Non-Fermi Liquids: Slave particles and beyond

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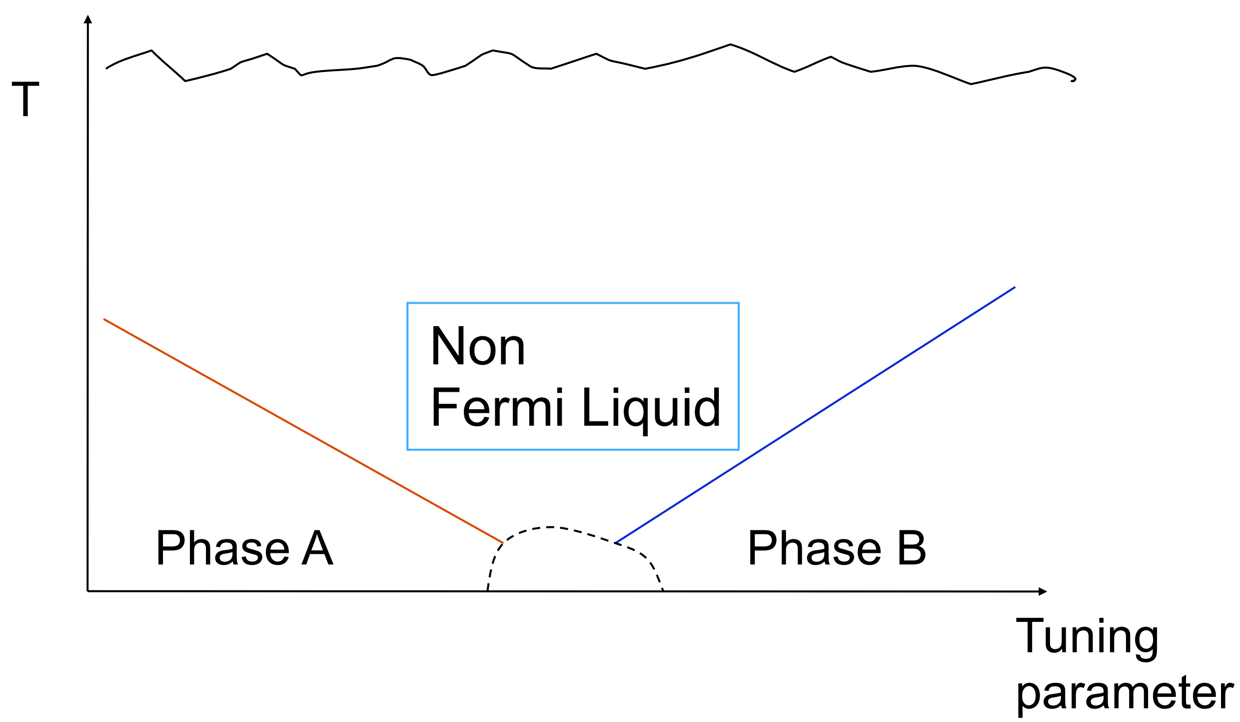
A common phase diagram



NFL due to quantum criticality?

Approach from Fermi liquid

Crucial: Fate of Fermi surface as a Fermi liquid metal undergoes a quantum phase transition?

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Two general possibilities

Mutilate

Fermi surface evolves continuously but is distorted in some way.
(Ferromagnet, nematic, SDW, CDW,.....)

Kill

Original Fermi surface completely disappears.
(Mott transition, Kondo breakdown,.....)

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Many talks at this meeting!

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Mutilate the Fermi surface

Model: Electron Fermi surface + X

X = critical order parameter fluctuations

Criticality and non-Fermi liquids at ``low'' energy scales?

Killing a Fermi surface

Expect

- strong NFL physics near transition

- change of electronic structure happens on electronic energy scales

- theory not described as electron Fermi surface + X

How does a Fermi surface die?

Fascinating question fundamental to many issues

1. Quasiparticle residue Z vanishes continuously on approaching transition. (Brinkman, Rice 1970).

Concrete examples: (d = 2,3) TS, Vojta, Sachdev 2004, TS, 2008, Nandkishore, Metlitski, TS (2012); DMFT in d = ∞

2. Fermi surface stays sharp at QCP even though there is no Landau quasiparticle (TS, 2008).

``Critical Fermi Surface"

Concrete examples: (d = 2) TS, 2008, Nandkishore, Metlitski, TS (2012).

Other related problems: Non-fermi liquid ***phases***, some (gapless) quantum spin liquids.........

Goal: Construct tractable, emergable effective field theories of these phenomena

(Emergable: *capable of emerging* from microscopic lattice models in the *`right' physical Hilbert space* with the *right symmetries*.)

A powerful approach

Slave particles (partons): Fractionalize spin/electron into partons which are then gapless.

Effective theory: Gapless partons + dynamical gauge fields.

An example
Orthogonal Metals: The simplest non-fermi liquids

Nandkishore, Metlitski, TS, 12

'Slave spin' representation:

$$c_{i\alpha} = \tau_i^x f_{i\alpha}$$

τ_i^x: Ising spin; $f_{i\alpha}$: charge-1, spin-1/2 fermion.
Z_2 gauge redundancy

$$\begin{aligned} \tau_i^x &\to -\tau_i^x \\ f_{i\alpha} &\to -f_{i\alpha} \end{aligned}$$

Phase where $\langle \tau_i^x \rangle = 0$, the $f_{i\alpha}$ form a Fermi surface, and Z_2 gauge field is deconfined:

Electron spectral function gapped (Z = 0) but still a metal: ``Orthogonal metal''
Charge carriers have no overlap with electrons.

(Prior literature: Mis-interpreted as Mott insulator)

Comments

1. Relation to usual slave bosons/rotors: $c_{i\alpha} = b_i f_{i\alpha}$

   Condense $\langle b_i^2 \rangle \neq 0$ with $\langle b_i \rangle = 0$.

2. OM Wavefunction

   $$\psi(\vec{r}_i \alpha_i) = \psi_{PSF}(\vec{r}_i)\psi_{Slater}(\vec{r}_i \alpha_i)$$

   ψ_{PSF}: wave function of paired boson superfluid

   $(\propto \mathcal{S}[g(\vec{r}_1 - \vec{r}_2)g(\vec{r}_3 - \vec{r}_4)\ldots..g(\vec{r}_{N-1} - \vec{r}_N)])$.

3. OM phase can exist at any density or even in continuum.

4. Exactly soluble lattice models easy to write down.

`Death' of a Fermi surface: FL- Orthogonal Metal transition

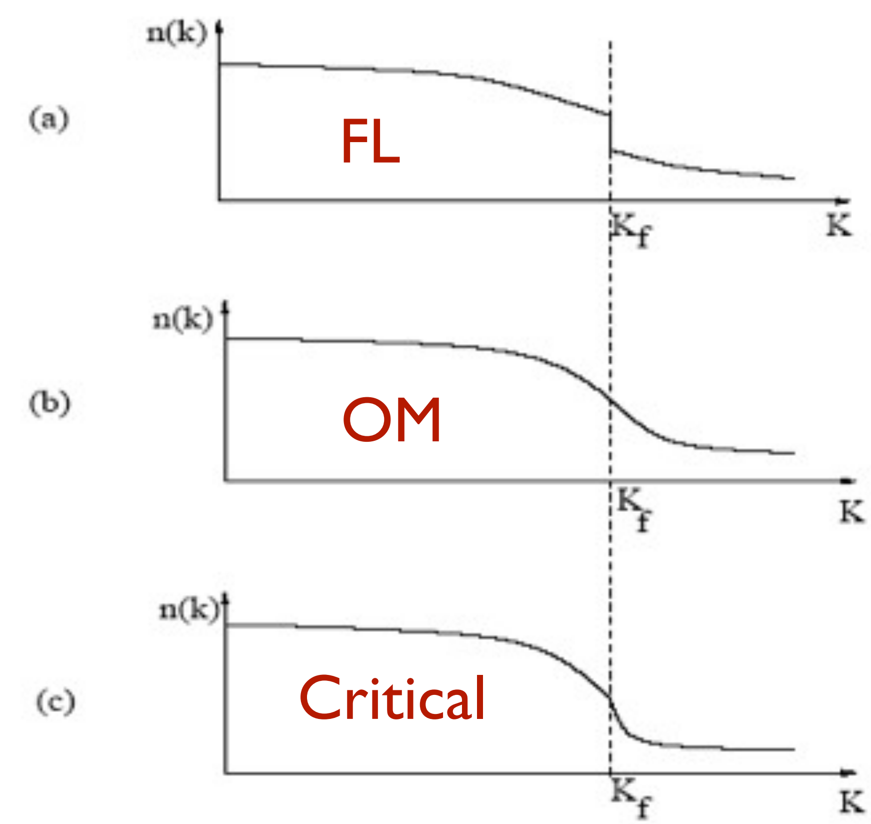
Approach from FL: electron Fermi surface dies.

Slave spin disordering transition

Critical theory: Critical slave spin + f-Fermi surface with energy-energy coupling.
(Second order in some situations).

Critical electron Fermi surface: exponents exactly determined by Ising exponents.

$$A(\vec{q},\omega) \sim \omega^{d-2+\eta} g\left(\frac{\omega}{v_f q_\perp}\right)$$

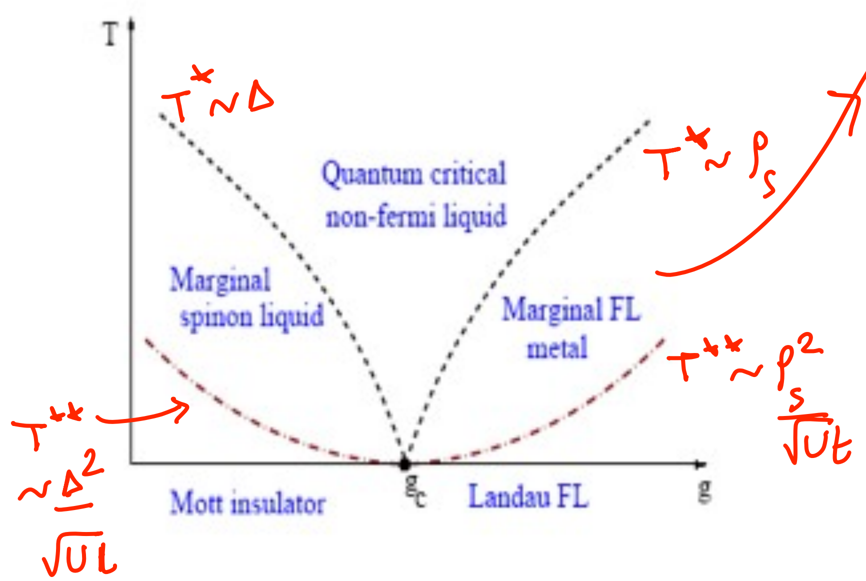


Critical theory ≠ electron Fermi surface + X

Similar story: Theory of continuous Mott transitions

Mott metal-insulator transition can be continuous in d = 2 or d = 3 if insulator is spin liquid.

Slave mean field: Florens, Georges 04; Fluctuations, field theory: TS 08; Numerics: Motrunich, Fisher, et al 14.



In this regime electron Greens function

$$G(\vec{k}, i\omega) \sim \frac{|\langle b \rangle|^2}{i\omega\left(1 + a \ln \frac{1}{|\omega|}\right) - \epsilon_k}$$

= Marginal FL form proposed for hiTc by Varma et al '89

Many interesting properties:

Eg: Two crossovers

Emergence from criticality ≠ Fermi liquid coherence

Comments on slave particle framework

1. **Conceptually important** construction of effective field theories of a class of quantum phases/phase transitions that are beyond standard quasiparticles

2. Slave particle effective field theories are emergable, and often tractable.

Theoretical demonstration of many unusual phenomena; some successful contact with experiments (eg, FQHE, 1/2-filled Landau level, some quantum spin liquids)

Essentially only ***available*** framework for field theory `beyond quasiparticles'.

Beyond (standard) slave particles?

Are there gapless quantum liquids that cannot be understood (easily) within the slave particle framework?

Yes!

Two examples in quantum spin/bose liquids

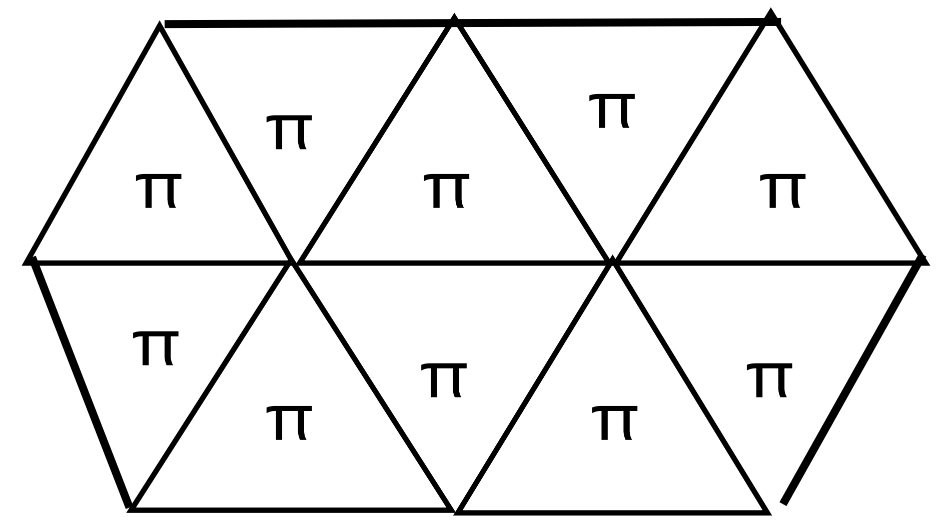
(i) Quantum vortex liquid phases of quantum XY magnets/frustrated boson systems

Chong Wang, TS to appear.

(ii) Quantum liquid of fluctuating spiral magnetic orders (inspired by MnSi).

A. Vishwanath, TS, to appear.

Example-I: Gapless quantum vortex liquids



d = 2 XY spin-S triangular lattice quantum antiferromagnet = bosons with frustrated hopping.

Duality: Vortices at 1/2 filling on honeycomb lattice.

Interesting early attempt: (Fisher et al)

Fermionize vortices through flux attachment and let then form a gapless fluid (``Algebraic Vortex Liquid")

Flaw (Wang, TS 13): With time reversal, this state cannot exist in strict d = 2 (but can exist at boundary of a 3d ``SPT" state)

Fractionalizing vortices

Chong Wang, TS 14;
also Hermele 10

Dual theory:

$$\mathcal{L} = \mathcal{L}[\Phi_v, a_\mu]$$

Φ_v: vortex field on honeycomb lattice
a_μ: non-compact $U(1)$ gauge field.
Dual partons:

$$\Phi_v = d_1 d_2$$

$d_{1,2}$: Fermionic $1/2$-vortices.
Dual parton field theory:
Put d in various 'mean field' states; analyze fluctuations.

Fractionalized vortex liquids

Dual parton mean field:
Can arrange to break all gauge structure associated with fractionalizing Φ_v.

d-fermions gapped: Paired superfluid (spin nematic) of original bosons (spins).

Dirac nodes for d-fermions => new gapless spin liquid phase.
Effective theory:

$$\mathcal{L} = \bar{d}\left(\gamma_\mu(i\partial_\mu - a_\mu\right) d + \frac{1}{g} f_{\mu\nu}^2$$

Dual massless *non-compact* QED_3.

Power law correlations for many quantities.

Physical picture

Excitations of paired boson superfluid:

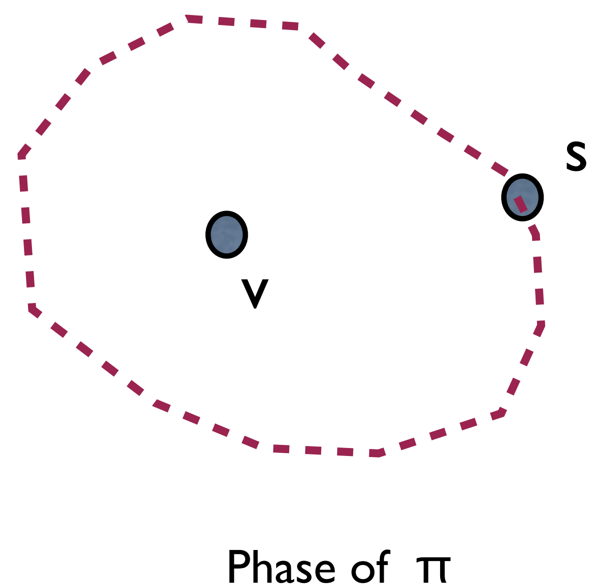
1. Gapless sound waves
2. Gapped neutralized unpaired boson s
3. Gapped 1/2-vortices v

s and v are mutual semions.

s-v bound state is a fermionic vortex.

Proliferate to get a gapless fractionalized vortex liquid.

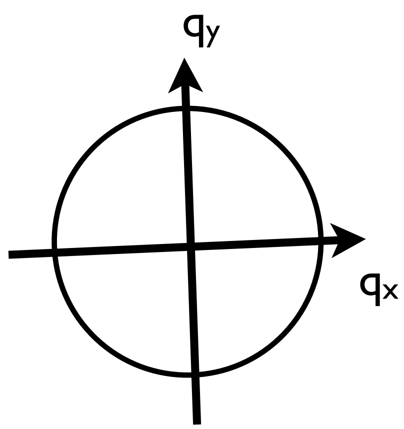
Beyond (standard) slave particles: in terms of original bosons, short distance physics is that of pairing, not fractionalization.



Example-II: Bose Luttinger Liquid in d > 1

Some situations in which bosonic degree of freedom has dispersion minima along surfaces in momentum space:

Eg: Incipient spiral magnetic order (eg MnSi), bosons with Rashba coupling,.......



Model action

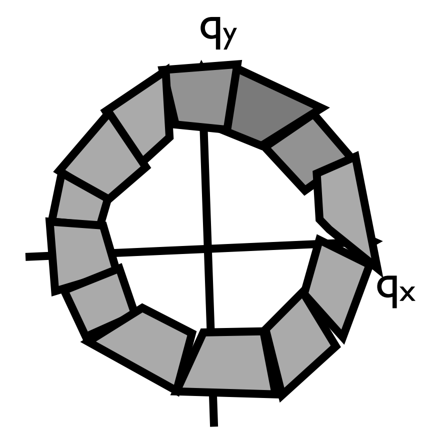
$$S = \int_{\vec{q},\omega} \left(\omega^2 + (q^2 - q_0^2)^2 + r\right) |b(\vec{q},\omega)|^2 + S_{int}$$

Usually interactions will spontaneously pick out some q-vectors for ordering.

This talk: a fluctuating quantum liquid state wihout any ordering by `bosonizing' the bosons.

Patch construction

Divide vicinity of Bose surface into discrete patches



Bose field

$$b(\vec{r},\tau) = \frac{1}{\sqrt{N}} \sum_S b_S(\vec{r},\tau) e^{i\vec{q}_S . \vec{r}}$$

Amplitude-phase representation

$$b_S = b_0 e^{i\phi_S}$$

Strategy:

1. Assume initially b_0 is uniform on Bose surface but ϕ_S may fluctuate.
2. Study "phase-only" theory of ϕ_S
3. Worry about vortices
4. Take patch continuum limit, thermodynamic limit.

Phase-only theory on Bose surface

First also ignore inter-patch coupling.

Quadratic action:

$$Ł_{\phi}^{(0)} = \frac{1}{N} \sum_{S=1}^{N} \rho_S \left\{ (\partial_\tau \phi_S)^2 + (\vec{v}_S \cdot \vec{\nabla} \phi_S)^2 \right\}$$

1+1-d XY field for each patch:

Boson has power law correlations centered at Bose surface with continuously varying exponent.

$$\langle \bar{b}_S(\vec{q}, i\omega) b_S(\vec{q}, i\omega) \rangle \sim \frac{1}{(v^2 q_\perp^2 + \omega^2)^{1-\frac{\eta}{2}}}$$

``Bose Luttinger Liquid"

Stability

Interpatch interactions: As in Fermi Liquid Theory, only ``forward'' scattering and ``BCS'' interactions survive.

``BCS'': Irrelevant by power counting if $\eta > 1$.

``Forward'' scattering: Couple together density and currents of XY fields at different patches.

Landau interactions with same qualitative effects as in Fermi Liquid

(similar renormalizations, collective zero sound modes, etc).

Vortices: Only legitimate vortex is in $l = 0$ phase mode.

This can be suppressed => Bose Luttinger Liquid exists as stable phase of matter.

Summary

1. Slave particle framework: effective field theories which concretely demonstrate many interesting phases/phase transitions beyond quasiparticles.

2. There are interesting states beyond slave particles.