# Quantum criticality beyond the Landau-Ginzburg-Wilson paradigm

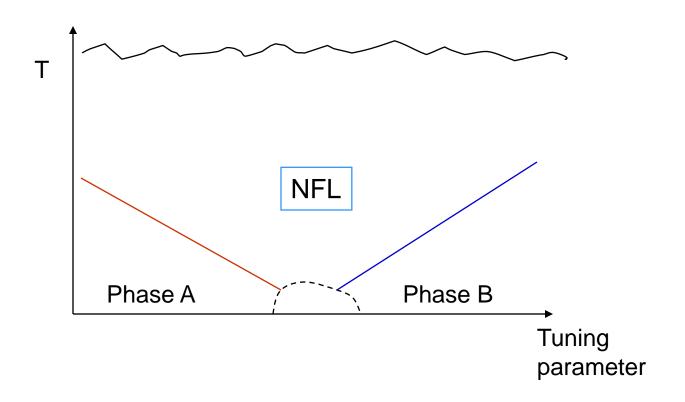
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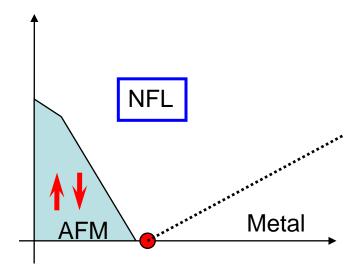
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# Competing orders and non-fermi liquids(NFL) in correlated electron systems

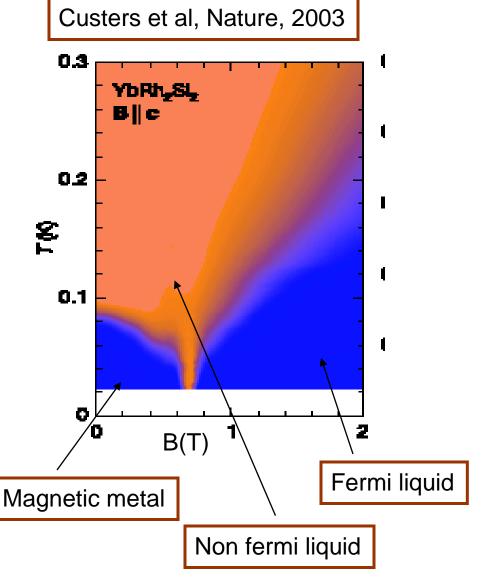


# Example: Magnetic ordering in heavy electron systems CePd<sub>2</sub>Si<sub>2</sub>, CeCu<sub>6-x</sub>Au<sub>x</sub>, YbRh<sub>2</sub>Si<sub>2</sub>,......

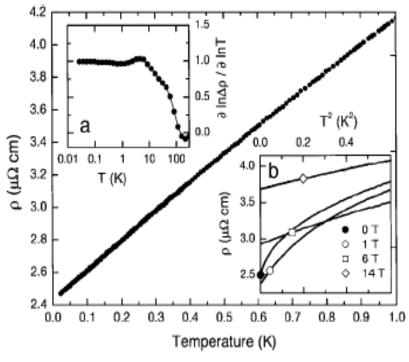


Model as lattice of localized magnetic moments coupled to conduction electrons by spin exchange (the Kondo lattice)

# Representative data on YbRh<sub>2</sub>Si<sub>2</sub>



Trovarelli et al, PRL 2000



T-dependence of resistivity at critical point:  $\rho(T) \sim T$  for three decades in temperature!

## "Classical" assumptions

 NFL: Universal physics associated with quantum critical point between phases A and B.

 Landau: Universal critical singularities ~ fluctuations of order parameter for transition between phases A and B.

Try to play Landau versus Landau.

 However ``classical' assumptions have difficulty with producing NFL at quantum critical points!!

Eg: Landauesque theory of magnetic ordering in metallic environment (``Moriya-Hertz-Millis theory'') spectacularly inconsistent with NFL near heavy fermion critical points.

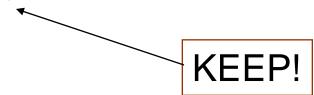
## Reexamine ``classical'' assumptions

1. NFL: Universal physics associated with quantum critical point between phases A and B.

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### Reexamine ``classical'' assumptions

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### Reexamine ``classical'' assumptions

1. NFL: Universal physics associated with quantum critical point between phases A and B.

KEEP!

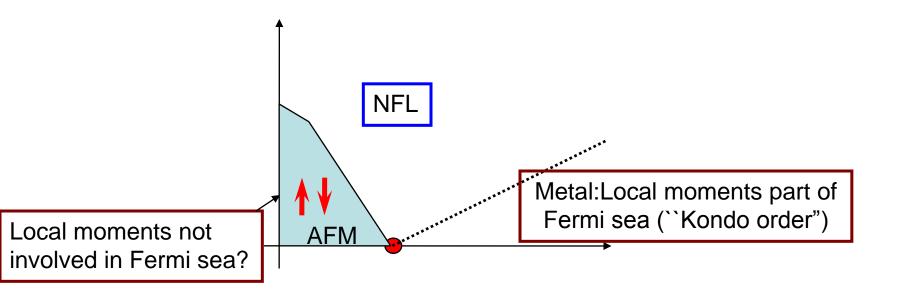
2. Landau: Universal critical singularities ~ fluctuations of order parameter for transition between phases A and B.

??IS THIS REALLY CORRECT??

### (Radical) alternate to classical assumptions

- Universal singularity at some quantum critical points: Not due to fluctuations of natural order parameter but due to some other competing effects.
- Order parameters/broken symmetries of phases A and B mask this basic competition.
- => Physics beyond Landau-Ginzburg-Wilson paradigm of phase transitions.

### NFL in heavy electron systems

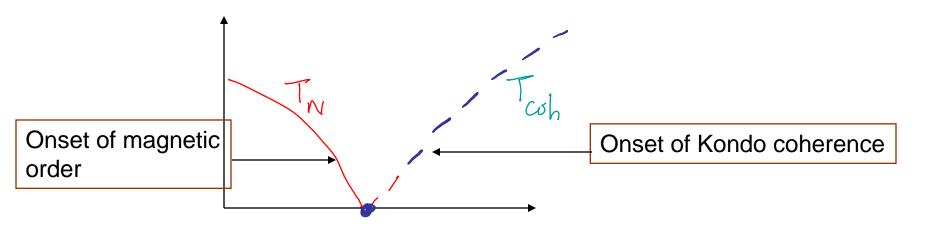


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Critical NFL physics: are fluctuations of loss of local moments from Fermi sea important? (Si, Coleman,.....)

Is magnetic ordering itself a distraction?? (TS, Vojta,Sachdev)

Perhaps NFL only due to fluctuations of ``Kondo order"??
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### Questions



- Is such a second order transition generically possible?
   (Loss of magnetic order happens at same point as onset of ``Kondo'' order)
- 2. Theoretical description?
- 3. Will it reproduce observed non-fermi liquid behaviour?

Answers not known!!

### General observations

f-moments drop out of Fermi surface ( change of electronic structure)

Associated time scale t<sub>e</sub>.

Onset of magnetic order

Associated time scale t<sub>m</sub>.

Both time scales diverge if there is a critical point.

### General observations

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Associated time scale t<sub>e</sub>.

Onset of magnetic order

Associated time scale t<sub>m</sub>.

Both time scales diverge if there is a critical point.

Suggestion: t<sub>m</sub> diverges faster than t<sub>e</sub>. (electronic structure change first, magnetic order comes later)

Separation between two competing orders as a function of scale (rather than tuning parameter) might make second order transition possible.

### Some implications

• <u>"Underlying"</u> transition: loss of participation of the f-electrons in forming the heavy fermi liquid.

(View as a Mott ``metal-insulator'' transition of f-band).

- Magnetic order: "secondary" effect a low energy complication once Kondo effect is suppressed.
- Non-fermi liquid due to fluctuations associated with change of electronic structure rather than those of magnetic order parameter.
- ⇒ PHYSICS BEYOND LANDAU-GINZBURG-WILSON PARADIGM FOR PHASE TRANSITIONS.

(Natural magnetic order parameter is a distraction).

# This talk – more modest goal

 Are there any clearly demonstrable theoretical instances of such strong breakdown of Landau-Ginzburg-Wilson ideas at quantum phase transitions?

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### Study phase transitions in insulating quantum magnets

- Good theoretical laboratory for physics of phase transitions/competing orders.

## Highlights

- Failure of Landau paradigm at (certain) quantum transitions
- Emergence of `fractional' charge and gauge fields near quantum critical points between two <u>CONVENTIONAL</u> phases.
- ``Deconfined quantum criticality'' (made more precise later).
- Many lessons for competing order physics in correlated electron systems.

### Phase transitions in quantum magnetism

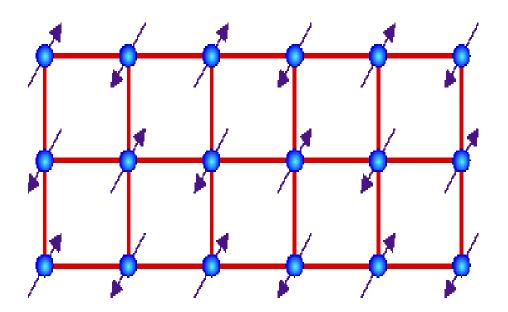
$$H = J \sum_{\langle rr \rangle} \vec{S}_r, + \cdots$$

- Spin-1/2 quantum antiferromagnets on a square lattice.
- ``.....' represent frustrating interactions that can be tuned to drive phase transitions.

(Eg: Next near neighbour exchange, ring exchange,.....).

# Possible quantum phases

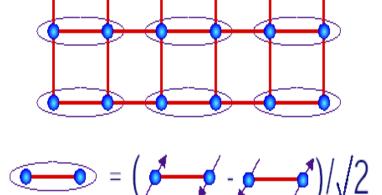
Neel ordered state



## Possible quantum phases (contd)

#### **QUANTUM PARAMAGNETS**

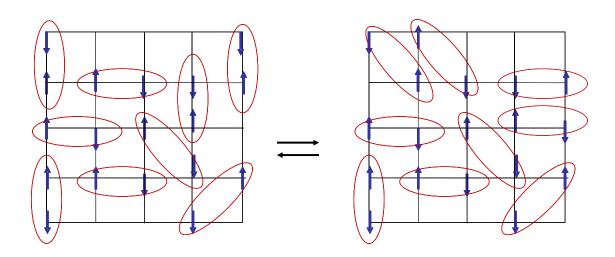
- Simplest: Valence bond solids.
- Ordered pattern of valence bonds breaks lattice translation symmetry.



 Elementary spinful excitations have S = 1 above spin gap.

## Possible phases (contd)

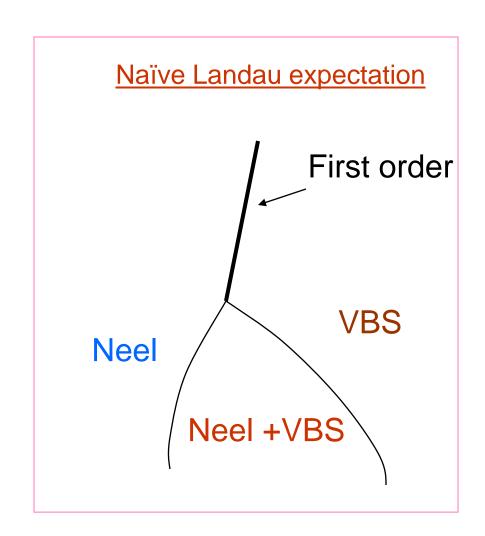
- Exotic quantum paramagnets ``resonating valence bond liquids''.
- Fractional spin excitations, interesting topological structure.



### Neel-valence bond solid(VBS) transition

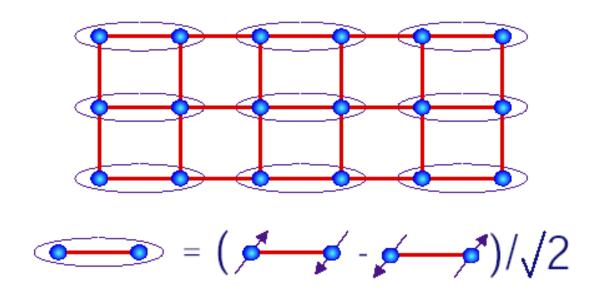
- Neel: Broken spin symmetry
- VBS: Broken lattice symmetry.
- Landau Two independent order parameters.
- no generic direct second order transition.
- either first order or phase coexistence.

This talk: Direct second order transition but with description not in terms of natural order parameter fields.

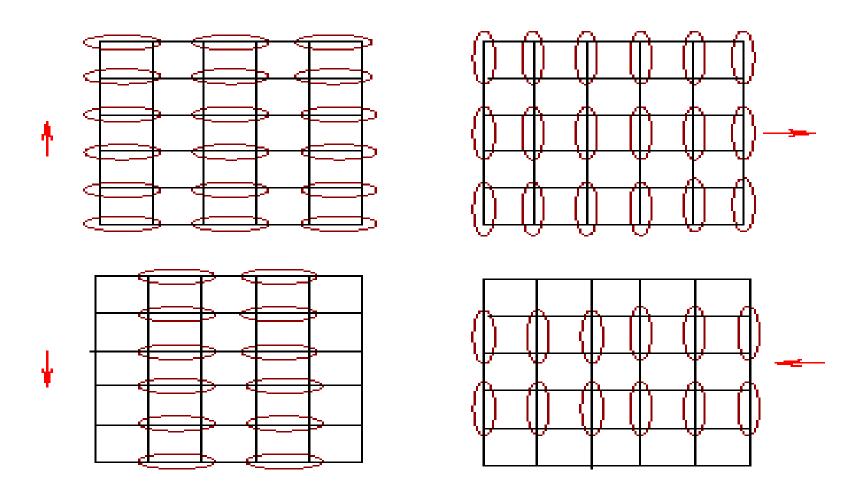


# Broken symmetry in the valence bond solid(VBS) phase

Valence bond solid with spin gap.



# Discrete Z<sub>4</sub> order parameter



### **Neel-Valence Bond Solid transition**

Naïve approaches fail

Attack from Neel  $\neq$ Usual O(3) transition in D = 3 Attack from VBS  $\neq$  Usual Z<sub>4</sub> transition in D = 3 (= XY universality class).

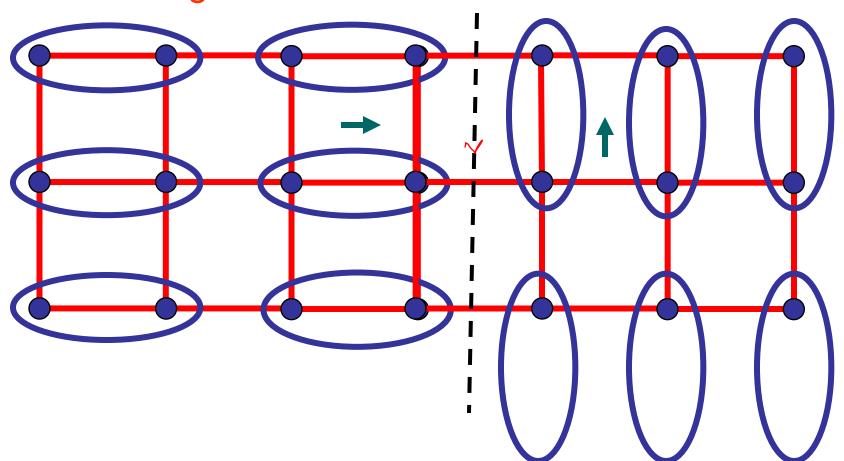
### Why do these fail?

Topological defects carry non-trivial quantum numbers!

Attack from VBS (Levin, TS, '04)

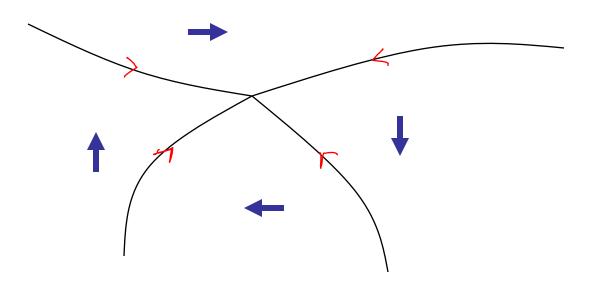
# Topological defects in Z<sub>4</sub> order parameter

 Domain walls – elementary wall has π/2 shift of clock angle



## Z<sub>4</sub> domain walls and vortices

- Walls can be oriented; four such walls can end at point.
- End-points are Z<sub>4</sub> vortices.

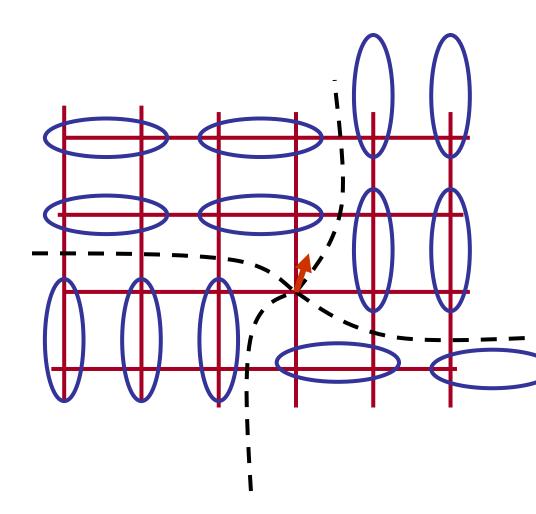


# Z<sub>4</sub> vortices in VBS phase

Vortex core has an unpaired spin-1/2 moment!!

Z<sub>4</sub> vortices are ``spinons".

Domain wall energy confines them in VBS phase.



# Disordering VBS order

 If Z<sub>4</sub> vortices proliferate and condense, cannot sustain VBS order.

Vortices carry spin =>develop Neel order

# Z<sub>4</sub> disordering transition to Neel state

 As for usual (quantum) Z<sub>4</sub> transition, expect clock anisotropy is irrelevant.

(confirm in various limits).

Critical theory: (Quantum) XY but with vortices that carry physical spin-1/2 (= spinons).

### Alternate (dual) view

Duality for usual XY model (Dasgupta-Halperin)
 Phase mode - ``photon''

Vortices – gauge charges coupled to photon.

Neel-VBS transition: Vortices are spinons

=> Critical spinons minimally coupled to fluctuating U(1) gauge field\*.

<sup>\*</sup>non-compact

# Proposed critical theory "Non-compact CP<sub>1</sub> model"

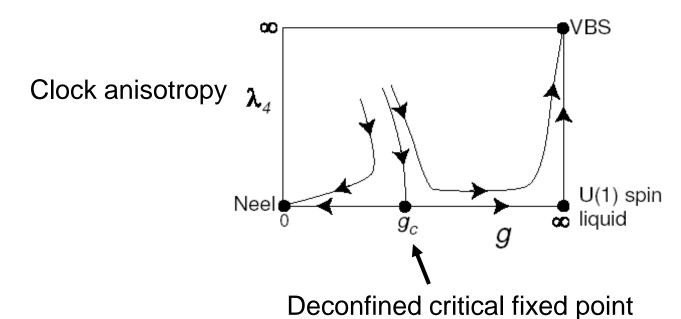
$$S = \int d^2x d\tau |(\partial_{\mu} - ia_{\mu})z|^2 + r|z|^2 + u|z|^4 + (\varepsilon_{\mu\nu\lambda}\partial_{\nu}a_{\lambda})^2$$

z = two-component spin-1/2 spinon field  $a_{\mu} =$  non-compact U(1) gauge field. <u>Distinct</u> from usual O(3) or  $Z_4$  critical theories\*.

Theory not in terms of usual order parameter fields but involve spinons and gauge fields.

\*Distinction with usual O(3) fixed point definitively established by detailed numerics (Motrunich, Vishwanath, '03)

### Renormalization group flows



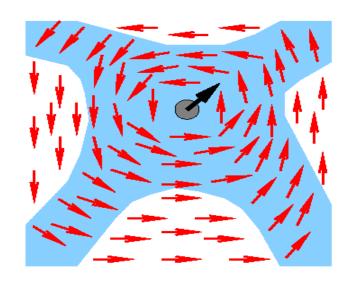
Clock anisotropy is ``dangerously irrelevant".

# Precise meaning of deconfinement

 Z<sub>4</sub> symmetry gets enlarged to XY

⇒ Domain walls get very thick and very cheap near the transition.

Domain wall energy not effective in confining Z<sub>4</sub> vortices (= spinons)



Formal: Extra global U(1) symmetry not present in microscopic model :

### Two diverging length scales in paramagnet

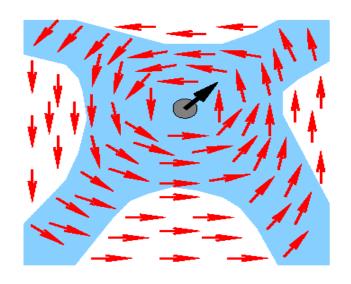


ξ: spin correlation length

 $\xi_{VBS}$ : Domain wall thickness.

 $\xi_{VBS} \sim \xi^{\kappa}$  diverges faster than  $\xi$ 

Spinons confined in either phase but `confinement scale' diverges at transition.



### Extensions/generalizations

- Similar phenomena at other quantum transitions of spin-1/2 moments in d = 2
- 1. VBS- spin liquid (Senthil, Balents, Sachdev, Vishwanath, Fisher, '03)
- 2. Neel -spin liquid (Ghaemi, Senthil)
- 3. Certain VBS-VBS (Vishwanath, Balents, Senthil, '03; Fradkin, Huse, Moessner, Oganesyan, Sondhi, '03)

### Apparently fairly common!

- Deconfined critical <u>phases</u> with gapless fermions coupled to gauge fields also exist in 2d quantum magnets (Hermele, Senthil, Fisher, Lee, Nagaosa, Wen, '04)
- interesting applications to cuprate theory.

# Numerical/experimental sightings of Landau-forbidden transitions

Weak first order/second order transitions between two phases with very different broken symmetry surprisingly common....

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Numerics (more in the next 2 talks)
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Antiferromagnet – superconductor

(Assaad et al 1996)

Superfluid – some kinds of charge density wave

(Sandvik et al 2002)

J1-J2 spin-1/2 quantum AF on square lattice:

Second order Neel -VBS?

(Singh, Sushkov,....)

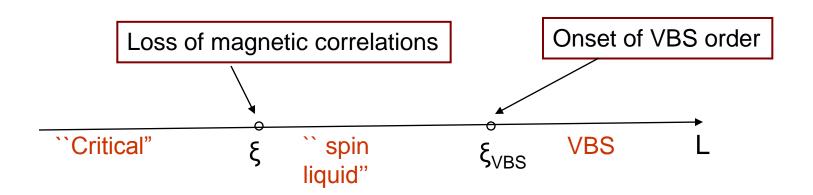
#### **Experiments:**

 $UPt_{3-x}Pd_x$  SC - AF with increasing x.

(Graf et al 2001)

### Some lessons-I

Separation between the two competing orders not as a function of tuning parameter but as a function of (length or time) scale (exactly as suggested near heavy fermion critical point)



### Some lessons-II

• Striking ``non-fermi liquid'' (morally) physics at critical point between two competing orders.

Eg: At Neel-VBS, magnon spectral function is anamolously broad (roughly due to decay into spinons) as compared to usual critical points.

#### Most important lesson:

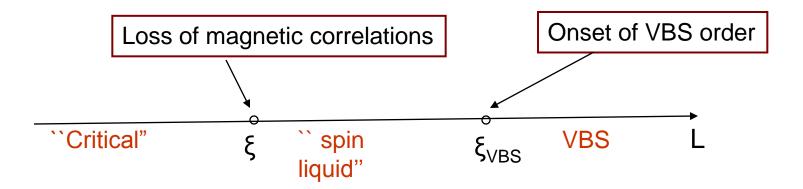
Failure of Landau paradigm – order parameter fluctuations do not capture true critical physics.

Caricature of phenomena suggested near heavy fermion critical points.

Experiments: Are there really two distinct time/length scales at heavy fermion critical points?

### Summary and some lessons-I

- Direct 2<sup>nd</sup> order quantum transition between two phases with different competing orders possible
- Separation between the two competing orders not as a function of tuning parameter but as a function of (length or time) scale



### Summary and some lessons-II

 Striking ``non-fermi liquid" (morally) physics at critical point between two competing orders.

Eg: At Neel-VBS, magnon spectral function is anamolously broad ( $\eta \sim 0.6$ )- roughly due to decay into spinons- as compared to usual critical points.