

Composite fermi liquids in the lowest Landau level

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C. Wang, and TS, arXiv:1505.03520 (PR X 2016), arXiv:1505.05141 (PR X 2015)

C. Wang and TS, arXiv:1507.08290 (PR B 2016): Synthesis and physical pictures

C. Wang and TS, arXiv: 1604.06807

Related work:

Son, arXiv:1502.03446 (PR X 2015); M. Metlitski and A. Vishwanath, arXiv:1505.05142.

Many other important contributions: (Experiments) Shayegan group; Barkeshli, Mulligan, Fisher 15; Geraedts, Zaletel, Mong, et al, 15; Murthy, Shankar, 15; Mross Alicea, Motrunich 15; Metlitski, 15; Potter, Serbyn, Vishwanath, 15.

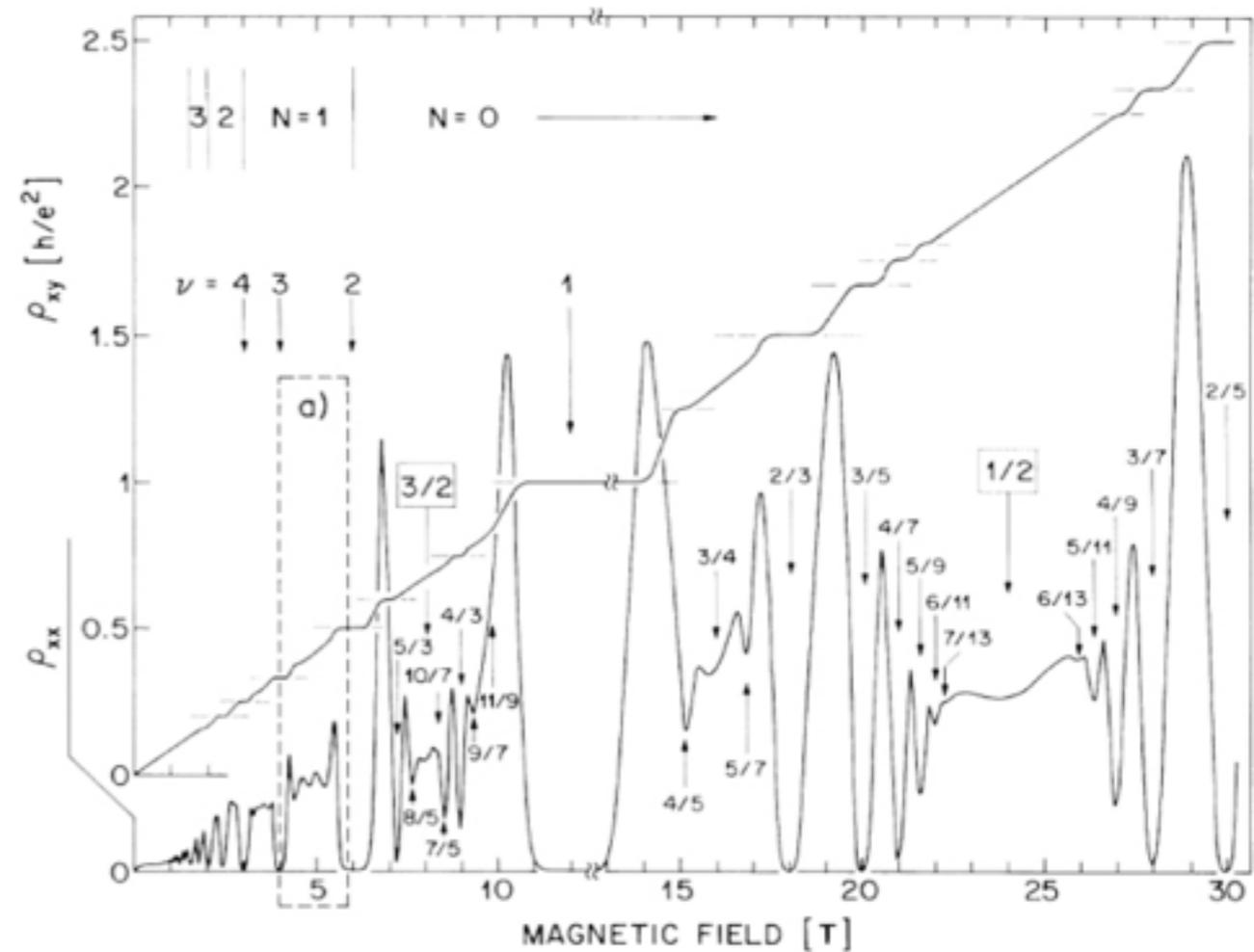
2d electrons in the “quantum Hall” regime

Filling factor $\nu = 1, 2, 3, \dots$ (IQHE)

$\nu = 1/3, 1/5, \dots$ (FQHE)

$\nu = 1/2, 1/4, \dots$???

Experiment: Metal with $\rho_{xx} \neq 0$, $\rho_{xy} \neq 0$,
but $\rho_{xx} \ll \rho_{xy}$.



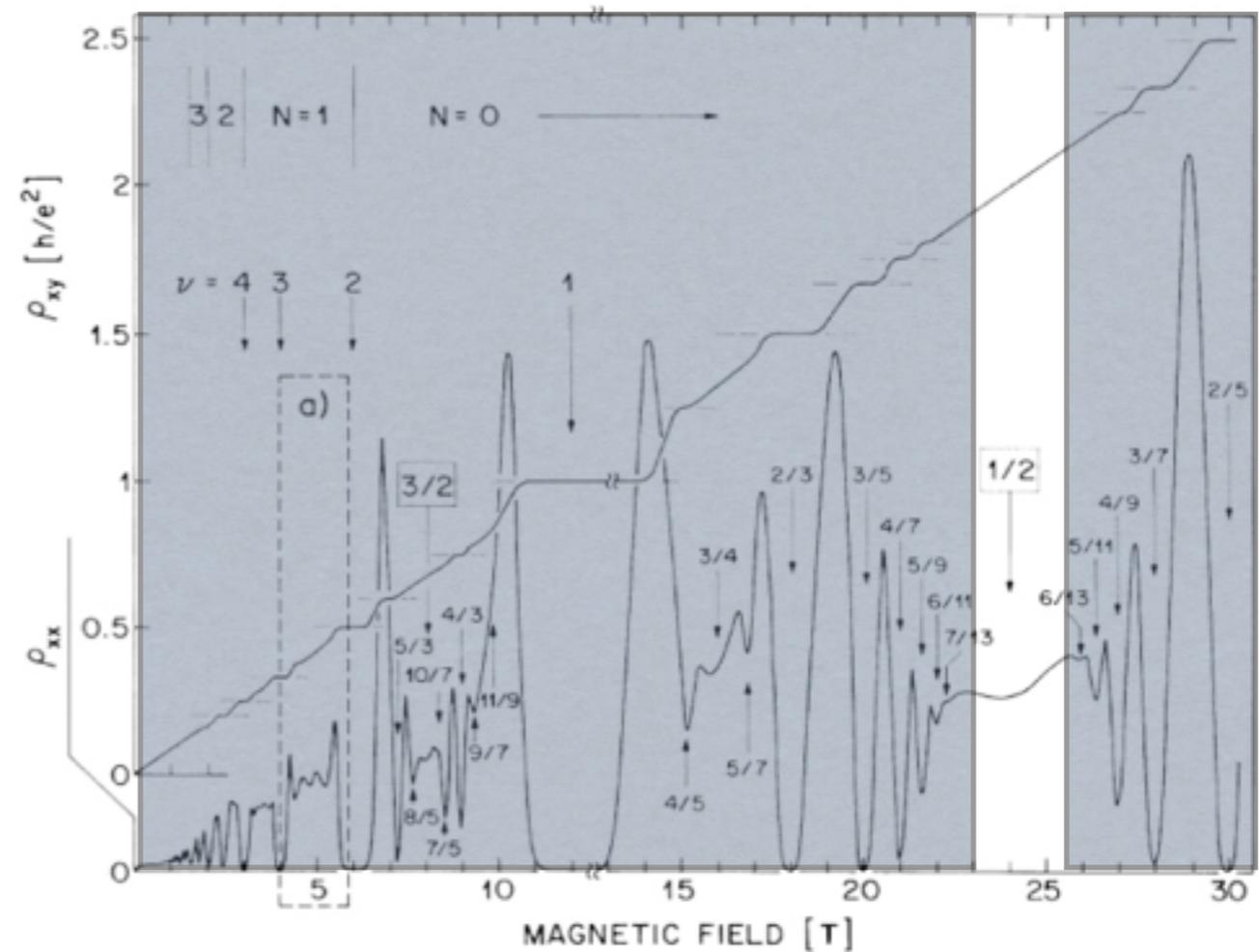
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The theoretical problem

Non-interacting electrons - highly degenerate Landau levels

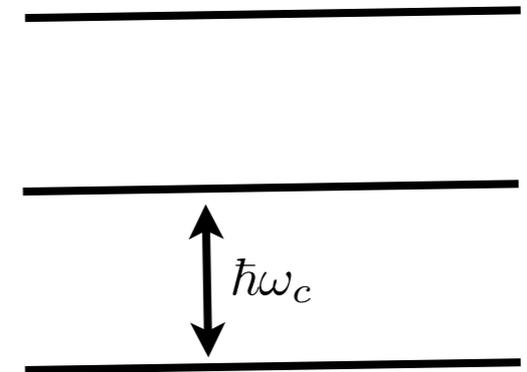
Integer effect: electrons fill integer number of Landau levels.

Incompressible FQHE states: fill certain rational fractions of a Landau level.

Large degeneracy is split by electron-electron interactions to give a gapped ground state.

Compressible metallic states: “unquantized quantum Hall effect”

?? How do interactions manage to produce a metal ??



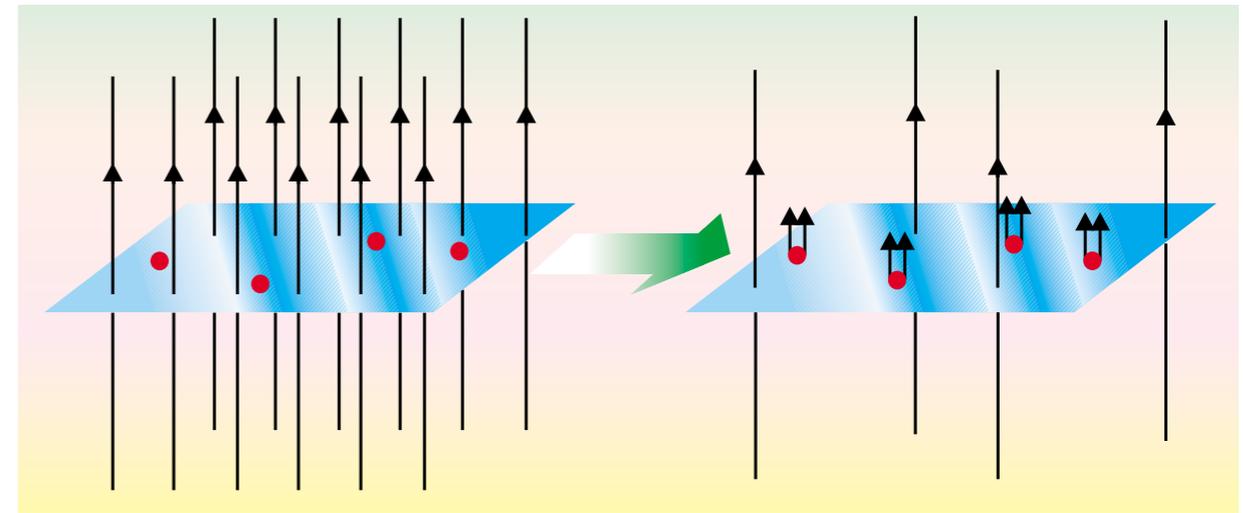
Composite fermi liquid theory (Halperin, Lee, Read (HLR) 1993)



Assume (Jain 89) each electron captures two flux quanta to form a new fermion
("Composite fermions")

See reduced effective field
 $B^* = B - (2h/e)\rho$

At $\nu = 1/2$, $B^* = 0$



=> form Fermi surface of composite fermions

Effective theory:

$$\mathcal{L} = \bar{\psi}_{CF} \left(i\partial_t - a_0 - iA_0^{ext} + \frac{(\vec{\nabla} - i(\vec{a} + \vec{A}))^2}{2m} \right) \psi_{CF} + \frac{1}{8\pi} a_\mu \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda \quad (1)$$

Some experimental verification of composite fermions

Many groups: Willett, Stormer, Tsui, Shayegan, Goldman,.....

Examples:

1. Slightly away from $\nu = 1/2$, $B^* = B - (2h/e)\rho \neq 0$ but much reduced from external field B .

=> composite fermions move in cyclotron orbits with radius \gg electron cyclotron radius

2. Confirmation of composite fermion Fermi surface (eg, Shubnikov-deHaas oscillations)

3. Successful description (Jain 89) of the prominent FQHE states at $\nu = n/(2n+1)$ as "integer quantum hall states" of the composite fermions.

Unsatisfactory aspects of the theory

1. Flux attachment mean field is uncontrolled (unreasonable?) approximation. Should we really trust it?

2. Theory should make sense within the Lowest Landau Level (LLL) but HLR not suited to projecting to LLL.

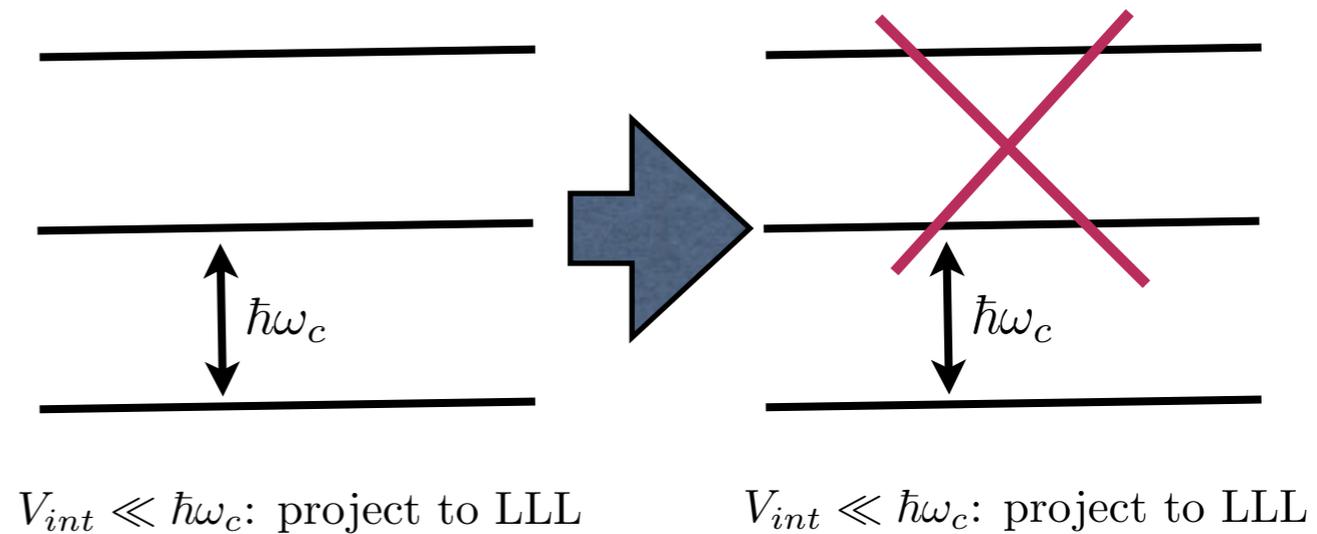
Mean field effective mass = bare electron mass in HLR

LLL limit: take m to zero; what happens??

Many refinements in the late 90s (Shankar, Murthy; Read; Halperin, Stern, Simon, van Oppen; D.-H. Lee, Pasquier, Haldane,.....) but dust never settled.

3. LLL theory has an extra symmetry (at $\nu = 1/2$) that HLR is blind to.

Issue identified in the 90s (Groth, Gan, Lee, Kivelson, 96; Lee 98) but no resolution.



Particle-hole symmetry in LLL

At $\nu = 1/2$, regard LLL as either “half-empty or half-full”:

Start from empty level, fill half the LLL

or start from filled LL and remove half the electrons



Particle-hole symmetry: formal implementation

Electron operator $\psi(x, y) \simeq \sum_m \phi_m(x, y) c_m$ after restriction to LLL.
($\phi_m(x, y)$: various single particle wave functions in LLL).

Particle-hole: **Antiunitary symmetry C**

$$C\psi C^{-1} = \psi^\dagger = \sum_m \phi_m^*(x, y) c_m^\dagger$$

Symmetry of 1/2-LLL with, eg, 2-body interaction is (at least) $U(1) \times C$

Numerical work: Metallic ground state at $\nu = 1/2$ preserves the C symmetry. (Rezayi, Haldane, 00, Geraedts et al, 16)

HLR theory: Not in Lowest Landau Level; p/h not within its scope.

Related theoretical problems

1. Electrons at $\nu = 1/4, 1/6, \dots$

No p/h but issue of nature of a LLL theory remains.

2. Useful to consider problem of bosons in the quantum Hall regime.

At fillings $\nu = 1/(2p+1)$, HLR procedure leads to a compressible metallic state.

Fate of such a state in the LLL?

For bosons, microscopically there is no p/h .

Progress - some old, some new

1. Bosons at $\nu = 1$

LLL theory of metallic Composite Fermi Liquid state (Read 1998)

2. Particle-hole symmetric theory for electronic CFL at $\nu = 1/2$

Suggestion by Son (2015): Is the composite fermion at $\nu = 1/2$ a Dirac particle?

- Field theoretic justification: Connection to surface of 3d topological insulators

(C.Wang, TS, [arXiv:1505.05141](https://arxiv.org/abs/1505.05141); M. Metlitski, A. Vishwanath, 1505.05142.)

- Simple physical picture of the particle-hole symmetric composite fermion

(C.Wang, TS, [arXiv:1507.08290](https://arxiv.org/abs/1507.08290))

- Numerical calculations

(Scott D. Geraedts, Michael P. Zaletel, Roger S. K. Mong, Max A. Metlitski, Ashvin Vishwanath, Olexei I. Motrunich, [arXiv:1508.04140](https://arxiv.org/abs/1508.04140))

Plan of talk

A. *Understanding the p/h symmetric composite fermi liquid of electrons at $\nu = 1/2$*

- physical picture; field theoretic derivation

B. Composite Fermi liquid of bosons at $\nu = 1$:

- review of Read's Lowest Landau Level theory
- comparison with electrons at $\nu = 1/2$

C. *Composite Fermi liquids in LLL at generic ν :*

*Quantum vortex liquids with Fermi surface Berry phases
(Wang, TS, 1604.06807)*

Particle-hole symmetric composite fermions at $\nu = 1/2$: a physical picture (Wang, TS, 2015)

Old physical picture of Composite Fermion (CF) in LLL

Read 94;.....

LLL wave function (Rezayi-Read 94)

$$\psi(z_1, \dots, z_N) = P_{LLL} \det(e^{i\vec{k}_i \cdot \vec{r}_j}) \prod_{i < j} (z_i - z_j)^2$$

Bind vortices to particles (as opposed to flux) (Jain 89; Read 89)

Amplitude of wavefunction suppressed in vortex core (unlike with flux attachment)

Vortex => correlation hole

Composite fermion = electron bound to 4π -vortex which has charge depletion $-e$

=> Composite fermions in LLL are electrically neutral

Old physical picture of Composite Fermion (CF) in LLL

Neutral dipolar fermions

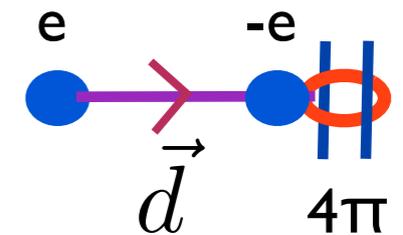
N. Read, 1994; many subsequent papers in late 90s
(Shankar, Murthy; Haperin, Stern; D.-H. Lee;
Pasquier-Haldane,.....)

LLL wave function (Rezayi-Read 94)

$$\psi(z_1, \dots, z_N) = P_{LLL} \det(e^{i\vec{k}_i \cdot \vec{r}_j}) \prod_{i < j} (z_i - z_j)^2$$

Neutral CF has a dipole moment perpendicular to its momentum.

(Classical drift of electric dipole in a magnetic field)



$$\vec{d} \perp \vec{k}$$

Heuristic wave function argument: Plane wave factors push vortex away from electron

However this picture misses some physics and further is not p/h symmetric (charge changes sign but vorticity does not).

New picture of composite fermion in LLL

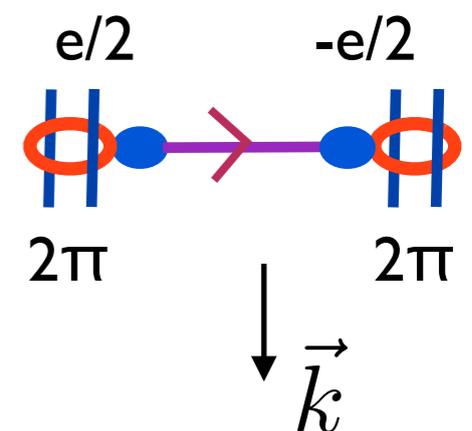
Fermion wavefunctions in LLL $\psi(z_1, z_2, \dots, z_N) = \prod_{i < j} (z_i - z_j) f(z_1, \dots, z_N)$

$f(z_1, \dots, z_N)$: a symmetric polynomial.

=> one vortex is exactly on electron due to Pauli.

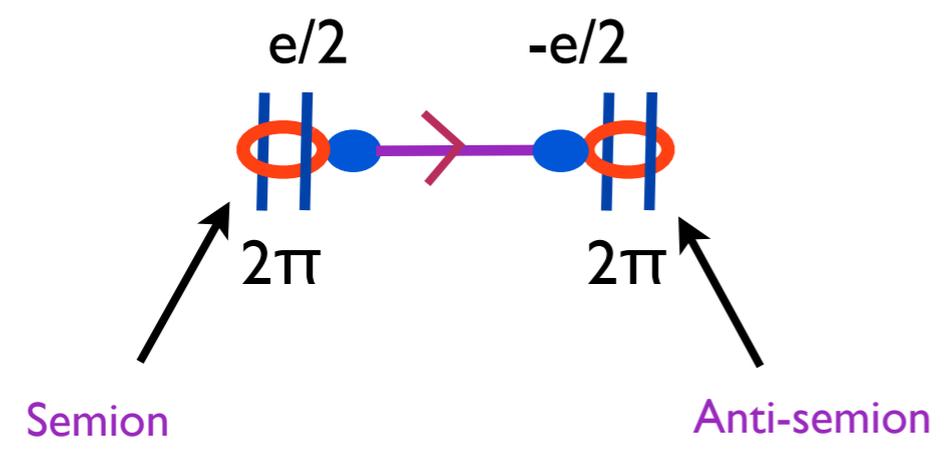
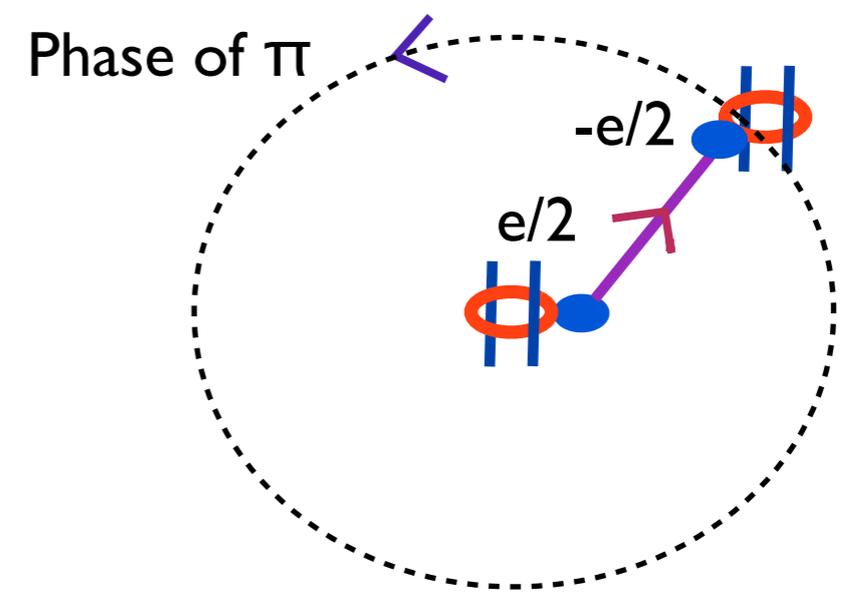
Only second vortex is displaced from first in direction perpendicular to CF momentum.

Each vortex has charge $-e/2$ => single vortex exactly on electron has charge $+e/2$ and the displaced vortex has charge $-e/2$.



Internal structure of composite fermion(*) in LLL

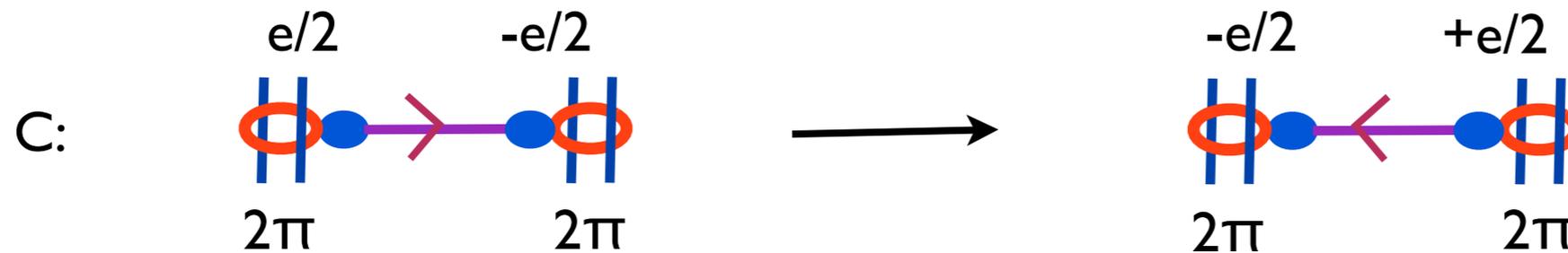
Two ends have mutual statistics of π



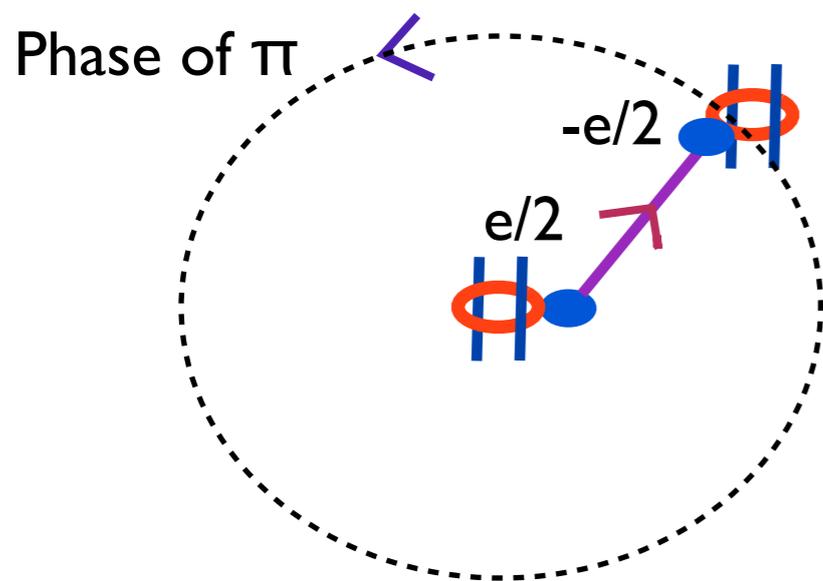
Bound state (of course) is a fermion.

* Assume ends are well separated compared to vortex size.

New picture of composite fermion in LLL (cont'd)



Anti-unitary C interchanges relative coordinates of the two ends.



Solve QM of relative motion =>

ground state ``spin''-1/2 doublet which is Kramers under C .

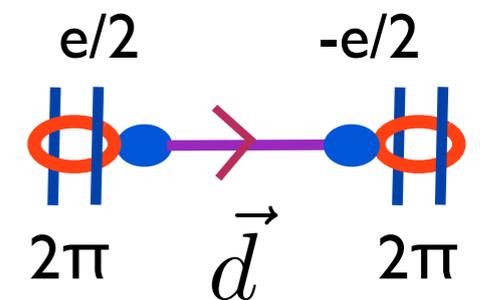
New picture of composite fermion in LLL (cont'd)

Non-zero CF momentum \Rightarrow non-zero dipole moment

\Rightarrow "spin" of composite fermion polarized perpendicular to its momentum

(spin-momentum locking expected of Dirac fermion).

Composite fermion goes around FS \Rightarrow momentum rotates by 2π
 \Rightarrow spin rotates by 2π \Rightarrow Berry phase of π as expected for a Dirac fermion.



$\downarrow \vec{k}$

$$\vec{d} \perp \vec{k}$$

$$\Rightarrow \langle \vec{\sigma} \rangle \perp \vec{k}$$

1/2-filled LL and correlated TI surfaces

Derivation of and more insight into p/h symmetric composite fermi liquid theory

(C.Wang, TS, 15; M. Metlitski, A.Vishwanath, 15)

ρ/h symmetric LL as a surface of a 3d fermionic topological insulator: Preliminaries

Consider (initially free) fermions with “weird” action of time-reversal (denote C):

$$C \rho C^{-1} = -\rho$$

ρ = conserved “charge” density.

Full symmetry = $U(1) \times C$

(called class AIII in Topological Insulator/Superconductor literature*)

Eg: Triplet time reversal-invariant superconductor where physical S_z is conserved and plays the role of such a ρ .

*distinct symmetry from usual spin-orbit coupled insulators which have $U(1) \rtimes C$ symmetry (i.e, ρ is usually even under time reversal).

p/h symmetric LL as a surface of 3d fermion SPT (cont'd)

Surface: Single massless Dirac fermion

C symmetry guarantees that surface Dirac cone is exactly at neutrality.

$$\mathcal{L} = \bar{\psi} (-i\partial + A) \psi + \dots$$



2-component fermion

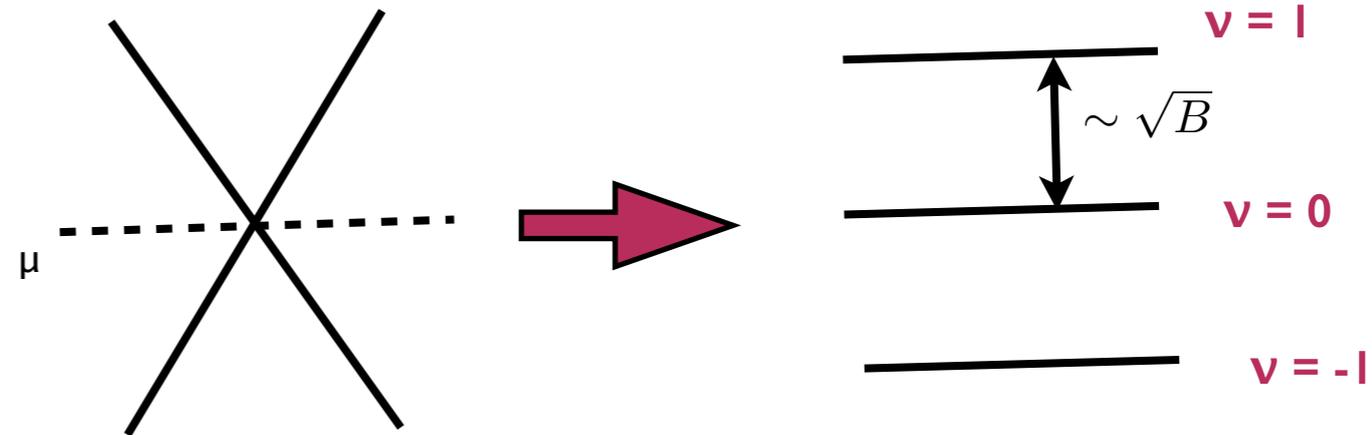
external probe gauge field

ρ/h symmetric LL as a surface of 3d fermion SPT (cont'd)

ρ is odd under $C \Rightarrow$ 'electric current' is even.

External E-fields are odd but external B-fields are even.

\Rightarrow Can perturb surface Dirac cone with external B-field.



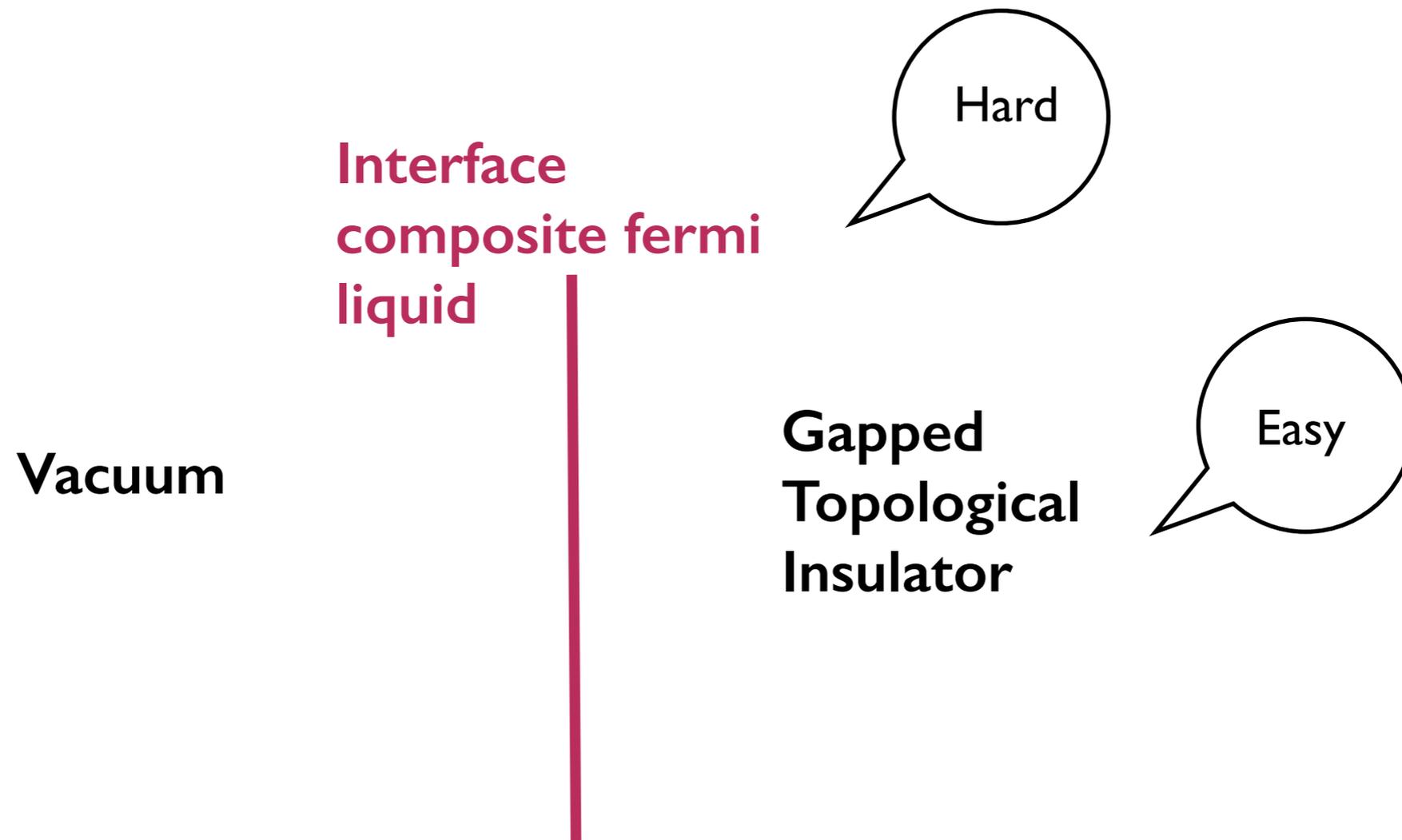
C-symmetry: $\nu = 0$ LL is exactly half-filled.

Low energy physics: project to 0LL

With interactions \Rightarrow map to usual half-filled LL

Comments

Implication: Study p/h symmetric half-filled LL level by studying correlated surface states of such 3d fermion topological insulators.



Exploit understanding of relatively trivial bulk TI to learn about non-trivial correlated surface state.

Theories of TI surface

Charge-vortex duality for Dirac fermions in 2+1-D

Wang, TS 15
Metlitski, Vishwanath 15
Mross et al 15

Two different surface theories for the topological insulator:

Standard surface theory:

$$\mathcal{L} = \bar{\psi} (-i\partial + A) \psi + \dots$$

2-component fermion

external probe gauge field

Dual surface theory(*):

$$\mathcal{L} = \bar{\psi}_v (-i\partial - a) \psi_v + \frac{1}{4\pi} \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda$$

dynamical gauge field

A new field theory duality

*Strictly speaking should work with a modified version which is more precisely defined.

Justification of Dirac composite fermion theory of 1/2-filled Landau level

1/2-filled Landau level: Obtain by turning on B-field in standard surface Dirac cone.

Dual description:

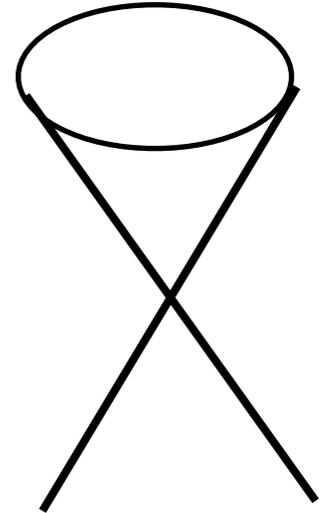
$B/(2h/e)$ = density of dual fermions

=> Theory of 1/2-filled Landau level: = dual Dirac fermions at finite density + U(1) gauge field
but no Chern-Simons term.

(exactly as proposed by Son (2015))

Effective action

$$\mathcal{L}[\psi, a_\mu] = \bar{\psi}(i\partial\!\!\!/ + \not{a} + \mu\gamma^0)\psi - \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu A_\lambda + \frac{1}{8\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$



Anti-unitary p/h (C) acts in same way as time reversal usually does on Dirac fermion:

$$C\psi_v C^{-1} = i\sigma_y\psi_v \Rightarrow C^2\psi_v C^{-2} = -\psi_v$$

Composite fermion is Kramers doublet under C .

Low energy theory: focus on states near Fermi surface.

Meaning of Dirac?

As CF goes around FS, pick up π Berry phase.

Comments

1. Low energy theory: focus on states near Fermi surface.

Meaning of Dirac?

As CF goes around FS, pick up π Berry phase.

2. Though particle-hole symmetric, as derived, this is not a LLL theory

(Projecting to LLL for Dirac electrons requires further limit $e^2/\nu \longrightarrow 0$)

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- review of Read's Lowest Landau Level theory

- comparison with electrons at $\nu = 1/2$

C. *Composite Fermi liquids in LLL at generic ν :*

Quantum vortex liquids with Fermi surface Berry phases

(Wang, TS, 1604.06807)

Composite Fermi liquid of bosons at $\nu = 1$

LLL approach: write Hamiltonian in terms of density operators projected to LLL.

Pasquier-Haldane (98): Represent density in terms of auxiliary fermions.

Read(98): Mean field + gauge fluctuations for CFL state.

Effective action different from HLR!

$$\mathcal{L}[\psi, a_\mu] = \mathcal{L}[\psi, a_\mu] - \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu A_\lambda + \frac{1}{4\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda \quad (1)$$

No Chern-Simons term in action.

Theory of neutral fermionic vortex

Comments

1. Similar formal structure to p/h symmetric theory for electrons at $\nu = 1/2$
Eg, CF density = flux density (and not charge density)

2. Relation to HLR for bosons at $\nu = 1$: postulate Fermi surface Berry phase -2π

3. Read's theory has an emergent anti-unitary particle hole symmetry.

$$\psi \rightarrow \psi$$

$$a_i \rightarrow -a_i$$

Same theory with exact p/h obtained at surface of 3d boson topological insulator (studied previously by Vishwanath, TS, 13)

Composite fermi liquids as vortex metals

HLR/Jain composite fermion: Charge - flux composites

Particle-hole symmetric composite fermion: Neutral vortex

Describe CFL as a vortex liquid metal formed by neutral fermionic vortices.

Vortex metal description:

- Simple understanding of transport
(similar to other 2d quantum vortex metals, eg, in Galitski, Refael, Fisher, TS, 06)
- Extensions to CFLs away from $\nu = 1/2$

Transport in the CFL

1. Longitudinal electrical conductivity \propto composite fermion resistivity (natural from vortex liquid point of view)

$$\text{Hall conductivity} = \frac{e^2}{2h} \text{ (exactly)}$$

2. Longitudinal thermal conductivity = composite fermion thermal conductivity

Wiedemann-Franz violation (Wang, TS, 15)

$$\frac{\kappa_{xx}}{L_0 T \sigma_{xx}} = \left(\frac{\rho_{xy}}{\rho_{xx}} \right)^2 > 10^3$$

L_0 : free electron Lorenz number

(Also actually in HLR)

3. Thermoelectric transport:

Vortex metal: Nernst effect from mobile vortices unlike HLR (Potter et al, 15)

Composite Fermi liquids in LLL at generic ν : Quantum vortex liquids with Fermi surface Berry phases

Wang, TS, 16

General $\nu = 1/(2q)$ but with LLL restriction:

Attach $2q$ vortices to electron.

LLL \Rightarrow neutral fermionic vortex distinct from Jain/HLR composite fermion (many people in late 1990s)

Effective theory must have no Chern-Simons term for internal gauge field to ensure this.

Trade Chern-Simons term for Fermi surface Berry phase $\phi_B = -2\pi\nu$

Effective Lagrangian

$$\mathcal{L} = \mathcal{L}_{\phi_B}[\psi_\nu, a_\mu] - \frac{1}{4q\pi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu A_\lambda + \frac{1}{8q\pi} \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda.$$

Proposal: Berry phases protected by LLL restriction (no need for p/h)

Comments/summary

1. Old issue of p/h symmetry in half-filled Landau level: simple, elegant answer

General viewpoint: Regard LLL composite fermi liquid as a quantum metal of neutral fermionic vortices.

2. Surprising, powerful connection to correlated 3d TI surfaces

3. Many other related results

- clarification of many aspects of correlated surfaces of 3d TIs

- particle-vortex duality for 2+1-d massless Dirac fermions

- classification of time reversal invariant 3d spin liquids with emergent gapless photon

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