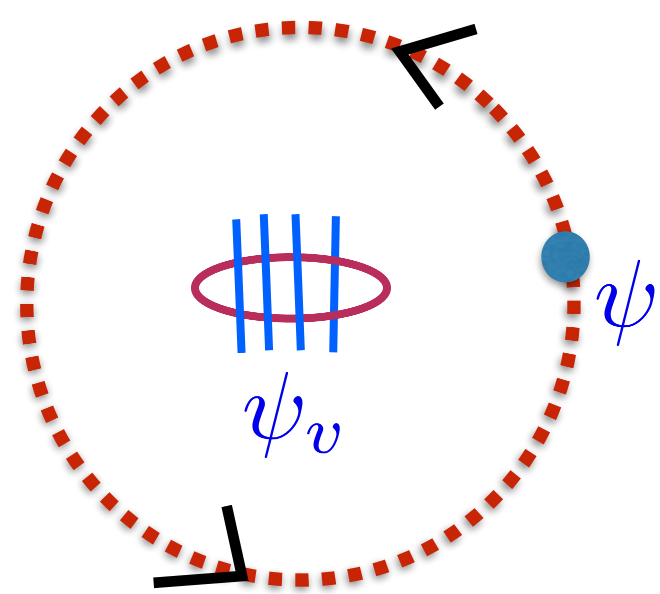

(*) For a more precise version see Seiberg, TS, Wang, Witten 16

# Charge-vortex duality for fermions

$$\mathcal{L} = \bar{\psi} \left( -i \gamma^\mu \partial_\mu \right) \psi \longleftrightarrow \mathcal{L}_v = \bar{\psi}_v \left( -i \gamma^\mu (\partial_\mu - i a_\mu) \right) \psi_v + \frac{1}{2e^2} \left( \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda \right)^2$$

Electron

Interpret: $\psi_v$ is a $4\pi$ "vortex" in the electron.




4π phase

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# Justifications

A consistency check:

Dual vortex theory has same operators and symmetries as the `charge' theory.

Derivations:

1. Through understanding of bulk 3d topological insulator (for which this theory is a surface) (Wang, TS 15; Metlitski, Vishwanath 15).

2. Construct both theories as systems of coupled 1d quantum wires (and use 1d dualities) (Mross, Alicea, Motrunich 16).

# Comment

This fermion-fermion duality is centrally connected to many frontier issues in modern condensed matter physics.

Examples:

1. Theory of composite fermions and quantum Hall physics in lowest Landau level in d = 2 space dimensions (connect to ideas of Son, 2015).

2. Theory of correlated 2d surface states of 3d topological insulators

3. Theory of quantum spin liquid states in 3 space dimensions

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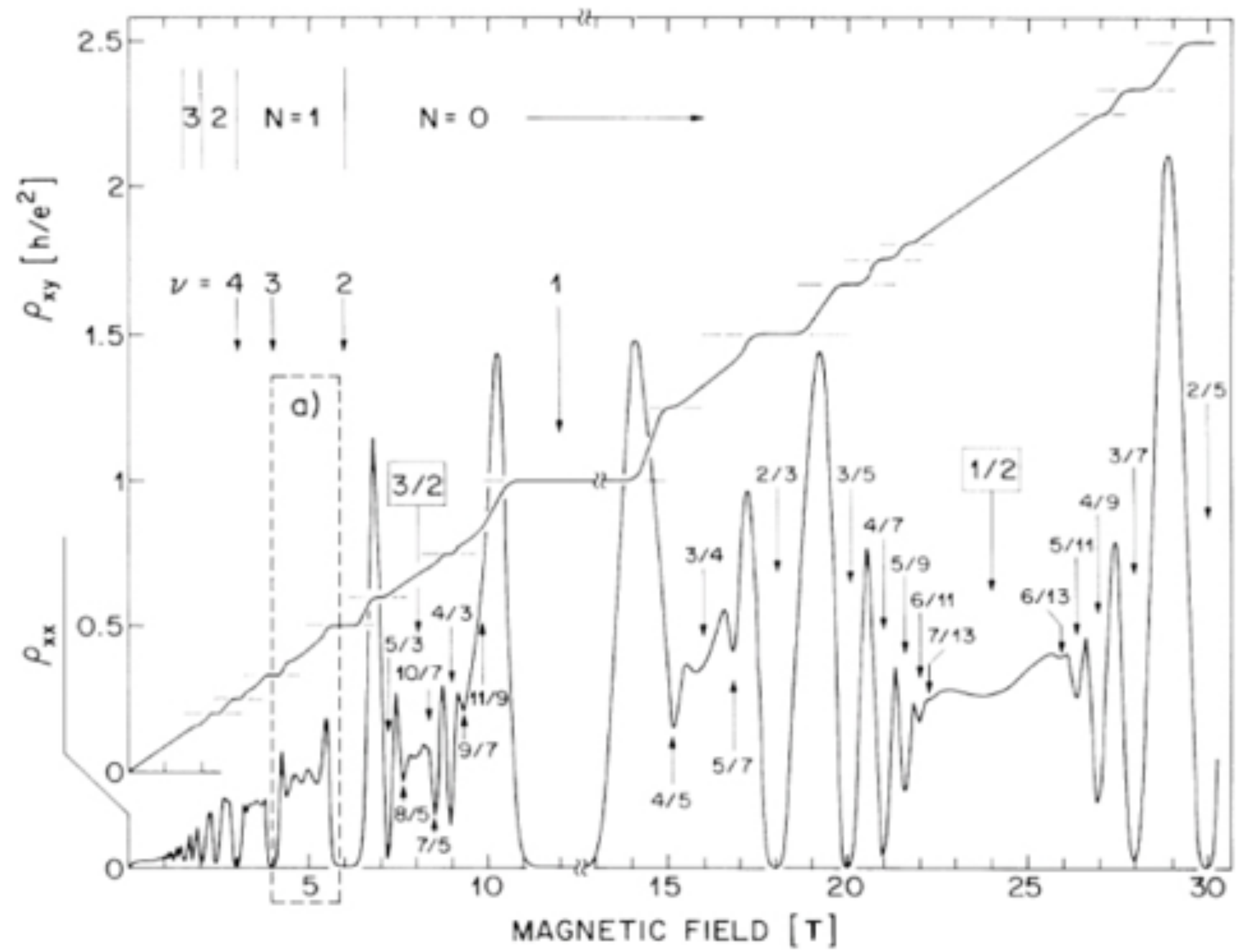
# 2d electrons in the ``quantum Hall" regime

Filling factor $\nu$ = 1,2., 3, ...... (IQHE)

$\nu$ = 1/3, 1/5, ...... (FQHE)

$\nu$ = 1/2, 1/4,…???

Experiment: Metal with $\rho_{xx} \neq 0$, $\rho_{xy} \neq 0$, but $\rho_{xx} \ll \rho_{xy}$.



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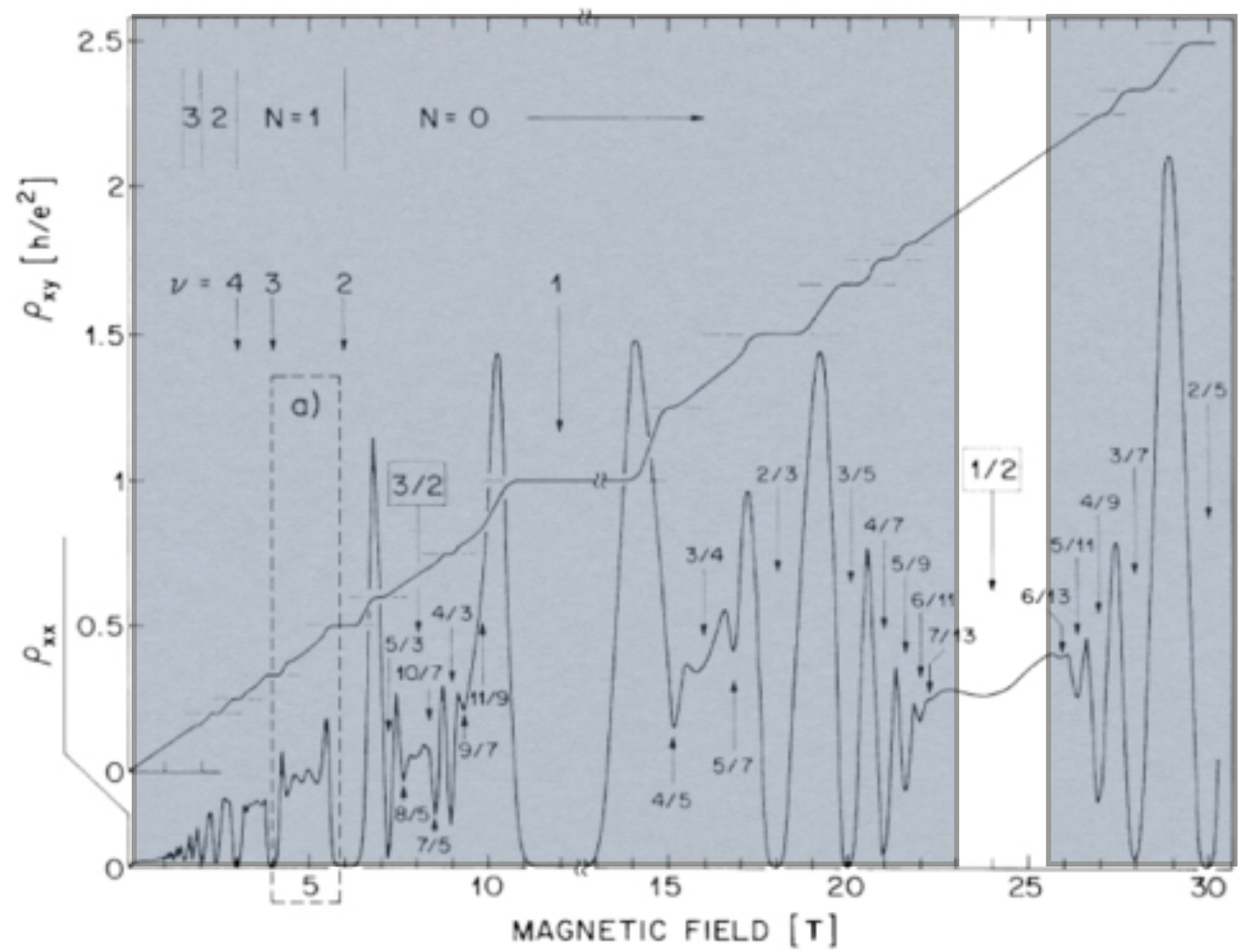
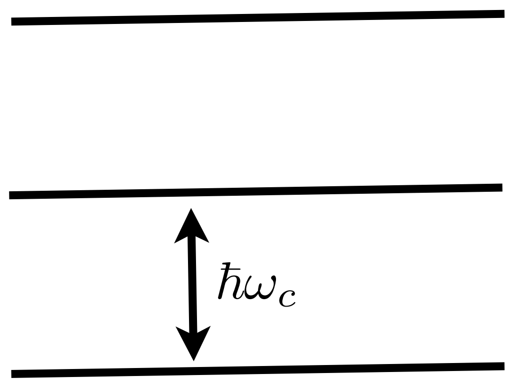


# The theoretical problem




Non-interacting electrons - highly degenerate Landau levels

Incompressible FQHE states: fill certain rational fractions of a Landau level.

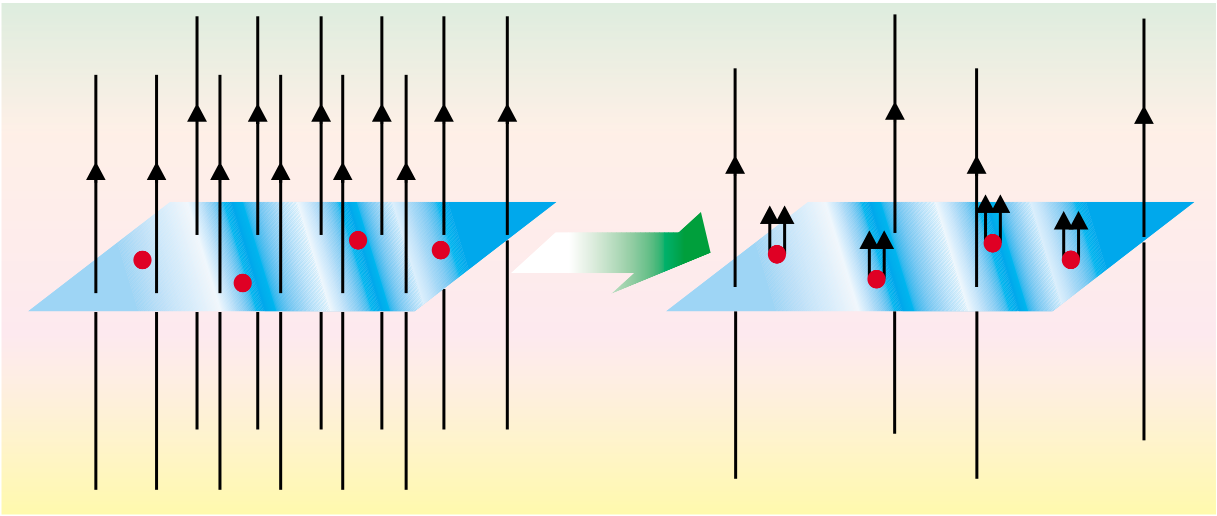
Large degeneracy is split by electron-electron interactions to give a gapped ground state.

Compressible metallic states: ``unquantized quantum Hall effect"

?? How do interactions manage to produce a metal ??

# Composite fermi liquid theory
# (Halperin, Lee, Read (HLR) 1993)

Assume (Jain 89) each electron captures two flux quanta to form a new fermion ("Composite fermions" )



See reduced effective field
$B^* = B - (2h/e)\rho$

At $\nu = 1/2$, $B^* = 0$

=> form Fermi surface of composite fermions

Effective theory:

$$\mathcal{L} = \overline{\psi}_{CF}\left(i\partial_t - a_0 - iA_0^{ext} + \frac{(\vec{\nabla} - i(\vec{a} + \vec{A}))^2}{2m}\right)\psi_{CF} + \frac{1}{8\pi}a_\mu \epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda \qquad (1)$$

# Some experimental verification of composite fermions

Many groups: Willett, Stormer, Tsui, Shayegan, Goldman,.....

Examples:

1. Slightly away from $\nu = 1/2$, $B^* = B - (2h/e)\rho \neq 0$ but much reduced from external field B.

=> composite fermions move in cyclotron orbits with radius >> electron cyclotron radius

2. Confirmation of composite fermion Fermi surface
(eg, Shubnikov-deHaas oscillations)

3. Successful description of the prominent FQHE states at $\nu = n/(2n+1)$ as ``integer quantum hall states'' of the composite fermions.

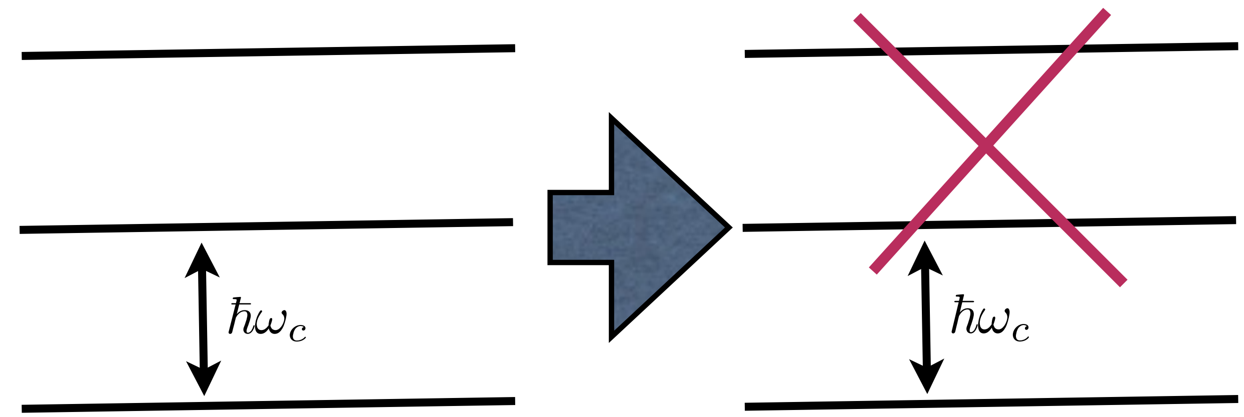
# Unsatisfactory aspects of the theory

1. Theory should make sense within the Lowest Landau Level (LLL) but HLR not suited to projecting to LLL.

Many refinements in the late 90s (Shankar, Murthy; Read; Halperin, Stern, Simon, van Oppen; D.-H. Lee, Pasquier, Haldane,.......) but dust never settled.

3. LLL theory has an extra symmetry that HLR is blind to.

Issue identified in the 90s (Grotov, Gan, Lee, Kivelson, 96; Lee 98) but no resolution.




$V_{int} \ll \hbar\omega_c$: project to LLL

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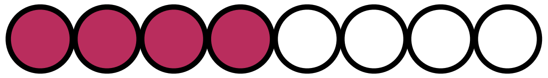
# Particle-hole symmetry in LLL

At $\nu = 1/2$, regard LLL as either ``half-empty or half-full'':

Start from empty level, fill half the LLL



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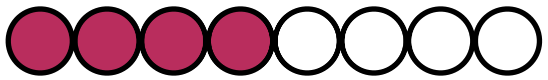


Numerical work: Metallic ground state at $\nu = 1/2$ preserves this symmetry. (Haldane, Rezayi, 00, Geraedts et al, 16)

HLR theory: Not in Lowest Landau Level; p/h not within its scope.

Recent intriguing suggestion for p/h symmetric theory(Son 15): Composite fermion as a Dirac particle?

Alternate Theory: Dirac composite fermions + U(1) gauge field but no Chern Simons term.

# Application of the new fermionic duality: Deriving the p/h symmetric composite Fermi liquid:

Wang, TS, 15; Metlitski, Vishwanath 15

# p/h symmetric LL in a system of Dirac fermions

Start with a single massless Dirac fermion with p/h symmetry (C).

C symmetry guarantees that surface Dirac cone is exactly at neutrality.

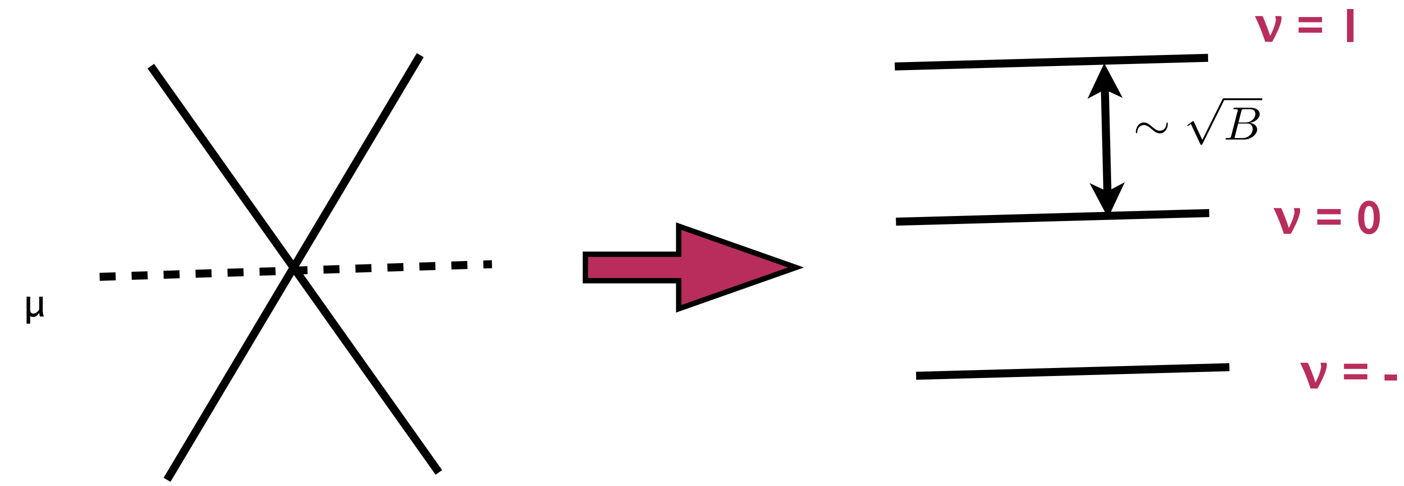
$$\mathcal{L} = \bar{\psi}\left(-i\not{\partial} + \not{A}\right)\psi + ....$$

external probe gauge field

2-component fermion

# p/h symmetric LL  in a system of Dirac fermions

Perturb Dirac cone with external B-field.



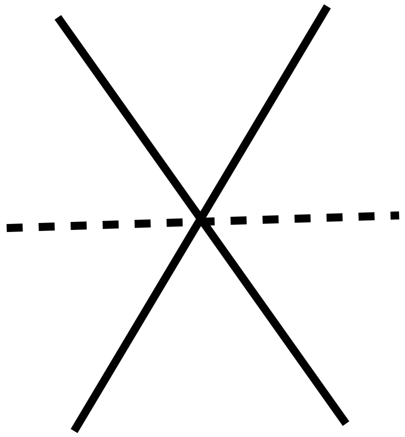

C-symmetry: $\nu = 0$ LL is exactly half-filled.

Low energy physics:  project to 0LL

With interactions => map to usual half-filled LL

# Use of the fermionic duality

Electrons



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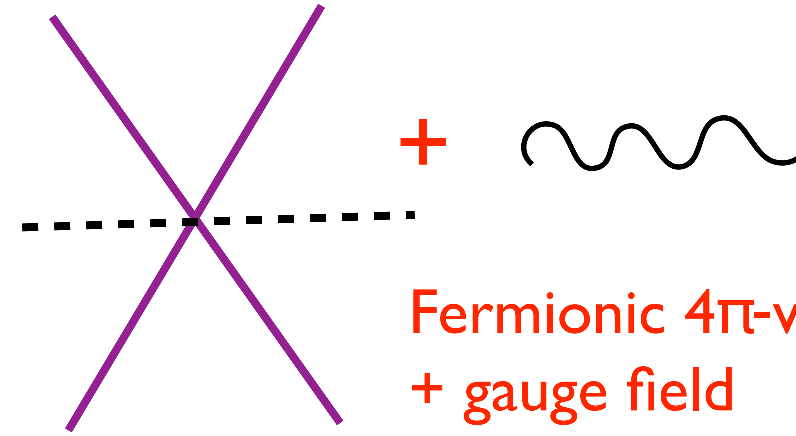
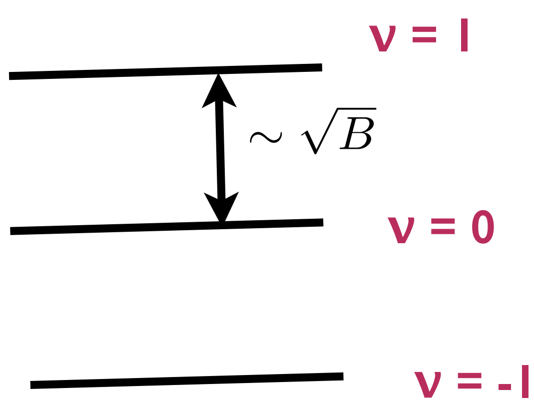



Fermionic $4\pi$-vortices
+ gauge field

Physical magnetic field B ⟷ Finite density of fermionic vortices

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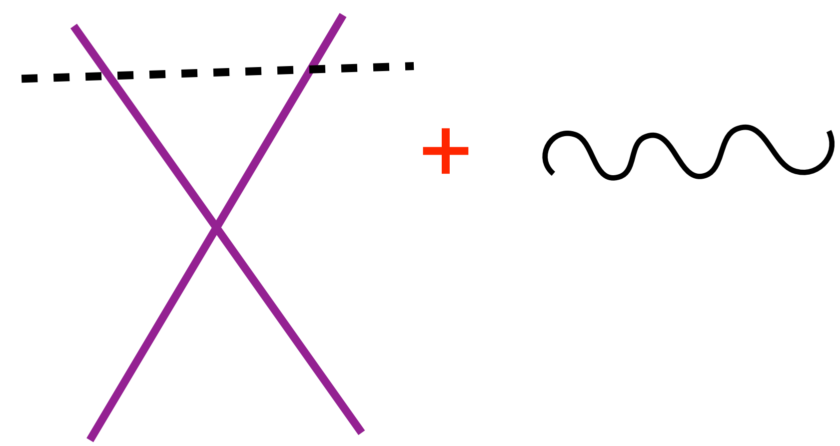


Exactly Son's proposed Dirac composite fermion theory.

# Composite fermi liquids as vortex metals

HLR/Jain composite fermion: Charge - flux composites

Particle-hole symmetric composite fermion: Neutral vortex

Describe CFL as a vortex liquid metal formed by neutral fermionic vortices.

Simple understanding of transport:
(similar to other 2d quantum vortex metals, eg, in Galitski, Refael, Fisher, TS, 06)

# Other applications of the duality/comments

1. Strongly correlated surface states of 3d topological insulators
(eg, gapped surface states with non-abelian anyons)

2. Many generalizations of this duality have been found in last 2 years and have been related to each other
(Seiberg, TS, Wang, Witten 16; Karch, Tong 16; You, Xu 16; ……..)

3. New progress in understanding field theories of some gapless quantum spin liquid magnets

4. Understanding of enlarged emergent symmetries at non-Landau quantum critical points in 2d magnets (Wang, Nahum, Metlitski, Xu, TS, 17)

Future…………….??

# Summary

Dualities provide a non-perturbative window into correlated many body systems.

Familiar and powerful for 1d physics, and for 2d bosons.

Dualities for 2d Dirac fermions: new window to view and solve some diverse SCES problems.

Many future challenges/opportunities:

3d fermions?? Metals with a Fermi surface??