Symmetry Protected Topological Phases of Matter

T. Senthil (MIT)

Review:
T. Senthil, Annual Reviews of Condensed Matter Physics, 2015

Topological insulators 1.0

Free electron band theory:
distinct insulating phases of electrons in the presence of symmetry (eg, time reversal)

(i) Conventional Band Insulator

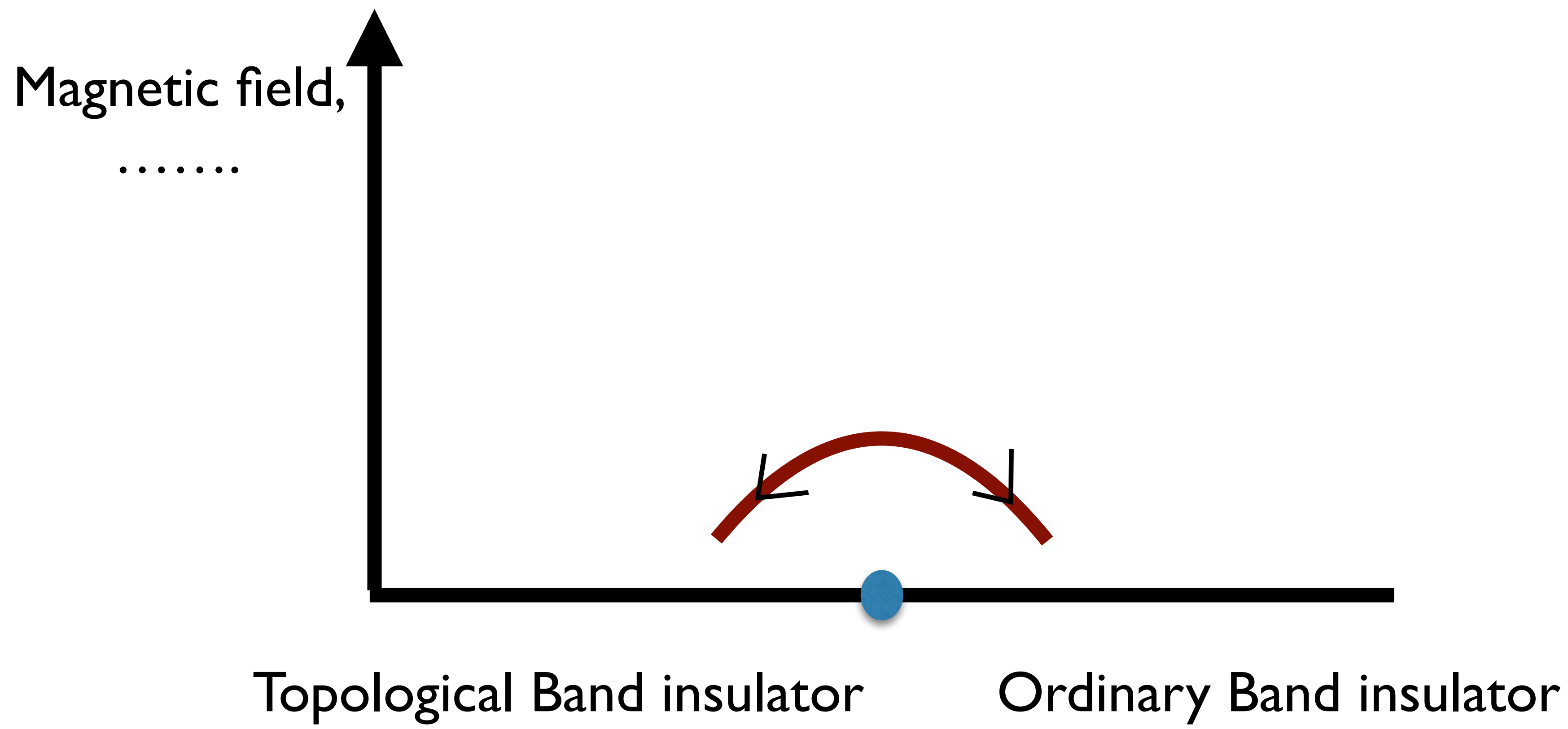
(ii) Topological Band Insulator (TBI)

Similar distinctions in `band structures' of superconductors: ``Topological superconductors''.

Complete classification of gapped phases of free fermions: Schnyder, Furusaki, Ryu, Ludwig 2009; Kitaev 2009.

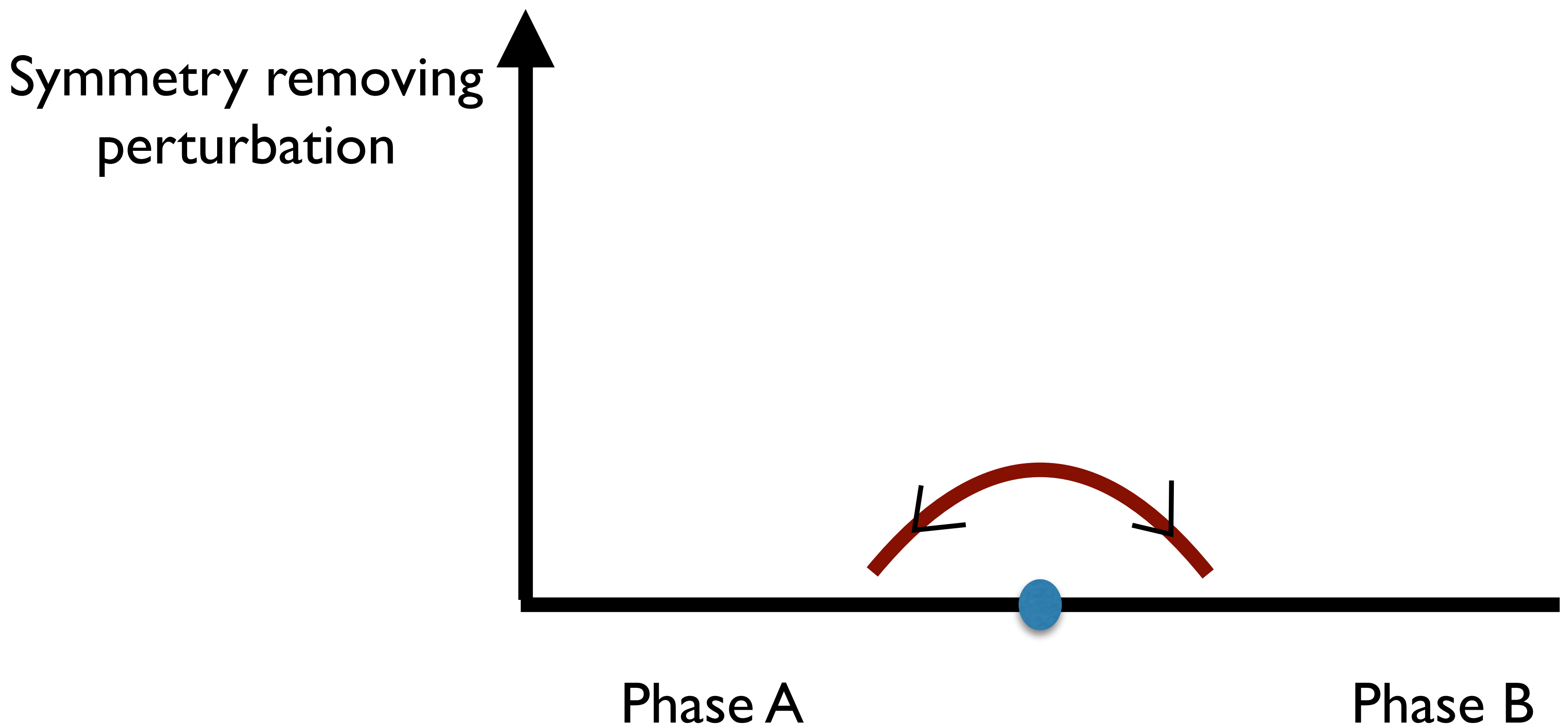
Lesson of topological insulators

Symmetry can protect distinction between two symmetry unbroken phases.



Strongly correlated quantum phases

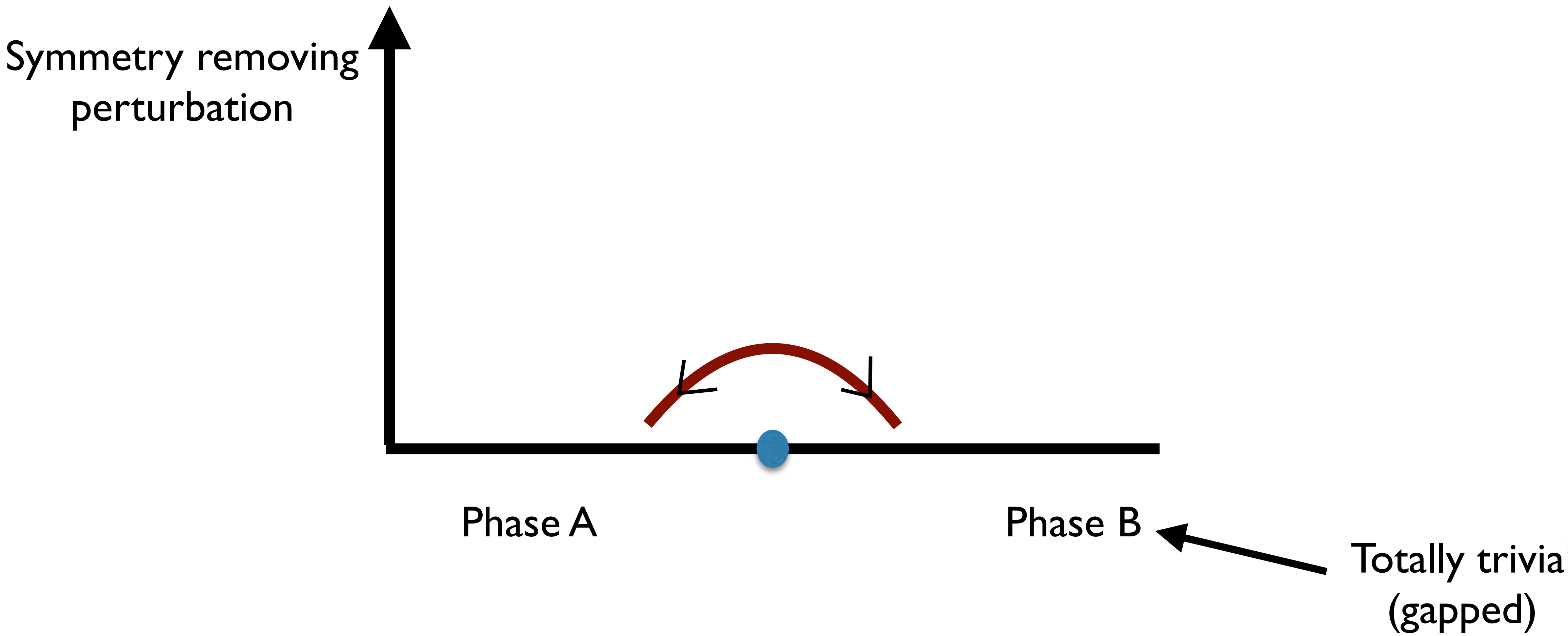
Are there similar phenomena in interacting quantum phases?



Symmetry protected distinctions between symmetry unbroken interacting phases?

Symmetry protected topological (SPT) phases

Specialize to case where phase B is totally trivial(*) (no exotic excitations, edge states, etc)



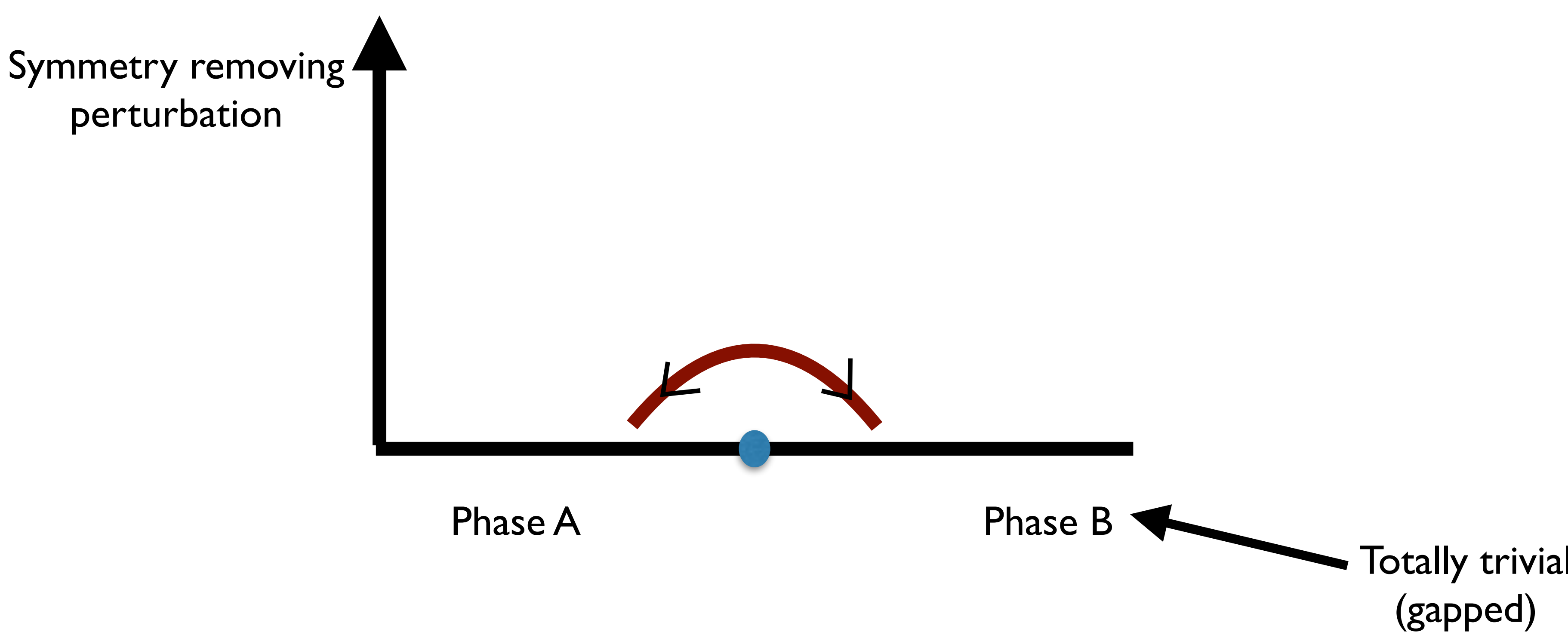
(*)Precise meaning: smoothly connected to wave function with no quantum entanglement between local degrees of freedom.

Some (obvious) properties of SPT phases

1. Gapped

2. No exotic excitations (no fractional statistics quasiparticles).

Essentially trivial in bulk but may support non-trivial edge modes.



An old example: antiferromagnetic spin-1 chain (``Haldane chain")

$$\mathcal{H} = J \sum_i \vec{S}_i . \vec{S}_{i+1}$$

Haldane, 1983; Affleck, Kennedy, Lieb, Tasaki, 1987.

$\vec{S}_i$: spin-1 operators

Gapped spectrum and no exotic excitations,
but dangling spin-1/2 moments at edge.

Edge states protected by symmetry (Eg, time reversal)

A symmetry protected topological paramagnet.

Antiferromagnetic spin-1 chain: Cartoon description

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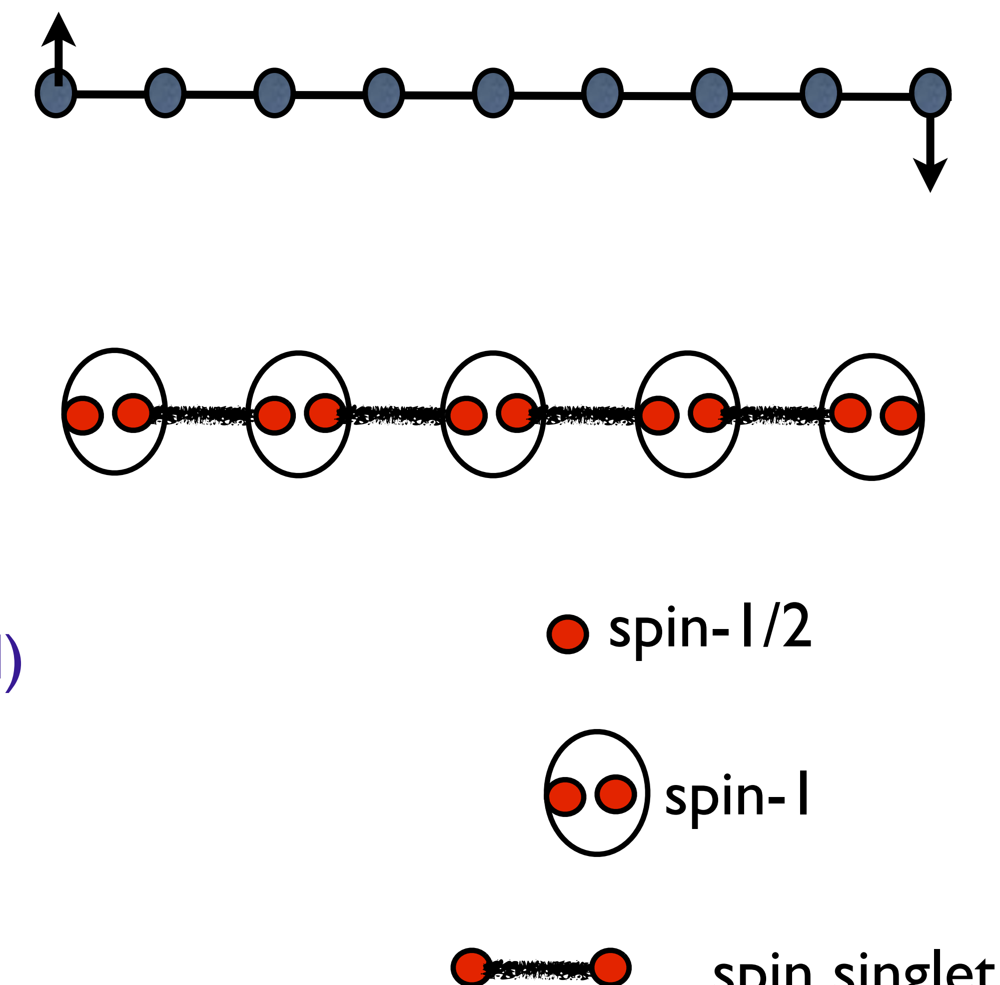
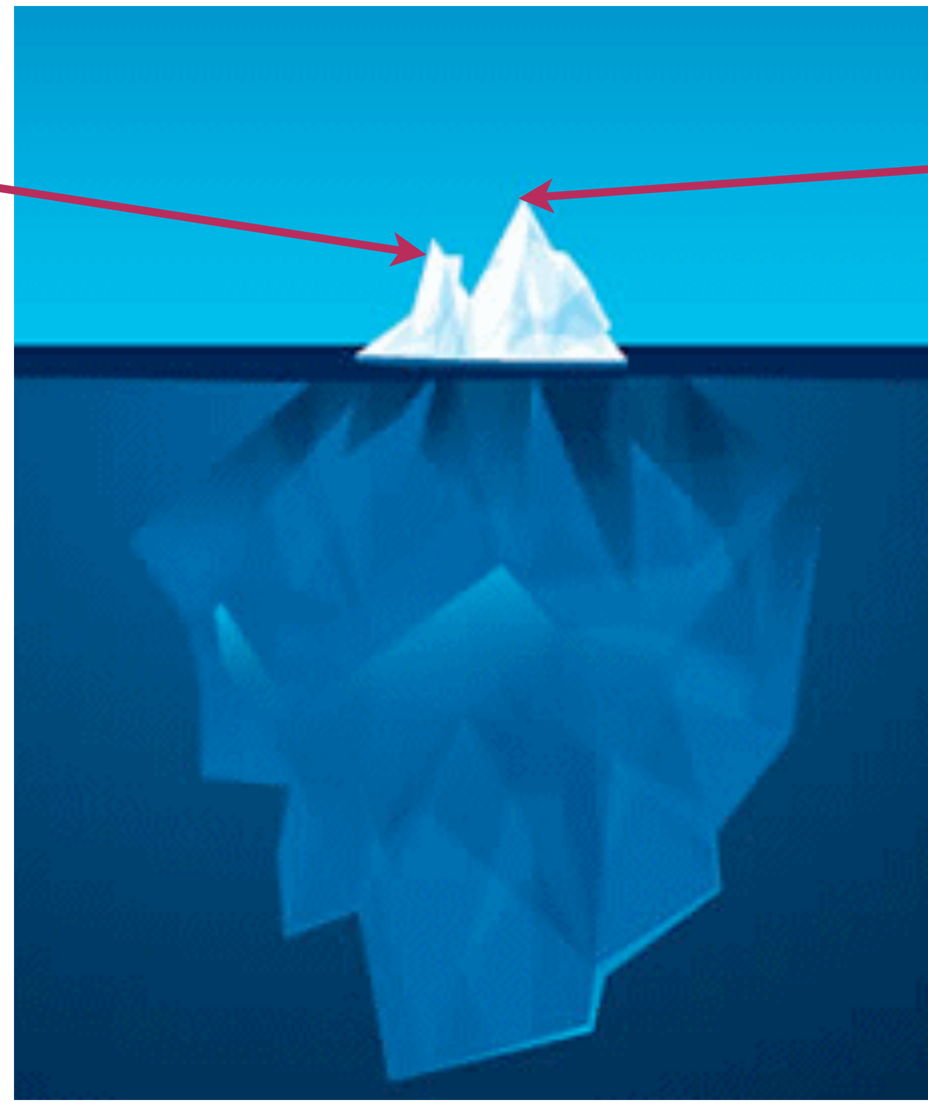


Haldane, 1983;
Affleck, Kennedy, Lieb, Tasaki, 1987.

A family of similar phases of matter

Haldane spin chain

Topological
Band Insulators



``Symmetry Protected Topological" (SPT) phases

Contrast with other interesting phenomena in modern quantum condensed matter physics

Exotic quasiparticles: Fractional charge, anyons (abelian/non-abelian).

Absence of ***any*** quasiparticles at some phases/phase transitions (eg, non-fermi liquid metals, etc).

Symmetry Protected Topological (SPT) phases:

Forbid all such exotic things in bulk.

Is anything interesting still left?

Why study Symmetry Protected Topological phases?

1. Because it may be there.......

Focus on systems with realistic symmetries in d = 3, 2, 1.

Eg: Electronic solids with same symmetries as topological band insulator (charge conservation, time reversal).

Many ongoing experiments looking for topological phenomena in such interacting insulators. What might they find?

Why study Symmetry Protected Topological phases?

2. Possibly simplest context to study interplay of symmetry, topology, and strong interactions.

Many profound and surprising insights into more complex systems.

- a deeper understanding of fractional charge/fractional statistics

- constraints on acceptable theories of 2d ``quantum spin liquid''/non-fermi liquid phases (eg, rule out some theories of Kagome magnets).

- many interesting connections to recent developments in field theory/math literature.

Questions about SPT phases

1. Are free fermion SPT phases stable in the presence of interactions?

2. Are there ***new*** phases that have no non-interacting counterpart?

- Physical properties?

- Experimental realization?

Questions about SPT phases

1. **Are free fermion SPT phases stable in the presence of interactions?**

2. Are there ***new*** phases that have no non-interacting counterpart?

- Physical properties?
- Experimental realization?

Stability of free fermion SPTs to interactions

Some free fermion SPTs are known to be unstable to interactions

Examples in d = 1: Fidkowski, Kitaev; d = 2: Qi, Yao, Ryu, Zhang,.....
d = 3: Fidkowski et al 13, Wang, TS, 14

But are usual spin-orbit coupled topological insulators stable to interactions?

Yes!

Illustrate in d= 3 (next slides)

Some trivial observations

Insulator with no exotic elementary excitations:

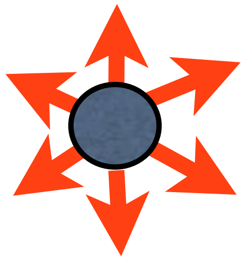
All excitations carry integer charge ne (e = electron charge).

n odd: fermion (eg: n = 1 is electron)
n even: boson. (eg: n= 2 is Cooper pair)

A powerful ***conceptual*** tool

A `gedanken' experiment:

Probe the fate of a magnetic monopole inside the material.

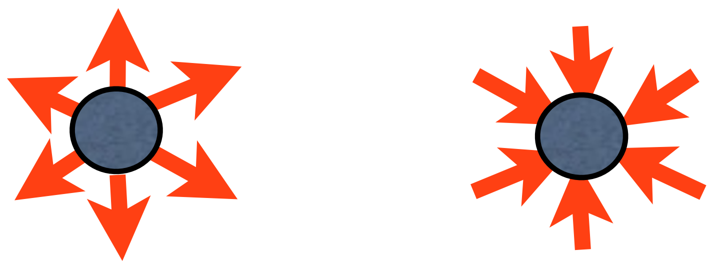


Thinking about the monopole is a profoundly simple way to non-perturbatively constrain the physics of the material.

Reminder: Elementary monopole is a source of hc/e magnetic flux.

Monopoles and symmetries

Time-reversal: Magnetic charge is odd
Electric charge is even

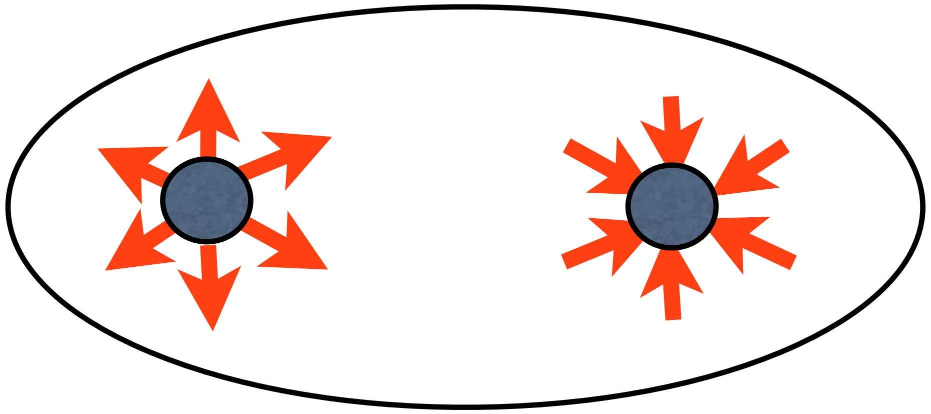


Suppose the monopole has electric charge q.

``Anti-monopole" also has electric charge q.

Monopoles and symmetries

Time-reversal: Magnetic charge is odd
Electric charge is even



Suppose the monopole has electric charge q.

Anti-monopole also has electric charge q.

Bring monopole and antimonopole together: result must be an excitation of the underlying material.

=> $2q = ne$ (n = integer)

Only two distinct possibilities consistent with time reversal
$q = 0, e, 2e, \ldots\ldots$ or $q = e/2, 3e/2, \ldots\ldots\ldots$

Fundamental distinction: $q = 0$ or $q = e/2$ (obtain rest by binding electrons)

Monopoles and topological insulators

Ordinary insulator: Monopoles have $q = 0$.

Topological band insulator: Monopoles have $q = e/2$.
(Qi, Hughes, Zhang 09)

Fractional charge on a probe monopole cannot be shifted by any perturbations (which preserve symmetry).

Topological band insulator stable to interactions.

Questions about SPT phases

1. Are free fermion SPT phases stable in the presence of interactions?

2. **<u>Are there *new* phases that have no non-interacting counterpart?</u>**

- Physical properties?
- Experimental realization?

Interacting SPT phases

First study SPT phases of bosons/spin systems.

1. Non-interacting bosons necessarily trivial - so must deal with an interacting theory right away

Necessitates thinking more generally about TI phases without the aid of a free fermion model.

2. Correlated bosons are stepping stone to correlated fermions.

Bosonic SPT phases

Reviews:
Turner, Vishwanath, 12,
TS, 14

d = 1: Classification/physics
Pollman, Turner, Berg, Oshikawa, 10; Fidkowski, Kitaev, 11; Chen, Gu, Wen 11; Schuh, Perez-Garcia, Cirac, 11.

d > 1: Progress in classification
1. Group Cohomology (Chen, Gu, Liu, Wen, 2012)
2. Chern-Simons approach in d =2 (Lu, Vishwanath, 2012).

Physics:
d = 2: Levin, Stern, 2012; Lu, Vishwanath, 2012; Levin, Gu, 2012; TS, Levin, 2013

d = 3: Vishwanath, TS, 2013; Wang, TS, 2013; Xu, TS, 2013;
Metlitski et al, 2013; Burnell, Chen, Fidkowski, Vishwanath, 2013; .....

I will focus on illustrating the physics with one example each in d = 2 and d = 3.

A simple example in d = 2

Integer quantum Hall state of bosons

(Bulk has no fractional statistics quasiparticles but has an integer quantum Hall effect).

TS, Levin 2013

Two component bosons in a strong magnetic field

Two boson species b_I each at filling factor ν = 1

$$H = \sum_I H_I + H_{int} \tag{1}$$

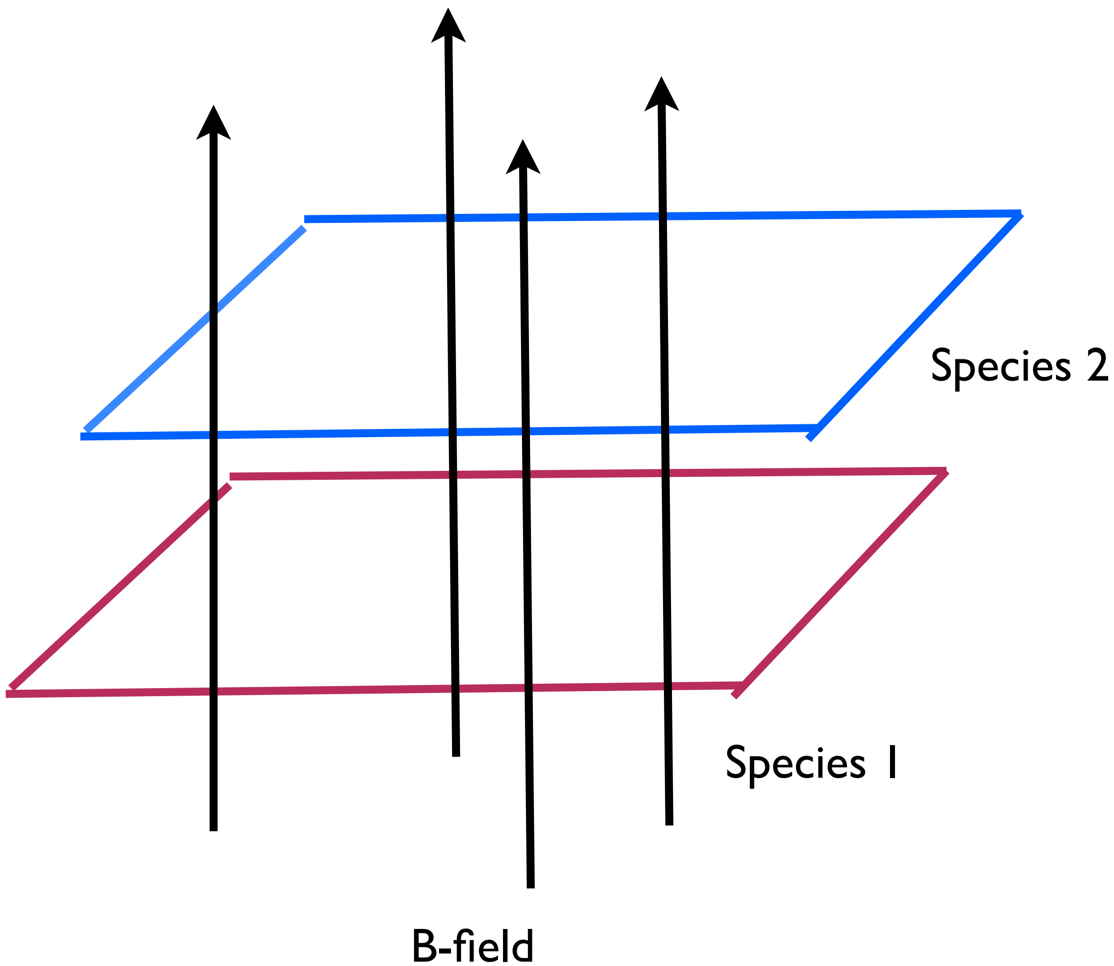
$$H_I = \int d^2x b_I^\dagger \left(-\frac{\left(\vec{\nabla} - i\vec{A}\right)^2}{2m} - \mu \right) b_I \tag{2}$$

$$H_{int} = \int d^2x d^2x' \rho_I(x) V_{IJ}(x - x') \rho_J(x') \tag{3}$$

External magnetic field $\vec{B} = \vec{\nabla} \times \vec{A}$.
$\rho_I(x) = b_I^\dagger(x) b_I(x) =$ density of species I

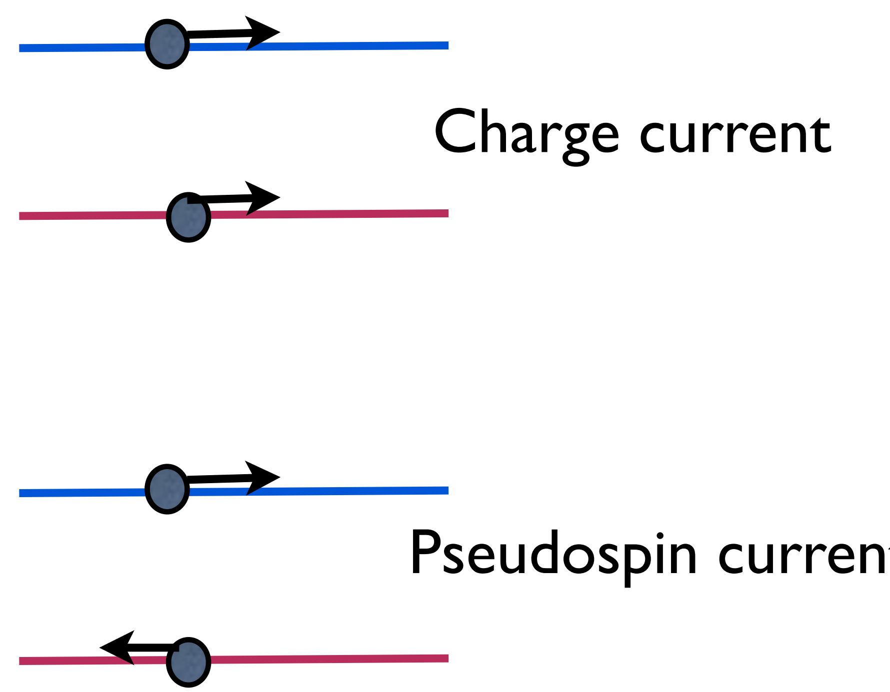
Symmetries and picture

Number of bosons N_1, N_2 of each species separately conserved: two separate global U(1) symmetries.



Total charge = N_1+N_2

Call $N_1 - N_2$ = total ``pseudospin”



Later relax to just conservation of total boson number

Guess for a possible ground state

If Interspecies repulsion V_{12} comparable or bigger than same species repulsion V_{11}, V_{22} particles of opposite species will try to avoid each other.

Guess that first quantized ground state wave function has structure

$$\psi = \text{``}\ldots\ldots \prod_{i,j} (z_i - w_j)\text{''} e^{-\sum_i \frac{|z_i|^2+|w_i|^2}{4l_B^2}} \tag{1}$$

z_i, w_i: complex coordinates of the two boson species.

Flux attachment mean field theory

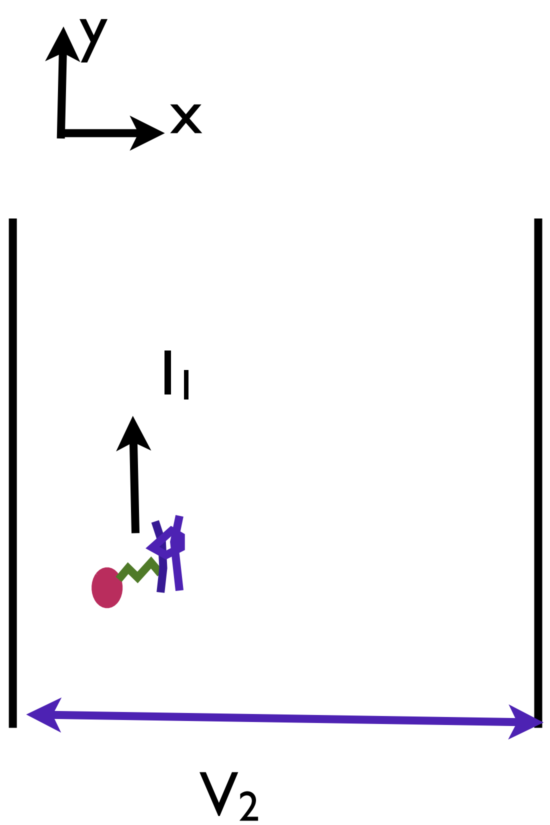
$\prod_{i.j}(z_i - w_j)$: particle of each species sees particle of the other species as a vortex.

Flux attachment theory:

Attach one flux quantum of one species to each boson of other species.
"Mutual composite bosons"

$\nu = 1 =>$ on average attached flux cancels external magnetic flux.
Mutual composite bosons move in zero average field.

Physical properties



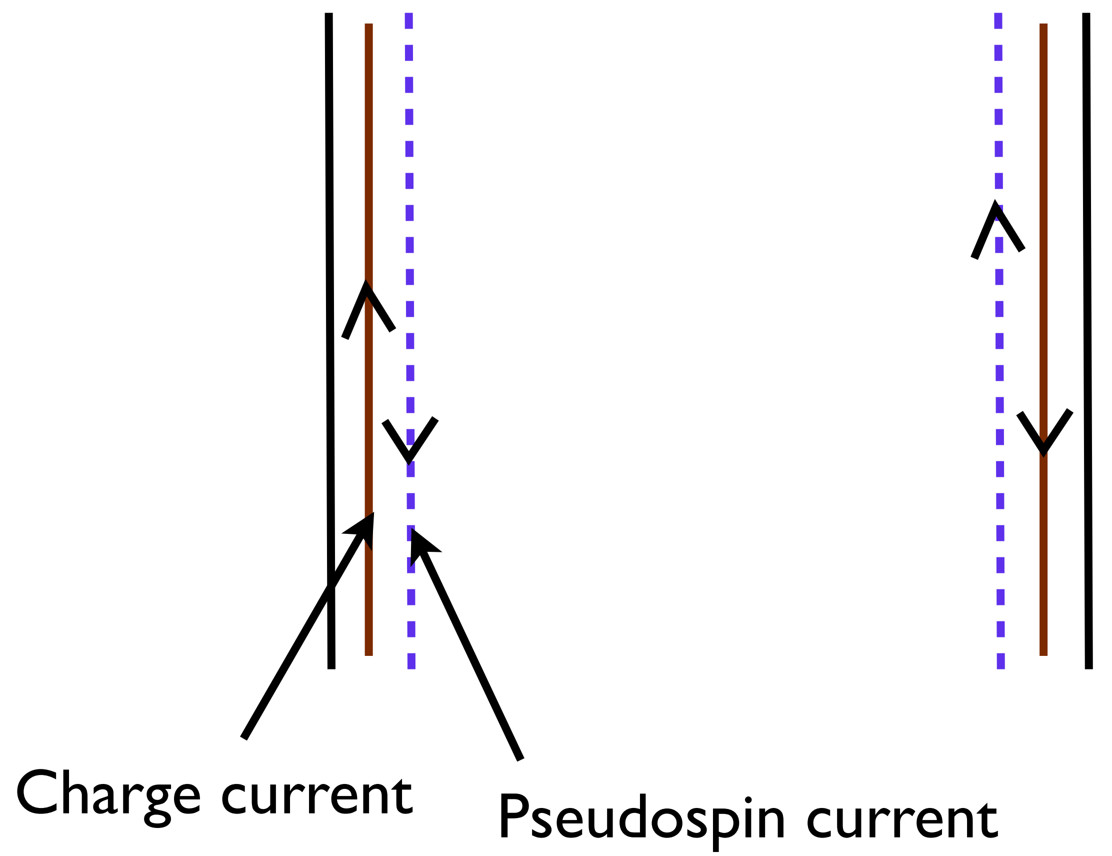
Analyse through usual Chern-Simons Landau Ginzburg theory for composite bosons

$$
\begin{aligned}
I_{1y} &= \frac{e^2}{h} V_{2x} \\
I_{2y} &= \frac{e^2}{h} V_{1x}
\end{aligned}
$$

Electrical Hall conductivity $\sigma_{xy} = 2$
Pseudospin Hall conductivity $\sigma^s_{xy} = -2.$

"Integer quantum Hall effect"

Edge states

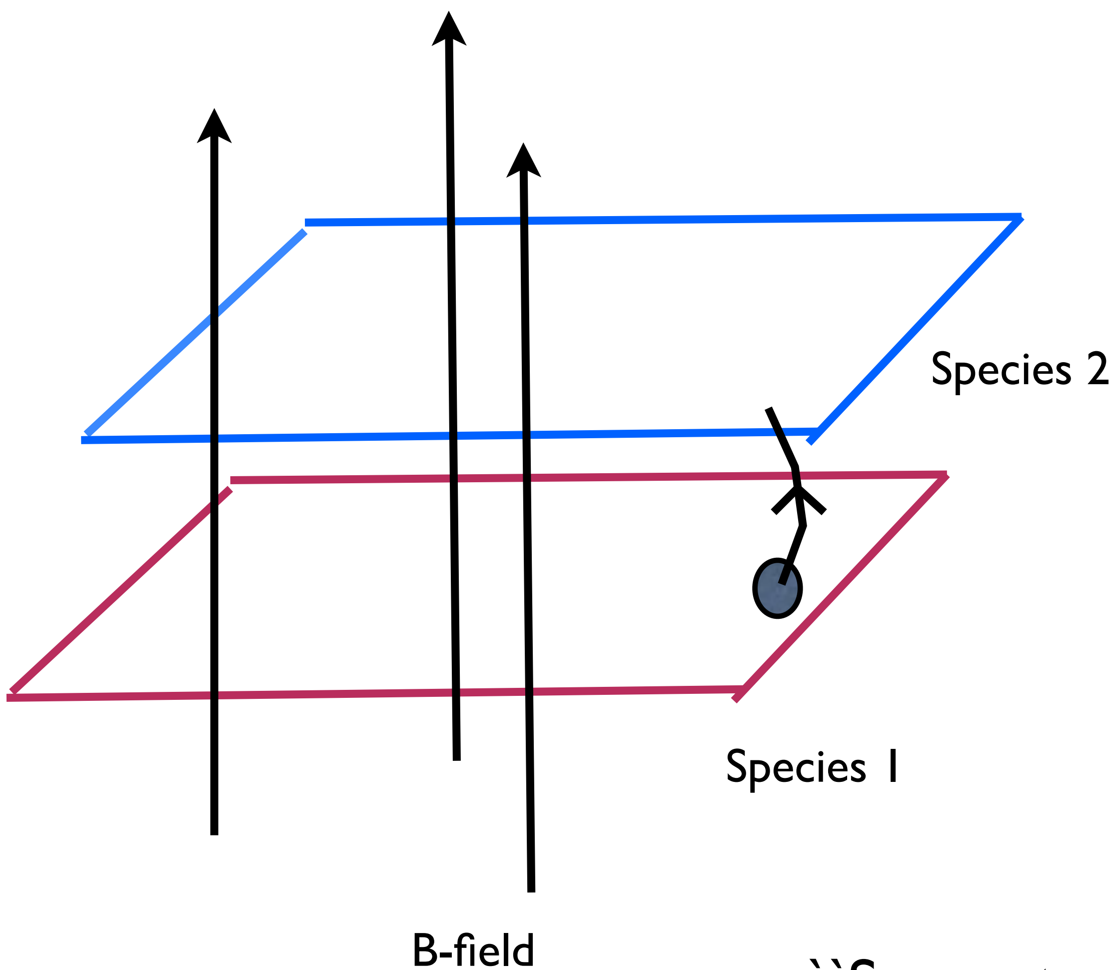


Comments:

1. Counterpropagating edge states but only one branch transports charge.

2. Thermal Hall conductivity = 0

Symmetry protection of edge states



Include interspecies tunneling:
Pseudospin not conserved,
only total particle number conserved.

Counterpropagating edge modes cannot backscatter due to charge conservation.

Edge modes are preserved so long as total charge is conserved.

``Symmetry Protected Topological Phase'' of bosons*

*Describe by two component Chern Simons theory with ``K-matrix'' with |det K| = 1: no bulk topological order/fractionalization.

Comments

- State described has $\sigma_{xy} = 2, \kappa_{xy} = 0$.

  Can obtain states with $\sigma_{xy} = 2n, \kappa_{xy} = 0$ by taking copies.

- For bosons, IQHE necessarily has σ_{xy} even.

Comments

- State described has $\sigma_{xy} = 2, \kappa_{xy} = 0$.

  Can obtain states with $\sigma_{xy} = 2n, \kappa_{xy} = 0$ by taking copies.

- For bosons, IQHE necessarily has σ_{xy} even.

Simple argument (TS, Levin 12).
Thread in 2π flux - pick up charge σ_{xy}.
Resulting particle has statistics $\pi\sigma_{xy}$.
No topological order => only boson excitations, so σ_{xy} even.

Example in 3d

Topological insulator of bosons

Symmetry: U(1) (charge conservation) and time reversal;
note boson charge is even under time reversal

Physics of 3d boson topological insulators

Vishwanath, TS, 2012

1. Quantized magneto-electric effect (eg: axion angle $\theta = 2\pi, 0$)

2. Emergent fermionic vortices at surface

3. Related exotic bulk monopole of external EM field (fermion, Kramers, or both) (Wang, TS, 2013; Metlitski, Kane, Fisher, 2013).

Explicit construction in systems of coupled layers: Wang, TS (2013).

Effective Field Theory of surface states of boson TI?

Fermion Topological Band Insulator:

Surface effective field theory: free fermions with odd number of Dirac cones + interactions

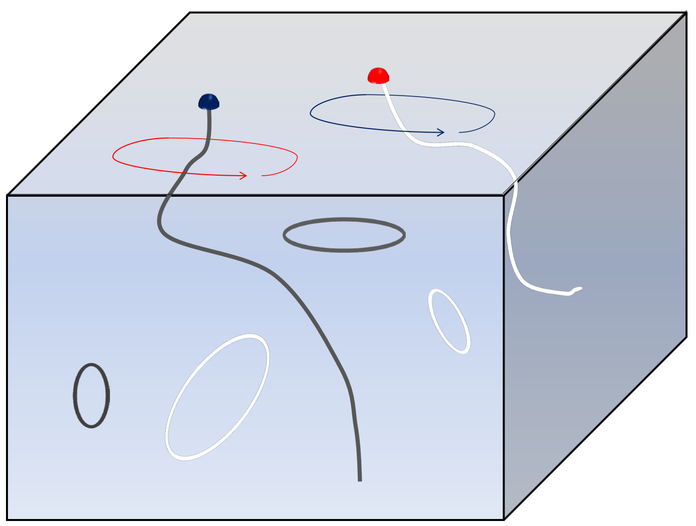
What is analog for bosonic TI?

Key requirement of surface theory

Trivial symmetry preserving insulator not possible at surface.

Convenient implementation through dual vortex point of view.

Description of surface in terms of point vortices = points where vortex lines of bulk penetrate surface.



Conventional 2d bosons: charge-vortex duality

$$\mathcal{L}_d = \mathcal{L}[\Phi, a_\mu] + \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda A_\mu$$

Dasgupta, Halperin, '80
Peskin, Stone, '80
Fisher, Lee, '89

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Bosonic vortex

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Physical boson current

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Bosonic vortex

Physical boson current

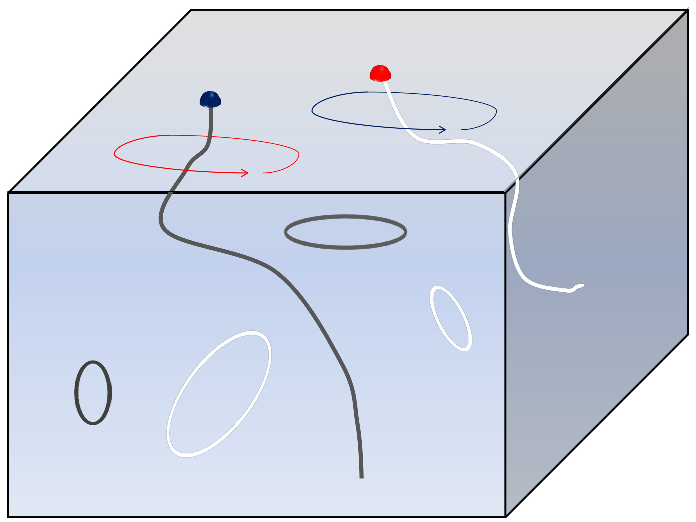
external probe gauge field

Boson superfluid = vortex insulator

Mott insulator = vortex condensate

Dasgupta, Halperin, '80
Peskin, Stone, '80
Fisher, Lee, '89

Dual description of boson TI surface



Mott insulator = vortex condensate

Boson TI surface: Demand that there is no trivial vortex that can condense to give a trivial insulator.

Implement: vortices are fermions!

Surface Landau-Ginzburg effective field theory

Vishwanath, TS, 12
Wang, TS, 13

$$\mathcal{L}_d = \mathcal{L}[c, a_\mu] + \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda A_\mu$$

c: fermionic vortex field

Comments:
1. ``Anomalous" implementation of global symmetry
2. Can use to describe surface phase diagram

Surface phase diagram

$$\mathcal{L}_d = \mathcal{L}[c, a_\mu] + \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda A_\mu$$

c: fermionic vortex field

(i) c gapped => surface superfluid but with a fermionic vortex

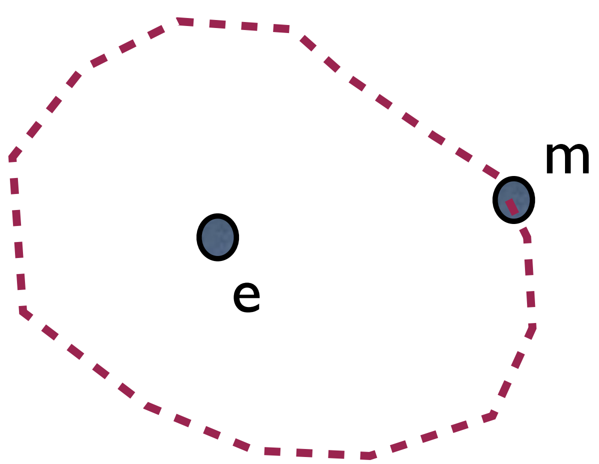
(ii) c in IQHE state with $\sigma_{xy} = 1$
=> T-broken surface state with physical $\sigma_{xy} = 1$ and $\kappa_{xy} = 0$

(``Half" of allowed boson IQHE in 2d)

(iii) Interesting possibility: pair condensate of c.
=> gapped insulator with fractional charge (of a kind not allowed in 2d).

Surface topological order of 3d SPTs

3d SPT surface can have intrinsic topological order* though bulk does not (Vishwanath, TS, 2012).



Phase of π

Resulting symmetry preserving gapped surface state realizes symmetry `anomalously' (cannot be realized in strict 2d; requires 3d bulk).

*Intrinsic topological order: excitations with anyonic statistics, ground state degeneracy on torus, ...

Electronic topological insulators

The problem: Phases and properties of correlated spin-orbit coupled electronic SPT insulators* in 3d ?

* Symmetry group $U(1) \rtimes Z_2^T$

The answer

Wang, Potter, TS, Science 2014

3d electronic insulators with charge conservation/T-reversal classified by Z_2^3
(corresponding to total of 8 distinct phases).

3 `root' phases:
Familiar topological band insulator, two new phases obtained as electron Mott insulators where spins form a spin-SPT (``topological paramagnets'').

Obtain all 8 phases by taking combinations of root phases.

Constraints from Monopoles

T-reversal symmetric insulators with no exotic excitations:

Monopoles have electric charge q = 0 or q = e/2.

Monopoles in such insulators must always be bosons (a non-trivial fact proven in Wang, Potter, TS, Science 14).

Possibilities other than the 2 band insulators (conventional and topological)?

A refined concrete question

Spin-orbit coupled insulators with

- a bulk gap

- no exotic excitations

- external `probe' monopole is an electrically neutral boson

Ordinary band insulator is an example but are there other distinct phases?

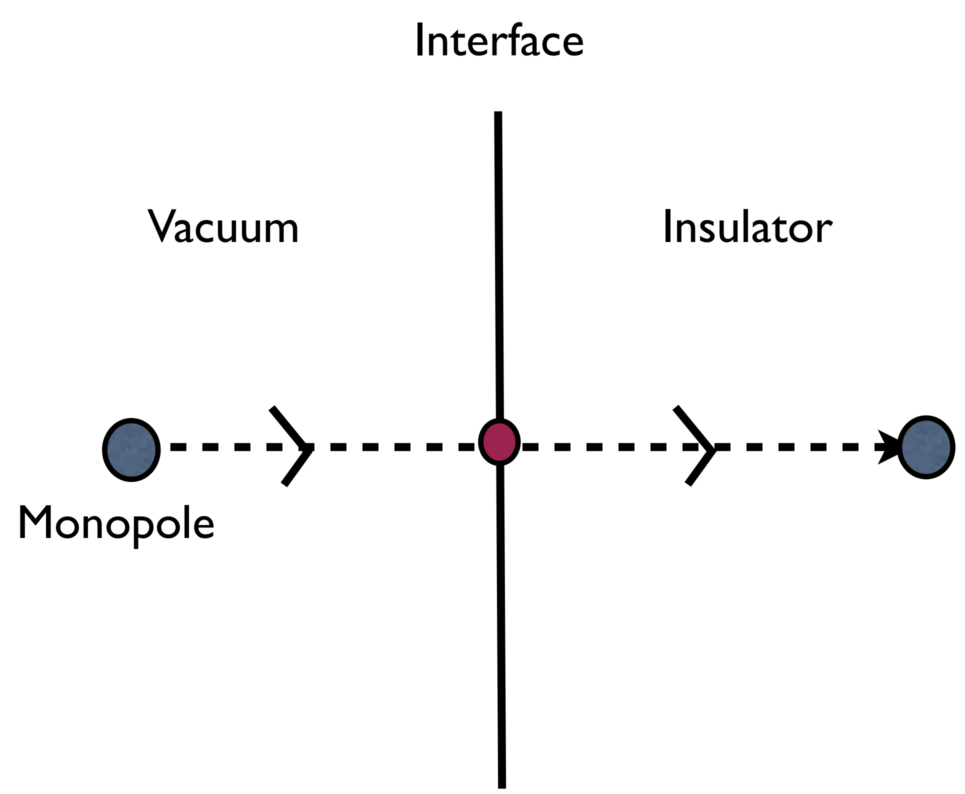
Bosonic magnetic monopole: implications for surface effective theory

Tunnel monopole from vacuum into bulk

Tunneling event leaves behind surface excitation which has charge-0 and is a boson.

A convenient surface termination- coat surface with superconductor.

Monopole tunneling leaves behind hc/e vortex which is a boson.



Consequences

Bosonic hc/e vortex in surface superconductor:

Enough to show that

1. the only possible non-trivial physics at the surface is for spin properties

2. Bulk insulator is at most an electronic Mott insulator where the spins have formed a Topological Paramagnet protected by time reversal.

Previous result (Vishwanath, TS 2013; Wang, TS, 2013):
There are precisely 3 such paramagnetic phases.

(Similarly also when monopole has electric charge e/2).

Some fundamental questions

0. Are topological band insulators distinct from ordinary insulators in the presence of interactions? YES! (implicit in early theory work)

1. Are there new phases that have no non-interacting counterpart?
YES! (Wang, Potter, TS, Science 14).
Exactly 6 new interacting topological insulators not contained in band theory.

 - Physical properties ?

 - Which materials ?

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- Physical properties ? (Illustrate with example)

- Which materials ?

Physical characterization

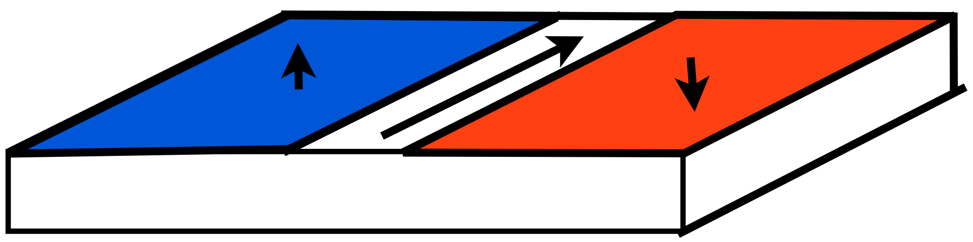
How to tell in experiments?

Break symmetry at surface to produce a simple state (eg: deposit ferromagnet or superconductor)

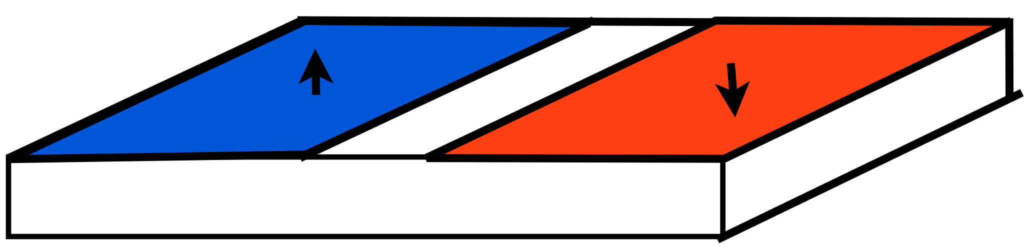
Unique experimental fingerprint!

Illustrate with one of the three allowed topological paramagnets (call it TopPara-I)

Depositing a ferromagnet: domain wall structure



Topological band insulator: Charged one-way mode



TopPara-I: Gapped domain wall

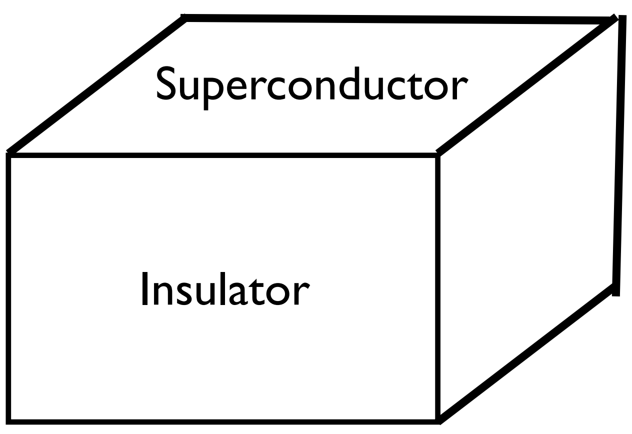
Depositing s-wave superconductor (SC): induced quasiparticle nodes

Topological band insulator: depositing s-wave SC leads to a **gapped** surface SC with interesting Majorana zero modes on vortices (Fu, Kane 08)

Top-Para-I:

Deposit s-wave SC => get a **gapless** SC with 4 gapless Dirac cones protected by time reversal symmetry! (Wang, Potter, TS 2014)

Probe by Angle Resolved Photoemission (and tunneling, etc)



Similar characterization for all 6 phases: unique experimental fingerprints.

Some fundamental questions

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YES! (Wang, Potter, TS, Science 14).
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   - Physical properties ? (Illustrate with example)

   - Which materials ? (some hints)

A suggestive direction: Frustrated spin-1 magnets in 3d

Spin-1 magnets seem likely candidates to host topological paramagnet phases in various dimensions.

Example: d = 1 Haldane spin-1 chain.

d = 3: Possible platform for TopPara-I

- Simple picture for ground state wavefunction

- a natural mean field theory.

Wang, Nahum, TS, 15

Spin-1 model wavefunction

Wang, Nahum, TS, 15

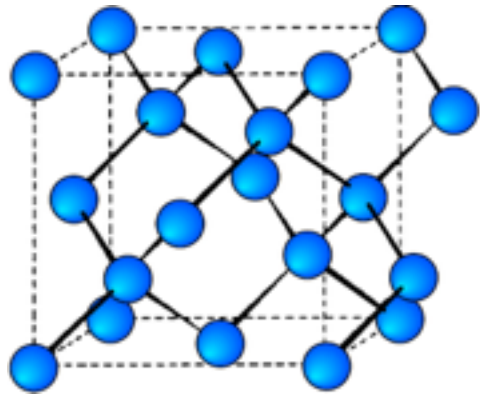
Diamond lattice with frustrating interactions.

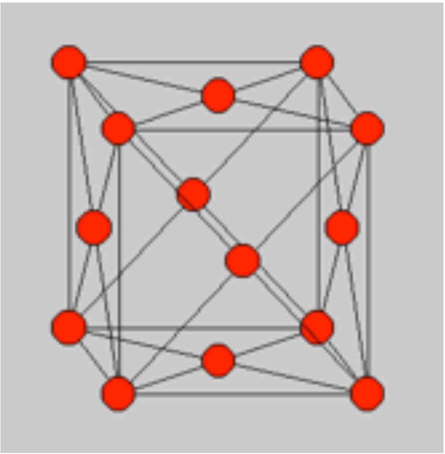
Diamond = two interpenetrating FCC lattices (label A and B sublattices)

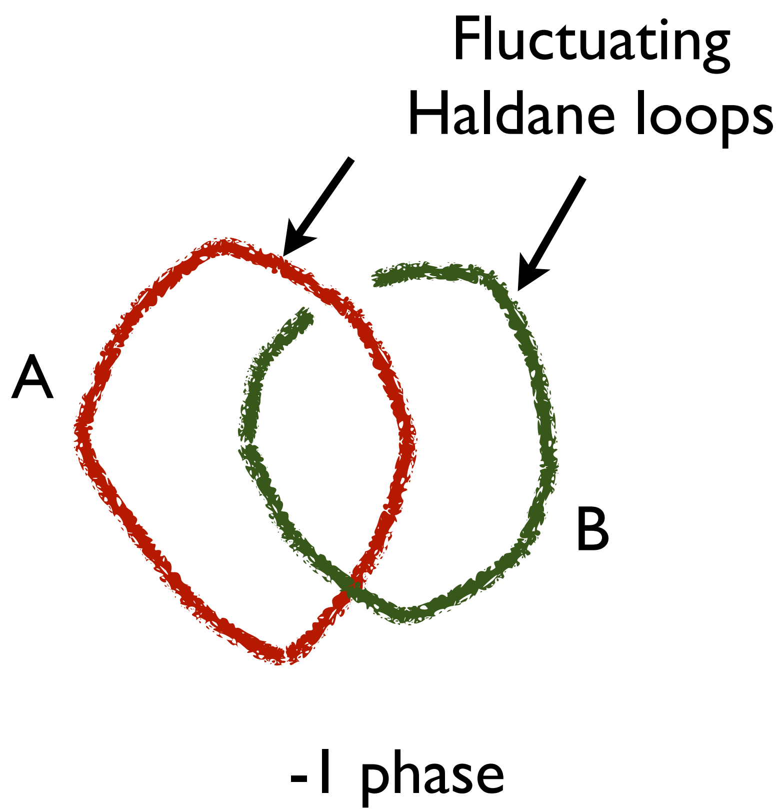
Close pack each FCC lattice with loops.

For each loop let the spin-1 form a Haldane chain.

Sum over all loop configurations with phase factors.



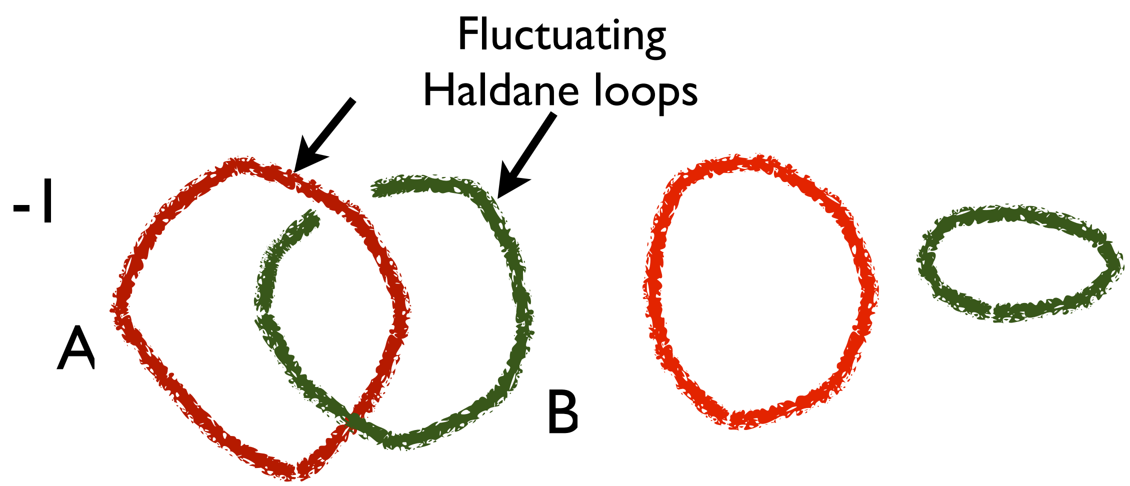




$$|\psi\rangle \sim \sum_{L_A, L_B} (-1)^{l(L_A, L_B)} |L_A, L_B\rangle$$

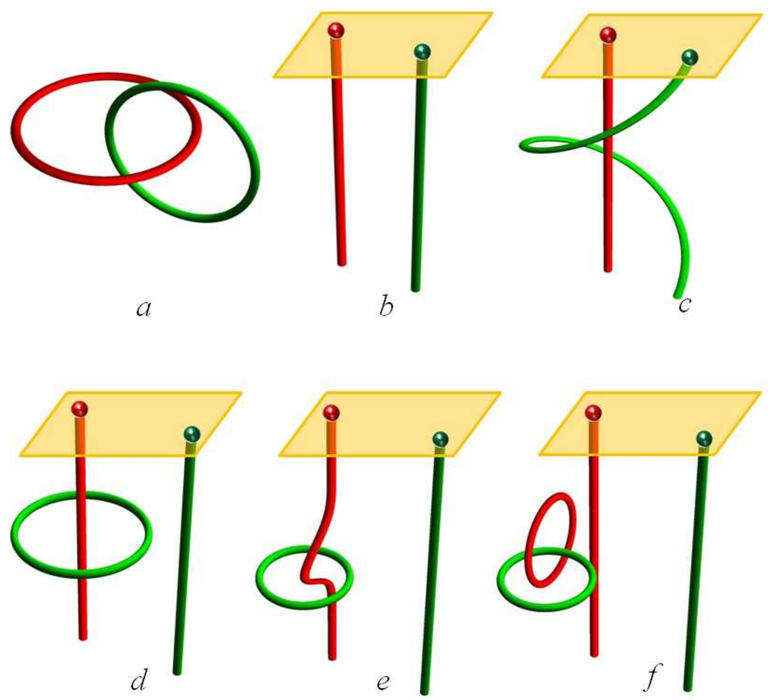
$l(L_A, L_B)$ = linking number of A loops with B loops.

Physics of the wavefunction



(-1) linking phase confines all
non-trivial bulk excitations
but what about surface?

End points at surface of both A and B loops are non-trivial.



Drag A-end point around B-end point => (-1) phase.

End points are open ends of Haldane loops => ``spin-1/2" objects.

Non-triviality precisely surface of Topological Paramagnet-I.

Wang, Nahum, TS, 15

Related models: Walker-Wang, 2011; Burnell, Chen, Fidkowski, Vishwanath 2013-2014.

Magnetic materials with $S > 1/2$ and with strong frustration known in 3d.

eg: ``Spinel'' oxides with ``A-site'' magnetism: $CoAl_2O_4$, ($S = 3/2$), $MnAl_2O_4$ ($S = 5/2$): frustrated diamond lattice magnets.

$S = 1$ examples???

Natural guess $NiAl_2O_4$ unfortunately does not work (Ni sites do not form diamond lattice)

Summary

1. SPTs are minimal generalizations of topological band insulators to interacting systems.

2. Simple but many interesting properties; deep connections to other problems (see colloquium Thursday).

3. Interacting electron TIs in 3d have a Z_2^3 classification

- apart from trivial and topological band insulators, 6 new TI phases with no non-interacting counterpart.

4. Progress in understanding interacting 3d fermion SPTs with many physically relevant symmetries (Wang, TS 14; Chen, Metlitski, Fidkowski, Vishwanath, 14).

3. Hints to important open question:

what kinds of real electronic insulators may be these new 3d TIs?